Re-examining $\sin 2\beta$ and $\Delta m_d$ from evolution of $B_d^0$ mesons with decoherence

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July 23, 2015, EPS-HEP Conference, Vienna
Plan of talk

1. Motivation
2. Determination of $\sin 2\beta$
3. Determination of $\Delta m_d$
4. An example of estimation of decoherence parameter
5. Conclusions

Based on the paper by Ashutosh Kumar Alok, Subhashish Banerjee and S. Uma Sankar, arXiv:1504.02893 (accepted for publication in Physics Letters B)
The time evolution of neutral mesons are used to measure a number of important parameters in flavor physics.

In the time evolution of neutral meson systems, a perfect quantum coherence is usually assumed.

However, any real system interacts with its environment and this interaction can lead to a loss of quantum coherence.

Hence with the inclusion of decoherence effects, the measured values of some of the parameters can get masked.

We study the effect of decoherence on the important observables in the $B_{d}^{0}$ meson system, such as the CP violating parameter $\sin 2\beta$ and the $B_{d}^{0} - \bar{B}_{d}^{0}$ mixing parameter $\Delta m_{d}$. 
Decoherence is an unavoidable phenomenon as any physical system is inherently open due to its inescapable interactions with a pervasive environment.

### Possible environment

- Environmental effects may arise at a fundamental level, such as the fluctuations in a quantum gravity space-time background.
- They may also arise due to the detector environment itself.

- The effect of environment on the neutral meson systems can be taken into account by using the ideas of open quantum systems.
- We use an effective description which is phenomenological in nature. It is independent of the details of the actual dynamics between the system and environment.

Our phenomenological approach provides a universal framework for the study of quantum decoherence effects.
We are interested in the decays of $B^0$ and $\bar{B}^0$ mesons as well as $B^0 \leftrightarrow \bar{B}^0$ oscillations.

To describe the time evolution of all these transitions, we need a basis of three states: $|B^0\rangle$, $|\bar{B}^0\rangle$ and $|0\rangle$, where $|0\rangle$ represents a state with no $B$ meson and is required for describing the decays.

We use the density matrix formalism to represent the time evolution of the $B^0$ system: $\rho_{B^0(\bar{B}^0)}(0)$ is the initial density matrix for the state which starts out as $B^0(\bar{B}^0)$.

The time evolution of these matrices is governed by the Kraus operators $K_i(t)$ as $\rho(t) = \sum_i K_i(t)\rho(0)K_i^\dagger(t)$.

The Kraus operators are constructed taking into account the decoherence in the system which occurs due to the evolution under the influence of the environment.
### Time dependent density matrices

\[
\begin{align*}
\rho_{B^0}(t) &= \begin{pmatrix}
a_{ch} + e^{-\lambda t} a_c & -a_{sh} - ie^{-\lambda t} a_s & 0 \\
-a_{sh} + ie^{-\lambda t} a_s & a_{ch} - e^{-\lambda t} a_c & 0 \\
0 & 0 & 2(e^{\Gamma t} - a_{ch})
\end{pmatrix}, \\
\rho_{\bar{B}^0}(t) &= \begin{pmatrix}
a_{ch} - e^{-\lambda t} a_c & -a_{sh} + ie^{-\lambda t} a_s & 0 \\
-a_{sh} - ie^{-\lambda t} a_s & a_{ch} + e^{-\lambda t} a_c & 0 \\
0 & 0 & 2(e^{\Gamma t} - a_{ch})
\end{pmatrix}.
\end{align*}
\]

- \(a_{ch} = \cosh \left(\frac{\Delta \Gamma}{2} t\right)\), \(a_{sh} = \sinh \left(\frac{\Delta \Gamma}{2} t\right)\), \(a_c = \cos (\Delta m t)\), \(a_s = \sin (\Delta m t)\).

- \(\Gamma = (\Gamma_L + \Gamma_H)/2\), \(\Delta \Gamma = \Gamma_L - \Gamma_H\): \(\Gamma_L\) and \(\Gamma_H\) are the respective decay widths of the decay eigenstates \(B^0_L\) and \(B^0_H\).

- \(\lambda\) is the decoherence parameter, due to the interaction between one-particle system and its environment.
CP asymmetry in $B^0_d \rightarrow J/\psi K_S$

We study the effect of decoherence on the important observables in the $B^0_d$ meson system, such as the CP violating parameter $\sin 2\beta$ and the $B^0_d - \bar{B}^0_d$ mixing parameter $\Delta m_d$.

To keep expressions simple, CP violation in mixing is neglected.

Hermitian operator describing decay $B^0 \rightarrow f$ and $\bar{B}^0 \rightarrow f$

$$O_f = \begin{pmatrix} |A_f|^2 & A_f^* \bar{A}_f & 0 \\ A_f \bar{A}_f^* & |\bar{A}_f|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$A_f \equiv A(B^0 \rightarrow f)$ and $\bar{A}_f \equiv A(\bar{B}^0 \rightarrow f)$.

The probability, $P_f(B^0/\bar{B}^0; t)$, of an initial $B^0/\bar{B}^0$ decaying into the state $f$ at time $t$ is given by $\text{Tr} \left[ O_f \rho_{B^0(\bar{B}^0)(t)} \right]$. 
CP asymmetry of $B^0_d \rightarrow J/\psi K_S$ decay

$$A_{J/\psi K_S}(t) = \frac{P_{J/\psi K_S}(\bar{B}^0_d; t) - P_{J/\psi K_S}(B^0_d; t)}{P_{J/\psi K_S}(\bar{B}^0_d; t) + P_{J/\psi K_S}(B^0_d; t)}$$

$$= \frac{(|\lambda_f|^2 - 1) \cos(\Delta m_d t) + 2\text{Im}(\lambda_f) \sin(\Delta m_d t)}{(1 + |\lambda_f|^2) \cosh \left( \frac{\Delta \Gamma_d t}{2} \right) - 2\text{Re}(\lambda_f) \sinh \left( \frac{\Delta \Gamma_d t}{2} \right)} e^{-\lambda t}$$

- $\lambda_f = A(B^0_d \rightarrow J/\psi K_S))/A(B^0_d \rightarrow J/\psi K_S)$.
- The standard expression for $A_{J/\psi K_S}(t)$ is obtained by putting $\lambda = 0$. With the approximations $\Delta \Gamma_d \approx 0$, $|\lambda_f| \approx 1$ and $\text{Im}(\lambda_f) \approx \sin 2\beta$, we get

$$A_{J/\psi K_S}(t) \approx \sin 2\beta e^{-\lambda t} \sin(\Delta m_d t)$$

The coefficient of $\sin(\Delta m_d t)$ is $\sin 2\beta e^{-\lambda t}$ and not $\sin 2\beta$!

The measurement of $\sin 2\beta$ is masked by the presence of decoherence.
Determination of $\Delta m_d$

In order to determine $\sin 2\beta$, we need to know $\Delta m_d$.

Is the measurement of $\Delta m_d$ also affected by the presence of decoherence?

**LHCb, CDF and D0** experiments determine $\Delta m_d$ by measuring rates that a state that is pure $B_d^0$ at time $t = 0$, decays as either as $B_d^0$ or $\bar{B}_d^0$ as function of proper decay time.

In the presence of decoherence, the survival (oscillation) probability of initial $B_d^0$ meson to decay as $B_d^0(\bar{B}_d^0)$ at a proper decay time $t$ is:

\[
P_{\pm}(t, \lambda) = \frac{e^{-\Gamma t}}{2} \left[ \cosh(\Delta \Gamma_d t/2) \pm e^{-\lambda t} \cos(\Delta m_d t) \right]
\]

The positive (negative) sign implies $B_d^0$ meson decaying with the same (opposite) flavor as its production.
\( \Delta m_d \) is determined from the following time dependent asymmetry:

\[
A_{\text{mix}}(t, \lambda) = \frac{P_+(t, \lambda) - P_-(t, \lambda)}{P_+(t, \lambda) + P_-(t, \lambda)} = e^{-\lambda t} \frac{\cos(\Delta m_d t)}{\cosh(\Delta \Gamma_d t/2)}.
\]

At **BaBar** and **Belle**, \( \Delta m_d \) is determined by measuring the \( B_d^0 \bar{B}_d^0 \) decay probability \( P_+ (\Delta t) \) and the \( B_d^0 B_d^0/\bar{B}_d^0 \bar{B}_d^0 \) decay probability \( P_- (\Delta t) \) for entangled neutral \( B_d \) meson pair produced by the decay of the \( \Upsilon(4S) \) resonance.

The expressions for \( P_\pm (\Delta t) \) are the same as those of \( P_\pm (t) \), except that the proper time \( t \) is replaced by proper decay-time difference \( \Delta t \) between the decays of the two neutral \( B_d \) mesons.
Determination of $\Delta m_d$

Determination of $\Delta m_d$ at LHCb, CDF, D0, Belle and BaBar experiments is masked by the presence of $\lambda$.

It can be shown that the time independent observables $r_d$ (measured by ARGUS and CLEO) and $\chi_d$ (measured by the LEP experiments), used to determine $\Delta m_d$, are also affected by the presence of decoherence.

The true value of $\Delta m_d$, along with $\Delta \Gamma_d$, can be determined by a three parameter ($\Delta m_d, \Delta \Gamma_d, \lambda$) fit to the time dependent mixing asymmetry $A_{\text{mix}}(t, \lambda)$. This in turn will enable a determination of true value of $\sin 2\beta$. 
Estimation of $\lambda$: An example

We make an attempt to determine $\lambda \Delta m_d$ and $\Delta \Gamma_d$ by performing a $\chi^2$ fit to $A_{\text{mix}}(\Delta t, \lambda)$, using the efficiency corrected distributions given by Belle Collaboration:

<table>
<thead>
<tr>
<th>$\Delta t$ bin</th>
<th>Window [ps]</th>
<th>$A$ and total error</th>
<th>Statistical error</th>
<th>Total</th>
<th>Event sel.</th>
<th>Systematic errors</th>
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<tr>
<td>2</td>
<td>0.5–1.0</td>
<td>0.916 ± 0.022</td>
<td>0.015</td>
<td>0.016</td>
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<td>0.009</td>
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<tr>
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<td>0.038</td>
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<tr>
<td>11</td>
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<td>0.243 ± 0.435</td>
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Talk@EPS-HEP 2015, Vienna  
July 23, 2015
First, the fit is done by assuming no decoherence, i.e., $\lambda = 0$. In this case, we find $\Delta m_d = (0.489 \pm 0.010) \text{ ps}^{-1}$ and $\Delta \Gamma_d = (0.087 \pm 0.054) \text{ ps}^{-1}$ with a $\chi^2/d.o.f. = 8.42/9$.

We then redo the fit including decoherence. This gives $\lambda = (-0.012 \pm 0.019) \text{ ps}^{-1}$, $\Delta m_d = 0.490 \pm 0.010 \text{ ps}^{-1}$ and $\Delta \Gamma_d = (0.144 \pm 0.088) \text{ ps}^{-1}$ with a $\chi^2/d.o.f. = 8.02/8$.

- Decoherence parameter $\lambda$ is very loosely bounded.
- The upper limit on $\lambda$ is 0.018 ps$^{-1}$ at 90% C.L.
- $\Delta m_d$ is numerically unaffected.

Given the wealth of data coming from LHCb and expected from the KEK Super B factory, a clear picture is expected to emerge.
The decoherence is expected to emerge from a scale much finer than that of the flavor physics.

Hence for an accurate determination, one should include all the known effects, such as CP violation in mixing, which are usually neglected in the extraction of $\sin 2\beta$.

The theoretical precision for the extraction of CP violating phase $\sin 2\beta$ from the CP asymmetry of $B_d^0 \rightarrow J/\psi K_S$ decay is limited by contributions from doubly Cabibbo-suppressed penguin topologies which cannot be calculated in a reliable way within QCD. However, $B_s^0 \rightarrow J/\psi K_S$ is related to $B_d^0 \rightarrow J/\psi K_S$ through the $U$-spin symmetry of strong interactions and it offers a tool to control the penguin effects.
The present analysis can easily be extended to the $B_s^0$ system as well.

The expression for the time dependent CP asymmetry in the mode $B_s^0 \rightarrow J/\psi\phi$ will be a function of four parameters: $\lambda$, $\sin 2\beta_s$, $\Delta m_s$ and $\Delta \Gamma_s$.

The time dependent mixing asymmetry will determine $\lambda$, $\Delta m_s$ and $\Delta \Gamma_s$.

These two time-dependent asymmetries should be re-analysed using a four parameter fit for a clean determination of $\sin 2\beta_s$, $\Delta m_s$, $\Delta \Gamma_s$ and $\lambda$.

Like $\sin 2\beta_d$, the extraction of $\sin 2\beta_s$ from time dependent CP asymmetry in the mode $B_s^0 \rightarrow J/\psi\phi$ is restricted due to penguin pollution. The penguin contribution to $B_s^0 \rightarrow J/\psi\phi$ can be estimated using decays $B_d^0 \rightarrow J/\psi\rho$ and $B_s^0 \rightarrow J/\psi K^*$. 
In this work, we have studied the effect of decoherence on two important observables $\sin 2\beta$ and $\Delta m_d$ in a neutral meson system. We find that the asymmetries which determine these quantities are also functions of the decoherence parameter $\lambda$. Hence it is imperative to measure $\lambda$ for a clean determination of these quantities. We suggest a re-analysis of the data on the above asymmetries for an accurate measurement of all the three quantities $\lambda$, $\sin 2\beta$ and $\Delta m_d$. The present analysis can easily be extended to the $B_s^0$ system as well.