On Bimixing and GUT's

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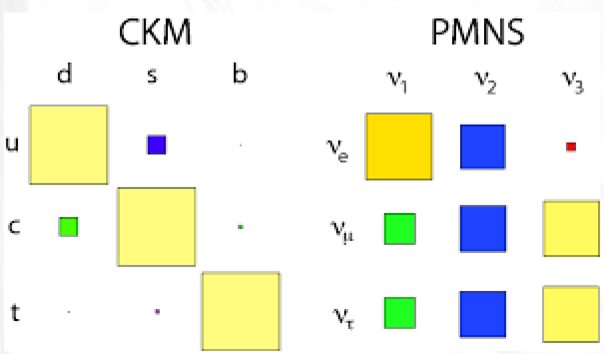
Standard oscillations

- Mixing matrix has the same structure in both contexts

$$U_{CKM,PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\times\begin{pmatrix}c_{13} & 0 & s_{13}e^{i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{-i\delta} & 0 & c_{13}\end{pmatrix}\times\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$



PMNS	CKM
all (but 1-3) matrix elements are of O(1)	matrix almost diagonal
one small and two large mixing angles	the three mixings are all small

(PMNS vs CKM)

in the Standard Model they do not talk to each other although the mechanism producing them is essentially the same

The need of New Physics

GOAL OF THIS TALK:

How to relate these two sectors ?

- Invoking GUT theories (different gauge groups):

  leptons and quarks sit in the same irreducible representations of the group

↓

Mass matrices are related

ex: SU(5)

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad m_d = m_e^T$$

⇒ Not enough for producing the correct mixings

The need of New Physics

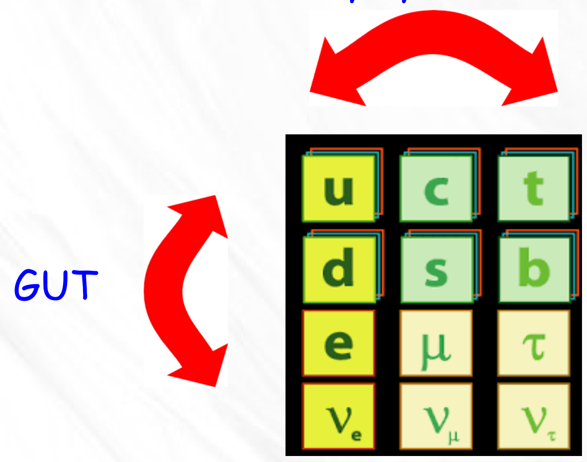
- to improve predictability: Invoke family symmetries:

different families sit in the same irreducible representations of the group

↓

Matrix elements of mass matrices are related

family symmetries



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Being less ambitious...QLC

- Numerically, one sees that: $\theta_{12} + \theta_c \sim \pi/4$ → quark-lepton complementarity (QLC)

  $\theta_{12} + O(\theta_c) \sim \pi/4$ is called *weak complementarity*

- Numerically, one also sees that: $\theta_{13} \sim \theta_c/\mathrm{sqrt}[2]$

this suggests that the Cabibbo is a key-role parameter

Where θ_c enters in the lepton sector?

Nature seems to help us !

- $m_\mu/m_\tau \sim \theta^2_c$
- $m_e/m_\mu \sim \theta_c^{3\text{-}4}$

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we have to deal with **mass matrices !**

Introducing θ_c into the charged lepton masses

- for large fermion masses, we can use renormalizable operators (d=4):

$$\overline{\psi_L} H \psi_R$$

- to generate hierarchies, we can use non-renormalizable operators (d >= 5):

$$\overline{\psi_L} H \psi_R \left(\frac{\varphi}{\Lambda}\right)^n$$

φ: new scalar fields, with vev = $\langle\phi\rangle$ transforming non-trivially under some flavor symmetry

Λ: cut-off of the theory

⇓

this number should be smaller than 1

breaking of the flavor symmetry →

$$\frac{\langle\varphi\rangle}{\Lambda} \sim \theta_C$$

⇒

- $m_\mu/m_\tau \sim$ (d=6) / (d=4)

Natural assumption: the vevs of the new scalar fields are all of the same order of magnitude

Getting the QLC relation

- The strategy:

BM in the neutrino sector ⟵ family symmetries

↓

Corrections from charged leptons:
QLC

↓

Connecting quarks and leptons: obtaining Vus ~ λ_c

GUT

Getting the QLC

Start with a model whose LO prediction in the neutrino sector is $\theta_{12} = \pi/4$

An easy task with family symmetries

Plethora of models in the literature

Frampton, Petcov and Rodejohann,
Nucl. Phys. B687 (2004) 31
T.Ohlsson,
Phys.Lett.B622, 159 (2005)
Altarelli, Feruglio and Merlo,
JHEP0905, 020 (2009)
D.Meloni,
JHEP1110, 010 (2011)
Altarelli, Machado and Meloni,
arXiv:1504.05514 [hep-ph]

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix} \quad \xrightarrow{\text{diagonalization}} \quad U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Maximal mixing

$$\sin^2\theta_{12} = \frac{1}{2} \quad \sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0$$

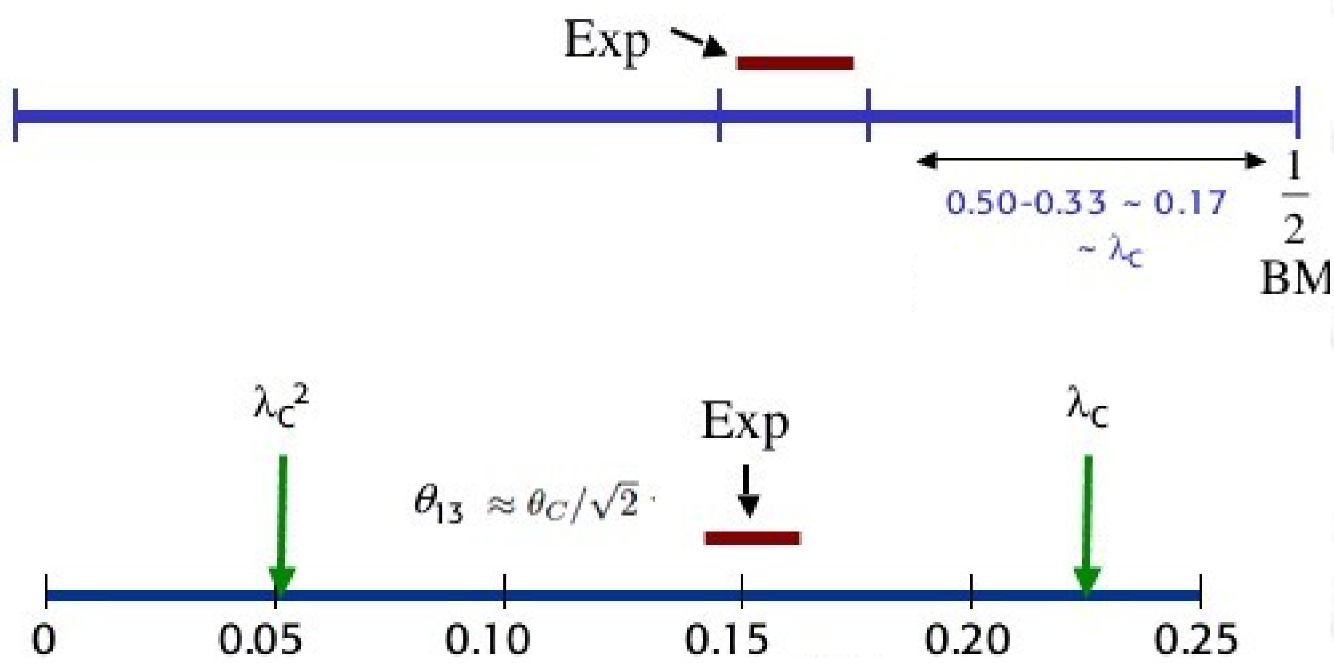
Corrections

- The strategy:

Now needs corrections to fall on the experimental value $\theta_{12} \sim 33°$

$\sin^2\theta_{12}$

$\sin\theta_{13}$



Corrections provided by the diagonalization of the charged leptons

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An SU(5) example

- Example from SU(5) x S_4 → group of permutation of 4 objects

Altarelli, Machado, Meloni
arXiv:1504.05514

$$m_e \sim \begin{bmatrix} a_{11}\lambda_C^5 & a_{12}\lambda_C^4 & a_{13}\lambda_C^4 \\ a_{21}\lambda_C^4 & -c\lambda_C^3 & 0 \\ a_{13}\lambda_C^4 & c\lambda_C^3 & a_{33}\lambda_C \end{bmatrix} \Rightarrow U_l \sim \begin{bmatrix} 1 & u_{12}\lambda_C & u_{13}\lambda_C \\ -u_{12}^*\lambda_C & 1 & 0 \\ -u_{13}^*\lambda_C & -u_{12}^*u_{13}^*\lambda_C^2 & 1 \end{bmatrix}$$

- a_{ij}, u_{ij} are O(1) coefficients
- u_{ij} is a linear combination of a_{ij}

$$U_{PMNS} = U_l^+ U_{BM}$$

this gives $\sin^2\theta_{12} = \frac{1}{2} - u_{12}\lambda_C$ which is perfectly OK

this relation is of the *weak complementarity* form **IF** the model also generates Vus ~ $O(\lambda_c)$

⇒ link with GUT

The Vus matrix element

- the down sector

$$m_d = m_e^T \Rightarrow U_d \sim \begin{bmatrix} 1 & d_{12}\lambda_C & d_{13}\lambda_C^3 \\ -d_{12}^*\lambda_C & 1 & d_{23}^*\lambda_C^2 \\ (d_{12}^* d_{23}^* - d_{13}^*)\lambda_C^3 & -d_{23}^*\lambda_C^2 & 1 \end{bmatrix}$$

d_{ij} are a different combination of a_{ij}

so mixings are different but the off-diagonal (1-2) element is again of $O(\lambda_c)$

(obviously we have to be sure that the up-quark sector does not destroy the scheme)

weak complementarity is realized in the context GUT + family symmetry

What about BM and SO(10) ?

- no SO(10) singlets for right-handed neutrinos → more difficult explanation of the difference between charged fermions and neutrinos
- see-saw type II is an useful ansatz to separate the neutrino masses from the dominant contribution to the charged fermions (given by the Yukawa h)

$$M_\nu \sim f \langle 126 \rangle_3 + \; type - I$$

f=Yukawa matrix of the fermion couplings to 126: can always be put on BM (or TBM) form

$$Y_u \sim m_{top}(f+h)$$

$$Y_d \sim m_b(f+h)$$

$$Y_e \sim m_\tau(-3f+h)$$

⇒ Corrections from Ye are typically of the same order as the largest quark mixing angle, i.e. λ_c

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Better BM or TBM in SO(10) ?

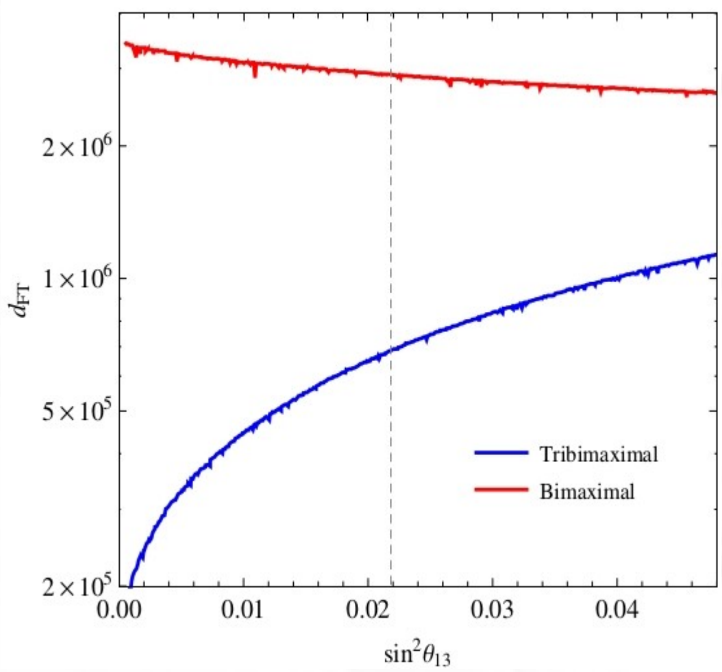
- The big question: which pattern is more favored by the data?

χ^2 analysis not useful, since f can be put in the desired form after an useful rotation of the 16 of fermions

- we have to use some estimator: the *fine-tuning parameter*

$$d_{FT} = \Sigma_i \left| \frac{par_i}{err_i} \right|$$

shift of the best-fit parameter that changes the χ^2 by 1 unit



the small first family masses dominate the fine-tuning

it turns out that the TBM fit to the data is slightly less fine-tuned than BM

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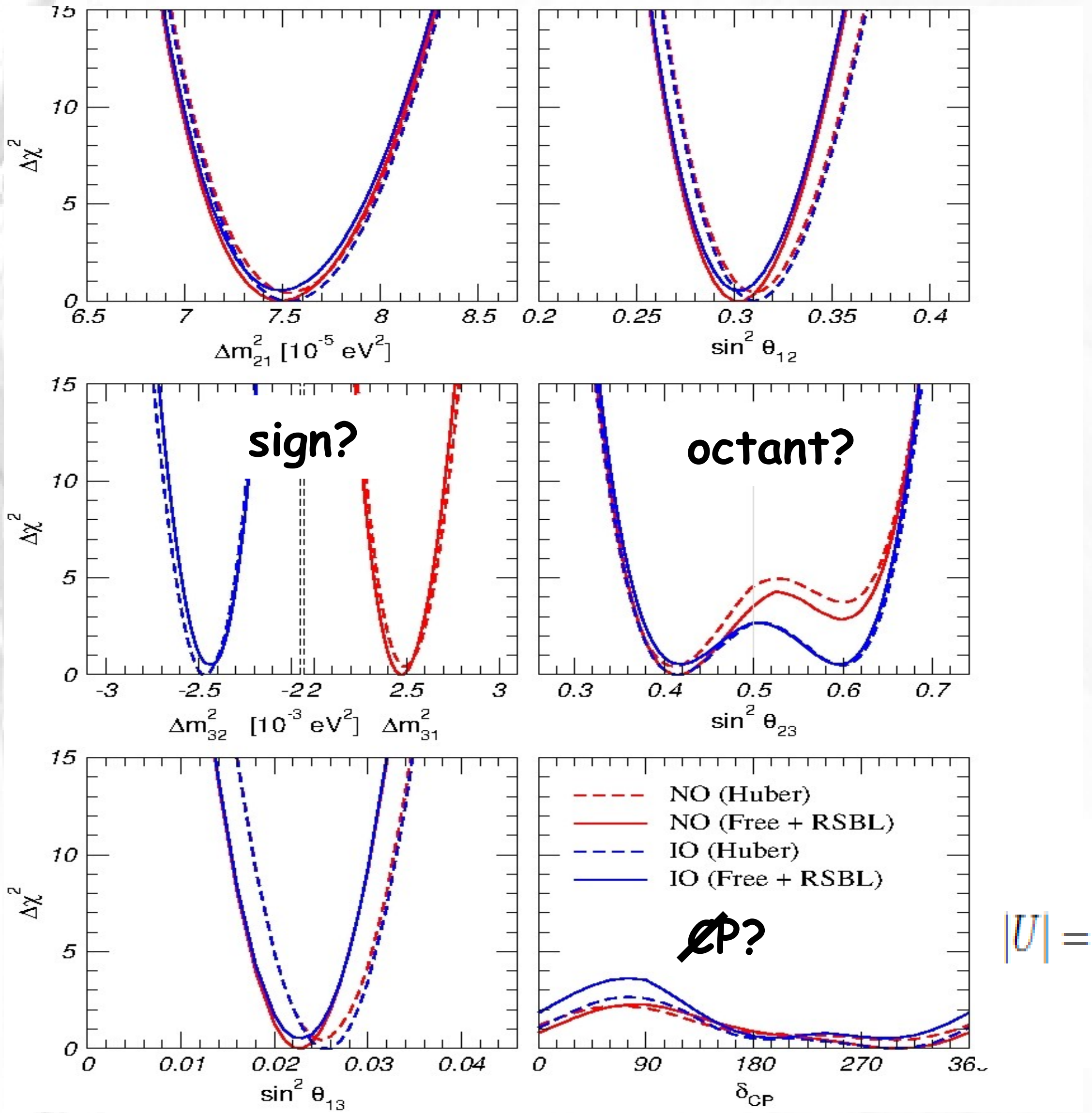
Conclusions

- Weak form of complementarity can be easily implemented in GUT context
- BM is good starting point in SU(5) + family symmetry framework
- No clear preference in the description of the data emerged from SO(10)

Backup

Global fit on neutrino data



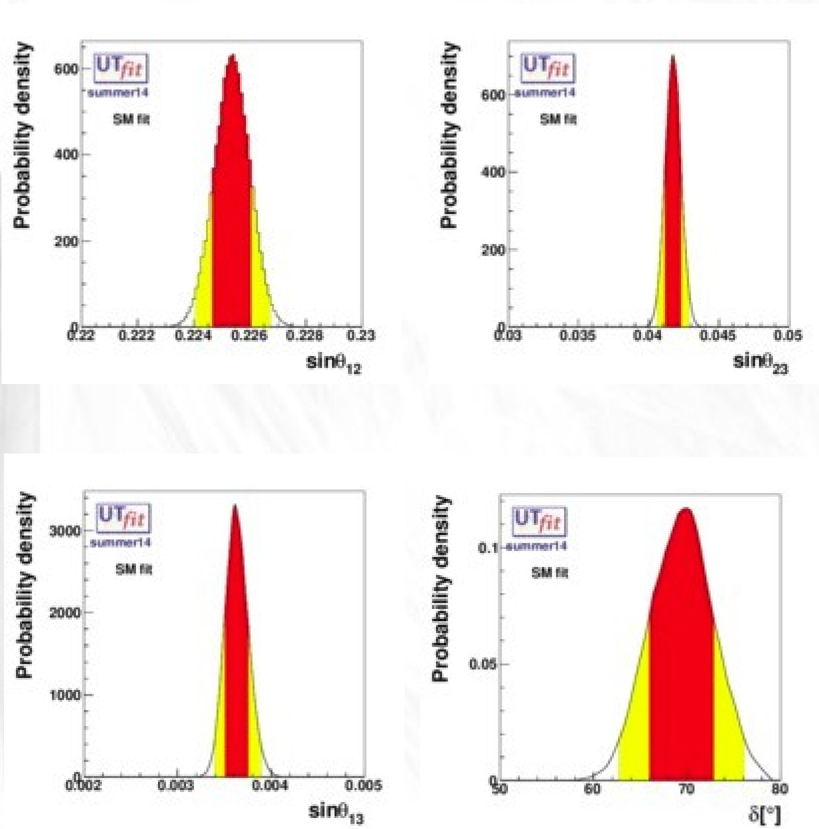
Gonzalez-Garcia et al. JHEP1212,(2012)123

Parameter	Result
θ_{12}	$33.36^{+0.81}_{-0.78}$
θ_{13}	$8.66^{+0.44}_{-0.46}$
θ_{23}	$40.0^{+2-1}_{-1.5}$
δ	300^{+66}_{-138}
Δm^2_{23} (10^{-3} eV^2)	$2.47^{+0.07}_{-0.07}$
Δm^2_{12} (10^{-5} eV^2)	$7.50^{+0.18}_{-0.19}$

$$|U| = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

PMNS mixing matrix

Global fit on quark data



http://www.utfit.org

Parameter	Result
$\sin\theta_{12}$	0.22523+-0.00065
$\sin\theta_{13}$	0.00363+-0.00012
$\sin\theta_{23}$	0.0417+-0.00057
$\delta^{(o)}$	69.4+-3.4

$$V_{CKM} = \begin{pmatrix} (0.97426 \pm 0.00015) & (0.22529 \pm 0.00061) & (0.00363 \pm 0.00012)e^{i(-69.3\pm3.3)^\circ} \\ (-0.22518 \pm 0.00066)e^{i(0.03509\pm0.00098)^\circ} & (0.97341 \pm 0.00015)e^{i(-0.00187\pm0.00005)^\circ} & (0.0417 \pm 0.00056) \\ (0.0088 \pm 0.00018)e^{i(-22.0\pm0.8)^\circ} & (-0.04092 \pm 0.00055)e^{i(1.069\pm0.042)^\circ} & (0.999119 \pm 0.000021) \end{pmatrix}$$

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CKM mixing matrix

Mixing matrices

- U_{PMNS} and V_{CKM} have contributions from two different sectors

<u>leptons</u>

$$U_{PMNS} = U^{+l}_{j\alpha} U^{\nu}_{\alpha i}$$

from the diagonalisation of the charged lepton mass matrix

from the diagonalisation of the neutrino mass matrix

<u>quarks</u>

$$V_{CKM} = U^{+d}_{j\alpha} U^{u}_{\alpha i}$$

How to relate these two sectors ?

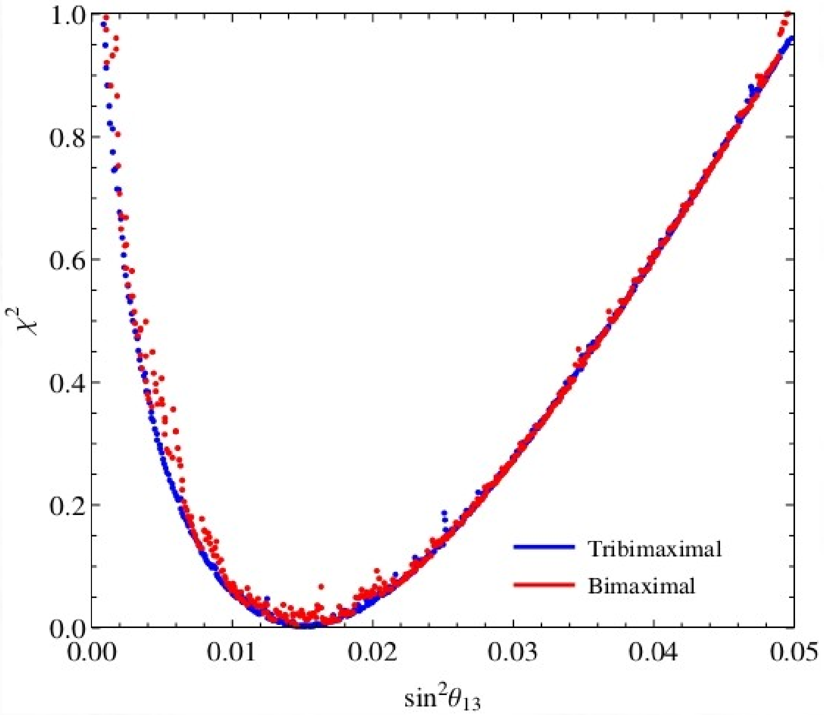
What about BM and SO(10) ?

- Are the data compatible with these constrains?

The answer is YES but not very conclusive

in fact, we could have started from f of the TBM form and still obtain a good description of the data, i.e., of θ_{13}

the set of parameters used in one fit are functions of the parameters of the other fit, so the χ^2 in the two cases are simply related to each other



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