Questioning the anomalies in $B \to K * \mu^+ \mu^-$ decays.

Ayan Paul

ERC Ideas: NPFlavour

INFN, Sezione di Roma. Roma, Italy.

EPS, Vienna. July 23rd 2015.

ANOMALY

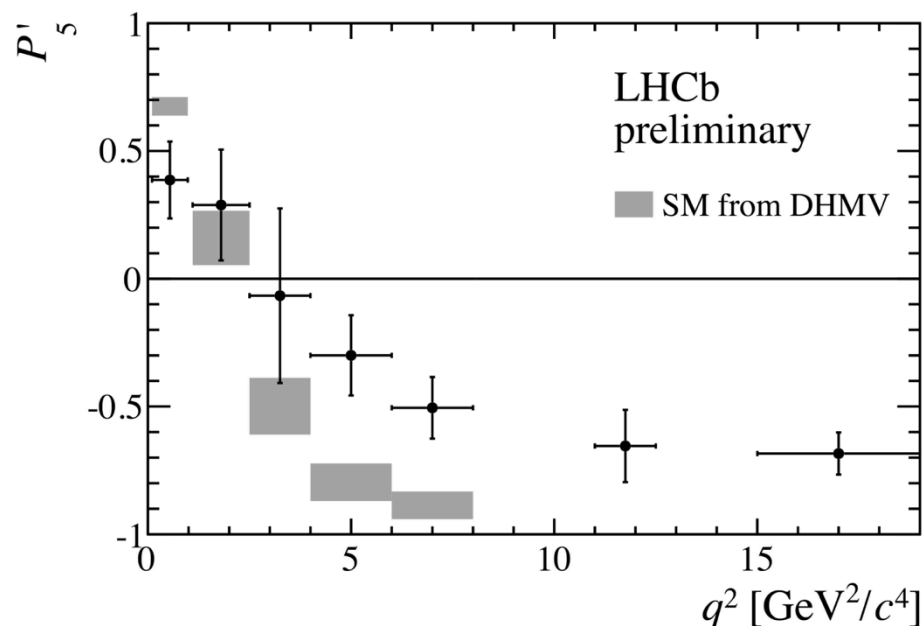anomaly |əˈnɒm(ə)li|
noun (pl.**anomalies**)
: something that deviates from what is standard, normal, or expected
: *there are a number of anomalies in the present system*

ingredients of an anomaly

LHCb-CONF-2015-002, March 23, 2015
DFMV: Descotes-Genon, Hofer, Matias and Virto: arXiv:1503.03328

✓ A set of Wilson coefficients defined at the weak scale and run down to the hadronic scale. (The Good)
✓ A set of form factors from the hadronic initial state to the hadronic final state defined at the hadronic scale. (The Bad)
✓ A non-factorizable (hadronic) contribution that cannot be computed from first principles. (The Ugly)

disclaimer: today I will talk about Standard Model dynamics ONLY.

the good, the bad and the ugly

The Helicity Amplitudes

$$H_V(\lambda) = -iN\left\{C_9^{\text{eff}}\tilde{V}_{L\lambda} + \frac{m_B^2}{q^2}\left[\frac{2\hat{m}_b}{m_B}C_7^{\text{eff}}\tilde{T}_{L\lambda} - 16\pi^2 h_\lambda\right]\right\},$$

$$H_A(\lambda) = -iNC_{10}\tilde{V}_{L\lambda},$$

$$H_P = iN\frac{2m_l m_B^2}{q^2}C_{10}\left(\tilde{S}_L - \frac{m_s}{m_B}\tilde{S}_R\right),$$

Good: The Wilson Coefficients

Bad: The Form Factors

Ugly: The non-factorizable Hadronic Contribution

the good

For now we ignore the chirality flipped operator and the scalar amplitude since we focus only on SM contributions.

Wilson coefficients can be computed at NNLO accuracy and all pieces of the evolutor necessary for the running from the weak scale to the hadronic scale are present.

Errors from the computation of Wilson coefficients is negligible.

the bad

$$V_\pm(q^2) = \frac{1}{2}\left[\left(1+\frac{m_V}{m_B}\right)A_1(q^2) \mp \frac{\lambda^{1/2}}{m_B(m_B+m_V)}V(q^2)\right],$$

$$V_0(q^2) = \frac{1}{2m_V\lambda^{1/2}(m_B+m_V)}\left[(m_B+m_V)^2(m_B^2-q^2-m_V^2)A_1(q^2) - \lambda A_2(q^2)\right]$$

$$T_\pm(q^2) = \frac{m_B^2-m_V^2}{2m_B^2}T_2(q^2) \mp \frac{\lambda^{1/2}}{2m_B^2}T_1(q^2),$$

$$T_0(q^2) = \frac{m_B}{2\,m_V\lambda^{1/2}}\left[(m_B^2+3m_V^2-q^2)T_2(q^2) - \frac{\lambda}{(m_B^2-m_V^2)}T_3(q^2)\right],$$

$$S(q^2) = A_0(q^2),$$

$$A_{12}(q^2) = \frac{(m_B+m_V)^2\left(m_B^2-m_V^2-q^2\right)A_1(q^2) - \lambda(q^2)A_2(q^2)}{16m_Bm_V^2\left(m_B+m_V\right)}$$

$$T_{23}(q^2) = \frac{\left(m_B^2-m_V^2\right)\left(m_B^2+3m_V^2-q^2\right)T_2(q^2) - \lambda(q^2)T_3(q^2)}{8m_Bm_V^2\left(m_B-m_V\right)}.$$

LCSR at large recoil (low q^2) [hep-ph/0412079 and arXiv:1503.05534]
LCSR at large recoil (low q^2) [hep-ph/0611193] (larger errors)
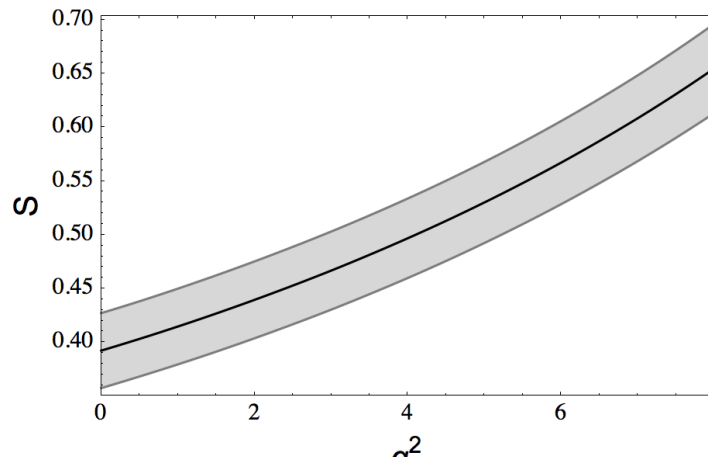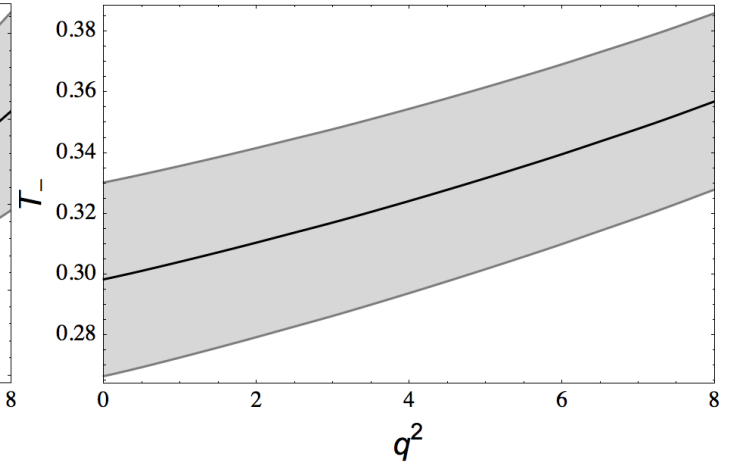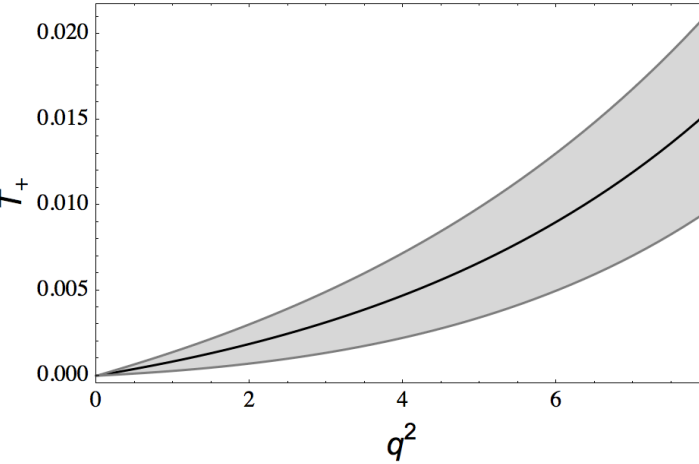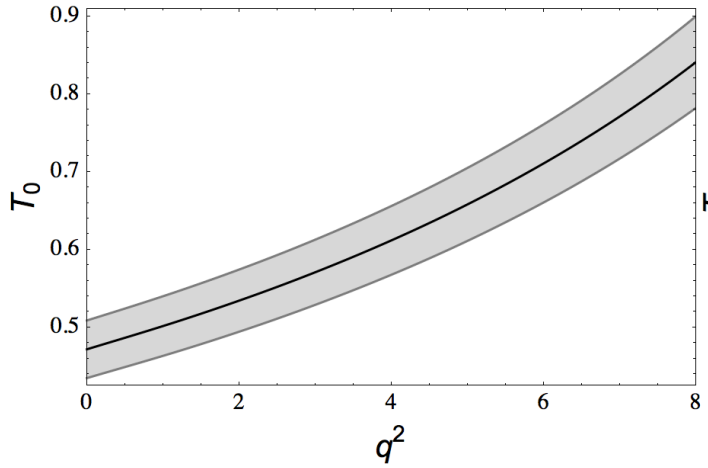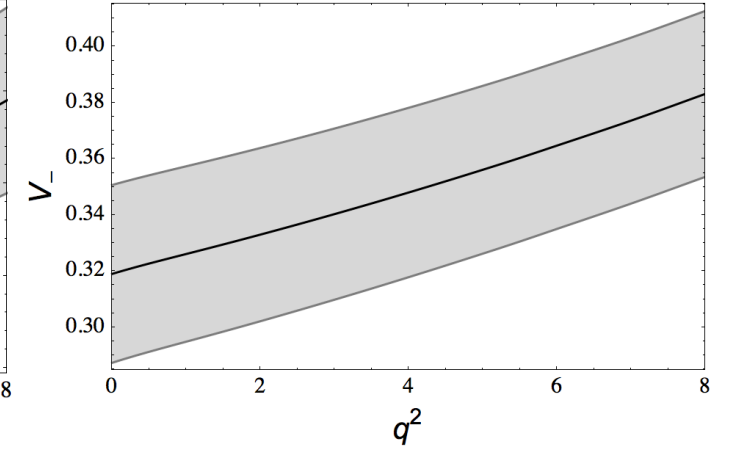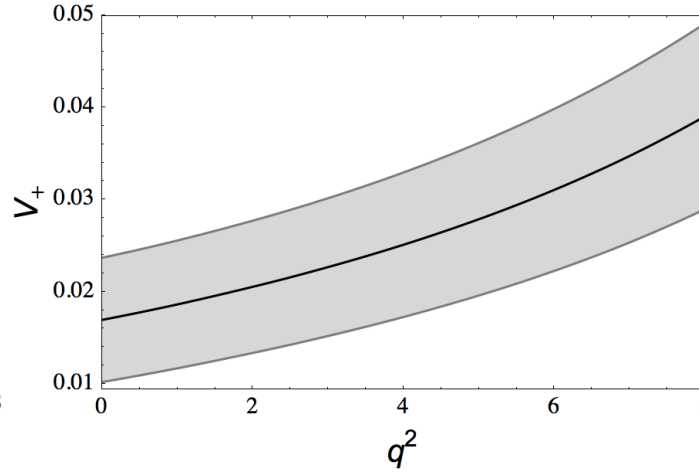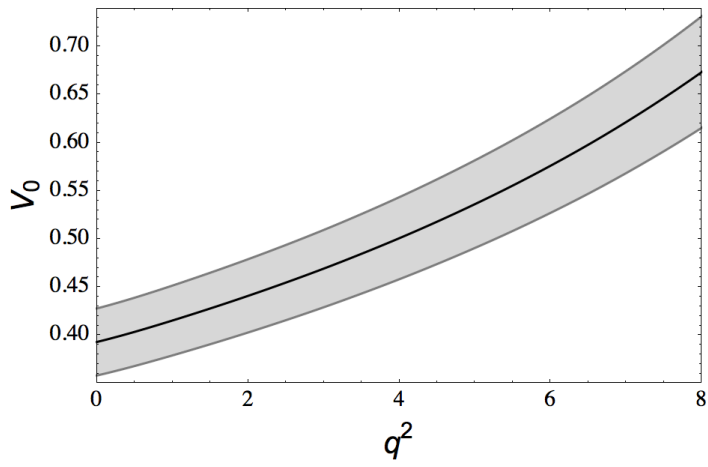Lattice at small recoil (high q^2) [arXiv:1501.00267]

In the infinite mass limit ignoring α_s corrections the number of independent form factors = 2 (soft form factors)

Exact symmetry relations at kinematic endpoint:

$$A_{12}(0) = \frac{m_B^2 - m_{K^*}^2}{8m_Bm_{K^*}}A_0(0) \qquad T_1(0) = T_2(0)$$

the bad

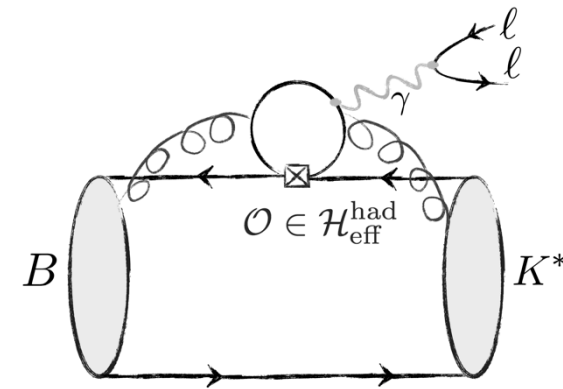Bharucha, Straub and Zwicky, arXiv:1505.05534

19 parameters. 19 x 19 correlation matrix

$$F_i(q^2) = P_i(q^2) \sum_k \alpha_k^i \left[z(q^2) - z(0) \right]^k$$

6

the ugly

$$h_\lambda(q^2) = \frac{\epsilon_\mu^*(\lambda)}{m_B^2} \int d^4x\, e^{iqx} \langle \bar{K}^* | T\{j_{\text{em}}^\mu(x) \mathcal{H}_{\text{eff}}^{\text{had}}(0)\} | \bar{B} \rangle$$

$$= h_\lambda^{(0)} + q^2 h_\lambda^{(1)} + q^4 h_\lambda^{(2)},$$

- The weakest link in the analysis is the estimates of the non-factorizable part.

- However, the estimates of the angular observables in the SM depend heavily on the estimate of the non-factorizable part. (EVEN the "clean ones")

- The nonlinear dependence of the angular observables on the hadronic contribution means that the central value *and* the error in the prediction depends on the size of this estimate.

- The *only* theory estimate available in the literature (arXiv:1006:4945) takes into account only a part of the possible contribution (soft gluon contribution)

- Other contributing diagrams can possible bring about corrections to this estimate that are as large or larger than the current estimate depending on the kinematic region one considers.

the angular analysis

$$\frac{d^{(4)}\Gamma}{dq^2 d(cos\theta_l)d(cos\theta_K)d\phi} = \frac{9}{32\pi}\Big(I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)cos2\theta_l$$

$$+I_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_l \cos\phi$$

$$+I_5 \sin 2\theta_K \sin\theta_l \cos\phi + (I_6^s \sin^2\theta_K + I_6^c \cos^2\theta_K)\cos\theta_l$$

$$+I_7 \sin 2\theta_K \sin\theta_l \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_l \sin\phi$$

$$+I_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi\Big).$$
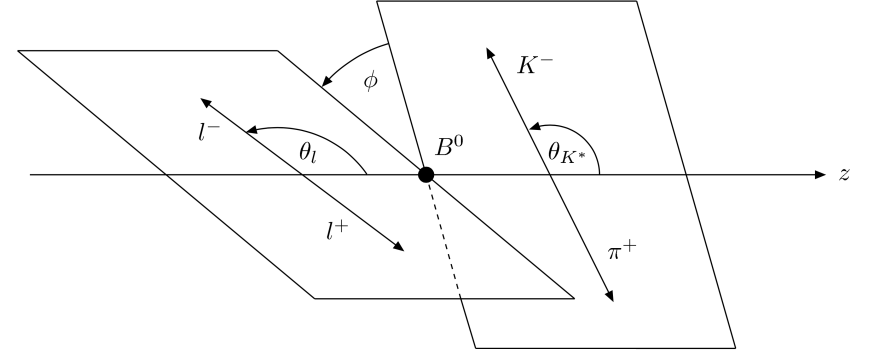
$$I_1^c = F\left(\frac{1}{2}\left(|H_V^0|^2 + |H_A^0|^2\right) + |H_P^0|^2 + \frac{2m_l^2}{q^2}\left(|H_V^0|^2 - |H_A^0|^2\right)\right),$$

$$I_1^s = F\left(\frac{\beta^2+2}{8}\left(|H_V^+|^2 + |H_V^-|^2 + |H_A^+|^2 + |H_A^-|^2\right) + \frac{m_l^2}{q^2}\left(|H_V^+|^2 - |H_V^-|^2 - |H_A^+|^2 + |H_A^-|^2\right)\right),$$

$$\Sigma_i = \frac{I_i + \bar{I}_i}{2}$$

$$I_2^c = -F\frac{\beta^2}{2}\left(|H_V^0|^2 + |H_A^0|^2\right),$$

$$I_2^s = F\frac{\beta^2}{8}\left(\left(|H_V^+|^2 + |H_V^-|^2\right) + \left(|H_A^+|^2 + |H_A^-|^2\right)\right),$$

$$I_3 = -\frac{F}{2}\Re\left[H_V^+(H_V^-)^* + H_A^+(H_A^-)^*\right],$$

$$I_4 = F\frac{\beta^2}{4}\Re\left[(H_V^+ + H_V^-)(H_V^0)^* + (H_A^+ + H_A^-)(H_A^0)^*\right],$$

$$I_5 = F\frac{\beta}{4}\Re\left[(H_V^- - H_V^+)(H_A^0)^* + (H_A^- - H_A^+)(H_V^0)^*\right],$$

$$I_6^s = F\beta\Re\left[H_V^-(H_A^-)^* - H_V^+(H_A^+)^*\right],$$

$$I_6^c = 0,$$

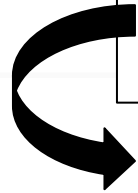$$I_7 = F\frac{\beta}{2}\Im\left[(H_A^+ + H_A^-)(H_V^0)^* + (H_V^+ + H_V^-)(H_A^0)^*\right],$$

$$I_8 = F\frac{\nu}{4}\Im\left[(H_V^- - H_V^+)(H_V^0)^* + (H_A^- - H_A^+)(H_A^0)^*\right],$$

$$I_9 = F\frac{\beta^2}{4}\Im\left[H_V^+(H_V^-)^* + H_A^+(H_A^-)^*\right],$$

$$\langle P_1\rangle = \frac{\langle\Sigma_3\rangle}{2\langle\Sigma_{2s}\rangle}, \qquad \langle P_2\rangle = \frac{\langle\Sigma_{6s}\rangle}{8\langle\Sigma_{2s}\rangle}, \qquad \langle P_3\rangle = -\frac{\langle\Sigma_9\rangle}{4\langle\Sigma_{2s}\rangle},$$

$$\langle P_4'\rangle = \frac{\langle\Sigma_4\rangle}{\sqrt{-\langle\Sigma_{2s}\Sigma_{2c}\rangle}}, \qquad \langle P_5'\rangle = \frac{\langle\Sigma_5\rangle}{2\sqrt{-\langle\Sigma_{2s}\Sigma_{2c}\rangle}},$$

$$\langle P_6'\rangle = -\frac{\langle\Sigma_7\rangle}{2\sqrt{-\langle\Sigma_{2s}\Sigma_{2c}\rangle}}, \qquad \langle P_8'\rangle = -\frac{\langle\Sigma_8\rangle}{2\sqrt{-\langle\Sigma_{2s}\Sigma_{2c}\rangle}},$$

$$= 0,$$ if hadronic contribution is switched off and C_7 and C_{10} are real

$$\langle\Gamma'\rangle = \langle\Sigma_{1c} + 4\Sigma_{2s}\rangle, \qquad \langle F_L\rangle = \frac{\langle 3\Sigma_{1c} - \Sigma_{2c}\rangle}{4\langle\Gamma'\rangle}, \qquad \langle A_{FB}\rangle = -\frac{3\langle\Sigma_{6s}\rangle}{4\langle\Gamma'\rangle}$$

the "optimized" observables

How optimized are the "optimized" observables?

In the infinite mass limit ignoring α_s corrections when the number of independent form factors = 2 (soft form factors) and the non-factorizable hadronic contribution is set to 0:

$$P_1 = 0,$$

$$P_5' = \frac{\mathrm{Re}[C_{10}^* C_{9,\perp} + C_{9,\parallel}^* C_{10}]}{\sqrt{(|C_{9,\parallel}|^2 + |C_{10}|^2)(|C_{9,\perp}|^2 + |C_{10}|^2)}},$$

$$\langle P_1 \rangle = \frac{\langle \Sigma_3 \rangle}{2 \langle \Sigma_{2s} \rangle}$$

$$\langle P_5' \rangle = \frac{\langle \Sigma_5 \rangle}{2\sqrt{-\langle \Sigma_{2s}\Sigma_{2c} \rangle}}$$

*

Hence, deviations from these limits make the observables less clean from form factor uncertainties.

Moreover, the observables are ratios of bin-averages and NOT bin-averages of ratios:
this means the optimization in the analytic expression is lost by bin-integration.

caveat: the "clean" observables are not as clean as advertised!

* S. Jäger and J. Martin Camalich (2014), 1412.3183.

3 fb⁻¹ @ LHCb
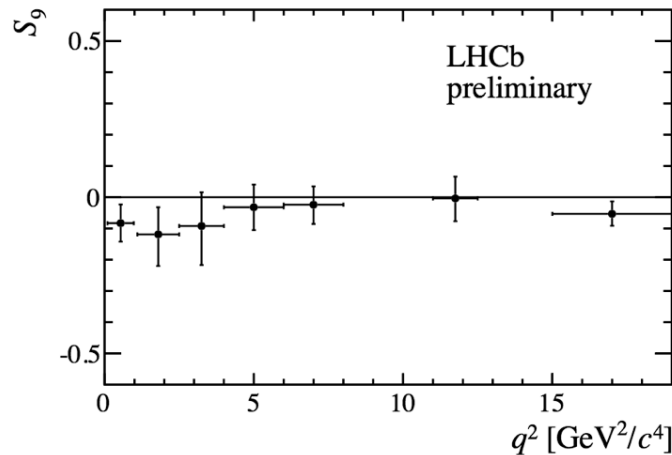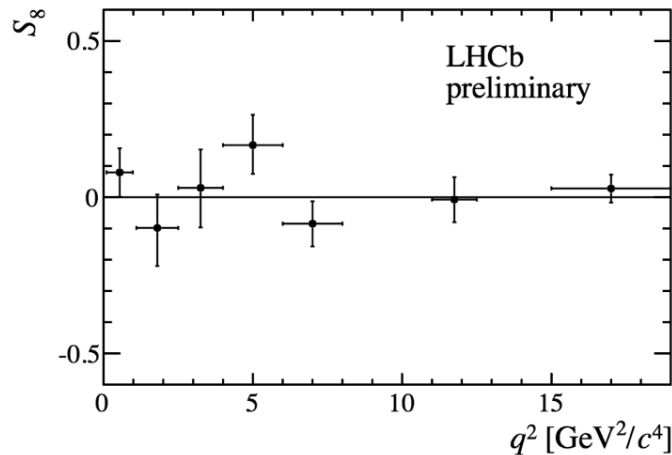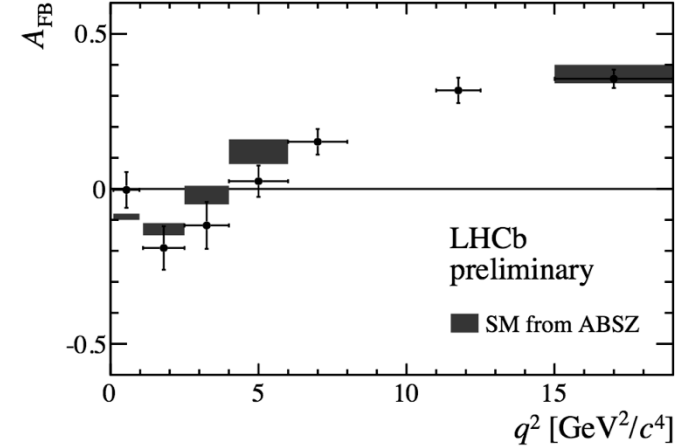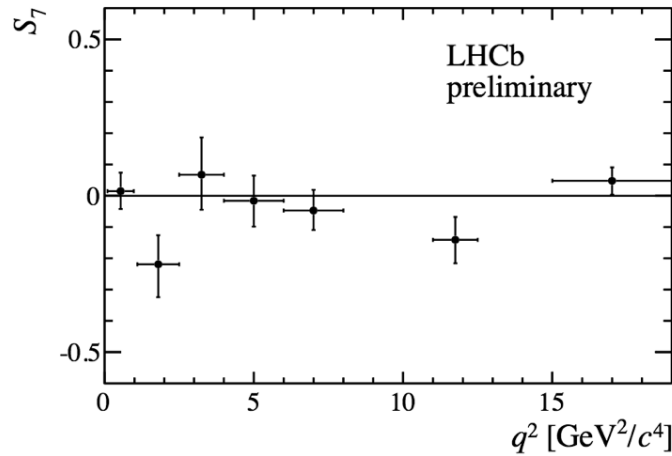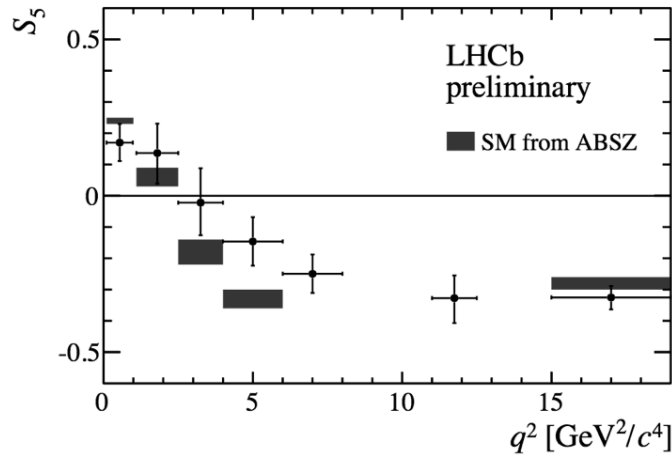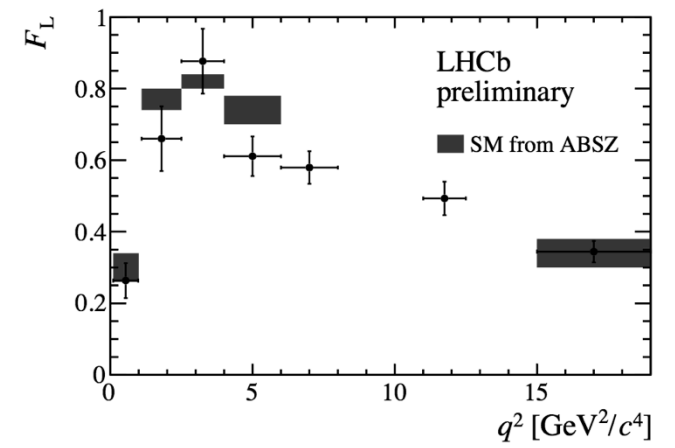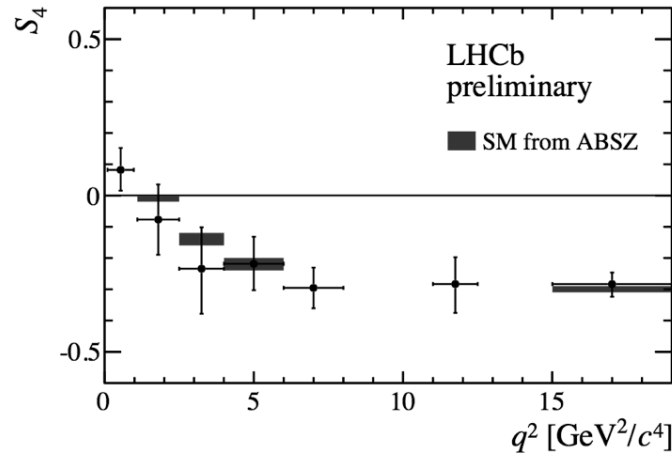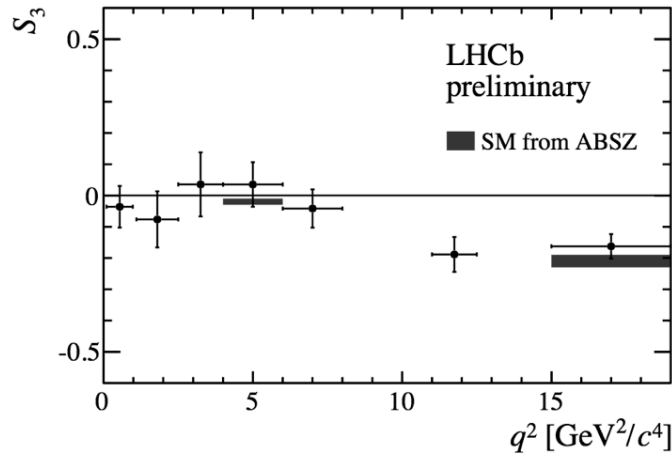
ABSZ: Bharucha, Straub and Zwicky,
arXiv:1505.05534

HEPfit

it is a public code and can be retrieved from the git
(https://github.com/silvest/SusyFit)

we have implemented the full kinematic distribution and all the
observables are ready to run with **flexible binning**

it is a fast MCMC based Bayesian analysis program and is completely
parallelized to run in clusters

full form factors can be used with complete correlations.

experimental likelihoods and correlations can be used to fit to data

experimentalist <u>do not</u> need to rely on theorists to give them binned
computation (full documentation will be available)

non-factorizable hadronic contribution can be set as a prior

ps. this is only a tiny fraction of what the code can do!

our analysis

observations for the first time. The simultaneous extraction of these observables enables the correlations between the measured quantities to be computed, enabling the observables to be included in global fits to theoretical models in a statistically correct way. This is critical to understand whether SM dynamics are sufficient to explain the above discrepancy, or if

LHCb, March 2015
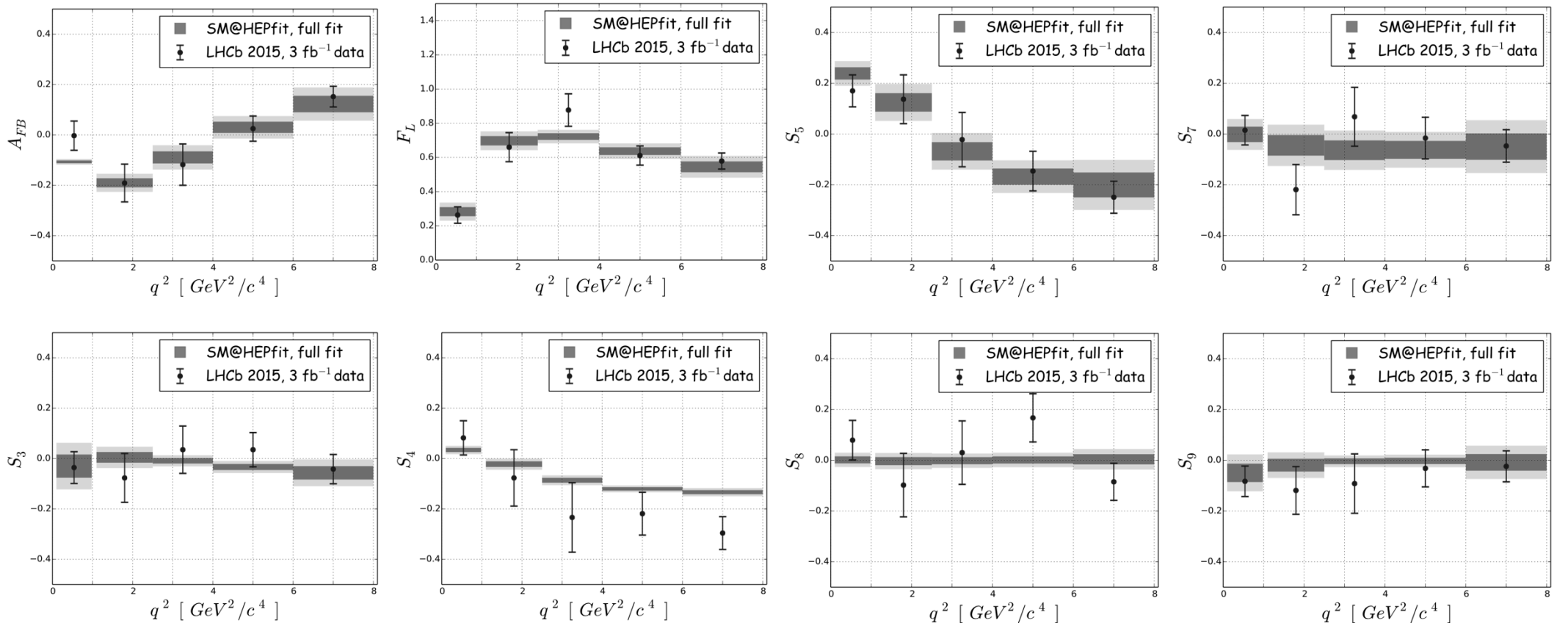
we use $B \to K^*\gamma$ to fix the hadronic contribution at $q^2 = 0$.

our prediction
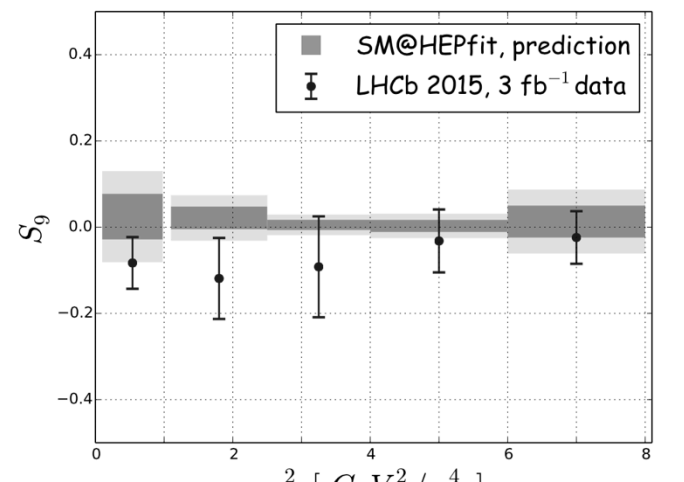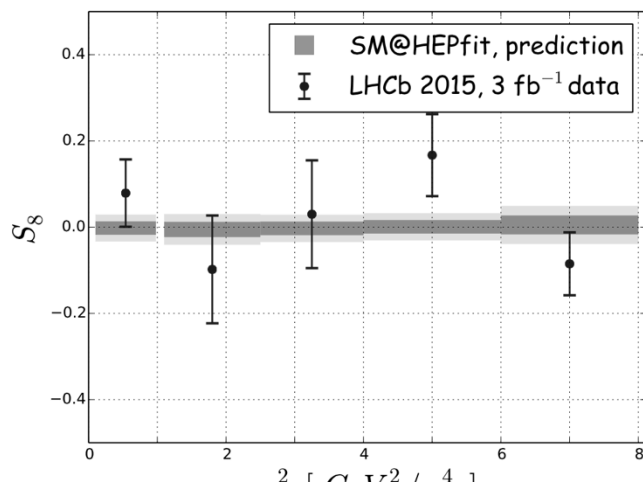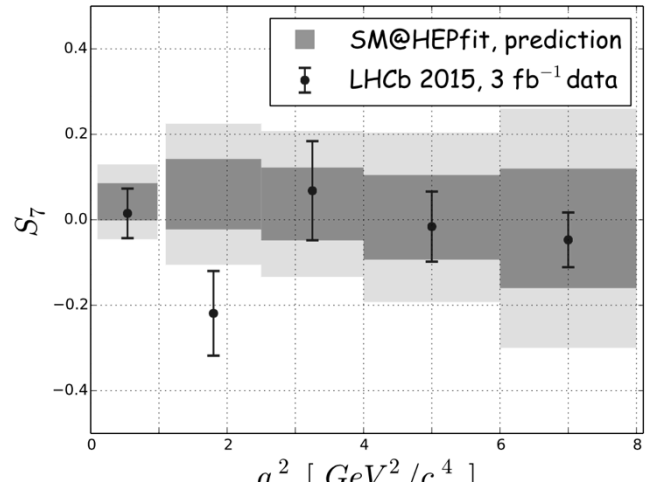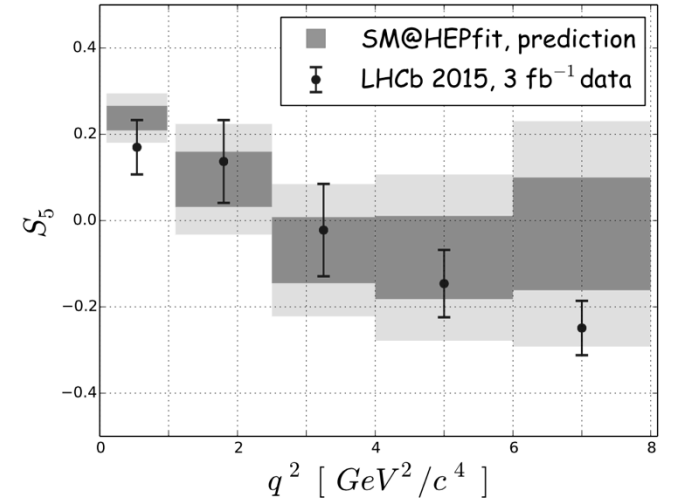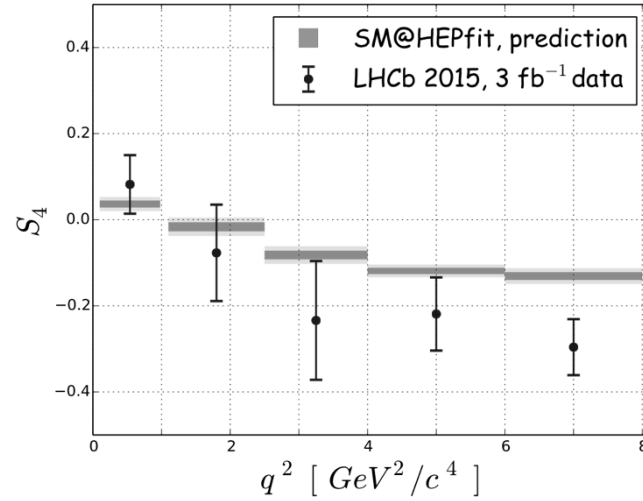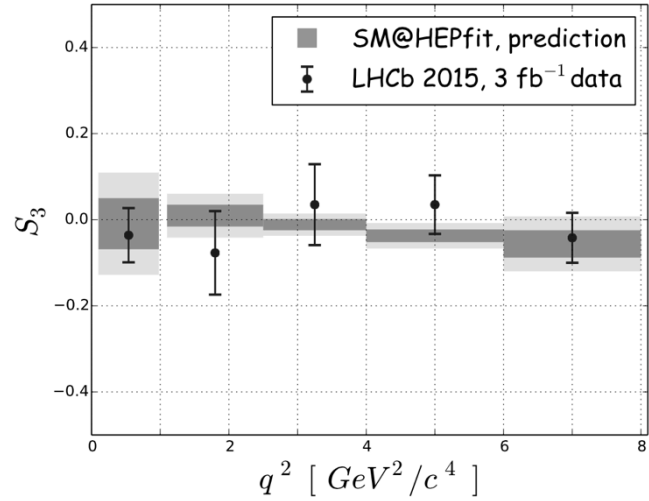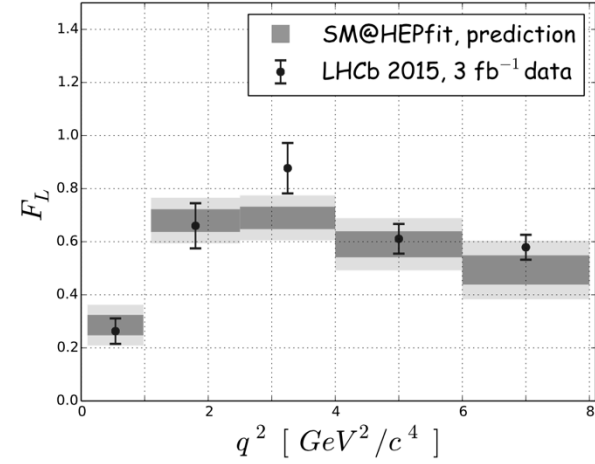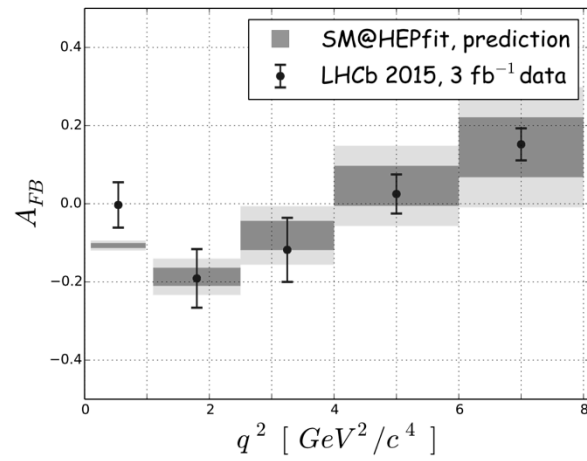
two sides of the story...
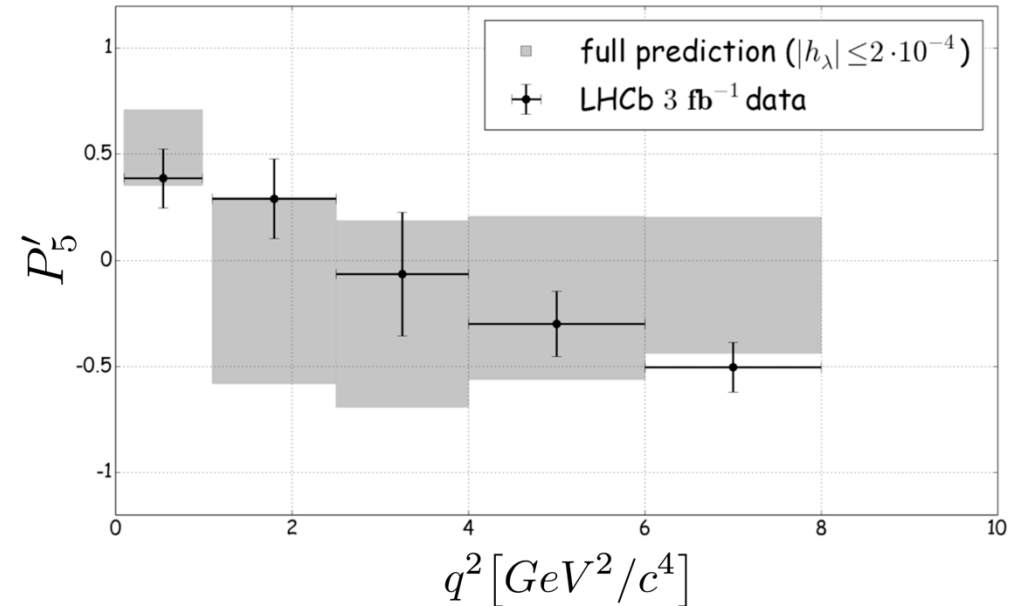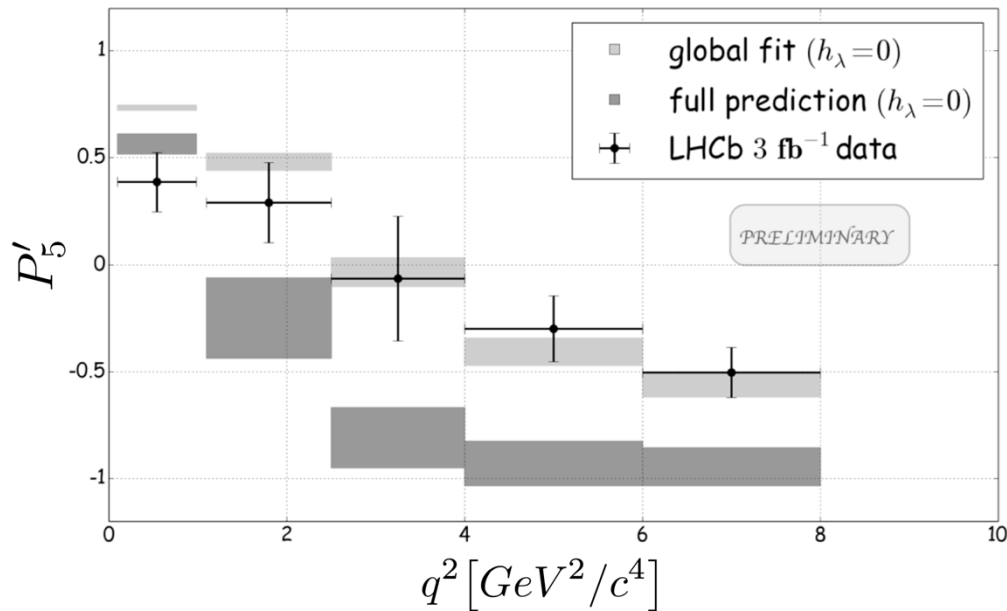
- data blind + underestimated hadronic contribution lead to incorrect estimates of the angular observables
- using data can seemingly lead to "correct" estimates...

however...

- data-blind estimations of the angular observables with large hadronic contributions can lead to a large shift in both the central values and inflation of errors in the angular observables.

the story
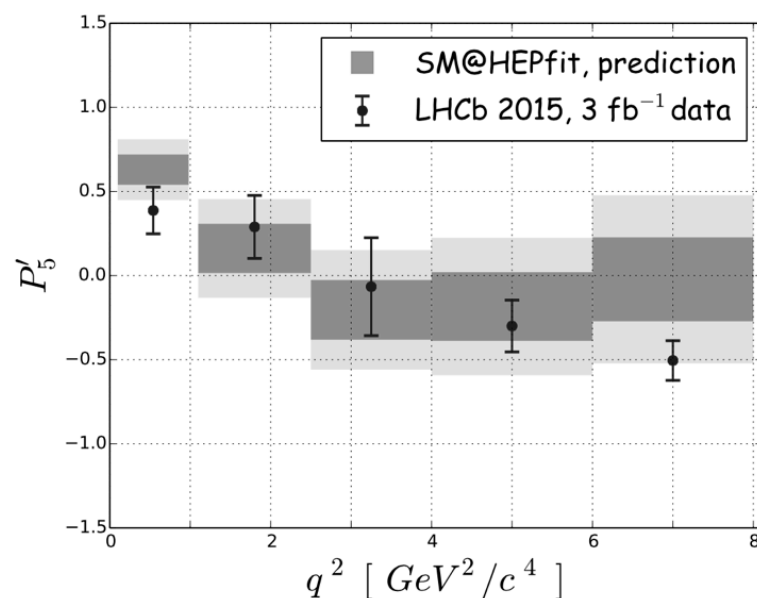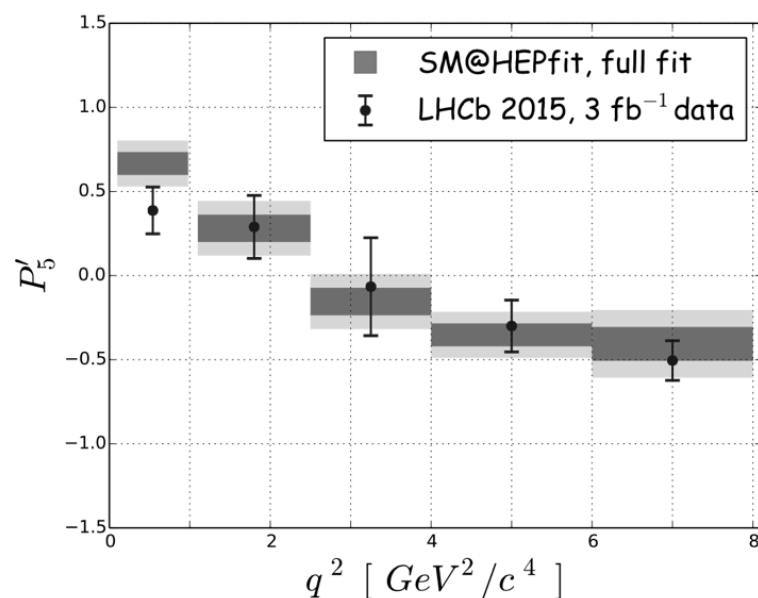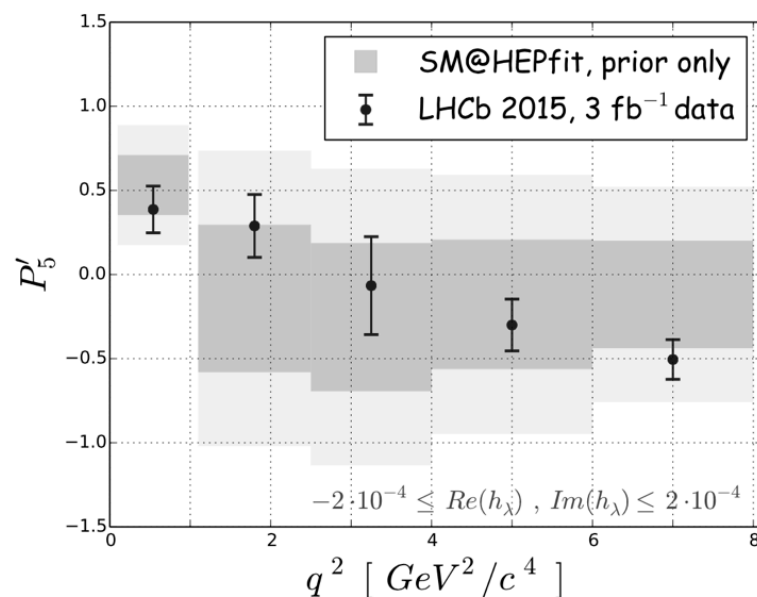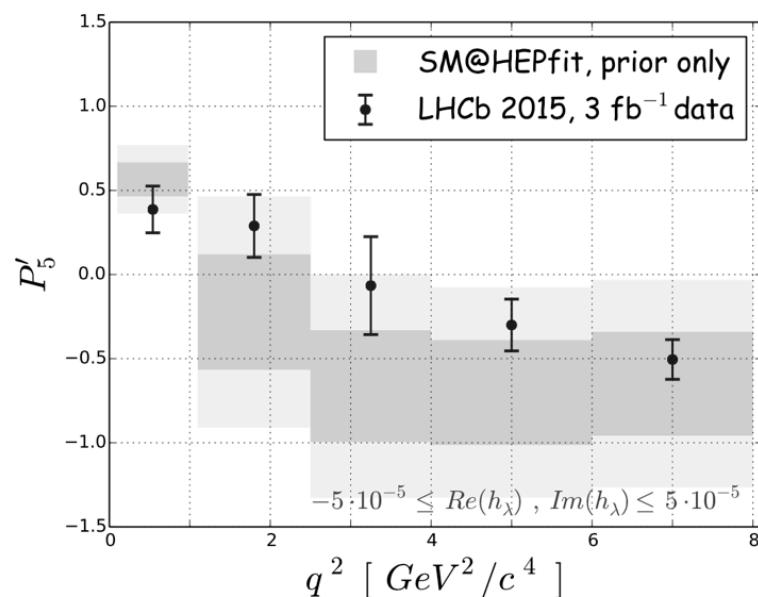
$$P_5' = \frac{S_5}{\sqrt{F_L(1-F_L)}}$$

We DO NOT use this expression for our fit or prediction. We use the helicity amplitudes.

the question of hadronic contributions

$$\Delta C_9^{(\bar{c}c, B \to K^*, \mathcal{M}_i)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) + 2C_1 \widetilde{g}^{(\bar{c}c, B \to K^*, \mathcal{M}_i)}(q^2)$$

$$C_9^{\text{eff}}(q^2) = C_9^{\text{eff}} + Y(q^2)$$

- in the very low q² regime the hadronic contributions extracted from data and theory estimates seem to be compatible

- in the region closer to the resonance hadronic contributions extracted from data seem to be larger than theory estimates, as they should be

caveat: a ΔC_9 or ΔC_7 would have a similar effect on the observables.

However, a ΔC_9 or ΔC_7 cannot have a q² dependence!

A. Khodjamirian, T. Mannel, A. Pivovarov and Y.-M. Wang, *Charm-loop effect in B → K⁽*⁾ℓ⁺ℓ⁻ and B → K*γ, JHEP* **09** (2010) 089 [arXiv:1006.4945]
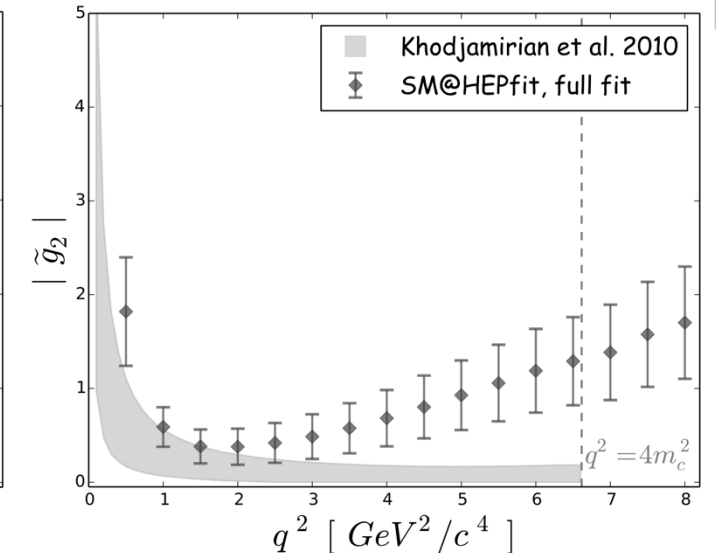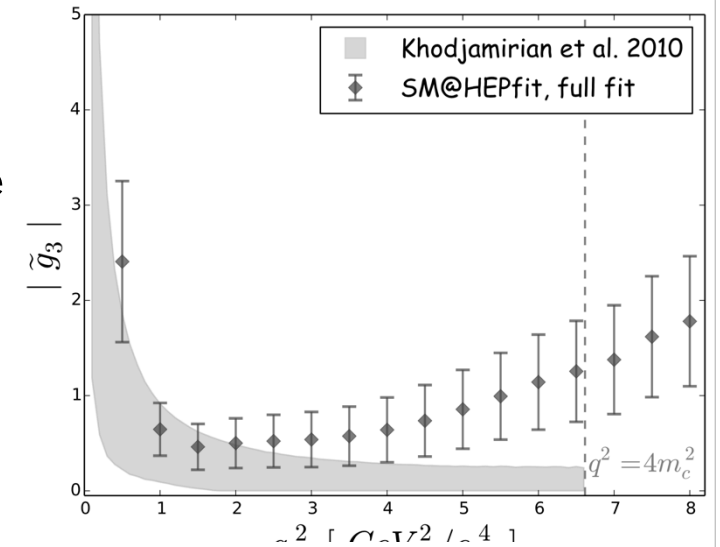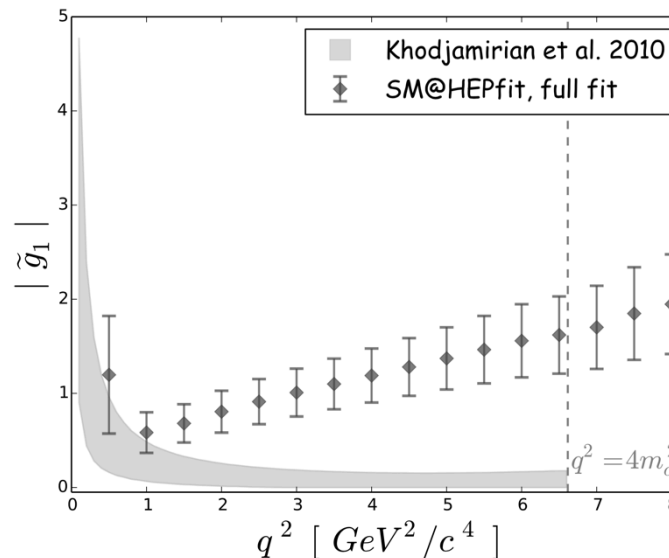
For now, no anomaly can be claimed with any level of human (or statistical) confidence.

Look, if you had, one shot or one opportunity
To seize everything you ever wanted in one moment
Would you capture it, or just let it slip?

Eminem

Thank you...!!

the angular observables

$$P_1 = -\frac{2}{1-4\frac{m_l^2}{q^2}} \frac{\Re\left[(C_{10}V_+)(C_{10}V_-)^*\right] + \Re\left[D_+D_-^*\right]}{|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2}$$

$$P_2 = \frac{1}{\sqrt{1-4\frac{m_l^2}{q^2}}} \frac{\Re\left[D_+(C_{10}V_+)^* + D_-(C_{10}V_-)^*\right]}{|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2}$$

$$P_3 = -\frac{\Im\left[(C_{10}V_+)(C_{10}V_-)^*\right] + \Im\left[D_+D_-^*\right]}{|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2}$$

$$D_0 = \frac{m_B^2}{q^2}\left(16\pi^2 h_0(q^2) - 2\frac{m_b}{m_B}C_7^{\text{eff}}\tilde{T}_0\right) - C_9^{\text{eff}}(q^2)\tilde{V}_0$$

$$D_+ = \frac{m_B^2}{q^2}\left(16\pi^2 h_+(q^2) - 2\frac{m_b}{m_B}C_7^{\text{eff}}T_+\right) - C_9^{\text{eff}}(q^2)V_+$$

$$D_- = \frac{m_B^2}{q^2}\left(16\pi^2 h_-(q^2) - 2\frac{m_b}{m_B}C_7^{\text{eff}}T_-\right) - C_9^{\text{eff}}(q^2)V_-$$

$$C_9^{\text{eff}}(q^2) = C_9^{\text{eff}} + Y(q^2)$$

$$P_4' = \frac{\Re\left[C_{10}(V_- + V_+)(C_{10}\tilde{V}_0)^*\right] + \Re\left[(D_- + D_+)D_0^*\right]}{\sqrt{\left(\left|C_{10}\tilde{V}_0\right|^2 + |D_0|^2\right)\left(|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2\right)}}$$

$$P_5' = -\frac{\Re\left[(D_- - D_+)\left(C_{10}\tilde{V}_0\right)^*\right] + \Re\left[C_{10}(V_- - V_+)(D_0)^*\right]}{\sqrt{\left(1-\frac{4m_l^2}{q^2}\right)\left(\left|C_{10}\tilde{V}_0\right|^2 + |D_0|^2\right)\left(|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2\right)}}$$

$$P_6' = -\frac{\Im\left[(D_- - D_+)\left(C_{10}\tilde{V}_0\right)^*\right] + \Im\left[C_{10}(V_- - V_+)D_0^*\right]}{\sqrt{\left(1-\frac{4m_l^2}{q^2}\right)\left(\left|C_{10}\tilde{V}_0\right|^2 + |D_0|^2\right)\left(|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2\right)}} \longrightarrow = 0$$

if hadronic contribution is switched off and C_7 and C_{10} are real

$$P_8' = \frac{\Im\left[C_{10}(V_- - V_+)(C_{10}\tilde{V}_0)^*\right] + \Im\left[(D_- - D_+)D_0^*\right]}{\sqrt{\left(\left|C_{10}\tilde{V}_0\right|^2 + |D_0|^2\right)\left(|C_{10}V_-|^2 + |C_{10}V_+|^2 + |D_+|^2 + |D_-|^2\right)}}$$

connecting S and P

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}}\bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K$$

$$+\tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_l$$

$$-F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$

$$+S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi$$

$$+\tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi$$

$$+S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$

$$S_i = \frac{I_i+\bar{I}_i}{\Gamma'} \longleftrightarrow \Sigma_i = \frac{I_i+\bar{I}_i}{2}$$

$$q^2 \gg m_l^2 \longrightarrow \Sigma_{1c} = -\Sigma_{2c} \; \textbf{+} \; \Sigma_{1s} = 3\Sigma_{2s}$$

$$F_L = \frac{\Sigma_{1c}}{\Gamma'} \quad \textbf{+} \quad \Gamma' = \Sigma_{1c} + 4\Sigma_{2s}$$

$$P_1 = A_T^{(2)} = \frac{2S_3}{1-F_L}, \qquad P_2 = -\frac{2}{3}\frac{A_{FB}}{1-F_L}, \qquad P_3 = -\frac{S_9}{1-F_L}$$

$$P_4' = \frac{2S_4}{\sqrt{F_L(1-F_L)}}, \qquad P_5' = \frac{S_5}{\sqrt{F_L(1-F_L)}}, \qquad P_6' = -\frac{S_7}{\sqrt{F_L(1-F_L)}},$$
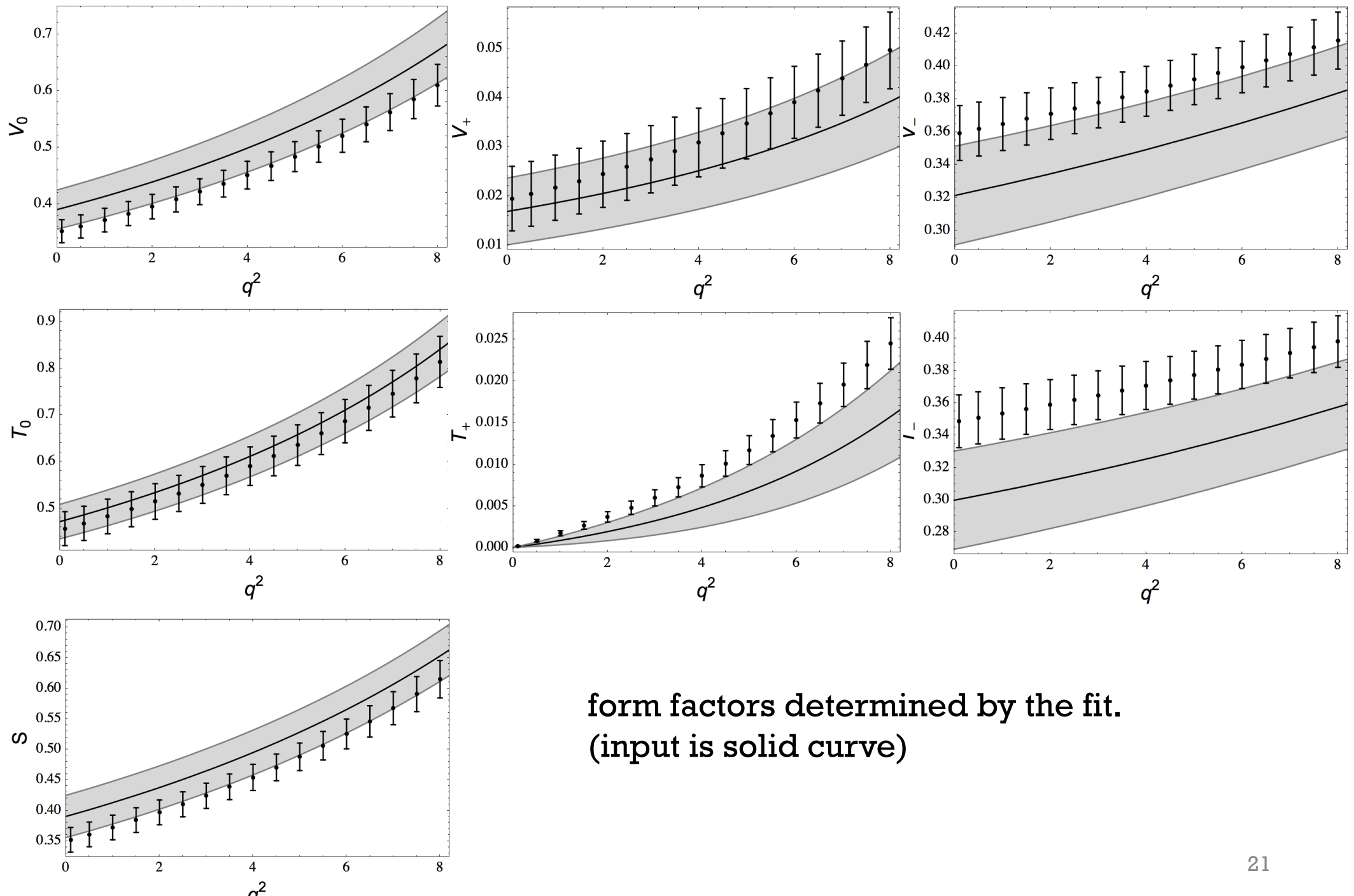
$$P_8' = -\frac{2S_8}{\sqrt{F_L(1-F_L)}}.$$

because of the definitions of the P observables and choice of kinematic angle:

$$P_2^{\mathrm{LHCb}} = -P_2^{\mathrm{T}}, \quad P_4'^{\mathrm{LHCb}} = -\tfrac{1}{2}P_4'^{\mathrm{T}}, \quad P_6'^{\mathrm{LHCb}} = -P_6'^{\mathrm{T}} \;\text{ and }\; P_8'^{\mathrm{LHCb}} = \tfrac{1}{2}P_8^{\mathrm{T}}$$

$$\theta_\ell \to \pi - \theta_\ell$$

the case of the form factors

form factors determined by the fit.
(input is solid curve)

To my Mother and Father, who showed me what I could do,

and to Ikaros, who showed me what I could not.

"To know what no one else does, what a pleasure it can be!"

– adopted from the words of

Eugene Wigner.