Origin of a large CP asymmetry in $B^{\pm} \to K^{+} K^{-} K^{\pm}$ decays

Work done in collaboration with Piotr Żenczykowski
(Institute of Nuclear Physics PAS, Kraków, Poland)

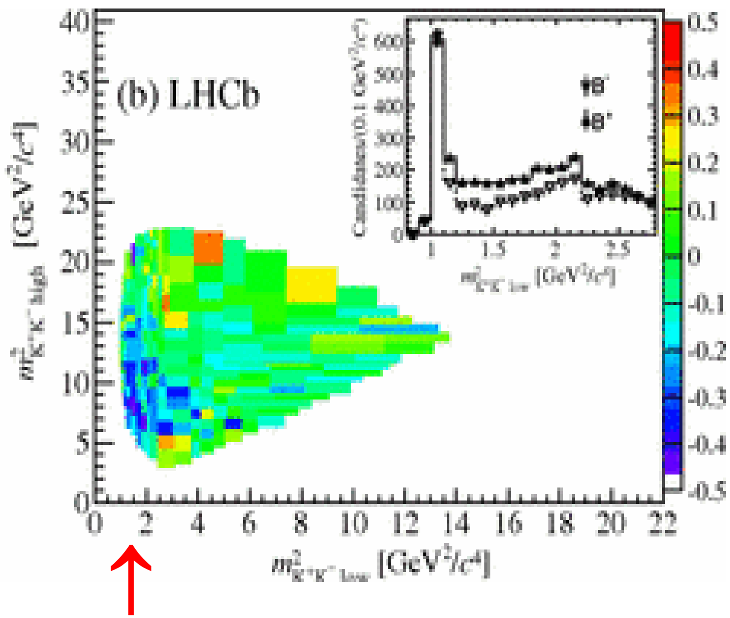
References:

1. L. Leśniak, P. Żenczykowski, Phys. Lett. B 737 (2014) 201,
Dalitz-plot dependence of CP asymmetry in $B^{\pm} \to K^{+} K^{-} K^{\pm}$ decays;

2. A. Furman, R. Kamiński, L. Leśniak, P. Żenczykowski,
Phys. Lett. B 699 (2011) 102,
Final state interactions in $B^{\pm} \to K^{+} K^{-} K^{\pm}$ decays.

Motivation

1. Large CP asymmetry in $B^{\pm} \to K^{+} K^{-} K^{\pm}$ decays for the $K^{+} K^{-}$ effective masses above the $\phi(1020)$ peak has been **predicted** in our model developed in **2011** (A. Furman et al., Phys. Lett. B 699 (2011) 102).
2. In **2013** the LHCb Collaboration has measured the CP-violating asymmetry and **found** a large negative asymmetry in the region $1.1 < m^2_{K^+K^- low} < 2.0$ GeV^2 and $m^2_{K^+K^- high} < 15$ GeV^2 (R. Aaij et al. (LHCb Coll.) PRL 111 (2013) 101801).



3. **Our aim**: deeper understanding of the **origin** of this large negative CP asymmetry.

S-wave amplitudes

$B^- \to K^- K^+ K^-$

$$A^- = \lambda_u F_u + \lambda_c F_c$$

Separation of **weak** transition factors **(λ_u , λ_c)**

$B^+ \to K^+ K^- K^+$

$$A^+ = \lambda_u^* F_u + \lambda_c^* F_c$$

$$\lambda_u = V_{ub} V_{us}^*$$

$$\lambda_c = V_{cb} V_{cs}^*$$

$V_{ub} = |V_{ub}| e^{-i\gamma}$

CKM angle $\gamma \approx 68^0$

$V_{us}, V_{cb}, V_{cs} \approx$ real

$\lambda_u = |\lambda_u| e^{i\varphi_u}$ $\varphi_u = -\gamma$ $\lambda_c = |\lambda_c| e^{i\varphi_c}$ $\varphi_c \approx 0$ $|\lambda_c| / |\lambda_u| \approx 51$

tree + penguin contributions:

$$F_u = u_n \Gamma_2^{n*} + u_s \Gamma_2^{s*}$$

only penguin contributions:

$$F_c = c_n \Gamma_2^{n*} + c_s \Gamma_2^{s*}$$

Γ_2^n and Γ_2^s are **scalar non-strange and strange kaon form factors**.

tree contribution **dominates** in ***F****$_u$*

$$|u_n \Gamma_2^{n*}| >> |u_s \Gamma_2^{s*}|$$

c_n and c_s are comparable

Coupled channel model of the scalar-isoscalar form factors

3 coupled channels: $\pi\ \pi\ \ (\pi^+\ \pi^-\ ,\ \pi^0\ \pi^0)$ - index 1

$K\ \overline{K}\ \ (K^+\ K^-\ ,\ K^0\ \overline{K}^0)$ - index 2

effective 4 π ($\sigma\ \sigma$ *or* $\rho\ \rho$) - index 3

6 scalar form factors:

$$\Gamma^n = \begin{pmatrix} \Gamma_1^n \\ \Gamma_2^n \\ \Gamma_3^n \end{pmatrix} \qquad \Gamma^s = \begin{pmatrix} \Gamma_1^s \\ \Gamma_2^s \\ \Gamma_3^s \end{pmatrix}$$

← **kaon** form factors

n - non-strange **s** - strange

9 amplitudes of meson-meson interactions:

	π	K	σ	

$$T = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} \begin{matrix} \pi \\ K \\ \sigma \end{matrix}$$

Refs: R. Kamiński, L. Leśniak, B. Loiseau, Phys. Lett. B413 (1997) 130 [**meson-meson amplitudes**]
J.-P. Dedonder *et al*., Acta Phys. Pol. B 42 (2011) 2013 [**scalar form factors**].

Matrix integral equations for the scalar form factors

3 coupled channels: $\pi\pi, \bar{K}K,$ effective 4π $(\sigma\sigma$ or $\rho\rho)$

$$\Gamma^{n*} = R^n + TGR^n$$

$$\Gamma^{s*} = R^s + TGR^s$$

$$G = \begin{pmatrix} G_1 & 0 & 0 \\ 0 & G_2 & 0 \\ 0 & 0 & G_3 \end{pmatrix}$$

Green's functions

$R_j^{n,s}$ – **production** functions, T – 3x3 matrix of **amplitudes**

$$R_j^{n,s}(E) = \frac{\alpha_j^{n,s} + \tau_j^{n,s} E + \omega_j^{n,s} E^2}{1 + cE^4} \qquad j = 1,2,3$$

$$E = m_{\pi\pi} = m_{KK} = m_{4\pi}$$

The parameters $\alpha_j^{n,s}, \tau_j^{n,s}, \omega_j^{n,s}$ are calculated using the **chiral perturbation model**.
The fitted parameter c controls the high energy behaviour of R_j.

origin of a large CP asymmetry

$$A_{CP} = \frac{|A^-|^2 - |A^-|^2}{|A^-|^2 + |A^+|^2} = -\frac{2R\sin(\varphi_c - \varphi_u)\sin(\delta_c - \delta_u)}{1 + R^2 + 2R\cos(\varphi_c - \varphi_u)\cos(\delta_c - \delta_u)}$$

$$R = \frac{|\lambda_c|}{|\lambda_u|}\frac{|F_c|}{|F_u|}$$

$$F_u = |F_u| e^{i\delta_u} \qquad F_c = |F_c| e^{i\delta_c} \qquad \Gamma_2^{s*} = |\Gamma_2^{s*}| e^{i\delta_s} \qquad \Gamma_2^{n*} = |\Gamma_2^{n*}| e^{i\delta_n}$$

$$\delta_c - \delta_u \approx \delta_s - \delta_n \equiv \Delta\delta$$

difference of strong amplitude phases

$\frac{|\lambda_c|}{|\lambda_u|} \approx 51 \quad \frac{|F_c|}{|F_u|} \approx 0.02$ for $m^2_{K^+K^-} \approx 1.2$ GeV^2

$R \approx 1 \rightarrow$ max $|A_{CP}|$

$\Delta\delta \approx 45^0$ for $1 \le m^2_{K^+K^-} \le 1.5$ GeV^2

$\varphi_c - \varphi_u = \gamma = 68^0$

Result: large $A_{CP} \sim -0.5$ for the S-wave amplitude

How are strong phases created?

$$\Gamma_2^{n*} = R_2^n + T_{22}G_2R_2^n + T_{21}G_2R_1^n + T_{23}G_2R_3^n$$

$$\Gamma_2^{s*} = R_2^s + T_{22}G_2R_2^s + T_{21}G_2R_1^s + T_{23}G_2R_3^s$$

T_{22} = KK → KK elastic scattering amplitude

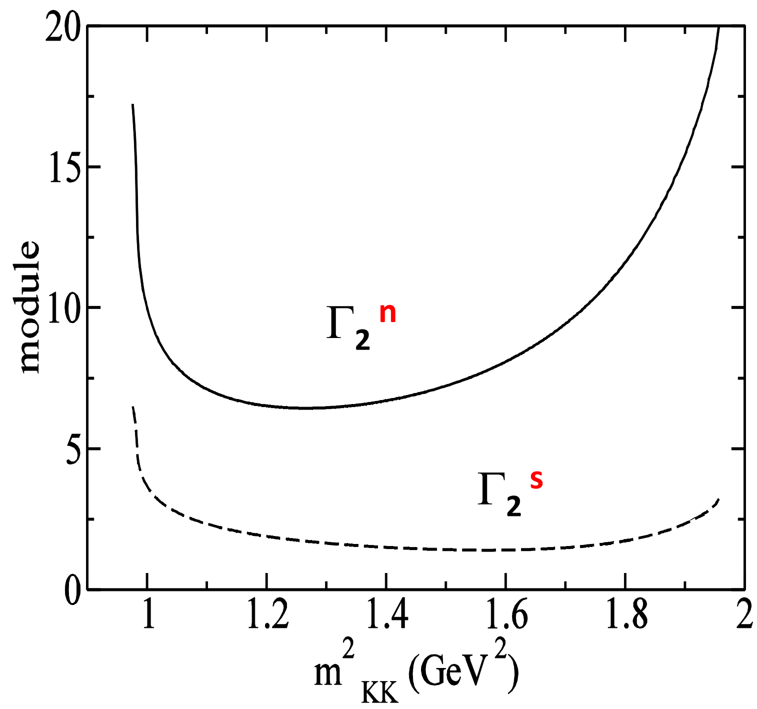
T_{12} = ππ → KK transition (rescattering) amplitude

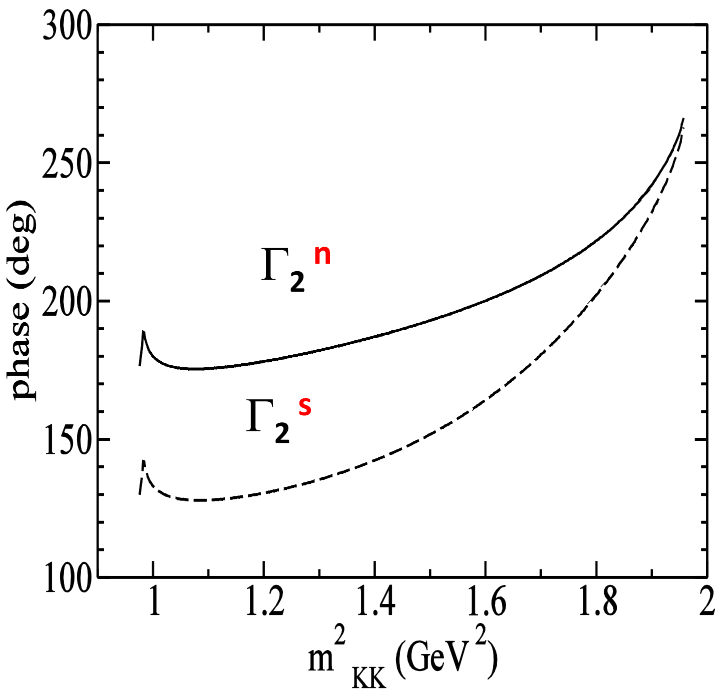
T_{23} = 4π → KK elastic transition (rescattering) amplitude

$R_j^n \neq R_j^s$ $T_{21} \neq 0$ and $T_{23} \neq 0$ → the **phases** of Γ_2^{n*} and Γ_2^{s*} are **different**.

Both ππ → KK and 4π → KK transition amplitudes are important.

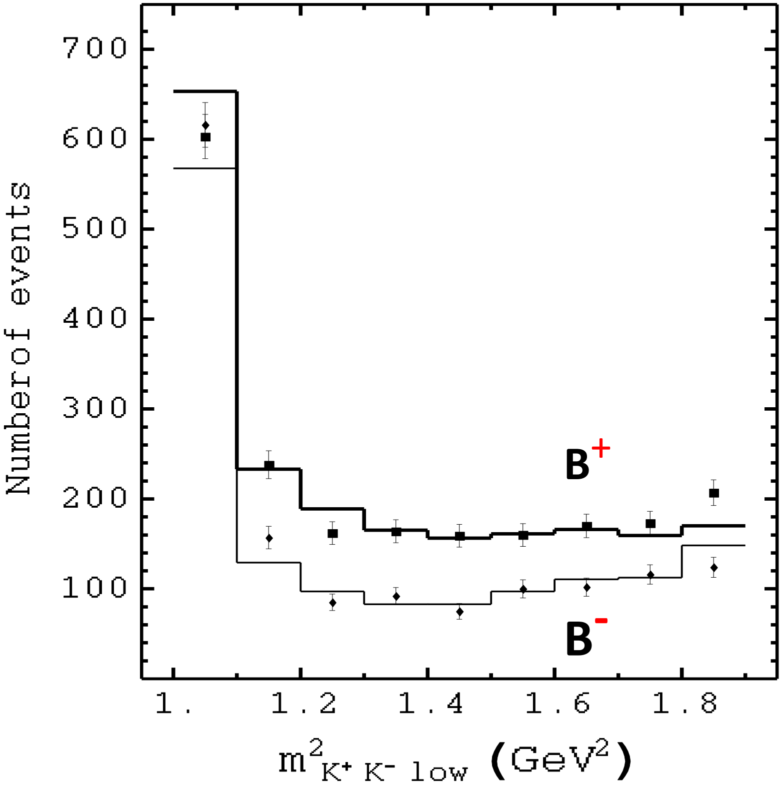
kaon scalar form factors

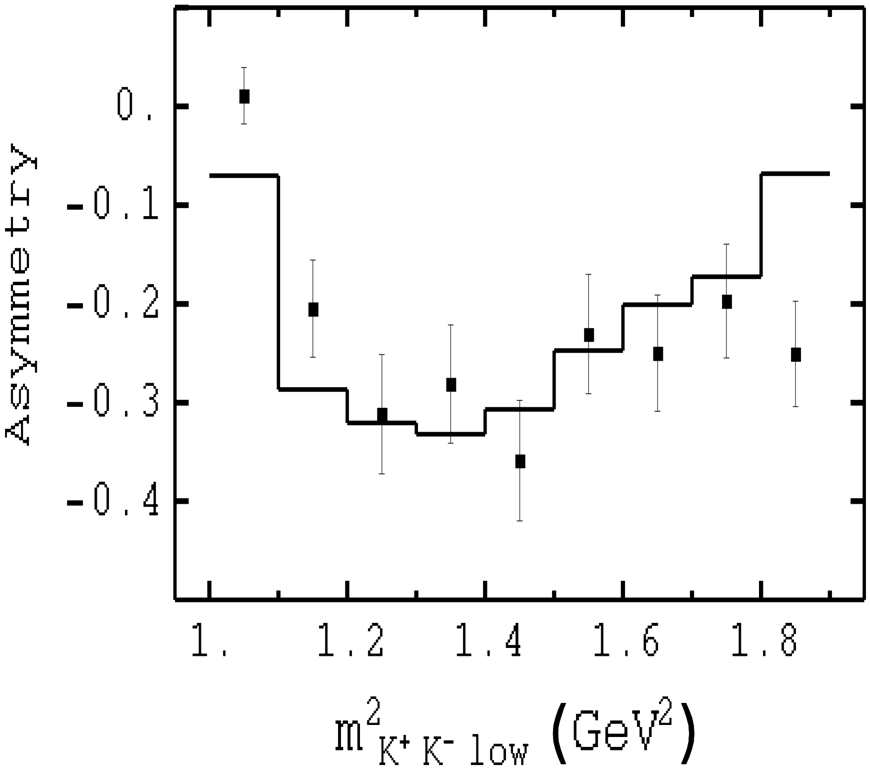




Γ_2^{n} **non-strange** form factor; Γ_2^{s} **strange** form factor

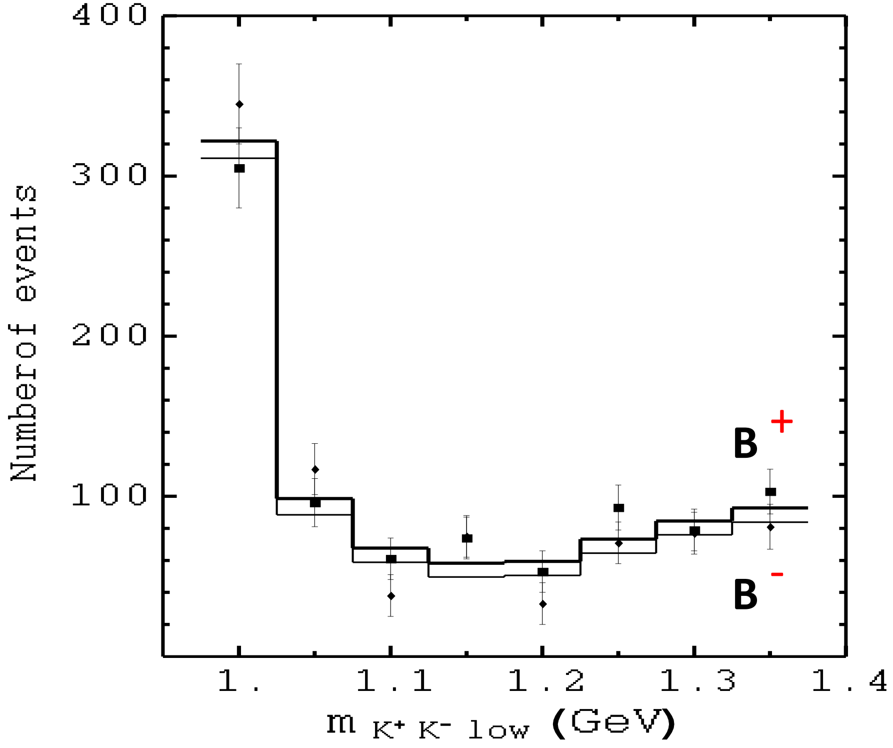
LHCb data

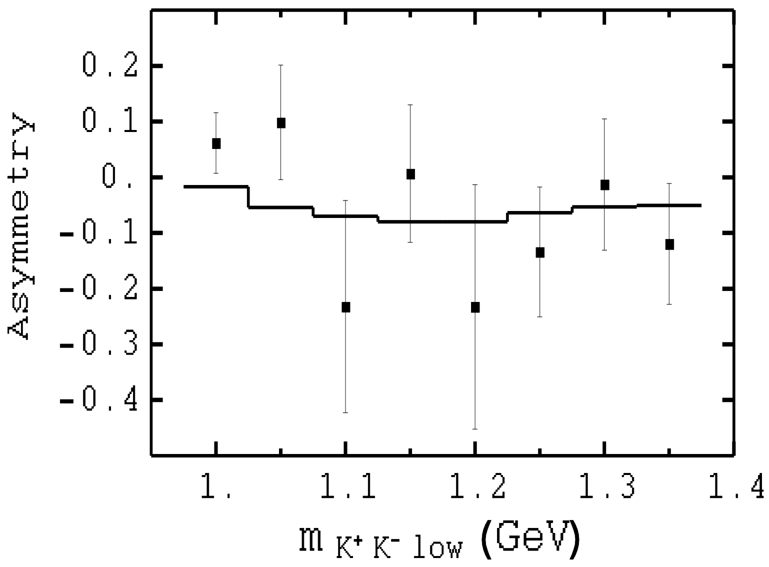




R. Aaij et al. (LHCb Coll.), Phys. Rev. Lett. 111 (2013) 101801

BABAR data





J.P. Lees et al. (BABAR Coll.), Phys. Rev. D 85 (2012) 112010

Conclusions

1. Large CP-violating asymmetry effects in the $B^{\pm} \to K^+ K^- K^{\pm}$ decays have been predicted in the QCD factorization model.

2. The model includes strong $K^+ K^-$ **final-state** long-distance **interactions** in the **S-, P-** and **D**- waves.

3. A **large negative CP asymmetry** appears for the $K^+ K^-$ effective mass squares between 1. 1 and 1.9 GeV^2 .

4. It originates from a presence of **two weak** phase differences together with the existence of **two** different **strong** phases of the **S**-wave amplitudes related to the phases of the kaon scalar strange and non-strange form factors.

5. Interchannel couplings of the **two pion** as well as the **four pion** states to the kaon-kaon states are essential to form two different strong phases. The $\pi\pi \to$ **KK** and **4**$\pi \to$ **KK rescattering** effects are naturally included in the model.

S-wave amplitude for $B^- \to K_1^- K_2^+ K_3^-$

$$A^- = -\tfrac{1}{2} G_F f_K (M_B^2 - m_{23}^2) F_0^{B^- \to (K^+K^-)_S}(m_K^2)\, U\, \chi \Gamma_2^{n*}(m_{23}) +$$
$$G_F \frac{\sqrt{2} B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{B^- \to K^-}(m_{23}^2)\, W\, \Gamma_2^{s*}(m_{23}) \equiv \lambda_u F_u + \lambda_c F_c$$

2 **weak** amplitudes (in ***U*** *both* tree and penguin terms, in ***W*** only penguin ones):

$$U = \lambda_u [a_1 + a_4^u + a_{10}^u - (a_6^u + a_8^u r)] + \lambda_c [a_4^c + a_{10}^c - (a_6^c + a_8^c r)]$$
$$W = \lambda_u (-a_6^u + \tfrac{1}{2} a_8^u) + \lambda_c (-a_6^c + \tfrac{1}{2} a_8^c), \qquad B_0 = m_\pi^2 / 2m_u$$
$$\lambda_u = V_{ub} V_{us}^*, \qquad \lambda_c = V_{cb} V_{cs}^*, \qquad r = 2m_K^2 / [(m_b + m_u)(m_s + m_u)]$$

a_i - Wilson coefficients; $\mathbf{a_1}$ - **tree** contribution; $\mathbf{a_4, a_6, a_8, a_{10}}$ - **penguin** terms

2 **strong interaction** functions:

Γ_2^n and Γ_2^s are **scalar non-strange and strange kaon form factors.**

Amplitudes for $B^- \to K_1^- K_2^+ K_3^-$ decay

S-wave:

$$A_S = -\tfrac{1}{2} G_F f_K (M_B^2 - m_{23}^2) F_0^{B^- \to (K^+K^-)_S}(m_K^2)\, y \chi \Gamma_2^{n*}(m_{23})$$
$$+ \tfrac{1}{2} G_F \frac{2\sqrt{2} B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{B^- \to K^-}(m_{23})\, v \Gamma_2^{s*}(m_{23})$$

P-wave:

$$A_P = 2\sqrt{2}\, \vec{p}_1 \cdot \vec{p}_2 G_F \{ \frac{f_K}{f_\rho} A_0^{B\to\rho}(m_K^2) F_u^{K^+K^-}(m_{23})\, y$$
$$- F_1^{BK}(m_{23}) [F_u^{K^+K^-}(m_{23}) w_u + F_d^{K^+K^-}(m_{23}) w_d + F_s^{K^+K^-}(m_{23}) w_s]\}$$

$F_u^{K^+K^-}, F_d^{K^+K^-}, F_s^{K^+K^-}$ are the kaon vector form factors.

$\vec{p}_1, \vec{p}_2$ are the K_1, K_2 momenta in the $K_2 K_3$ c.m. frame.

y, ν, w_u, w_d, w_s – weak decay coefficients

$B^{\pm} \to K^+ K^- K^{\pm}$ decays

B to light meson **transition form factors**: $F_0^{BS(K^+K^-)}(m_K^2), A_0^{B\rho}(m_K^2),$

$F_0^{BK}(s_{23}), F_1^{BK}(s_{23})$

kaon non-strange and strange **scalar form factors**: $\Gamma_2^{n*}(s_{23}), \Gamma_2^{s*}(s_{23})$

kaon vector form factors: $F_u^{K^+K^-}(s_{23}), F_d^{K^+K^-}(s_{23}), F_s^{K^+K^-}(s_{23})$

$$\Gamma_2^{n*} = \frac{1}{\sqrt{2}B_0}\langle [K^+K^-]_{S,I=0} | \bar{n}n | 0\rangle \qquad \Gamma_2^{s*} = \frac{1}{\sqrt{2}B_0}\langle [K^+K^-]_{S,I=0} | \bar{s}s | 0\rangle$$

$$\langle K^+(p_2)K^-(p_3) | \bar{q}\gamma_\mu q | 0\rangle = (p_2 - p_3)_\mu F_q^{K^+K^-}(s_{23})$$

q=u,d,s

Contributions to kaon vector form factors

8 vector mesons: ρ(770), **ρ'**=ρ(1450), **ρ''**=ρ(1700)

ω(782), **ω'**=ω(1420), **ω''**=ω(1650)

ϕ=ϕ(1020), **ϕ'**=ϕ(1680)

Isospin:

$$\frac{1}{2}(F_u^{K^+K^-} + F_d^{K^+K^-}) = F_\rho + F_{\rho'} + F_{\rho''}$$

I = 1

$$\frac{1}{6}(F_u^{K^+K^-} - F_d^{K^+K^-}) = F_\omega + F_{\omega'} + F_{\omega''}$$

I = 0

$$-\frac{1}{3}F_s^{K^+K^-} = F_\phi + F_{\phi'}$$

I = 0

Model of form factors by Bruch, Khodjamirian and Kuhn (2005)

Model of the scalar-isoscalar amplitudes

The three-channel **model of the meson-meson elastic scattering and transitions** is based on:

1. coupled channel **Lippmann-Schwinger** equations,

2. separable form of the meson-meson potentials,

3. constraints from $\pi\,\pi$ and $K\bar{K}$ phase shift analyses including data of the CERN-Cracow- Munich measurements of the $\pi^- p \to \pi^+ \pi^- n$ reaction on a **polarized hydrogen target**

   (R. Kamiński, L. Leśniak, B. Loiseau, Phys. Lett. B 413 (1997) 130, Eur. Phys. J. C9 (1999) 141).

Chiral symmetry constraints

Low energy constraints on the scalar form factors:

$$\Gamma_{1,2}^{n,s}(E) \approx a_{1,2}^{n,s} + b_{1,2}^{n,s} E^2, \qquad \Gamma_3^{n,s} \approx 0, \qquad E \to 0.$$

At first the parameters a_j and b_j are calculated using the **chiral** model of **Meissner** and **Oller** (Phys. Rev. D 65 (2002) 094004) and the **lattice QCD** results (from RBC and UKQCD Collaborations, Phys. Rev. D 78 (2008) 114509).

Then the $\alpha_j, \tau_j, \omega_j$ coefficients are calculated in terms of a_j and b_j.

Unitarity conditions

Unitarity relations for the meson - meson amplitudes :

$$\mathrm{Im}\, T_{ik}(E) = \sum_{j=1}^{3} T_{kj}^{*}(E) r_j T_{ij}(E) \theta(E - 2m_j), \qquad i,k = 1,2,3$$

$$\mathrm{Im}\, \Gamma_i^{*}(E) = \sum_{j=1}^{3} T_{ji}^{*}(E) r_j \Gamma_j^{*}(E) \theta(E - 2m_j), \qquad i = 1,2,3$$

$$r_j = -\frac{k_j E}{8\pi}, \quad k_j - \text{channel momenta}$$

Watson's theorem: for a single channel (i=1) the **phases** of T_{11} and $\Gamma_1{}^*$ are **equal**.