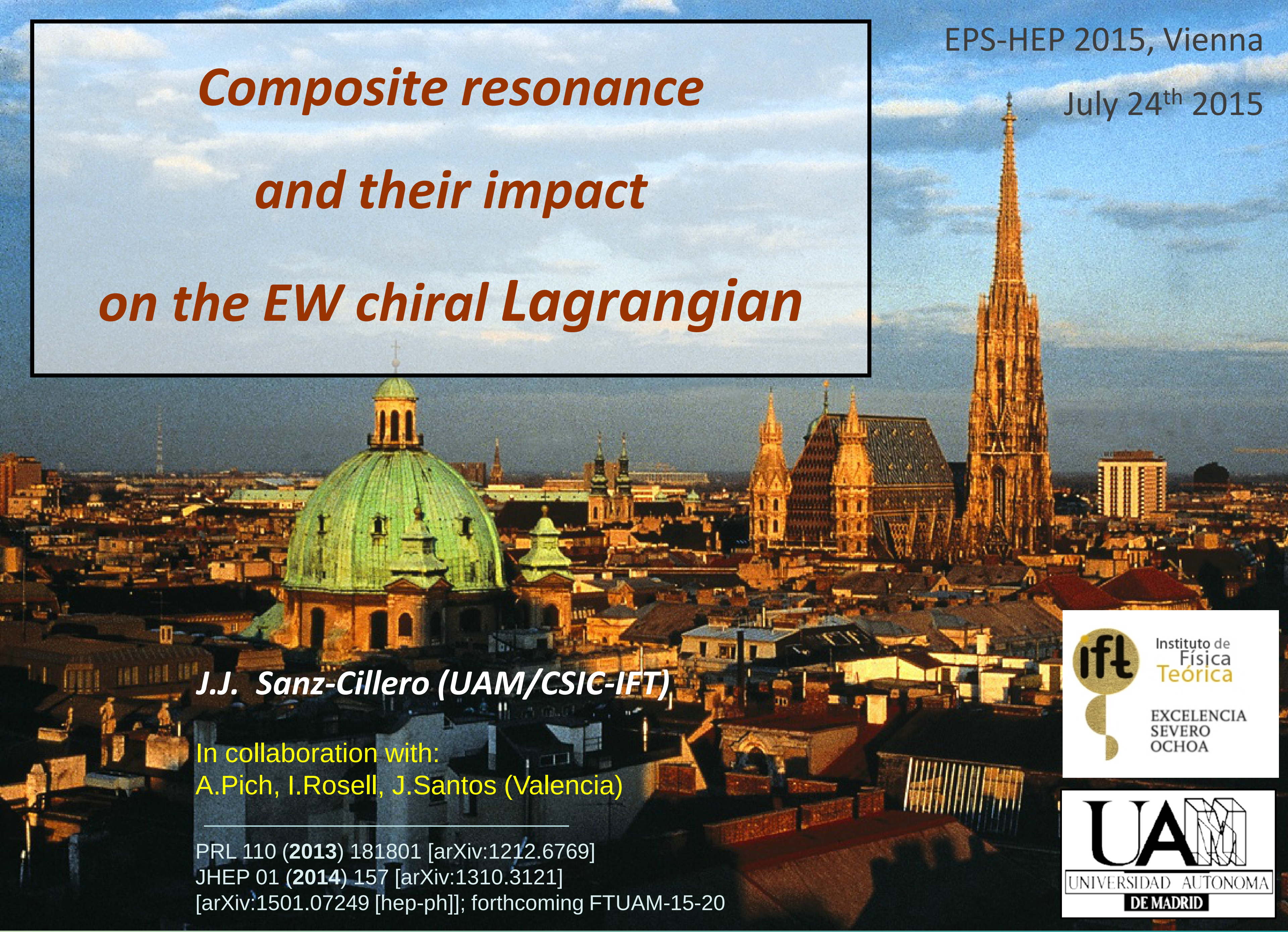

EPS-HEP 2015, Vienna

July 24th 2015

Composite resonance and their impact on the EW chiral Lagrangian

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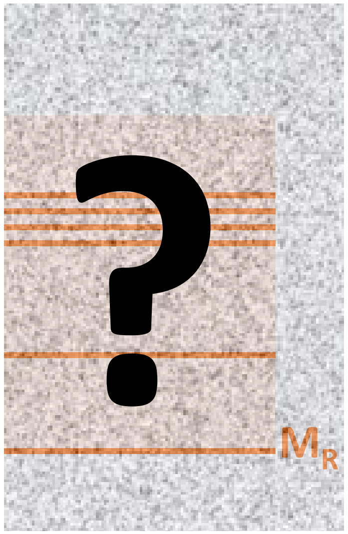
In collaboration with:
A.Pich, I.Rosell, J.Santos (Valencia)

PRL 110 (**2013**) 181801 [arXiv:1212.6769]
JHEP 01 (**2014**) 157 [arXiv:1310.3121]
[arXiv:1501.07249 [hep-ph]]; forthcoming FTUAM-15-20

CURRENT SITUATION



$\uparrow$ M > 1 TeV

Mass gap

t

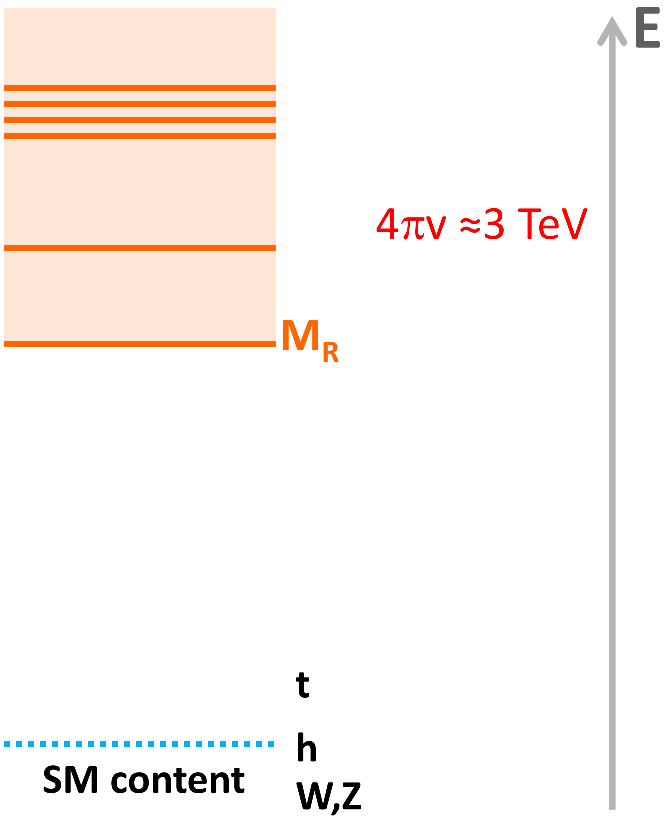
h

W,Z

SM content

- **Bosons χ:**
  singlet h,
  EW Goldstones $U(\omega^a)$,
  gauge bosons
- **Fermion doublets $\psi_{L,R}$**

Goal #1:
the Resonance + EFT program



SM content

- **Bosons χ:**
  singlet h,
  EW Goldstones $U(\omega^a)$,
  gauge bosons
- **Fermion doublets $\psi_{L,R}$**

Custodial symmetry — $SU(2)_L \otimes SU(2)_R / SU(2)_{L+R}$

\+

Resonance Lagrangian — V,A,S,P singlet & triplet

\+

UV completion hypothesis — Sum-rules

⇓

THIS WORK

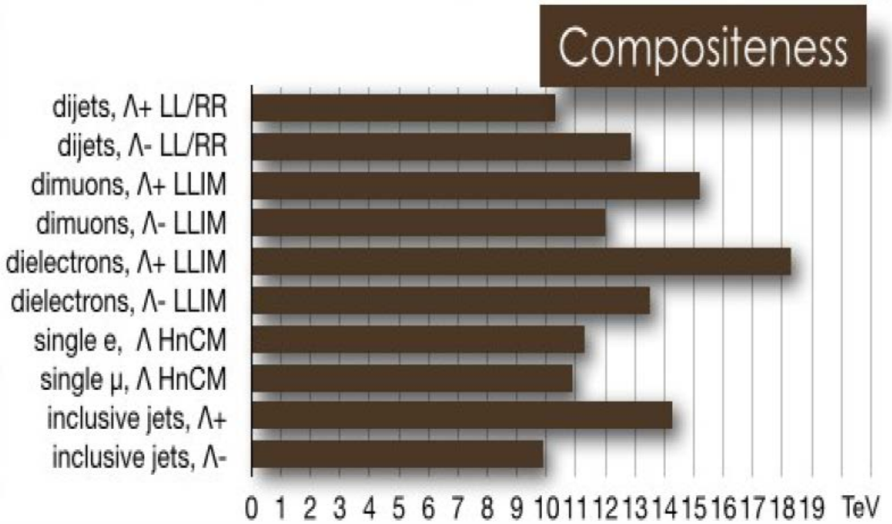
Resonance contributions to the NLO low-energy couplings

⇓

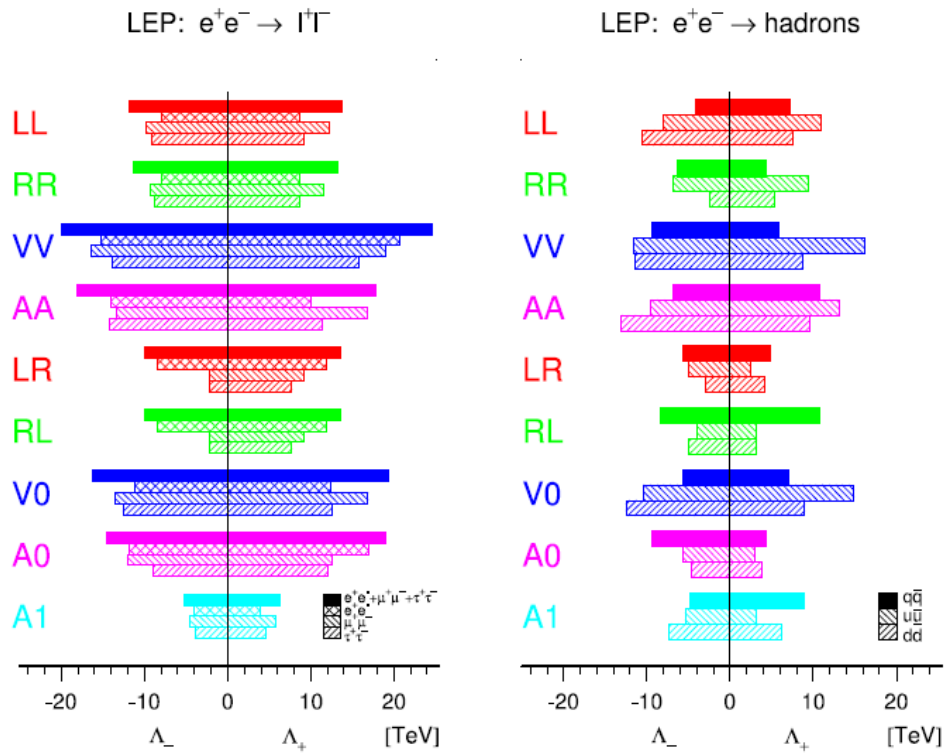
Prediction for low-energy observables

Goal #2:
Does Λ_{BSM} need to be > 10 TeV?

CMS Exotica Physics Group Summary – Moriond, 2015



LEP, Phys.Rep.532 (2013) 119



In both cases the scale refers to the $\Delta\mathcal{L}_{\rm eff} \sim \frac{4\pi}{\Lambda_\pm^2}(\bar f\,\Gamma\, f)^2$ inspired on weakly interacting BSM

Reply: NO *in the appropriate EFT and counting*

Low-energy chiral expansion

- If Higgs (pseudo) Goldstone boson ➔ **Non-linearity for h + ωᵃ**

- **Expansion** in non-linear EFT's: *

Finite pieces from loops (amplitude dependent) (+)

$$\mathcal{M}(2\to 2) \approx \frac{p^2}{v^2}\left[\underbrace{1}_{\text{LO (tree)}} + \left(\underbrace{\frac{c_k^r\, p^2}{v^2}}_{\text{NLO (tree)}} - \underbrace{\frac{\Gamma_k\, p^2}{16\pi^2 v^2}\ln\frac{p}{\mu} + \dots}_{\text{NLO (1-loop)}}\right) + \mathcal{O}(p^4)\right]$$

LO (tree) | **NLO (tree)** | **NLO (1-loop)**

Related to suppression
$\sim 1/M_R^2$
(other states)

THIS WORK

Typical loop suppression
$\sim 1/(16\pi^2 v^2)$
(non-linearity)

100% determined by $\mathcal{L}_{\rm LO}$
[Guo,Ruiz-Femenia,SC,[1506.04204]

** Catà, EPJC74 (2014) 8, 2991
** Pich,Rosell,Santos,SC, [1501.07249]; 'forthcoming FTUAM-15-20
** Pich,Rosell and SC, JHEP 1208 (2012) 106;
PRL 110 (2013) 181801
(x) Contino,Salvarezza, [1504.02750]
(x) Contino,Marzocca,Pappadopulo,Rattazzi,
JHEP 1110 (2011) 081

- Espriu,Yencho, PRD 87 (2013) 055017
- Espriu,Mescia, Yencho, PRD88 (2013) 055002
- Delgado,Dobado,Llanes-Estrada, JHEP1402 (2014) 121
- Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149
- Gavela,Kanshin,Machado,Saa, JHEP 1503 (2015) 043
- Azatov, Contino,Di Iura,Galloway, PRD88 (2013) 7, 075019
- Azatov,Grojean,Paul,Salvioni, Zh.Eksp.Teor.Fiz. 147 (2015) 410, J.Exp.Theor.Phys. 120 (2015) 354

* Weinberg '79
* Manohar,Georgi, NPB234 (1984) 189
* Urech '95
* Buchalla,Catà,Krause '13
* Hirn,Stern '05
* Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149
* Pich,Rosell,Santos,SC, forthcoming FTUAM-15-20

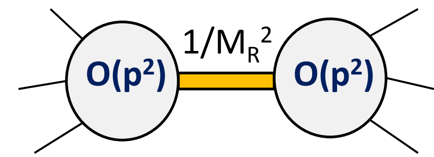
R contributions to the NLO EFT couplings   *[i.e., $O(p^4)$]*

$$\Delta\mathcal{L}_{\mathbf{R}} = \boxed{\mathbf{F_R}\,\mathbf{R}\,\mathcal{O}_{\mathbf{p^2}}[\chi,\psi]} + \ \dots$$

- **High-energy theory for Resonances + χ + ψ**

  General $\Delta\mathcal{L}_R$ `up to O(p²)' **

  R= singlet and triplet V, A, S, P [also $J^{PC}=1^{+-}$ at (x)]



- **Low-energy EFT (ECLh) ***

$$e^{i\,S[\chi,\psi]_{\rm EFT}} = \int [dR]\, e^{i\,S[\chi,\psi,R]} \stackrel{\rm tree-level}{=} e^{i\,S[\chi,\psi,R_{\rm cl.}]}$$

$$\mathcal{L}_{ECLh} = \mathcal{L}_{p^2} + \boxed{\mathcal{L}_{p^4}} + \ \dots$$

$$\supset \mathcal{L}^{\rm SM}$$

$$\begin{aligned}
\Delta\mathcal{L}_{p^4}^{\rm EFT} &= \frac{1}{2M_R^2}\left(\langle\mathcal{O}_R\mathcal{O}_R\rangle - \frac{1}{N}\langle\mathcal{O}_R\rangle^2\right) && (R=S,\,P)\,,\\
\Delta\mathcal{L}_{p^4}^{\rm EFT} &= -\frac{1}{M_R^2}\left(\langle\mathcal{O}_R^{\mu\nu}\mathcal{O}_{R\,\mu\nu}\rangle - \frac{1}{N}\langle\mathcal{O}_R^{\mu\nu}\rangle^2\right) && (R=V,\,A)\,,\\
\Delta\mathcal{L}_{p^4}^{\rm EFT} &= \frac{1}{2M_{R_1}^2}(\mathcal{O}_{R_1})^2 && (R_1=S_1,\,P_1)\,,\\
\Delta\mathcal{L}_{p^4}^{\rm EFT} &= -\frac{1}{M_{R_1}^2}(\mathcal{O}_{R_1}^{\mu\nu}\mathcal{O}_{R_1\,\mu\nu}) && (R_1=V_1,\,A_1)\,.
\end{aligned}$$

(classification of the operators according to `chiral' dimension)

- Impose UV-completion assumptions on $\mathcal{L}_R$, (sum-rules,unitarity…)

  ➔ **EFT predictions ****

* Weinberg, 79; Manohar,Georgi, NPB234 (1984) 189
* Gasser,Leutwyler '84 '85
* Hirn,Stern '05
* Buchalla,Catà,Krause '13
* Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149
* Apelquist,Bernard '80
* Longhitano '80, '81
* Alonso,Gavela,Merlo,Rigolin,Yepes '12
* Brivio,Corbett,Eboli,Gavela,Gonzalez–Fraile,Gonzalez–Garcia,Merlo,Rigolin '13
(x) Catà, EPJC74 (2014) 8, 2991
** Ecker et al. '89
** Cirigliano et al., NPB753 (2006) 139
** Pich,Rosell,Santos,SC, [1501.07249]; forthcoming FTUAM-15-20

<u>BASIC TREE-LEVEL EXAMPLE:</u> V form-factors & $Z\to f\bar{f}$

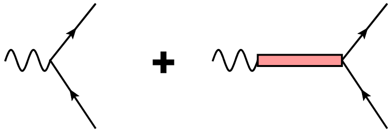
- Triplet V contribution in C & P-even strongly couple theories (full R Lagrangian in *)

$$\mathcal{L}_V = Tr\{V_{\mu\nu}\left(\frac{F_V}{2\sqrt{2}}f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}}[u^\mu,u^\nu] + c_1^V\nabla^\mu J_V^\nu/v^2 + \dots\right)\}$$

- **Contribution from V,**

to the EFT @ NLO:
$$\mathcal{L}_{p^4}^{\text{from V}} = \underbrace{-i\frac{F_V G_V}{4M_V^2}}_{i\,(a_2-a_3)/2} Tr\{f_+^{\mu\nu}[u^\mu,u^\nu]\} - \underbrace{\frac{F_V c_1^V}{\sqrt{2}M_V^2}}_{C^{\psi^2 h_0}_{10}} Tr\{f_+^{\mu\nu}\nabla_\mu J_{V\,\nu}/v^2\} + \dots$$

- UV constraints on $\mathcal{L}_R$ (S-parameter Peskin-Takeuchi sum-rule) ➔ **EFT predictions *, ****



$$\left.\begin{aligned}\mathcal{F}^v_{\omega\omega}(q^2) &= 1 + \frac{F_V G_V}{v^2}\frac{q^2}{M_V^2-q^2}\\ \mathcal{F}^v_{\psi\overline{\psi}}(q^2) &= 1 - \frac{\sqrt{2}F_V c_1^V}{v^2}\frac{q^2}{M_V^2-q^2}\end{aligned}\right\} \xrightarrow{q^2\to\infty} 0$$

$$\Rightarrow \boxed{\begin{aligned}(a_2-a_3) &= -v^2/(2M_V^2)\\ c^{\psi^2 h_0}_{10} &= v^2/(2M_V^2)\end{aligned}}$$

* Pich,Rosell,Santos,SC, [1501.07249]; forthcoming FTUAM-15-20

** Peskin,Takeuchi, PRL65 (1990) 96; PRD46 (1992) 381
** Orgogozo,Rychkov, JHEP1203 (2012) 046; 1306 (2013) 014
** Pich,Rosell, SC, PRL110 (2013) 181801; JHEP01 (2014) 157

- Contributions to $Z \to f\bar{f}$ (with f=t,b) ? *

$$\mathbf{M_V} \gtrsim \mathbf{1.5}\ \mathrm{TeV}$$

V contribution to the EFT

UV completion (EM form-factor)

$$\mathbf{c_{10}^{\psi^2 h^0}}\Big|_{\mathbf{V}} = -\frac{\mathbf{F_V c_{V1}}}{\sqrt{\mathbf{2}}\mathbf{M_V^2}} = \frac{\mathbf{v^2}}{\mathbf{2M_V^2}} \lesssim \mathbf{1.4\cdot 10^{-2}}$$

$$|\delta \mathbf{g_{R,L}^{Zf\bar{f}}}|\Big|_{\mathbf{V}} = \frac{\mathbf{\cos(2\theta_W)\, c_{10}^{\psi^2 h^0}\, m_Z^2}}{\mathbf{2v^2}} \lesssim \mathbf{10^{-3}}$$

(P) Observable estimate

easily in agreement with O(10^{-3}) measurements of $\delta g_{R,L}$ **

* Pich,Rosell,Santos,SC, fothcoming
** Agashe,Contino,Da Rold,Pomarol, PLB641 (2006) 62
** Efrati ,Falkowski,Soreq, [1503.07872]
** LEP [0511027]

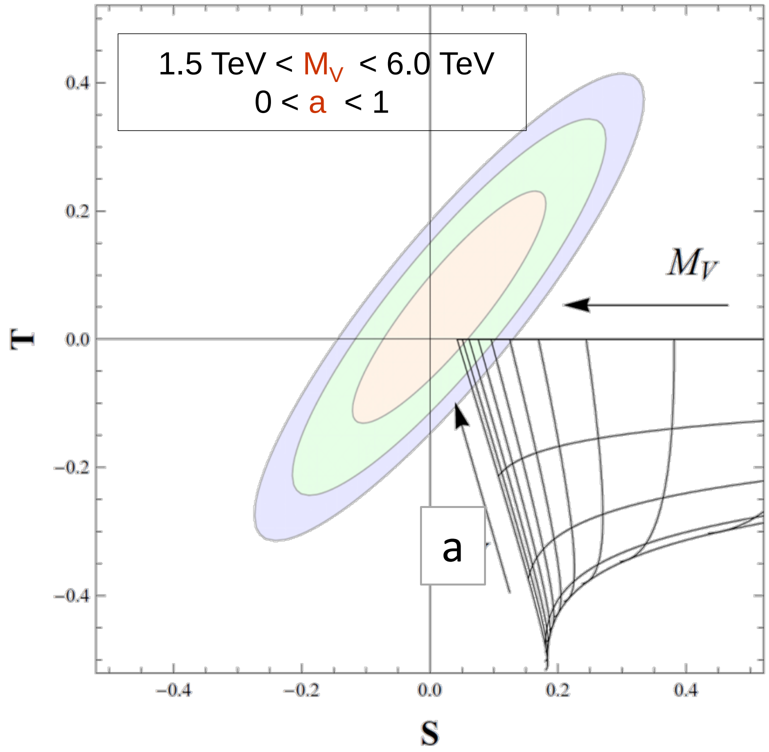
(S , T) oblique parameters:
ONE-LOOP results + UV-constraint

C & P-even

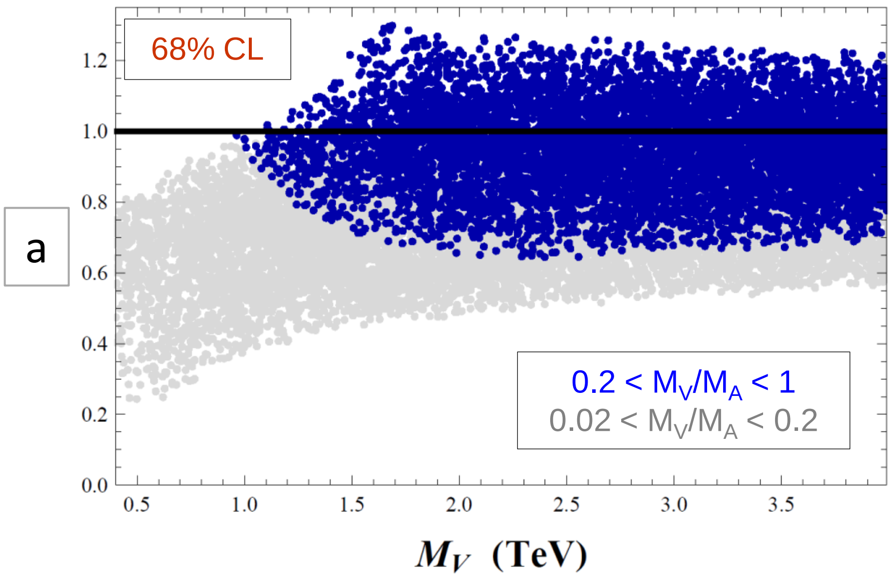
ii) NLO results: 1st and 2nd WSRs*

$1 > a > 0.97$
$M_A \approx M_V > 5$ TeV
(68%CL)

iii) NLO results: 1st WSR and $M_V < M_A$*



Similar conclusions, but softened



For $M_V<M_A$:

✓ $M_R > 1$ TeV at 68% CL.

Only BSM boson loops in these plots.
Preliminary fermion loop studies➔ Similar result

* Pich,Rosell, SC, PRL 110 (2013) 181801; JHEP 01 (2014) 157

• Similar exp. agreement/suppression in other observables:

For $M_R \sim 4\pi v \approx 3$ TeV,

WW-scattering	(a_4, a_5)
$\gamma\gamma$-WW scattering	(a_1,a_2,a_3,c_γ)
$h \to \gamma\gamma$	(c_γ)
4-fermion operators	$(c^{\psi 4})$

ruled by NLO EFT couplings $c_j = F_R^2/M_R^2 \sim v^2/M_R^2 < 10^{-2}$

(<1% correction to the amplitudes for $p \sim v$=246 GeV)

Conclusions

1) **Build a custodial-invariant Lagrangian with R + light dof ($\chi + \psi$)**

2) Low-energy expansion and matching: **R** contributions to the EFT @ $O(p^4)$

➔ **NLO eff. couplings in terms of R parameters**

GOAL #1

3) High-energy constraints on **R** theory ➔ stronger low-energy predictions

✓ **<u>This framework allows us in a natural way:</u>**

i) to incorporate resonance at $M_R \sim 4\pi v \approx 3$ TeV **(direct searches)**

ii) to fulfill the low-energy EWPO (chiral suppression) **GOAL #2** **(indirect searches)**

Z→ff: Ok for $M_V > 1.5$ TeV

S & T: Ok for a≈1 and $M_V > 1$ TeV *(5 TeV)* with only 1st WSR *(1st+2nd WSR)*

BACKUP

PREDICTIONS:
ONE-LOOP results + UV-constraint

Higgsless
C & P-even

$$T = \frac{3}{16\pi\cos^2\theta_W}\left[1+\log\frac{m_H^2}{M_V^2} - \kappa_W^2\left(1+\log\frac{m_{S_1}^2}{M_A^2}\right)\right]$$

LO

$$S = \boxed{4\pi v^2\left(\frac{1}{M_V^2}+\frac{1}{M_A^2}\right)} + \frac{1}{12\pi}\left[\left(\log\frac{M_V^2}{m_H^2}-\frac{11}{6}\right) - \kappa_W^2\left(\log\frac{M_A^2}{m_{S_1}^2}-\frac{11}{6}-\frac{M_A^2}{M_V^2}\log\frac{M_A^2}{M_V^2}\right)\right]$$

[terms $O(m_S^2/M^2_{V,A})$ neglected]

(k_W=a)

✓ 1st and 2nd WSRs at LO and NLO + ππ-VFF:

→ 2nd WSR: $0 < a = M_V^2/M_A^2 < 1$

* Pich,Rosell, SC, PRL 110 (2013) 181801; JHEP 01 (2014) 157

PREDICTIONS:
TREE-LEVEL results + UV-constraint

Higgsless part CP conserving
Only P-even interactions

- Higgsless R Lagrangian with only bosons

*(general custodial R Lagrangian in *)*

$$
\begin{aligned}
\chi_V^{\mu\nu} &= \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}}[u^\mu, u^\nu] + \cancel{\frac{\widetilde{F}_V}{2\sqrt{2}} f_-^{\mu\nu}} \\
\chi_A^{\mu\nu} &= \frac{F_A}{2\sqrt{2}} f_-^{\mu\nu} + \cancel{\frac{\widetilde{F}_A}{2\sqrt{2}} f_+^{\mu\nu}} + \cancel{\frac{i\widetilde{G}_A}{2\sqrt{2}}[u^\mu, u^\nu]}, \\
\chi_{S_1} &= \frac{c_d}{\sqrt{2}} \langle\, u_\mu u^\mu \,\rangle
\end{aligned}
$$

- Integrate out V and A:

$$
\gamma^* \to \omega^+\omega^- \ \mathrm{EM-FF}: \qquad \mathrm{F_V G_V = v^2}
$$

$$
1 + 2\,\mathrm{WSR\,on\,\Pi_{W_3B}}: \qquad \mathrm{F_V^2 - F_A^2 = v^2,\ F_V^2 M_V^2 - F_A^2 M_A^2 = 0}
$$

$$
\begin{aligned}
a_1 &= -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}\,, \\
(a_2 - a_3) &= -\frac{F_V G_V}{2M_V^2}\,, \\
a_4 &= \frac{G_V^2}{4M_V^2}\,, \\
a_5 &= \frac{c_d^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2}\,,
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
a_1 &= -\frac{v^2}{4}\left(\frac{1}{M_V^2} + \frac{1}{M_A^2}\right)\,, \\
(a_2 - a_3) &= -\frac{v^2}{2M_V^2}\,, \\
a_4 &= \frac{v^2}{4}\left(\frac{1}{M_V^2} - \frac{1}{M_A^2}\right)\,, \\
a_5 &= -\frac{v^2}{4}\left(\frac{1}{M_V^2} - \frac{1}{M_A^2}\right) + \frac{c_d^2}{4M_{S_1}^2}\,,
\end{aligned}
}
$$

(x) Longhitano '80, '81

* Ecker et al. '89
* Pich,Rosell,Santos,SC,[1501.07249]; forthcoming FTUAM-15-20

RESONANCE LAGRANGIAN

- Introduce light dof **+ Resonances** *,**

- Lightest SU(2) triplets V, A, S, P and singlets V_1, A_1, S_1, P_1 **

*(antisymetric-tensor formalism $R_{\mu\nu}$ for spin-1 Resonances *)*

- To extract their contribution to $\mathcal{L}_{p4}$

(NOTICE that this avoids contributions to $\mathcal{L}_{p2}$, avoiding large low-energy corrections to SM)

➔ We need only **R operators O(p²)**

$$\mathcal{L}_R = \frac{1}{2}\langle \nabla^\mu R \nabla_\mu R - M_R^2 R^2 \rangle + \langle R\chi_R \rangle \qquad (R = S,\, P)\,,$$

$$\mathcal{L}_R = -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu} \nabla_\sigma R^{\sigma\mu} - \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu} \rangle + \langle R_{\mu\nu}\chi_R^{\mu\nu} \rangle \qquad (R = V,\, A)\,,$$

$$\mathcal{L}_{R_1} = \frac{1}{2}\left(\partial^\mu R_1 \partial_\mu R_1 - M_{R_1}^2 R_1^2\right) + R_1 \chi_{R_1} \qquad (R_1 = S_1,\, P_1)\,,$$

$$\mathcal{L}_{R_1} = -\frac{1}{2}\left(\partial^\lambda R_{1\,\lambda\mu} \partial_\sigma R_1^{\sigma\mu} - \frac{1}{2} M_{R_1}^2 R_{1\,\mu\nu} R_1^{\mu\nu}\right) + R_{1\,\mu\nu}\chi_{R_1}^{\mu\nu} \qquad (R_1 = V_1,\, A_1)\,,$$

* Ecker et al. '89
** Cirigliano et al., NPB753 (2006) 139
** Pich,Rosell,SC '12, '13
** Pich,Rosell,Santos,SC, [1501.07249]; forthcoming FTUAM-15-20

BOSONIC SECTOR : χ

- Building blocks with bosons χ (x):

EW Goldstones **(ω^a)** →

$$D_\mu U = \partial_\mu U - i\hat{W}_\mu U + iU\hat{B}_\mu \,,$$
$$u^\mu = iu_R^\dagger(\partial_\mu - i\hat{B}_\mu)u_R - iu_L^\dagger(\partial_\mu - i\hat{W}_\mu)u_L = iu(D^\mu U)^\dagger u \,,$$

EW gauge bosons **(B, W^a)** →

$$\hat{W}_{\mu\nu} = \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu - i[\hat{W}_\mu, \hat{W}_\nu] \,, \qquad \hat{B}_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu - i[\hat{B}_\mu, \hat{B}_\nu] \,,$$
$$f_\pm^{\mu\nu} = u_L^\dagger \hat{W}^{\mu\nu} u_L \pm u_R^\dagger \hat{B}^{\mu\nu} u_R \,.$$

Higgs **(singlet h)** → **h** via polynomials **$\mathcal{F}$(h/v)** & derivatives

soft-scale!!!

- "Chiral" counting*,**:

$$\partial_\mu \,, \quad m_W \,, \quad m_Z \,, \quad m_h \quad \sim \quad \mathcal{O}(p)$$
$$D_\mu U \,, \quad V_\mu \,, \quad g' v \mathcal{T} \,, \quad \hat{W}_\mu \,, \quad \hat{B}_\mu \quad \sim \quad \mathcal{O}(p) \,,$$
$$\hat{W}_{\mu\nu} \,, \quad \hat{B}_{\mu\nu} \quad \sim \quad \mathcal{O}(p^2) \,.$$

(x) Apelquist,Bernard '80
(x) Longhitano '80, '81
(x) Herrero,Morales '95
(x) Pich,Rosell,SC '12 '13
(x) Alonso et al., PLB722 (2013) 330
...etc

* Buchalla,Catà,Krause '13
* Hirn,Stern '05
* Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149

** Urech '95

‘CHIRAL’ COUNTING

- “Chiral” counting *

$$\frac{\chi}{v} \sim \mathcal{O}\left(p^0\right) , \qquad \frac{\psi}{v} \sim \mathcal{O}\left(p^{\frac{1}{2}}\right) , \qquad \partial_\mu ,\, m_\chi ,\, m_\psi \sim \mathcal{O}(p)$$

and for the building blocks,

$$\begin{aligned}
u(\varphi/v),\, U(\varphi/v),\, \frac{h}{v},\, \frac{W^a_\mu}{v},\, \frac{B_\mu}{v} &\sim \mathcal{O}\left(p^0\right) , \\
D_\mu U,\, u_\mu,\, \hat{W}_\mu,\, \hat{B}_\mu &\sim \mathcal{O}\left(p\right) , \\
\hat{W}_{\mu\nu},\, \hat{B}_{\mu\nu},\, f_{\pm\,\mu\nu} &\sim \mathcal{O}\left(p^2\right) , \\
\partial_{\mu_1}\partial_{\mu_2}...\partial_{\mu_n}\, \mathcal{F}(h/v) &\sim \mathcal{O}\left(p^n\right) , \\
\frac{\xi}{v} &\sim \mathcal{O}\left(p^{\frac{1}{2}}\right)
\end{aligned}$$

- Assignment of the ‘chiral’ dimension: *

$$\mathcal{L}_{p^{\hat{d}}} \sim a_{(\hat{d})}\; p^{\hat{d}-N_F/2} \left(\frac{\overline{\psi}\psi}{v^2}\right)^{N_F/2} \sum_j \left(\frac{\chi}{v}\right)^j$$

* Manohar,Georgi, NPB234 (1984) 189
* Hirn,Stern ‘05
* Buchalla,Catà,Krause ‘13
* Pich,Rosell,Santos,SC, forthcoming FTUAM-15-20

'CHIRAL' expansion in ECLh

- EFT Lagrangian at LO and NLO in chiral exp. *

$$\mathcal{L}_{ECLh} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \ldots$$

SM

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + h.c. + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

Examples of BSM terms:

$$\mathcal{L}_{p^2}^{\rm BSM} = \frac{(a-1)\,h}{2v} \mathrm{Tr}\{D_\mu U^\dagger D^\mu U\} + \ldots$$

$$\mathcal{L}_{p^4}^{\rm BSM} = \frac{i}{2}(a_2 - a_3)\,\mathrm{Tr}\{f_+^{\mu\nu}[u_\mu, u_\nu]\} + \mathcal{F}_{X\psi\psi}\,\mathrm{Tr}\{f_{+\,\mu\nu} d^\mu J_V^\nu\} + \ldots$$

which leads to a chiral exp. in the sctatering

$$T(2\to 2) = \frac{p^2}{v^2} + \underbrace{\frac{a_{(4)} p^4}{v^4}}_{tree-NLO} + \underbrace{\frac{p^4}{16\pi^2 v^4}}_{1\ell oop-NLO} + \ldots$$

* Weinberg '79
* Manohar,Georgi, NPB234 (1984) 189
* Urech '95
* Georgi,Manohar NPB234 (1984) 189
* Buchalla,Catà,Krause '13
* Hirn,Stern '05
* Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149
* Pich,Rosell,Santos,SC, forthcoming

Integrating out the RESONANCES

$$e^{iS[\chi,\psi]_{\rm EFT}} \quad = \quad \int [dR]\; e^{iS[\chi,\psi,R]}$$

- **At the practical level, ***

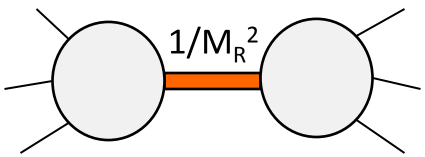
1.) Compute the Resonance EoM for p<<M_R:

$$R_{c\ell} \;=\; \frac{1}{M_R^2}\left(\chi_R - \frac{1}{N}\langle\chi_R\rangle\right) \quad + \quad \dots \qquad (R=S,\,P)\,,$$

$$R^{\mu\nu}_{c\ell} \;=\; -\frac{2}{M_R^2}\left(\chi^{\mu\nu}_R - \frac{1}{N}\langle\chi^{\mu\nu}_R\rangle\right) \quad + \quad \dots \qquad (R=V,\,A)\,,$$

$$R_{1\;c\ell} \;=\; \frac{1}{M_{R_1}^2}\chi_{R_1} \quad + \quad \dots \qquad (R_1=S_1,\,P_1)\,,$$

$$R^{\mu\nu}_{1\;c\ell} \;=\; -\frac{2}{M_{R_1}^2}\chi^{\mu\nu}_{R_1} \quad + \quad \dots \qquad (R=V,\,A)\,,$$



2.) Tree-level contribution to the O(p^4) ECLh for p<<M_R:

$$S[\chi,\psi]_{\rm EFT} \quad = \quad S[\chi,\psi,R_{c\ell}]$$

$$\Delta\mathcal{L}_R^{\mathcal{O}(p^4)} \;=\; \frac{1}{2M_R^2}\left(\langle\chi_R\chi_R\rangle - \frac{1}{N}\langle\chi_R\rangle^2\right) \qquad (R=S,\,P)\,,$$

$$\Delta\mathcal{L}_R^{\mathcal{O}(p^4)} \;=\; -\frac{1}{M_R^2}\left(\langle\chi^{\mu\nu}_R\chi_{R\,\mu\nu}\rangle - \frac{1}{N}\langle\chi^{\mu\nu}_R\rangle^2\right) \qquad (R=V,\,A)\,,$$

$$\Delta\mathcal{L}_{R_1}^{\mathcal{O}(p^4)} \;=\; \frac{1}{2M_{R_1}^2}(\chi_{R_1})^2 \qquad (R_1=S_1,\,P_1)\,,$$

$$\Delta\mathcal{L}_{R_1}^{\mathcal{O}(p^4)} \;=\; -\frac{1}{M_{R_1}^2}(\chi^{\mu\nu}_{R_1}\chi_{R_1\,\mu\nu}) \qquad (R_1=V_1,\,A_1)\,.$$

* Ecker et al. '89

- I could show you this,

$$\chi_V^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}} [u^\mu, u^\nu]$$
$$+ \; c_{V0} v J_T^{\mu\nu}$$
$$+ \; \frac{c_{V1}}{2} (\nabla^\mu J_V^\nu - \nabla^\nu J_V^\mu) + \frac{ic_{V2}}{2} ([J_A^\mu, u^\nu] - [J_A^\nu, u^\mu])$$
$$+ \; \frac{c_{V3}}{2} \left(\frac{(\partial^\mu h)}{v} J_V^\nu - \frac{(\partial^\nu h)}{v} J_V^\mu \right)$$
$$+ \; c_{V4} \epsilon^{\mu\nu\alpha\beta} \{ J_{V\,\alpha}, u_\beta \} + c_{V5} \epsilon^{\mu\nu\alpha\beta} J_{A'\,\alpha\beta} \, ,$$

$$\chi_A^{\mu\nu} = \frac{F_A}{2\sqrt{2}} f_-^{\mu\nu} + \frac{\lambda_1^{hA}}{\sqrt{2}} ((\partial^\mu h) u^\nu - (\partial^\nu h) u^\mu)$$
$$+ \; \frac{c_{A1}}{2} (\nabla^\mu J_A^\nu - \nabla^\nu J_A^\mu) + \frac{ic_{A2}}{2} ([J_V^\mu, u^\nu] - [J_V^\nu, u^\mu])$$

…

etc.

Full Higgsless result
(Longhitano [(x)])

Higgsless part CP conserving
But P-even & P-odd terms

- Higgsless R Lagrangian with only bosons

*(general custodial R Lagrangian in *)*

$$\chi_V^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}}[u^\mu, u^\nu] + \frac{\widetilde{F}_V}{2\sqrt{2}} f_-^{\mu\nu}$$

$$\chi_A^{\mu\nu} = \frac{F_A}{2\sqrt{2}} f_-^{\mu\nu} + \frac{\widetilde{F}_A}{2\sqrt{2}} f_+^{\mu\nu} + \frac{i\widetilde{G}_A}{2\sqrt{2}}[u^\mu, u^\nu],$$

$$\chi_{S_1} = \frac{c_d}{\sqrt{2}} \langle u_\mu u^\mu \rangle$$

P-odd

- Integrate out V and A:

$$\mathcal{L}_4 \supset \frac{1}{4} a_1 \langle f_+^{\mu\nu} f_{+\,\mu\nu} - f_-^{\mu\nu} f_{-\,\mu\nu} \rangle + \frac{i}{2}(a_2 - a_3)\langle f_+^{\mu\nu}[u_\mu, u_\nu]\rangle + \frac{i}{2}(a_2 + a_3)\langle f_-^{\mu\nu}[u_\mu, u_\nu]\rangle + a_4 \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + a_5 \langle u_\mu u^\mu \rangle^2$$

$$a_1 = -\frac{F_V^2}{4M_V^2} + \frac{\widetilde{F}_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2} - \frac{\widetilde{F}_A^2}{4M_A^2}$$

$$a_2 - a_3 = -\frac{F_V G_V}{2M_V^2} - \frac{\widetilde{F}_A \widetilde{G}_A}{2M_A^2}$$

$$a_2 + a_3 = -\frac{\widetilde{F}_V G_V}{2M_V^2} - \frac{F_A \widetilde{G}_A}{2M_A^2}$$

$$a_4 = \frac{G_V^2}{4M_V^2} + \frac{\widetilde{G}_A^2}{4M_A^2}$$

$$a_5 = \frac{c_{d\,1}^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2} - \frac{\widetilde{G}_A^2}{4M_A^2}$$

(x) Longhitano ‘80, ‘81

* Ecker et al. ‘89
* Pich,Rosell,Santos,SC,[1501.07249]; forthcoming

PREDICTIONS:

TREE-LEVEL results + UV-constraint

Higgsless part CP conserving
Only P-even interactions

- Higgsless R Lagrangian with only bosons

*(general custodial R Lagrangian in *)*

$$\chi_V^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}}[u^\mu, u^\nu] + \cancel{\frac{\widetilde{F}_V}{2\sqrt{2}}} f_-^{\mu\nu}$$

$$\chi_A^{\mu\nu} = \frac{F_A}{2\sqrt{2}} f_-^{\mu\nu} + \cancel{\frac{\widetilde{F}_A}{2\sqrt{2}} f_+^{\mu\nu}} + \cancel{\frac{i\widetilde{G}_A}{2\sqrt{2}}[u^\mu, u^\nu]},$$

$$\chi_{S_1} = \frac{c_d}{\sqrt{2}}\langle u_\mu u^\mu \rangle$$

- Integrate out V and A:

$$\gamma^* \to \omega^+\omega^- \ \mathbf{EM-FF}: \qquad \mathbf{F_V G_V = v^2}$$

$$\mathbf{1 + 2\,WSR\,on\,\Pi_{W_3B}}: \qquad \mathbf{F_V^2 - F_A^2 = v^2,\ F_V^2 M_V^2 - F_A^2 M_A^2 = 0}$$

$$a_1 = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2},$$

$$(a_2 - a_3) = -\frac{F_V G_V}{2M_V^2},$$

$$a_4 = \frac{G_V^2}{4M_V^2},$$

$$a_5 = \frac{c_d^2}{4M_{S_1}^2} - \frac{G_V^2}{4M_V^2},$$

$\Rightarrow$

$$a_1 = -\frac{v^2}{4}\left(\frac{1}{M_V^2} + \frac{1}{M_A^2}\right),$$

$$(a_2 - a_3) = -\frac{v^2}{2M_V^2},$$

$$a_4 = \frac{v^2}{4}\left(\frac{1}{M_V^2} - \frac{1}{M_A^2}\right),$$

$$a_5 = -\frac{v^2}{4}\left(\frac{1}{M_V^2} - \frac{1}{M_A^2}\right) + \frac{c_d^2}{4M_{S_1}^2},$$

(x) Longhitano '80, '81

* Ecker et al. '89
* Pich,Rosell,Santos,SC,[1501.07249]; forthcoming

$SU(2)_L \otimes SU(2)_R / SU(2)_{L+R}$ Resonance Theory (P-even)

$$\mathcal{L} = \mathcal{L}_{\rm EW}^{(2)} + \mathcal{L}_{\rm GF} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VV}^{\rm kin} + \mathcal{L}_{AA}^{\rm kin} + \ldots$$

w/ field content: $SU(2)_L \otimes SU(2)_R / SU(2)_{L+R}$ EW Goldstones + SM gauge bosons

+ one $SU(2)_L \otimes SU(2)_R$ singlet Higgs-like scalar S_1 with m_{S1}=126 GeV ***

+ lightest V and A resonances -triplets- *(antisym. tensor formalism)* (x)

•Relevant resonance Lagrangian (x), **

NOTATIONS:
$\omega = \mathbf{a} = \kappa_{\mathbf{W}} = \kappa_{\mathbf{Z}}$

$$\mathcal{L} = \frac{v^2}{4}\langle u_\mu u^\mu\rangle \left(1 + \frac{2\,a}{v}\,h\right)$$
h + ω sector

$$+\frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$
V + ω sector

$$+\frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + \sqrt{2}\,\lambda_1^{hA}\,\partial_\mu h\,\langle A^{\mu\nu} u_\nu\rangle$$
A+h+ω sector

We will have 7 resonance parameters:
F_V, G_V, F_{AW}, κ_W, λ_1^{SA}, M_V and M_A

High-energy constraints will be crucial

(x) SD constraints: Ecker et al. '89
(x) EoM simplifications: Xiao, SC '07
(x) EoM simplifications: Georgi '91
(x) EoM simplification: Pich,Rosell,SC '13

** Appelquist, Bernard '80
** Longhitano '80 '81
** Dobado,Espriu,Herrero '91
** Dobado et al. '99
** Espriu,Matias '95 …

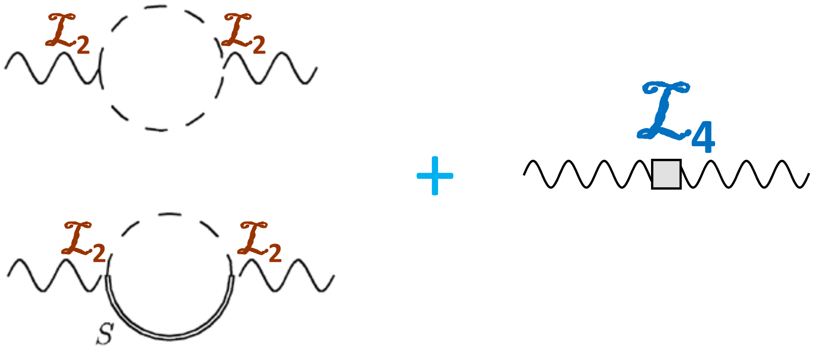
*** Alonso et al. '13
*** Manohar et al. '13
*** Elias-Miro et al. '13…

	P	C	CP	h.c.
S	S	S^T	S^T	S
P	$-P$	P^T	$-P^T$	P
$V^{\mu\nu}$	$V_{\mu\nu}$	$-V^{\mu\nu\,T}$	$-V_{\mu\nu}^T$	$V^{\mu\nu}$
$A^{\mu\nu}$	$-A_{\mu\nu}$	$A^{\mu\nu\,T}$	$-A_{\mu\nu}^T$	$A^{\mu\nu}$

	P	C	CP	h.c.
U	$U^\dagger$	U^t	U^*	$U^\dagger$
u	$u^\dagger$	u^t	u^*	$u^\dagger$
u^μ	$-u_\mu$	$u^{\mu\ t}$	$-u_\mu^t$	u^μ
$(d^\mu X)$	$(d_\mu X')$	$(d^\mu X)'$	$(d_\mu X)'$	$(d^\mu X^\dagger)$
$f_\pm^{\mu\nu}$	$\pm f_{\pm\,\mu\nu}$	$\mp f_\pm^{\mu\nu\ t}$	$-f^t_{\pm\,\mu\nu}$	$f_\pm^{\mu\nu}$
$\partial^\mu h$	$\partial_\mu h$	$\partial^\mu h$	$\partial_\mu h$	$\partial^\mu h$

$$
\begin{aligned}
(J_S)_{mn} &\equiv -Tr_D\{\xi_m\bar{\xi}_n\} = \bar{\xi}_n\xi_m\,, \\
(J_P)_{mn} &\equiv -i\,Tr_D\{\xi_m\bar{\xi}_n\gamma_5\} = i\,\bar{\xi}_n\gamma_5\xi_m\,, \\
(J_V^\mu)_{mn} &\equiv -Tr_D\{\xi_m\bar{\xi}_n\gamma^\mu\} = \bar{\xi}_n\gamma^\mu\xi_m\,, \\
(J_A^\mu)_{mn} &\equiv -Tr_D\{\xi_m\bar{\xi}_n\gamma^\mu\gamma_5\} = \bar{\xi}_n\gamma^\mu\gamma_5\xi_m\,, \\
(J_T^\mu)_{mn} &\equiv -Tr_D\{\xi_m\bar{\xi}_n\sigma^{\mu\nu}\} = \bar{\xi}_n\sigma^{\mu\nu}\xi_m\,, \\
(J_{V'}^{\mu\nu})_{mn} &\equiv -iTr_D\{(d^\mu\xi)_m\bar{\xi}_n\gamma^\nu - \xi_m(d^\mu\bar{\xi})_n\gamma^\nu\}\,, \\
(J_{A'}^{\mu\nu})_{mn} &\equiv -iTr_D\{(d^\mu\xi)_m\bar{\xi}_n\gamma^\nu\gamma_5 - \xi_m(d^\mu\bar{\xi})_n\gamma^\nu\gamma_5\}\,, \\
(J_{S'})_{mn} &\equiv -iTr_D\{(d_\mu\xi)_m\bar{\xi}_n\gamma^\mu - \xi_m(d_\mu\bar{\xi})_n\gamma^\mu\} = J^\mu_{V'\,\mu}\,, \\
(\tilde{J}_{S'})_{mn} &\equiv -iTr_D\{(d_\mu\xi)_m\bar{\xi}_n\gamma^\mu\gamma_5 - \xi_m(d_\mu\bar{\xi})_n\gamma^\mu\gamma_5\} = J^\mu_{A'\,\mu}
\end{aligned}
$$

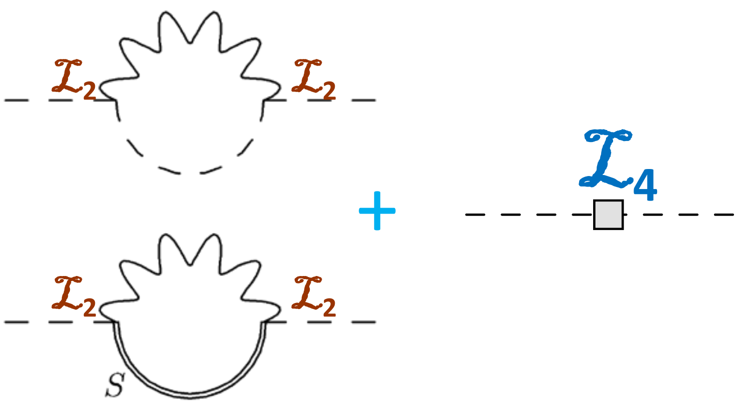
→ W^3B correlator*



$$S = -16\pi\, \mathbf{a}_1^{\mathbf{r}}(\mu) + \frac{(1-a^2)}{12\pi}\left(\frac{5}{6} + \ln\frac{\mu^2}{m_h^2}\right)$$

→ NGB self-energy *

3 eff. couplings



$$T = \frac{8\pi}{c_W^2}\, \mathbf{a}_0^{\mathbf{r}}(\mu) - \frac{3\,(1-a^2)}{16\pi c_W^2}\left(\frac{5}{6} + \ln\frac{\mu^2}{m_h^2}\right)$$

* Dobado et al. '99
* Pich, Rosell, SC '12, '13
* Delgado,Dobado,Herrero,SC [in prep]

→Similar in linear models:
Masso,Sanz,PRD87 (2013) 3, 033001
Chen,Dawson,Zhang,PRD89 (2014) 015016

• More observables* can over-constrain the $a_i(\mu)$

BUT not (S,T) alone!!!

• Taking just tree-level is incomplete $\longrightarrow \left[\quad S = -16\pi\, a_1(\mu?)\,, \qquad T = \frac{8\pi}{c_W^2} a_0(\mu?) \quad \right]$

and similar if only loops $\longrightarrow \left[\quad S = \frac{(1-a^2)}{12\pi} \ln\frac{\mu^2}{m_h^2}\,, \qquad T = -\frac{3\,(1-a^2)}{16\pi c_W^2} \ln\frac{\mu^2}{m_h^2} \quad \right]$

•Otherwise, one may resource to models**:

→Resonances *(lightest V + A)*

→ UV-completion assumptions *(high-energy constraints)*

* Delgado,Dobado,Herrero,SC [in prep.] ** Pich, Rosell, SC '12, '13

also notice the subtlety*,** $g^{(')} \sim m_{W,Z}/v \sim p/v$ *[notice $e \sim p/v$ too]*

* Buchalla,Catà,Krause '13
* Hirn,Stern '05
* Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149

** Urech '95

(x) Apelquist,Bernard '80
(x) Longhitano '80, '81
(x) Herrero,Morales '95
(x) Pich,Rosell,Sc '12 '13
(x) Brivio et al. '13
(x) Gavela,Kanshin,Machado,Saa '14, etc.

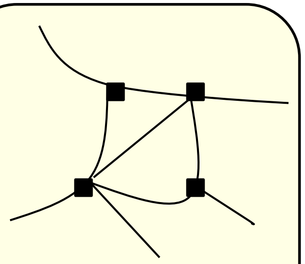
Counting, loops & renormalization

- In general, the O(p^d) Lagrangian has the symbolic form (χ=W,B,π,h),

$$\mathcal{L}_d \;=\; \sum_k f_k^{(d)}\, p^d \left(\frac{\chi}{v}\right)^k$$

$$f_k^{(2)} \sim v^2$$
$$f_k^{(4)} \sim a_i$$
...

leading to a general scaling* of a diagram with
- *L loops*
- *E external legs*
- *N_d vertices of $\mathcal{L}_d$*



E.g.: L=2, E=5

$$\mathcal{M} \sim \left(\frac{\mathbf{p^2}}{\mathbf{v^{E-2}}}\right)\left(\frac{\mathbf{p^2}}{\mathbf{16\pi^2 v^2}}\right)^{\mathbf{L}} \prod_{\mathbf{d}}\left(\frac{\mathbf{f_k^{(d)} p^{(d-2)}}}{\mathbf{v^2}}\right)^{\mathbf{N_d}}$$

[scaling of individual diagrams; cancellations & higher suppressions for the total amplitude]

- O(p^d) loop divergence + O(p^d) chiral coupling = UV-finite

E.g. W_LW_L-scat:**

LO O(p^2)➔ $\frac{\mathbf{p^2}}{\mathbf{v^2}}$ (tree)

NLO O(p^4)➔ $\mathbf{a_i}\frac{\mathbf{p^4}}{\mathbf{v^4}}$ (tree) + $\frac{\mathbf{p^4}}{\mathbf{16\pi^2 v^4}}\left(\frac{1}{\epsilon}+\log\right)$ (1-loop)

* Weinberg '79
* Urech '95
* Georgi,Manohar NPB234 (1984) 189
* Buchalla,Catà,Krause '13
* Hirn,Stern '05
* Delgado,Dobado,Herrero,SC,JHEP1407 (2014) 149
** Espriu,Mescia,Yencho '13
** Delgado,Dobado '13

S-parameter sum-rule *

✓ In this work, dispersive representation introduced by Peskin and Takeuchi*.

$$S = \frac{16}{g^2 \tan\theta_W} \int_0^\infty \frac{\mathrm{dt}}{t} \left(\mathrm{Im}\widetilde{\Pi}_{30}(t) - \mathrm{Im}\widetilde{\Pi}_{30}(t)^{\mathrm{SM}} \right)$$

$$= \int_0^\infty \frac{\mathrm{dt}}{t} \left(\frac{16}{g^2 \tan\theta_W} \mathrm{Im}\widetilde{\Pi}_{30}(t) - \frac{1}{12\pi} \left[1 - \left(1 - \frac{m^2_{H,ref}}{t} \right)^3 \theta(t - m^2_{H,ref}) \right] \right)$$

→ The convergence of the integral requires $\rho_S(t) \equiv \frac{1}{\pi} \mathrm{Im}\widetilde{\Pi}_{30}(t) \xrightarrow{t\to\infty} 0$

→ S-parameter defined for an arbitrary reference value $m_{H,ref}$

→ Higher threshold cuts in $\mathrm{Im}\Pi_{30}$ will be suppressed in the dispersive integral

→ At tree-level: $S_{\mathrm{LO}} = 4\pi \left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right)$

* Peskin and Takeuchi '92.

High-energy constraints

- ✓ We will have 7 resonance parameters: F_V, G_V, F_A, κ_W, λ_1^{SA}, M_V and M_A.
- ✓ The number of unknown couplings can be reduced by using short-distance information.
- ✓ In contrast with the QCD case, we ignore the underlying dynamical theory.

0) Once-subtracted dispersion* relation for $\Pi_{30}(s) = \frac{g^2 \tan\theta_W}{4}\, s\, [\Pi_{VV}(s) - \Pi_{AA}(s)]$

- ✓ Once-subtract. dispersive relation from tree+1-loop spectral function**

  $\pi\pi$, $h\pi$ … (higher cuts suppressed) $\quad \Pi_{30}(s) = \Pi_{30}(0) + \frac{s}{\pi}\int_0^\infty \frac{\mathrm{d}t}{t\,(t-s)} \mathrm{Im}\Pi_{30}(t)$

- ✓ F_R^r and M_R^r are *renormalized* couplings which define the resonance poles at the one-loop level.

$$\Pi_{30}(s)|_{\mathrm{NLO}} = \frac{g^2\tan\theta_W}{4}\, s\, \left(\frac{v^2}{s} + \frac{F_V^{r\,2}}{M_V^{r\,2} - s} - \frac{F_A^{r\,2}}{M_A^{r\,2} - s} + \overline{\Pi}(s) \right)$$

* Peskin,Takeuchi ‘90, ‘91

** Pich, Rosell, SC ‘08

i) Weinberg Sum Rules (WSR)*

$$\Pi_{30}(s) = \frac{g^2 \tan\theta_W\, s}{4}\,[\,\mathbf{\Pi_{VV}(s)} - \mathbf{\Pi_{AA}(s)}\,]$$
$$= \frac{g^2 v^2 \tan\theta_W}{4} + s\,\widetilde{\mathbf{\Pi}}_{\mathbf{30}}(\mathbf{s})$$

1ST WSR:

$$\left|\mathbf{s} \times \mathbf{\Pi_{V-A}}(\mathbf{s})\right| \quad \stackrel{\mathbf{s}\to\infty}{\longrightarrow} \quad \mathbf{0} \qquad \Longrightarrow \qquad \int_{\mathbf{0}}^{\infty} \mathbf{dt}\ \rho_{\mathbf{S}}(\mathbf{t}) = \frac{\mathbf{g^2 v^2} \tan\theta_{\mathbf{W}}}{\mathbf{4}}$$

2ND WSR:

$$\left|\mathbf{s^2} \times \mathbf{\Pi_{V-A}}(\mathbf{s})\right| \quad \stackrel{\mathbf{s}\to\infty}{\longrightarrow} \quad \mathbf{0} \qquad \Longrightarrow \qquad \int_{\mathbf{0}}^{\infty} \mathbf{dt}\ \mathbf{t}\,\rho_{\mathbf{S}}(\mathbf{t}) = \mathbf{0}$$

* Weinberg'67
* Bernard et al.'75.

$$\rho_{\mathbf{S}}(\mathbf{s}) = \frac{\mathbf{1}}{\pi}\mathrm{Im}\widetilde{\mathbf{\Pi}}_{\mathbf{30}}(\mathbf{s})$$

i.i) LO

$$F_V^2 - F_A^2 = v^2$$
$$F_V^2\, M_V^2 - F_A^2\, M_A^2 = 0$$

(1 / 2 constraints)

i.ii) Imaginary NLO

$$\mathrm{Im}\Pi_{V-A}(s) \sim \mathcal{O}\left(\frac{1}{s^{\Delta/2}}\right)$$

(1 / 2 constraints)

i.iii) Real NLO: fixing of $F_{V,A}^r$ or lower bounds**

$$F_V^{r\,2} - F_A^{r\,2} = v^2\,(1 + \delta^{(1)}_{\rm NLO})$$
$$F_V^{r\,2}\, M_V^{r\,2} - F_A^{r\,2} M_A^{r\,2} = v^2 M_V^{r\,2} \delta^{(2)}_{\rm NLO}$$

(constraints on $F_{V,A}^r$)

F_R^r and M_R^r are *renormalized* couplings which define the resonance poles at the one-loop level**

$$\Pi_{30}(s) = \Pi_{30}(0) + \frac{s}{\pi}\int_0^\infty \frac{\mathrm{d}t}{t\,(t-s)}\mathrm{Im}\Pi_{30}(t)$$

$$\Pi_{30}(s)|_{\rm NLO} = \frac{g^2 \tan\theta_W}{4}\; s\;\left(\frac{v^2}{s} + \frac{F_V^{r\,2}}{M_V^{r\,2}-s} - \frac{F_A^{r\,2}}{M_A^{r\,2}-s} + \overline{\Pi}(s)\right)$$

* Weinberg'67
* Bernard et al.'75.

** Pich, Rosell, SC '08

LO results***

i.i) 1st and 2nd WSRs **

$$S_{\rm LO} \;=\; \frac{4\pi v^2}{M_V^2}\left(1+\frac{M_V^2}{M_A^2}\right)$$

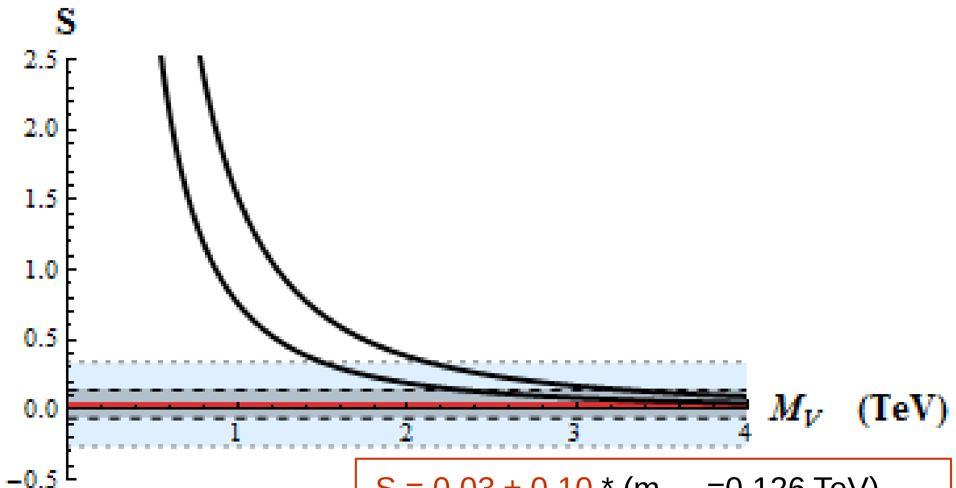
$$\boxed{\frac{4\pi v^2}{M_V^2} < S_{\rm LO} < \frac{8\pi v^2}{M_V^2}}$$

$$S_{\rm LO} = 4\pi\left(\frac{F_V^2}{M_V^2}-\frac{F_A^2}{M_A^2}\right)\,, \qquad T_{\rm LO}=0$$

i.ii) Only 1st WSR *** (lower bound for $M_A > M_V$)

$$S_{\rm LO} \;=\; 4\pi\left\{\frac{v^2}{M_V^2}+F_A^2\left(\frac{1}{M_V^2}-\frac{1}{M_A^2}\right)\right\}$$

$$\boxed{S_{\rm LO} > \frac{4\pi v^2}{M_V^2}}$$



S = 0.03 ± 0.10 * ($m_{H,ref}$=0.126 TeV)

At LO M_V > 2.4 TeV at 68% CL

(M_V > 3.6 TeV if T_{LO}=0 also considered)

* Gfitter
* LEP EWWG
* Zfitter

** Peskin and Takeuchi '92.

*** Pich, Rosell, SC '12

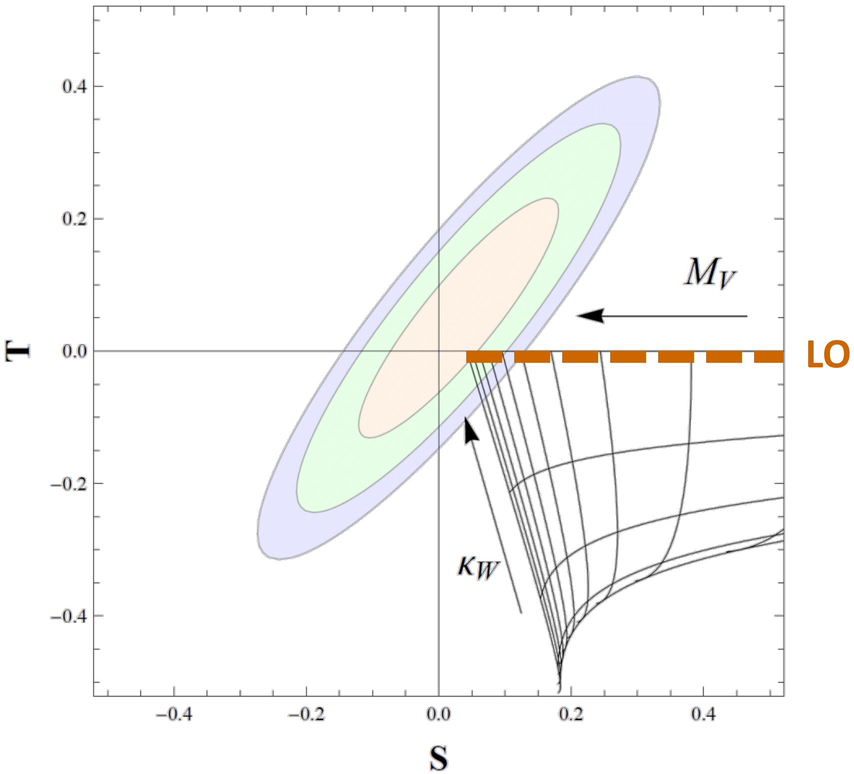
*NLO results:** 1st and 2nd WSRs in Π_{30}

(*asymptotically-free theories*)

$$T = \frac{3}{16\pi\cos^2\theta_W}\left[1+\log\frac{m_H^2}{M_V^2} - \kappa_W^2\left(1+\log\frac{m_{S_1}^2}{M_A^2}\right)\right]$$

$$S = \underbrace{4\pi v^2\left(\frac{1}{M_V^2}+\frac{1}{M_A^2}\right)}_{\text{LO}} + \frac{1}{12\pi}\left[\left(\log\frac{M_V^2}{m_H^2}-\frac{11}{6}\right) - \kappa_W^2\left(\log\frac{M_A^2}{m_{S_1}^2}-\frac{11}{6}-\frac{M_A^2}{M_V^2}\log\frac{M_A^2}{M_V^2}\right)\right]$$

[terms $O(m_S^2/M^2_{V,A})$ neglected]



✓ 1st and 2nd WSRs at LO and NLO + ππ-VFF:

→ 2nd WSR: $0 < \kappa_W = M_V^2/M_A^2 < 1$

At NLO with the 1st and 2nd WSRs

$M_V > 5.4$ TeV, $0.97 < \kappa_W < 1$ at 68% CL

Small splitting $(M_V/M_A)^2 = \kappa_W$

* Pich,Rosell,SC ‘12, ‘13

*NLO Results:** Only 1st WSRs in Π_{30}

(*walking & conformal TC, extra dimensions,…*)**

$$T = \frac{3}{16\pi\cos^2\theta_W}\left[1+\log\frac{m_H^2}{M_V^2} - \kappa_W^2\left(1+\log\frac{m_{S_1}^2}{M_A^2}\right)\right]$$

$$S > \underbrace{\boxed{\frac{4\pi v^2}{M_V^2}}}_{\text{LO}} + \frac{1}{12\pi}\left[\left(\ln\frac{M_V^2}{m_H^2} - \frac{11}{6}\right) - \kappa_W^2\left(\log\frac{M_A^2}{m_{S_1}^2} - \frac{17}{6} + \frac{M_A^2}{M_V^2}\right)\right]$$

[terms $O(m_S^2/M^2_{V,A})$ neglected]

- ✓ **Assumption** $M_A > M_V$ for the S lower-bound
- ✓ Only 1st WSR at LO and NLO + ππ-VFF:

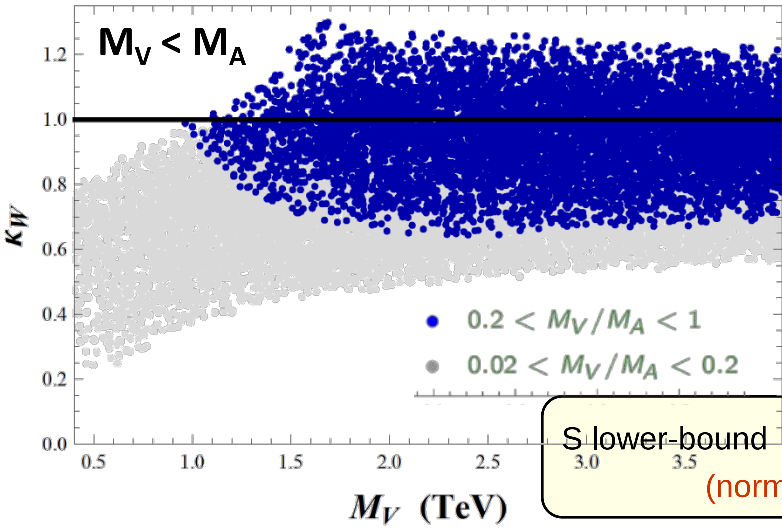
  → Free parameters: M_V, M_A and κ_W

* Pich,Rosell,SC ‘12, ‘13

** Orgogozo,Rychkov ‘11

*NLO Results:** Only 1st WSRs in Π_{30}

(*walking & conformal TC, extra dimensions,…*)**



S lower-bound for $M_A > M_V$
(normal hierarchy)

$$T = \frac{3}{16\pi\cos^2\theta_W}\left[1+\log\frac{m_H^2}{M_V^2} - \kappa_W^2\left(1+\log\frac{m_{S_1}^2}{M_A^2}\right)\right]$$

LO

$$S > \boxed{\frac{4\pi v^2}{M_V^2}} + \frac{1}{12\pi}\left[\left(\ln\frac{M_V^2}{m_H^2} - \frac{11}{6}\right) - \kappa_W^2\left(\log\frac{M_A^2}{m_{S_1}^2} - \frac{17}{6} + \frac{M_A^2}{M_V^2}\right)\right]$$

At NLO with only 1st WSRs

$M_V > 1$ TeV, **$\kappa_W \in (0.6\ ,\ 1.3)$** at 68% CL

for moderate splitting $0.2 < M_V/M_A < 1$

$\kappa_W = g_{HWW}/g_{HWW}^{SM}$

very different from the SM

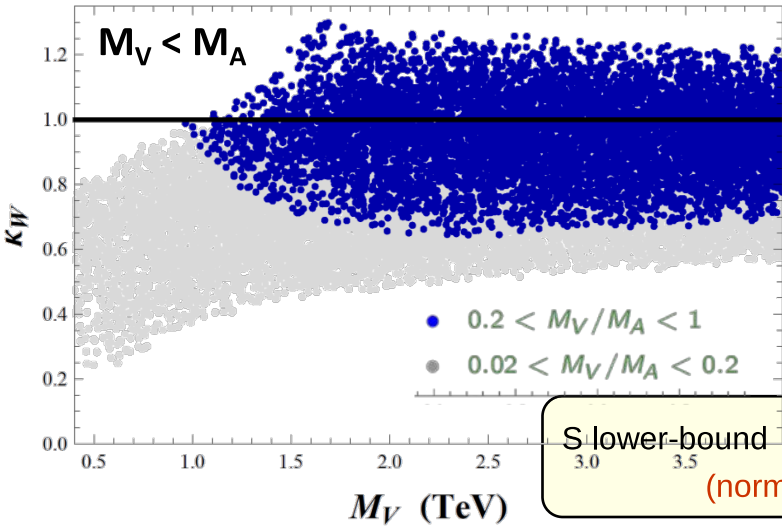
if one requires large (unnatural) splittings

* Pich,Rosell,SC '12, '13

** Orgogozo,Rychkov '11

*NLO Results:** Only 1st WSRs in Π_{30}

(*walking & conformal TC, extra dimensions,…*)**



S lower-bound for $M_A > M_V$ (normal hierarchy)

$$T = \frac{3}{16\pi\cos^2\theta_W}\left[1+\log\frac{m_H^2}{M_V^2}-\kappa_W^2\left(1+\log\frac{m_{S_1}^2}{M_A^2}\right)\right]$$

$$S > \underbrace{\boxed{\frac{4\pi v^2}{M_V^2}}}_{\text{LO}} + \frac{1}{12\pi}\left[\left(\ln\frac{M_V^2}{m_H^2}-\frac{11}{6}\right)-\kappa_W^2\left(\log\frac{M_A^2}{m_{S_1}^2}-\frac{17}{6}+\frac{M_A^2}{M_V^2}\right)\right]$$

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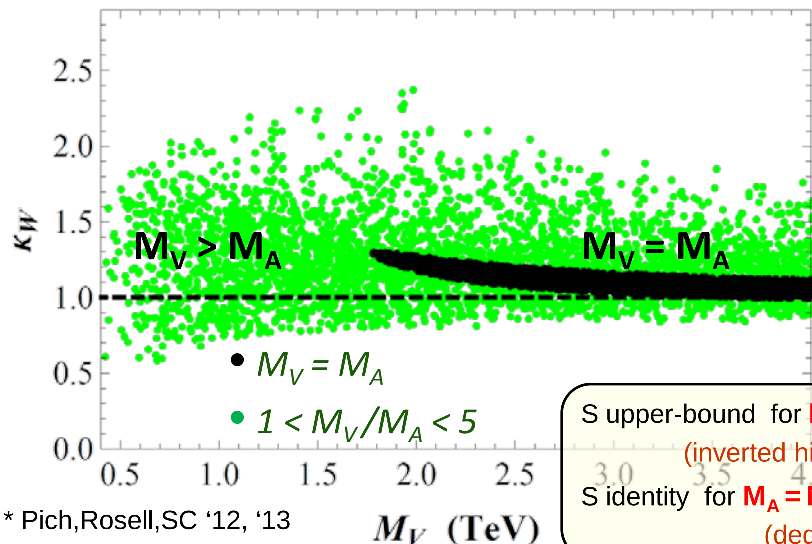


S upper-bound for $M_A < M_V$ (inverted hierarchy)

S identity for $M_A = M_V$ (degenerate)

* Pich,Rosell,SC '12, '13

** Orgogozo,Rychkov '11

Further comments:

- ✓ 1< M_A/M_V< 2 yields **M_V > 1.5 TeV**, **$\kappa_W \in$[0.84 , 1.30]**

- ✓ The limit $\kappa_W \to 0$ only reached for $M_V/M_A \to 0$

  κ_W=0 incompatible with data *(independently of whether 1st+2nd WSR's or just 1st WSR)*

- ✓ Predictions for ECLh low-energy couplings

1st+2nd WSRs →

$$a_1(\mu) = \underbrace{\boxed{-\frac{v^2}{4}\left(\frac{1}{M_V^2}+\frac{1}{M_A^2}\right)}}_{\text{LO}} + \frac{1}{192\pi^2}\left(\frac{8}{3}+\ln\frac{\mu^2}{M_V^2}\right) - \frac{\kappa_W^2}{192\pi^2}\left(\frac{8}{3}+\ln\frac{\mu^2}{M_A^2}\right) + \kappa_W \ln\kappa_W^2$$

$$a_0(\mu) = \frac{3}{128\pi^2}\left(\frac{11}{6}+\ln\frac{\mu^2}{M_V^2}\right) - \frac{3\,\kappa_W^2}{128\pi^2}\left(\frac{11}{6}+\ln\frac{\mu^2}{M_A^2}\right)$$

- ✓ Calculation valid for particular models with this symmetry:

  E.g., in SO(5)/SO(4) with $\kappa_W=\cos\theta < 1$ *

* Agashe,Contino,Pomarol '05
* Barbieri et al '12
* Marzocca,Serone,Shu '12 ...

•O(p^d) loop divergence + O(p^d) chiral coupling = UV-finite

•In OUR case, renormalization at O(p4): $a_1, a_2, a_3, c_\gamma \to a_1^r, a_2^r, a_3^r, c_\gamma^r$

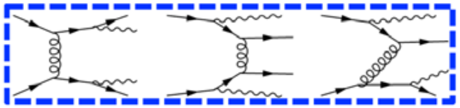
$$\boxed{\begin{aligned} C^r(\mu) &= C^{(B)} + \frac{\Gamma_C}{32\pi^2}\frac{1}{\hat{\epsilon}} \\ \frac{dC^r}{d\ln\mu} &= -\frac{\Gamma_C}{16\pi^2} \end{aligned}}$$

•Naively, our EFT range of validity given by $\mathbf{p^2} \ll \min\left\{\mathbf{16\pi^2 v^2}, \frac{\mathbf{v^2}}{\mathbf{a_i}}\right\}$

•Previous May: WW-scattering

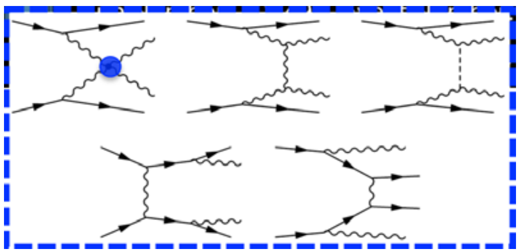
[ATLAS 1405.6241: $W^{\pm}W^{\pm}jj$]

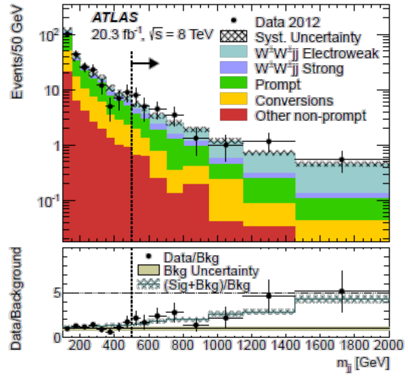
strong production

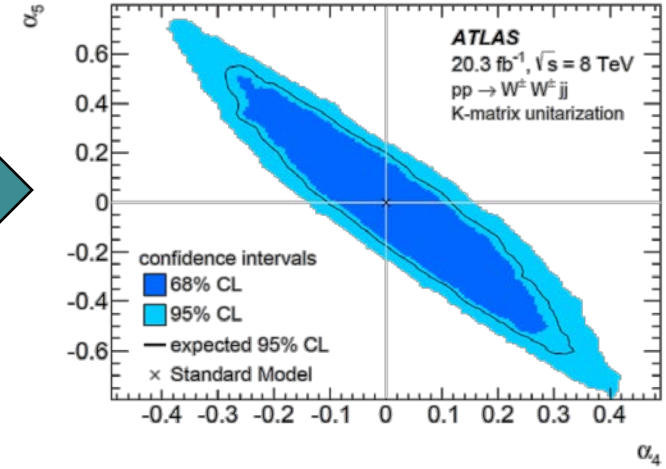


+

electroweak production







Theory side: ** Espriu,Mescia,Yencho '13
** Delgado,Dobado '13

More experiment!!
+
More theory!!

•Bounds on eff. vertices →

JUST EFF. VERTICES
NOT ENOUGH

(stronger than LEP & Tevatron)

Low-energy EFT calculation

FERMIONIC SECTOR: ψ

- Custodial $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ framework (+)

- (t,b)-type doublets ψ: $\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $\psi_R = \begin{pmatrix} t_R \\ b_R \end{pmatrix}$

turned into a covariant doublet ξ with the help of Goldstones u(x)

$$\xi_m^a = \frac{1}{2}(\delta^{ab} - \gamma_5^{ab}) u_{mn} \psi_n^b + \frac{1}{2}(\delta^{ab} + \gamma_5^{ab})(u^\dagger)_{mn} \psi_n^b$$

$$\left[\begin{array}{lll} \xi & = & \xi_L + \xi_R \,, \\ \xi_L & = & u_L^\dagger \, \psi_L = u \, \psi_L \,, \qquad \xi_R = u_R \, \psi_R = u^\dagger \psi_R \end{array}\right]$$

- Breaking down to $SU(2)_L \otimes U(1)_Y$ in $d_\mu \xi$ only through spurions

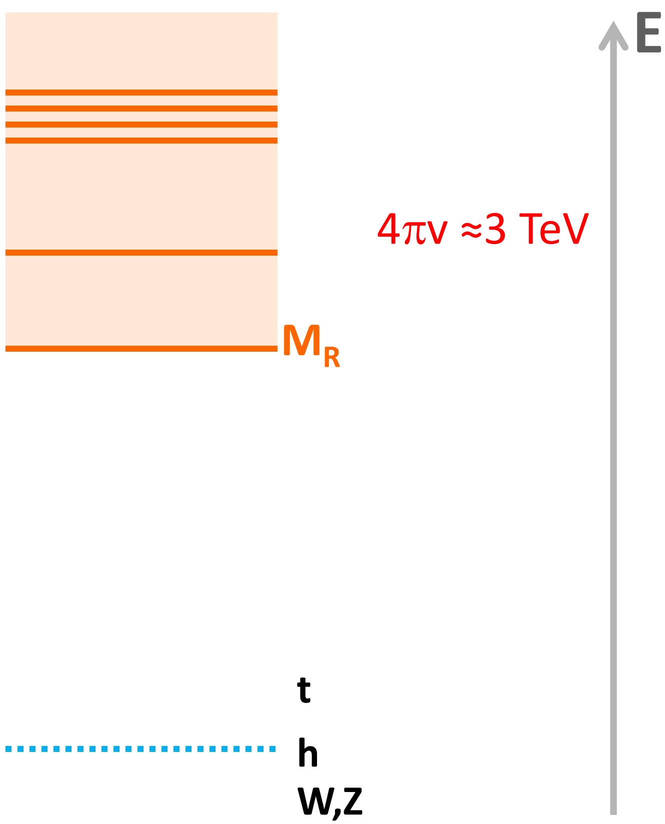
$$\boxed{\begin{array}{lll} \hat{W}_\mu & = & -\frac{g}{2} W_\mu^a \sigma^a \,, \\ \hat{B}_\mu & = & -\frac{g'}{2} B_\mu \sigma^3 \,, \\ X_\mu & = & -B_\mu \,, \end{array}}$$

- More general EFT based on $SU(2)_L \otimes U(1)_Y$ also possible *

(+) Pich,Rosell,Santos,SC, forthcoming

* Buchalla,Catà,Krause ‘13
* Hirn,Stern ‘05

Energy scales in a strong-int. Higgs theory?



Naïve rescaling
from QCD to EW scale

QCD		EW
F_π=0.090 GeV	→	v=0.246 TeV
$\Lambda_{\chi PT}=4\pi F_\pi \approx$ 1.2 GeV	→	$\Lambda_{EW}=4\pi v \approx$ 3.1 TeV
M_ρ=0.770 GeV	→	M_{V1}=2.1 TeV
M_{a1}=1.260 GeV	→	M_{A1}=3.4 TeV

EFT general considerations

1. "SM" content: - Bosons χ: Higgs h + EW Golsdtones $\omega^{\pm}$,z + gauge bosons A^a_μ,B_μ,

   - Fermions ψ: (t,b)-type doublets

2. Applicability: $E \ll \Lambda_{\mathrm{EClh}} \sim \min\{4\pi v, M_R\}$ $(4\pi v \sim 3$ TeV)

3. EW would-be Goldston bosons ➔ Non-linear realization $U(\omega^a)$

4. **Custodial symmetry:** $SU(2)_L \otimes SU(2)_R / SU(2)_{L+R}$ pattern

5. Gauge symmetry: $SU(2)_L \otimes U(1)_Y$

Additionally it is appropriate to work with

6. Renormalizable R_ξ gauge: Landau gauge convenient $(\, m_{\omega\pm,z} = 0\,)$

R contributions to the O(p^4) EFT couplings

1.) Only R operators O(p^2) :

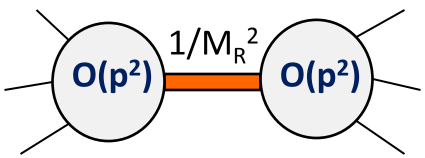
$$\mathcal{L}_R = \frac{1}{2}\langle \nabla^\mu R\, \nabla_\mu R - M_R^2 R^2 \rangle + \langle R\chi_R \rangle \qquad (R = S,\ P)\,,$$

$$\mathcal{L}_R = -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu}\, \nabla_\sigma R^{\sigma\mu} - \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu} \rangle + \langle R_{\mu\nu}\chi_R^{\mu\nu} \rangle \qquad (R = V,\ A)\,,$$

$$\mathcal{L}_{R_1} = \frac{1}{2}\left(\partial^\mu R_1\, \partial_\mu R_1 - M_{R_1}^2 R_1^2\right) + R_1 \chi_{R_1} \qquad (R_1 = S_1,\ P_1)\,,$$

$$\mathcal{L}_{R_1} = -\frac{1}{2}\left(\partial^\lambda R_{1\,\lambda\mu}\, \partial_\sigma R_1^{\sigma\mu} - \frac{1}{2} M_{R_1}^2 R_{1\,\mu\nu} R_1^{\mu\nu}\right) + R_{1\,\mu\nu}\chi_{R_1}^{\mu\nu} \qquad (R_1 = V_1,\ A_1)\,,$$

*[antisymetric-tensor formalism $R_{\mu\nu}$ for spin-1 Resonances *]*



2.) Tree-level contribution to the O(p^4) ECLh for p<<M_R:

$$e^{i\,S[\chi,\psi]_{\rm EFT}} = \int [dR]\, e^{i\,S[\chi,\psi,R]} \overset{\rm tree-level}{=} e^{i\,S[\chi,\psi,R_{\rm cl.}]}$$

$$\Delta\mathcal{L}_R^{\mathcal{O}(p^4)} = \frac{1}{2M_R^2}\left(\langle \chi_R\chi_R\rangle - \frac{1}{N}\langle\chi_R\rangle^2\right) \qquad (R = S,\ P)\,,$$

$$\Delta\mathcal{L}_R^{\mathcal{O}(p^4)} = -\frac{1}{M_R^2}\left(\langle \chi_R^{\mu\nu}\chi_{R\,\mu\nu}\rangle - \frac{1}{N}\langle\chi_R^{\mu\nu}\rangle^2\right) \qquad (R = V,\ A)\,,$$

$$\Delta\mathcal{L}_{R_1}^{\mathcal{O}(p^4)} = \frac{1}{2M_{R_1}^2}(\chi_{R_1})^2 \qquad (R_1 = S_1,\ P_1)\,,$$

$$\Delta\mathcal{L}_{R_1}^{\mathcal{O}(p^4)} = -\frac{1}{M_{R_1}^2}(\chi_{R_1}^{\mu\nu}\chi_{R_1\,\mu\nu}) \qquad (R_1 = V_1,\ A_1)\,.$$

** Ecker et al. '89
** Cirigliano et al., NPB753 (2006) 139
** Pich,Rosell,Santos,SC, [1501.07249]; forthcoming

- I could show you this,

$$\chi_V^{\mu\nu} = \frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}}[u^\mu, u^\nu]$$
$$+ \; c_{V0} v J_T^{\mu\nu}$$
$$+ \; \frac{c_{V1}}{2}\left(\nabla^\mu J_V^\nu - \nabla^\nu J_V^\mu\right) + \frac{ic_{V2}}{2}\left([J_A^\mu, u^\nu] - [J_A^\nu, u^\mu]\right)$$
$$+ \; \frac{c_{V3}}{2}\left(\frac{(\partial^\mu h)}{v} J_V^\nu - \frac{(\partial^\nu h)}{v} J_V^\mu\right)$$
$$+ \; c_{V4}\epsilon^{\mu\nu\alpha\beta}\{J_{V\,\alpha}, u_\beta\} + c_{V5}\epsilon^{\mu\nu\alpha\beta} J_{A'\,\alpha\beta}\,,$$

$$\chi_A^{\mu\nu} = \frac{F_A}{2\sqrt{2}} f_-^{\mu\nu} + \frac{\lambda_1^{hA}}{\sqrt{2}}\left((\partial^\mu h)u^\nu - (\partial^\nu h)u^\mu\right)$$
$$+ \; \frac{c_{A1}}{2}\left(\nabla^\mu J_A^\nu - \nabla^\nu J_A^\mu\right) + \frac{ic_{A2}}{2}\left([J_V^\mu, u^\nu] - [J_V^\nu, u^\mu]\right)$$

…

etc.

BASIC TREE-LEVEL EXAMPLE

C and P-even theories

- But it is more instructive to focus on a case (full R Lagrangian in *)

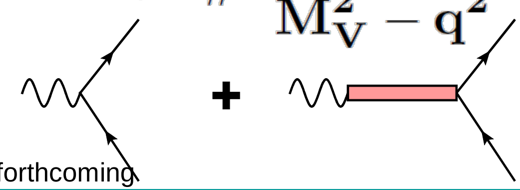
$$\mathcal{L}_V = Tr\{V_{\mu\nu}\ \underbrace{\left(\frac{F_V}{2\sqrt{2}} f_+^{\mu\nu} + \frac{iG_V}{2\sqrt{2}}[u^\mu, u^\nu] + c_{V1}\nabla^\mu J_V^\nu + \ldots\right)}_{\chi_{\mathbf{V}}^{\mu\nu}}\}$$

- Integrate out V:

$$\mathcal{L}^{\mathrm{from\,V}}_{\mathcal{O}(p^4)} = \underbrace{-i\frac{F_V G_V}{4M_V^2}}_{\downarrow\ i\,(a_2-a_3)/2} Tr\{f_+^{\mu\nu}[u^\mu, u^\nu]\} \underbrace{- \frac{F_V c_{V1}}{\sqrt{2}M_V^2}}_{\downarrow\ \mathcal{F}_{X\psi\psi}} Tr\{f_+^{\mu\nu}\nabla_\mu J_{V\,\nu}\} + \ldots$$

- UV constraints:

$$\mathbf{F(q^2)} \sim \mathbf{1} + \# \frac{\mathbf{q^2}}{\mathbf{M_V^2 - q^2}}$$



$\sim 1/q^2$

$(a_2-a_3) = -v^2/(2M_V^2)$

$\mathcal{F}_{X\psi\psi} = 1/(2M_V^2)$

* Pich,Rosell,Santos,SC, [1501.07249]; forthcoming