# Subleading P-wave, Higgs and nonresonant contributions to top-pair production near threshold

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based on work in collaboration with

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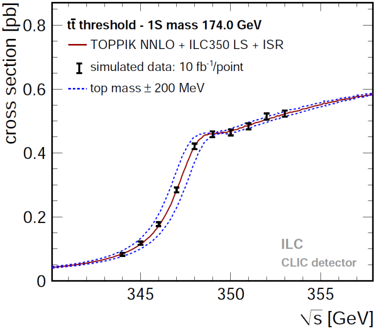
# Motivation

Motivation for studying $e^+e^- \to t\bar{t}$ near threshold:

- Threshold scan at future linear collider
  - Ultra-precise measurement of top quark mass: $\delta m_t^{\overline{\mathrm{MS}}} \sim \mathcal{O}(50\ \mathrm{MeV})$
  - High sensitivity to top width and $\alpha_s$
  - Possibility to measure top Yukawa coupling
- Technically very interesting computation




[Seidel, Simon, Tesar, Poss]

# Introduction

- Near threshold tops are nonrelativistic with velocity $v \sim \alpha_s$
  - Multiple scales are relevant:

| | | |
|---|---|---|
| hard scale | $m_t$ | mass |
| soft scale | $m_t v$ | momentum |
| ultrasoft scale | $m_t v^2$ | energy |

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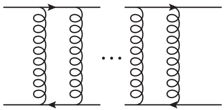
  - Conventional perturbation theory in $\alpha_s$ fails
  - Coulomb singularities $(\alpha_s/v)^n$ from $n$ exchanges of potential gluons $(k^0, \mathbf{k}) \sim (m_t v^2, m_t v)$ have to be summed to all orders

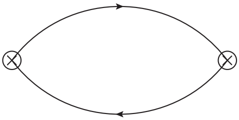


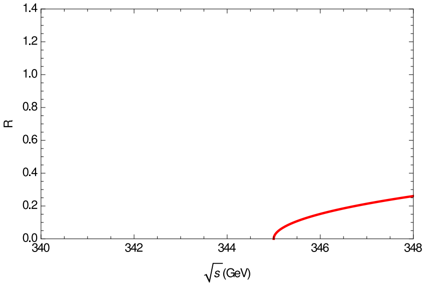
  - This resummation can be organized systematically using nonrelativistic effective theories, see review [Beneke, Kiyo, Schuller: 1312.4791]

# Cross section in pure QCD

- Normalized cross section $R(s) = \frac{\sigma(e^+e^- \to t\bar{t}X)}{\sigma_0(e^+e^- \to \mu^+\mu^-)} = 12\pi e_t^2 f(s)\ \mathrm{Im}\left[\Pi^{(v)}(s)\right]$

Born cross section:





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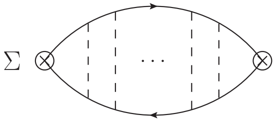
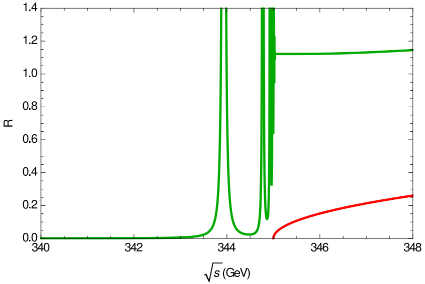
Resummed cross section at LO:

$$\equiv$$



$\Gamma_t = 0$



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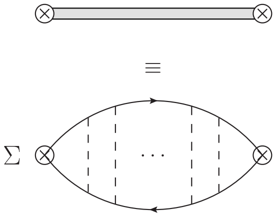
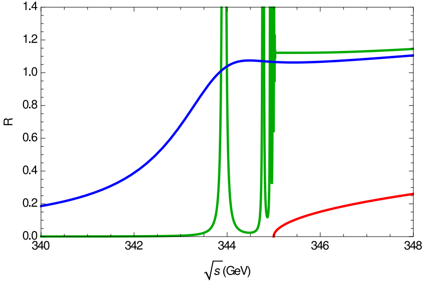


$\Gamma_t \neq 0$



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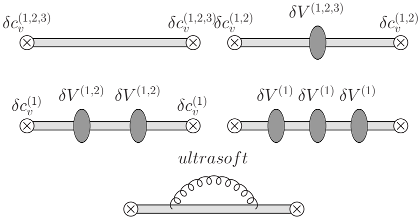
Resummed cross section at NNNLO:

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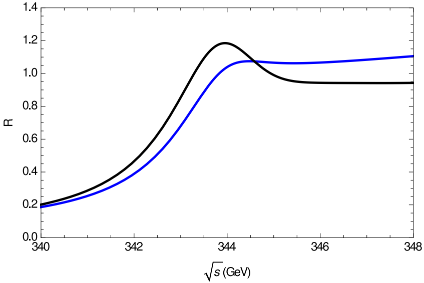
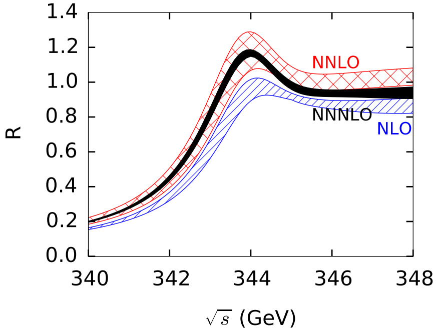


# Full third order in QCD

- NNNLO QCD result completed this year (NNLO from late 90's)




Inputs:

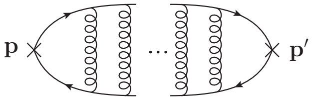
$$m_t^{\rm PS}(\mu_f = 20\ \text{GeV}) = 171.5\ \text{GeV}$$
$$\Gamma_t = 1.33\ \text{GeV}$$
$$\alpha_s(m_Z) = 0.1185$$
$$\alpha(m_Z) = 1/128.944$$
$$\sin^2\theta_W = 0.223$$

Scale variation: $\mu \in [50\ \text{GeV}, 350\ \text{GeV}]$
with $\mu^{\rm cent} = 80$ GeV

- Plot from [Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser: 1506.06864]
  NNNLO ingredients: [Anzai, Beneke, Kiyo, Kniehl, Marquard, Penin, Piclum, Schuller, Seidel, Smirnov, Smirnov, Steinhauser, Sumino, Wüster]
- NNLL results [Pineda, Signer; Hoang, Stahlhofen]

# P wave contribution

- P-wave contribution to the cross section starting at NNLO
  - Top pair is produced dominantly in an S wave, since produced by vector current
  - Axial-vector coupling from Z boson yields different production operator $\rightarrow$ P wave
  - Operator contains top momentum $\rightarrow$ suppressed by $v \sim \alpha_s \rightarrow$ NNLO effect




  - NNNLO correction computed in [Beneke, Piclum, TR: 1312.4792]
  - Older results exist [Penin, Pivovarov], but are not in dimensional regularization, which is required for a consistent combination with nonresonant effects

# Higgs and QED effects

- Higgs exchange leads to two modifications
  - Matching coefficients of the vector current [Eiras, Steinhauser]
  - Additional local (not Yukawa) potential for $m_H \sim m_t$

$$\frac{1}{\mathbf{q}^2 + m_H^2} \sim \frac{1}{m_H^2} + \mathcal{O}\left(\frac{\mathbf{q}^2}{m_H^2} \sim v^2\right) \quad \xrightarrow{\text{FT}} \quad \frac{\delta^{(3)}(\mathbf{r})}{m_H^2}$$

  - Higgs effects up to NNNLO included in [Beneke, Maier, Piclum, TR: 1506.06865]

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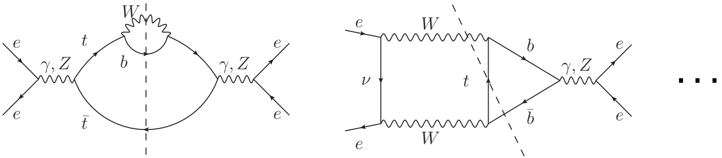
- Leading QED effect is QED Coulomb potential (NLO)
  - Included up to NNNLO
  - But only complete at NLO, further resonant electroweak effects arise at NNLO [Grzadkowski, Kühn, Krawczyk, Stuart; Guth, Kühn; Hoang, Reißer], but are not included yet

# Nonresonant contribution

- Due to top instability the physical final state is $W^+W^-b\bar{b}$
  - Dominantly produced through resonant (i.e. on-shell) top pair
  - At higher orders: Production with just one or no resonant top
  - Both contributions are separately divergent, only the sum is physical
  - Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi]
  - Known at NLO [Beneke, Jantzen, Ruiz-Femenía]




  - Partial results at NNLO [Penin, Piclum; Jantzen, Ruiz-Femenía] not yet included

# P wave contribution

- P wave gives a small effect $\lesssim 1\%$
- Complete NNNLO QCD result (incl. NLO P wave) will be used as reference prediction for the study of subleading effects in the following

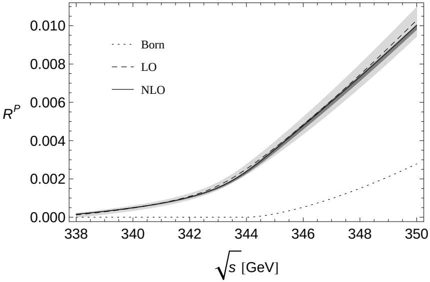
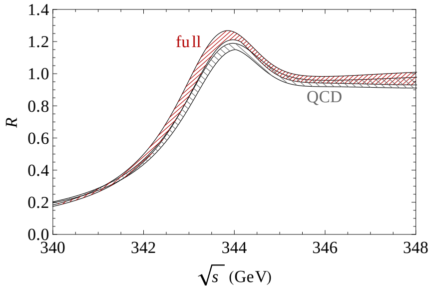
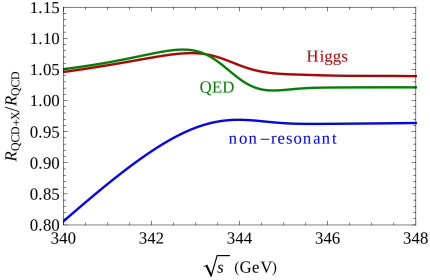


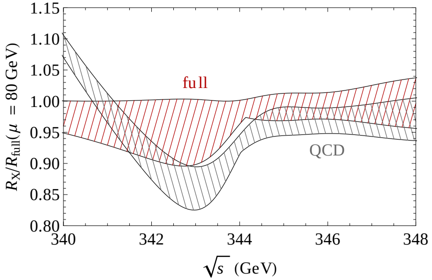

Figure from [Beneke, Piclum, TR: 1312.4792]

# Subleading effects

- Relative size of Higgs, QED and nonresonant contributions (down)
- Impact on the cross section (right)
- Effects significantly larger than QCD uncertainty, particularly in the important region at and below threshold

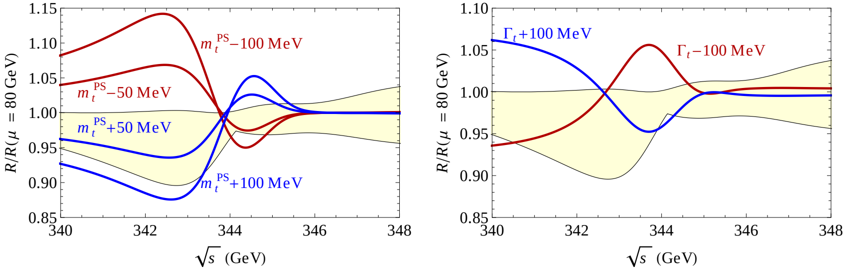


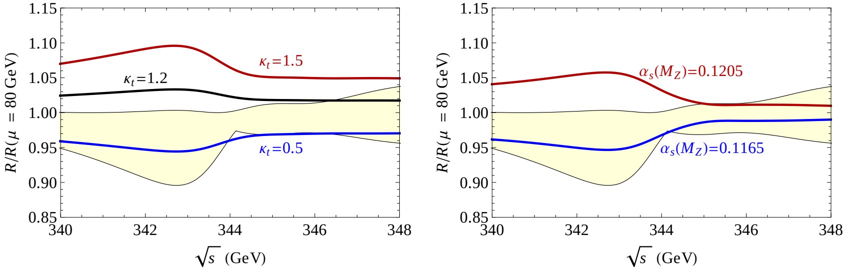
# Parameter dependence

- The region at and below the peak is very sensitive to variations of $m_t$ and $\Gamma_t$
  - Increase (decrease) of $m_t$ shifts the peak to the right (left)
  - Increase (decrease) of $\Gamma_t$ makes the peak less (more) pronounced
  - Allows ultra-precise measurements in theoretically well-defined mass schemes (unlike reconstructions of the top mass at LHC)



# Yukawa coupling dependence

- Assume that some new physics modify the SM Yukawa coupling, parametrization through $y_t = \kappa_t \frac{\sqrt{2} m_t}{v}$
  - Changes normalization of cross section
  - Variation of $\alpha_s$ has a similar effect
  - Degeneracy possibly restricts measurement of $y_t$, but $\alpha_s$ should be known sufficiently well by the time a measurement is possible



# Experimental studies

- Reliable estimate on achievable uncertainties requires experimental study
- Several analyses have been performed

| | MM | SSTP (stat.) | HISFSKY (stat.) |
|---|---|---|---|
| $\delta m_t$ [MeV] | 20 | 27 | 16 |
| $\delta\Gamma_t$ [MeV] | 30 | - | 21 |
| $\delta\alpha_s$ | 0.0012 | 0.0008 | - |
| $\delta y_t$ [%] | 35 | - | 4.2 |

[Martinez, Miquel; Seidel, Simon, Tesar, Poss;
Horiguchi, Ishikawa, Suehara, Fujii, Sumino, Kiyo, Yamamoto]

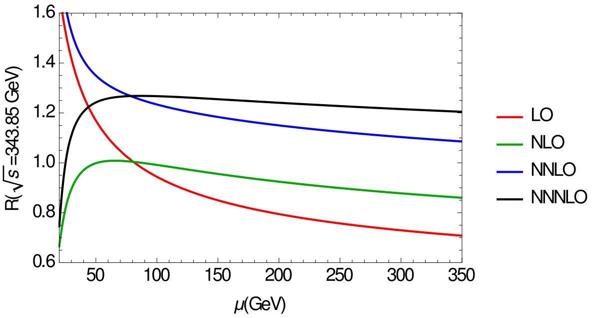
$\longrightarrow$ See talk by Roman Poeschl tomorrow at 12:10 !

- However: Analysis using full available theory prediction not available yet

# Summary and Outlook

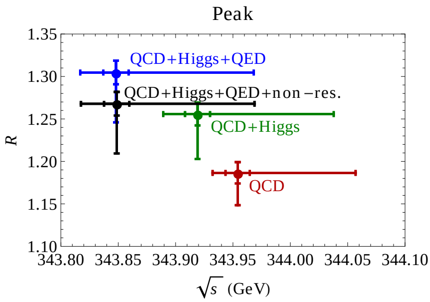
- Strong dynamics in $e^+e^- \to t\bar{t}X$ near threshold are under control at the level of $\sim \pm 3\%$
- Non-QCD effects are important and must be included, the first steps are completed
- Threshold scan at a future linear collider will give an ultra-precise measurement of $m_t$ and $\Gamma_t$ and be sensitive to $\alpha_s$ and $y_t$
- Experimental studies will give a clear picture of what to expect, from the theory point of view things look very promising (with maybe a grain of salt for $y_t$)
- Still many things to do, complete knowledge of non-QCD effects at NNLO desirable
  - Complete nonresonant and electroweak effects at NNLO
  - Initial state radiation
  - NNNLO+NNLL

# Backup: Scale variation

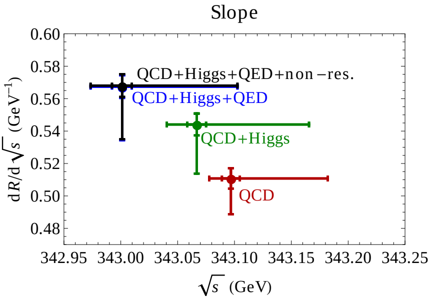



- No sign of convergence below $\sim$ 50 GeV

# Backup: Peak and maximal slope




Peak height and position



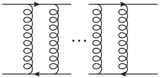

Maximum of slope

# Backup: Effective field theory setup

- Use EFTs that subsequently integrate out the hard and soft scale

| | |
|---|---|
| QCD | Full theory |
| ↓ | Integrated out hard modes $k \sim m_t$<br>Hard subgraphs become point-like vertices |
| NRQCD | Contains nonrelativistic modes |
| ↓ | Integrated out soft modes $k \sim m_t v$<br>Soft subgraphs become non-local vertices |
| PNRQCD | Contains potential heavy quarks and ultrasoft gluons |

- PNRQCD [Pineda, Soto] is a spatially non-local theory, where the LO Coulomb potential is part of the LO Lagrangian. The heavy-quark pair propagator in PNRQCD is given by the sum of ladder diagrams involving arbitrary numbers of potential gluon exchanges. Higher corrections follow from Rayleigh-Schrödinger perturbation theory.



- For a detailed account of the EFT setup see [Beneke, Kiyo, Schuller: 1312.4791]