QCD Analysis of the combined HERA structure function data - HERAPDF2.0

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on behalf of H1 and ZEUS
Today’s data on proton structure

The cleanest way to probe Proton Structure is via Deep Inelastic Scattering [DIS]:

- Neutrinos, muons, electrons

→ probes linear combination of quarks: sea quarks, gluon

HERA provides the basis of any PDFs

Precision of PDFs can be complemented by the Drell Yan [DY] processes at the collider experiments

\[ \sigma_{hh \to X} = f_{h \to a} \otimes \sigma_{ab \to X} \otimes f_{h \to b} \]

→ can provide flavour separation and more insight into gluons

→ probes bilinear combination of quarks
HERA ep collider (1992-2007) @ DESY

- H1 and ZEUS experiments at HERA collected \( \sim 1/fb \) of data
- \( \text{Ep}=460/575/820/920 \text{ GeV and Ee}=27.5 \text{ GeV} \)
- 4 types of processes accessed at HERA: Neutral Current and Charged Current \( e+p, e-p \)

\[
\frac{d\sigma^+_{\text{NC}}}{dx dQ^2} = \frac{2\pi\alpha^2}{x} \left[ \frac{1}{Q^2} \right]^2 \left[ Y_x \tilde{F}_2 + Y_x x \tilde{F}_3 - y^3 \tilde{F}_L \right]
\]

\[
\frac{d\sigma^+_{\text{CC}}}{dx dQ^2} = \frac{G_F^2}{4\pi x} \left[ \frac{M_w^2}{M_w^2 + Q^2} \right]^2 \left[ Y_x \tilde{W}_2^z + Y_x x \tilde{W}_3^z - y^2 \tilde{W}_L^z \right]
\]

\( Y_\pm = 1 \pm (1-y)^2 \)

\[ \tilde{F}_2 \propto \sum (xq_i + x\bar{q}_i) \] dominant contribution (all \( Q^2 \) plane)

\[ x\tilde{F}_3 \propto \sum (xq_i - x\bar{q}_i) \] significant contributions at high \( Q^2 \)

\[ \tilde{F}_L \propto \alpha_s \cdot xg(x,Q^2) \] high \( y \)
HERA ep collider (1992-2007) @ DESY

- H1 and ZEUS experiments at HERA collected ~1/fb of data
- $E_p = 460/575/820/920$ GeV and $E_e = 27.5$ GeV
- 4 types of processes accessed at HERA: **Neutral Current** and **Charged Current** $e+p$, $e-p$

**Neutral Current (NC):**
$$e p \rightarrow e' X$$

**Charged Current (CC):**
$$e p \rightarrow \nu_e X$$

**DIS data**

- $d\sigma^{+}_{NC}/dx dQ^2 = \frac{2\pi\alpha^2}{x} \left[ \frac{1}{Q^2} \right]^2 \left[ Y_1 \bar{F}_2 + Y_2 x \bar{F}_3 - y^2 \bar{F}_L \right]$
- $Y_1 = 1 \pm (1-y)^2$

- $d\sigma^{+}_{CC}/dx dQ^2 = \frac{G_F^2}{4\pi x} \left[ \frac{M_W^2}{M_W^2 + Q^2} \right]^2 \left[ Y_1 \bar{W}_2^+ + Y_2 x \bar{W}_3^+ - y^2 \bar{W}_L^+ \right]$

- $\bar{F}_2 \propto \sum (xq_i + x\bar{q}_i)$
- $x\bar{F}_3 \propto \sum (xq_i - x\bar{q}_i)$
- $\bar{F}_L \propto \alpha_s \cdot xg(x, Q^2)$

- dominant contribution (all Q2 plane)
- significant contributions at high Q2
- high y
HERA ep collider (1992-2007) @ DESY

41 data sets: 2927 data points are combined to 1307 averaged measurements with 169 sources of correlated systematic uncertainties.

HERAPDF1.0
JHEP01 (2010) 109

HERAPDF1.5
(prelim)

HERAPDF2.0
[arxiv:1506.06042]
Combination of the H1 and ZEUS Measurements

Ultimate precision is obtained by combining the H1 and ZEUS measurements.

The combination procedure is performed before QCD analysis using $\chi^2$ minimisation:
- $\chi^2 / \text{dof} = 1687/1620$

Improvement on Statistical precision:
- Improvement of Systematic precision:
  - H1 and ZEUS are different detectors and use different analysis techniques;
  - The H1 and ZEUS cross sections have different sensitivities to similar sources of correlated systematic uncertainty.

--- total uncertainty < 1.3% for $Q^2$ up to 400 GeV$^2$

$0.045 < Q^2 < 50000 \text{ GeV}^2$  $6.10^{-7} < x_{\text{Bj}} < 0.65$

Combination of data is now actively used at LHC for ex W, Z for muon and electron channels.

Voica Radescu | EPS 2015
Extraction of PDFs through QCD fits

[see V.R. HERAFitter talk]

- Extraction of PDFs relies on the factorisation:
  \[ \sigma = \hat{\sigma} \otimes \text{PDF} \]

- Typical measurements sensitive to PDFs are precise, with statistical uncertainties < 10\%, so they follow normal distribution \( \rightarrow \) use of \( \chi^2 \) minimisation for PDF extraction.

Main Steps:
- Parametrise PDFs at a starting scale
- Evolve PDFs to the scale corresponding to data point
- Calculate the cross section
- Compare with data via \( \chi^2 \)
- Minimise \( \chi^2 \) with respect to PDF parameters which takes about ~2000 iterations:

herafitter.org: open source QCD platform  arxive:1503.05221
QCD Settings for HERAPDF2.0

The QCD settings are optimised for HERA measurements of proton structure functions: PDFs are parametrised at the starting scale $Q_0^2=1.9$ GeV$^2$ as follows:

\[
\begin{align*}
x_g(x) &= A_g x^B (1-x)^C - A'_g x'^B (1-x)^C', \\
x_{u,v}(x) &= A_{u,v} x^B (1-x)^C (1 + E_{u,v} x^2), \\
x_{d,v}(x) &= A_{d,v} x^B (1-x)^C, \\
x_{\bar{U}}(x) &= A_{\bar{U}} x^B (1-x)^C (1 + D_{\bar{U}} x), \\
x_{\bar{D}}(x) &= A_{\bar{D}} x^B (1-x)^C.
\end{align*}
\]

Due to increased precision of data, more flexibility in functional form is allowed —> 14 free parameters

- PDFs are evolved via evolution equations (DGLAP) to NLO and NNLO ($\alpha_s(M_Z)=0.118$)
- Thorne-Roberts GM-VFNS for heavy quark coefficient functions – as used in MMHT
- $\chi^2$ definition used in the minimisation [MINUIT] accounts for correlated uncertainties:

\[
\chi^2_{\text{exp}} (m, s) = \sum_i \left[ m^i - \sum_j \gamma_j m^i s_j - \mu^i \right]^2 \delta^2_{i, \text{stat}} + \delta^2_{i, \text{uncor}} (m^i)^2 + \sum_j s_j^2 + \sum_i \ln \frac{\delta^2_{i, \text{stat}} \mu^i m^i + (\delta_{i, \text{uncor}} m^i)^2}{(\delta^2_{i, \text{stat}} + \delta^2_{i, \text{uncor}}) (\mu^i)^2}
\]

$m$ - th prediction
$\mu$ - data
$s$ - sys shift

NC structure functions

\[
\begin{align*}
F_2 &= \frac{4}{9} (xU + x\bar{U}) + \frac{1}{9} (xD + x\bar{D}) \\
xF_3 &\sim xu_v + xd_v
\end{align*}
\]

CC structure functions

\[
\begin{align*}
W_2^- &= x(U + \bar{D}), & W_2^+ &= x(U + D) \\
xW_3^- &= x(U - \bar{D}), & xW_3^+ &= x(D - \bar{U})
\end{align*}
\]
Modern understanding of PDFs

Different types of PDF uncertainties are considered:

- **Experimental:**
  - Hessian method used: MMHT, CT, ...
  - Consistent data sets $\rightarrow$ use $\Delta \chi^2 = 1$
  - Monte Carlo Method: replicas of data (NNPDF)

- **Model:** variations of all assumed input parameters in the fit

- **Parametrisation:** only HERAPDF includes this as an additional uncertainty

- NNPDFs use neural network approach based on data driven regularisation

<table>
<thead>
<tr>
<th>Variation</th>
<th>Standard Value</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^2_{\text{min}}$ [GeV$^2$]</td>
<td>3.5</td>
<td>2.5</td>
<td>5.0</td>
</tr>
<tr>
<td>$Q^2_{\text{min}}$ [GeV$^2$] HiQ2</td>
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<td>7.5</td>
<td>12.5</td>
</tr>
<tr>
<td>$M_e$ (NLO) [GeV]</td>
<td>1.47</td>
<td>1.41</td>
<td>1.53</td>
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<tr>
<td>$M_e$ (NNLO) [GeV]</td>
<td>1.43</td>
<td>1.37</td>
<td>1.49</td>
</tr>
<tr>
<td>$M_b$ [GeV]</td>
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<td>4.25</td>
<td>4.75</td>
</tr>
<tr>
<td>$f_s$</td>
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<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_s(M_Z^2)$</td>
<td>0.118</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\mu_f$ [GeV]</td>
<td>1.9</td>
<td>1.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

[arxiv:1506.06042]
QCD scaling and EW effects

EW effects clearly seen at high $Q^2$:

QCD scaling violations nicely seen:

<table>
<thead>
<tr>
<th>$x_B$</th>
<th>$Q^2$/$GeV^2$</th>
<th>$\sigma_{r, NC}$</th>
</tr>
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<tbody>
<tr>
<td>0.0005</td>
<td>575</td>
<td>(x575)</td>
</tr>
<tr>
<td>0.0008</td>
<td>400</td>
<td>(x400)</td>
</tr>
<tr>
<td>0.0013</td>
<td>800</td>
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<tr>
<td>0.0020</td>
<td>170</td>
<td>(x170)</td>
</tr>
<tr>
<td>0.0032</td>
<td>20</td>
<td>(x20)</td>
</tr>
<tr>
<td>0.005</td>
<td>6</td>
<td>(x6)</td>
</tr>
<tr>
<td>0.008</td>
<td>2</td>
<td>(x2)</td>
</tr>
</tbody>
</table>

$\sigma_{r, NC} = \frac{d^2\sigma^{e^+p}}{dx_BdQ^2} \cdot \frac{Q^4x_B}{2\alpha^2Y_+} = \bar{F}_2 + \frac{Y_+}{Y_+}x\bar{F}_3 - \frac{y^2}{Y_+}\bar{F}_L$

The $x$ region relevant for the Higgs production is at 10-2.
**Q^2 cut dependence on PDFs**

- HERA data provides a unique access to the low x, low Q^2 region to investigate:
  - the validity of the DGLAP mechanism

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**Figure 54:** The parton distribution functions \( x_u, x_d, x_S \) of HERAPDF2.0 NLO at \( \mu_r^2 = 10 \text{ GeV}^2 \) compared to those of HERAPDF2.0HiQ2 NLO on logarithmic (top) and linear (bottom) scales. The bands represent the total uncertainties.

---

low Q^2 data very important to constrain low x PDFs!
Q² cut dependence

- HERA data provides a unique access to the low x, low Q² region to investigate:
  - the validity of the DGLAP mechanism
  - the various scheme dependence (fixed vs variable flavours)

Low Q² remains an interesting region to investigate (low x phenomenology)

[arxiv:1506.06042]
HERAPDF2.0 vs other PDF sets

- HERAPDF sets are extracted solely from ep data and require no assumptions or corrections, hence provide an important cross check of PDF universality (process independence):

\[ \mu_f^2 = 10 \text{ GeV}^2 \]

H1 and ZEUS

- At NNLO gluon and sea quarks are both compatible with other PDFs

\[ \text{high } x \text{ valence different: new high- } x \text{ data and use of proton target only} \]

[arxiv:1506.06042]
PDFs are very important as they still limit our knowledge of cross sections whether SM or BSM.

- HERA has finalised its separate measurements relevant to PDFs and has combined them into final measurements to reach its ultimate precision:
  - PDFs, mc, mb, alphas

other related HERA talks at EPS:
- O. Turkot
- K. Wichmann
- A. Geiser
back-up slides
not necessarily useful ...
Longitudinal Structure Function

Longitudinal structure function FL is a pure QCD effect:
—> an independent way to probe sensitivity to gluon

Direct measurement of FL at HERA required differential cross sections at same $x$ and $Q^2$ but different $y$ —> different beam energies: $E_p = 460, 575, 920$ GeV

\[
F_L = \frac{\alpha_s}{4\pi} x^2 \int \frac{dz}{z^3} \left[ \frac{16}{3} F_2 + 8 \sum_q e_q^2 (1 - \frac{x}{z}) z g(z) \right]
\]

\[
\sigma_{NC}(x, Q^2, y) \propto F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)
\]

\[
x g(x, Q^2) \approx 1.77 \frac{3\pi}{2\alpha_S(Q^2)} F_L(ax, Q^2)
\]


**F2 charm Structure Function**

- Rates at HERA in DIS regime $\sigma(b) : \sigma(c) \approx O(1\%) : O(20\%)$ of $\sigma_{TOT}$
- Charm data combination is performed at charm cross sections level:
  - they are obtained from xsec in visible phase space and extrapolated to full space

\[ \sigma_{red}^{c}(x, Q^2, s) = F_2^{c}(x, Q^2) - \frac{Y}{Y_{+}} F_L^{c}(x, Q^2) \]

**QCD Fits**

**HERA I+charm**

Different calculation schemes prefer different $M_c$

**H1 and ZEUS**

**measurements help reduce uncertainties of predictions for the LHC**
The running of the charm mass in the $\overline{\text{MS}}$ scheme is measured for the first time from the same HERA combined charm data:

- Extract $m_c(m_c)$ in 6 separate kinematic regions
- Translate back to $m_c(\mu)$ [with $\mu=\sqrt{Q^2+4m_c^2}$] using OpenQCDrad [S.Alekhin's code].

The scale dependence of the mass is consistent with QCD expectations.
Running beauty mass from F2b

- The value of the running beauty mass is obtained using HERAFitter (via OPENQCDRAD):
  - chi2 scan method from QCD fits in FFN scheme to the combined HERA I inclusive data + beauty measurements, beauty-quark mass is defined in the $\overline{\text{MS}}$ scheme.

The extracted $\overline{\text{MS}}$ beauty-quark mass is in agreement with PDG average and LEP results.
**DIS Cross Sections**

- Differential cross section is experimentally measured: **theory meets the experiment**

- Factorisable nature of interaction: Inclusive scattering cross section is a product of leptonic and hadronic tensors times propagator characteristic of the exchanged particle:

\[
\frac{d^2 \sigma}{dx dQ^2} = \frac{2\pi \alpha^2}{Q^4 x} \sum_j \eta_j L_j^{\mu\nu} W_j^{\mu\nu}
\]

For NC: \(j=\gamma, Z, \gamma Z\)
For CC: \(j=W^+, W^-\)

**Leptonic tensor**: related to the coupling of the lepton with the exchanged boson
- contains the electromagnetic or the weak couplings
- can be calculated exactly in the standard electroweak \(U(1) \times SU(2)\) theory.

**Hadronic tensor**: related to the interaction of the exchanged boson with proton
- can't be calculated, but only be reduced to a sum of structure functions:

\[
W^{\alpha\beta} = -g^{\alpha\beta} W_1 + \frac{p^\alpha p^\beta}{M^2} W_2 - \frac{ie^{\alpha\beta\gamma\delta} p_\gamma q_\delta}{2M^2} W_3 + \frac{q^\alpha q^\beta}{M^2} W_4 + \frac{p^\alpha q^\beta + p^\beta q^\alpha}{M^2} W_5 + \frac{i(p^\alpha q^\beta - p^\beta q^\alpha)}{2M^2} W_6
\]

\[
\frac{d^2 \sigma}{dx dQ^2} = A^i \left\{ \left( 1 - y - \frac{x^2 y^2 M^2}{Q^2} \right) F_2^i + y^2 x F_1^i \mp \left( y - \frac{y^2}{2} \right) x F_3^i \right\}
\]

\(A^i\): process dependent

\(\eta_\gamma = 1; \quad \eta_{\gamma Z} = \left( \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \right) \left( \frac{Q^2}{Q^2 + M_Z^2} \right)^2; \quad \eta_Z = \eta_{\gamma Z}^2; \quad \eta_W = \frac{1}{2} \left( \frac{G_F M_W^2}{4\pi\alpha} \frac{Q^2}{Q^2 + M_W^2} \right)^2\)