

V_{cb} AND V_{ub}

WHERE DO WE STAND?

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IMPORTANCE OF $|V_{xb}|$

V_{cb} plays an important role in the determination of UT

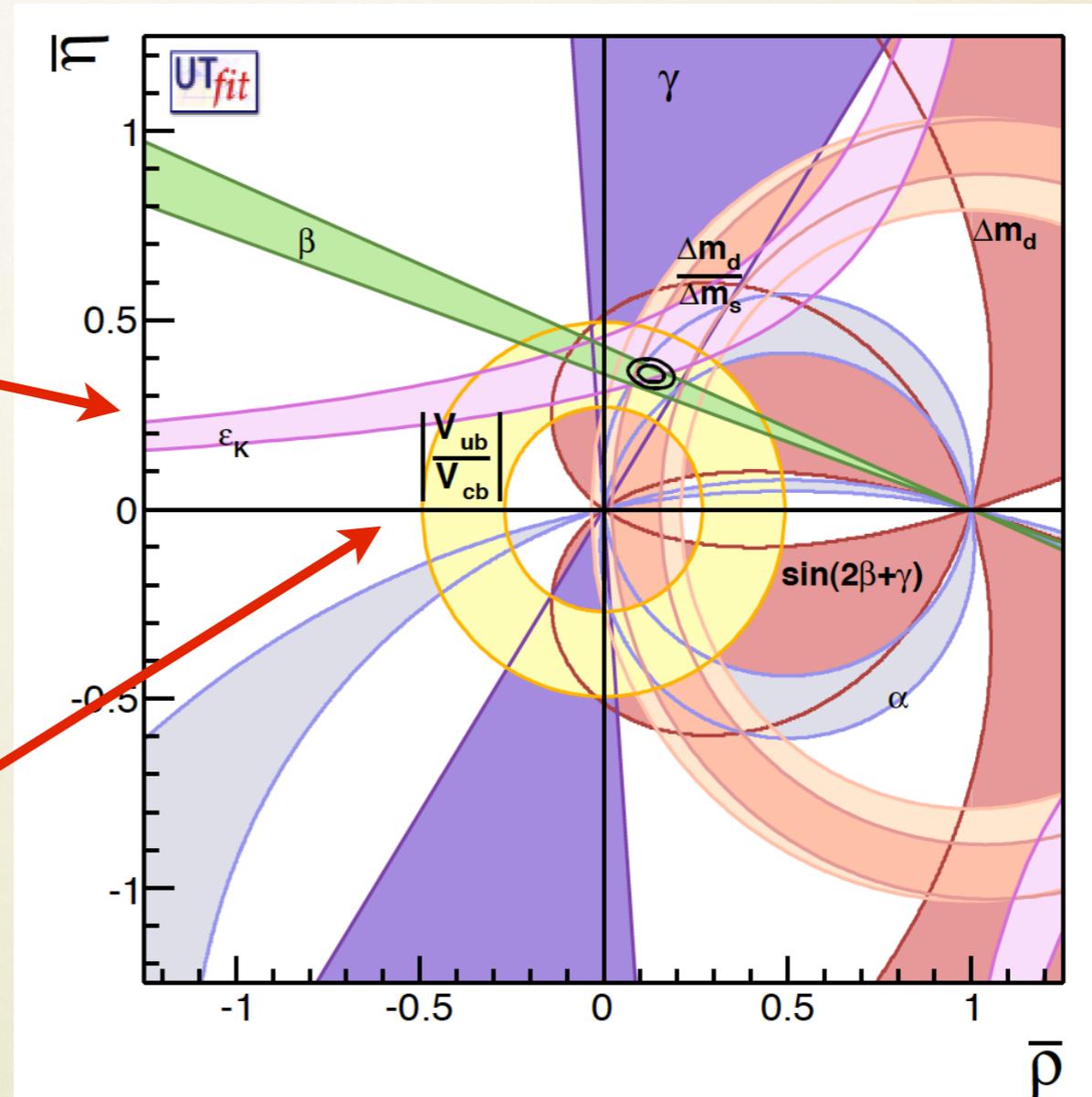
$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 \left[1 + O(\lambda^2) \right]$$

where it often dominates the theoretical uncertainty.

V_{ub}/V_{cb} constrains directly the UT



Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$

INCLUSIVE SEMILEPTONIC B DECAYS

OPE allows us to write inclusive observables as double series in $1/m_b$ and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s(\mu)}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s(\mu)}{\pi} M_i^{(\pi,1)} \right) \frac{\mu_\pi^2}{m_b^2} + \left(M_i^{(G,0)} + \frac{\alpha_s(\mu)}{\pi} M_i^{(G,1)} \right) \frac{\mu_G^2}{m_b^2} + M_i^{(D)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS)} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

OPE valid for inclusive enough measurements, away from perturbative singularities \Rightarrow semileptonic width, moments

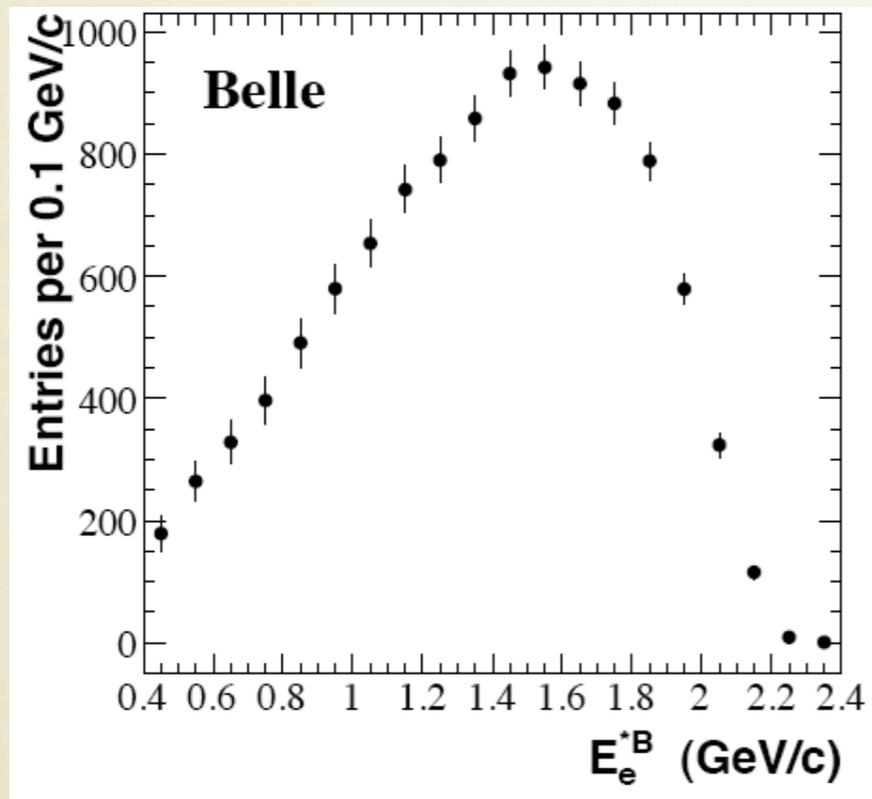
Current fits includes 6 non-pert parameters

$$m_{b,c} \quad \mu_{\pi,G}^2 \quad \rho_{D,LS}^3$$

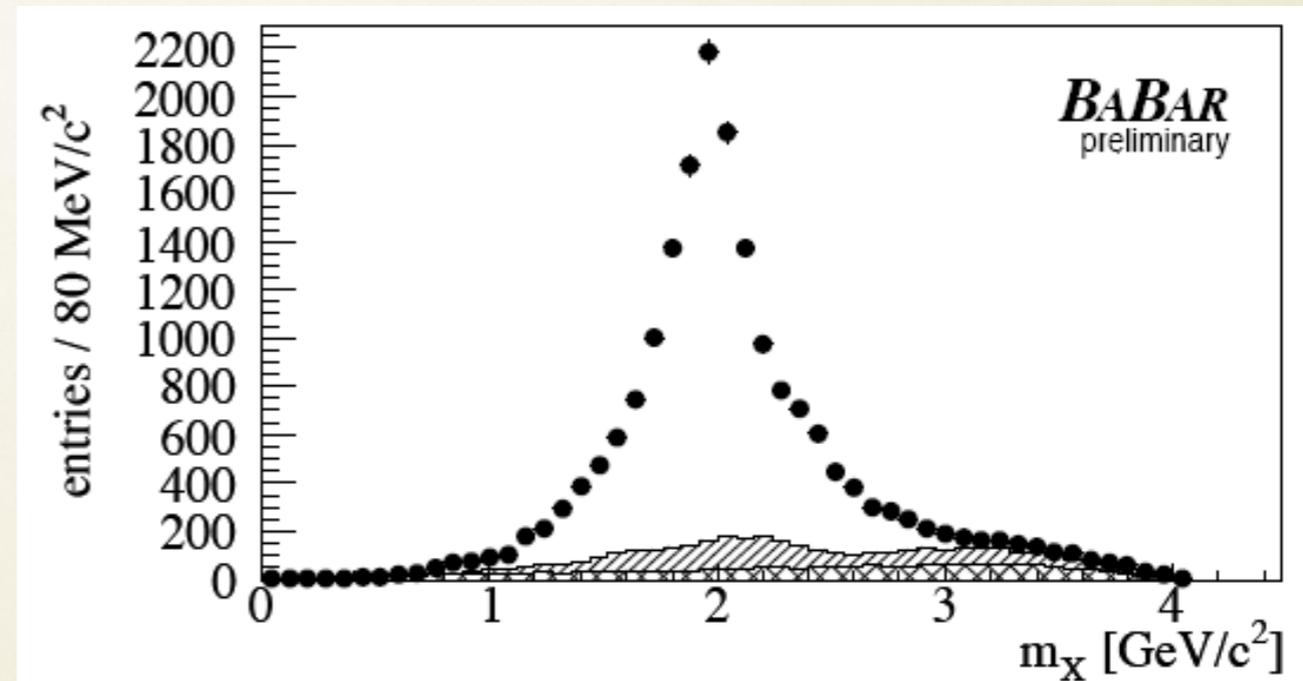
and all known corrections up to $O(\Lambda^3/m_b^3)$

EXTRACTION OF THE OPE PARAMETERS

E_1 spectrum



hadronic mass spectrum



Global **shape** parameters (first moments of the distributions) tell us about m_b , m_c and the B structure, total **rate** about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks \rightarrow useful in many applications (rare decays, V_{ub} ,...)

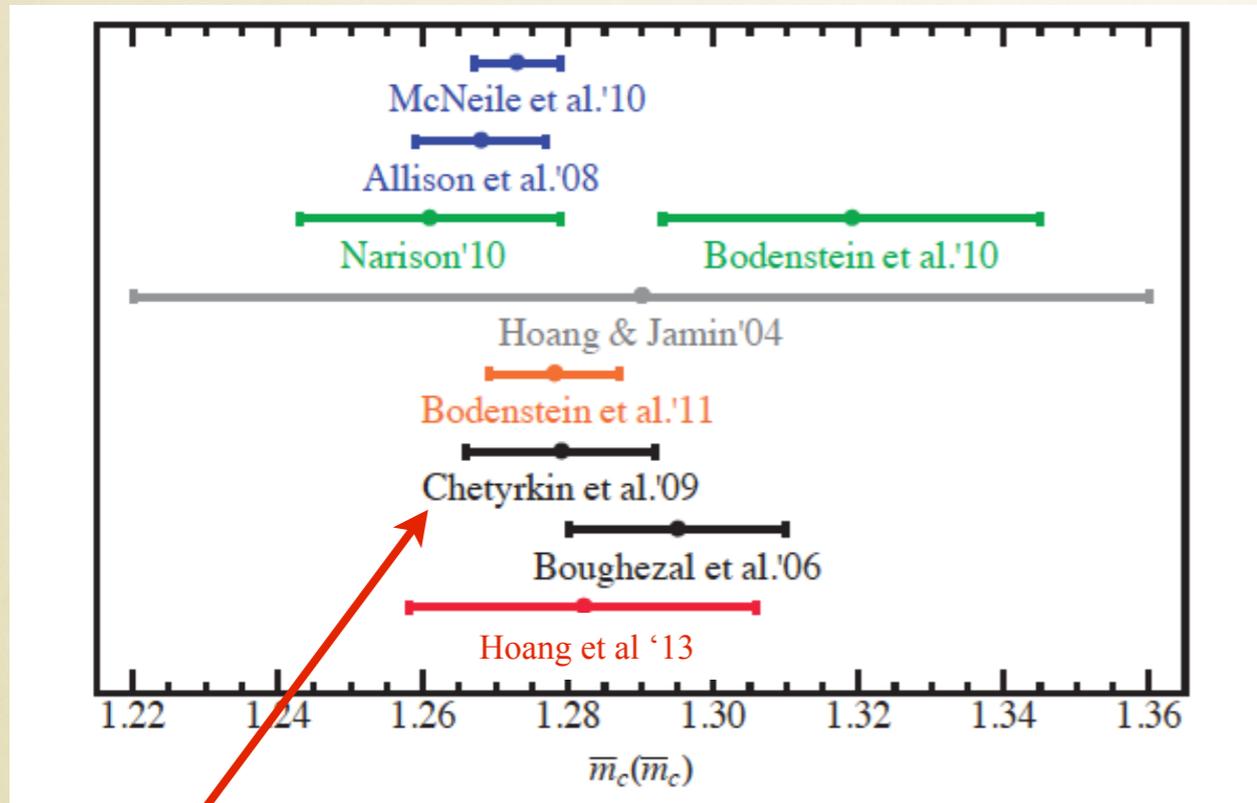
LATEST SEMILEPTONIC FIT

Alberti, Healey, Nandi, PG 1411.6560

- **updates** the fit in Schwanda, PG, 1307.4551
- **kinetic scheme** calculation based on 1107.3100; hep-ph/0401063
- includes all $O(a_s^2)$ and $O(a_s/m_b^2)$ corrections
- reassessment of theoretical errors, realistic correlations
- **external constraints:** precise heavy quark mass determinations, plus mild constraints on μ^2_G from hyperfine splitting and Q^3_{LS} from sum rules

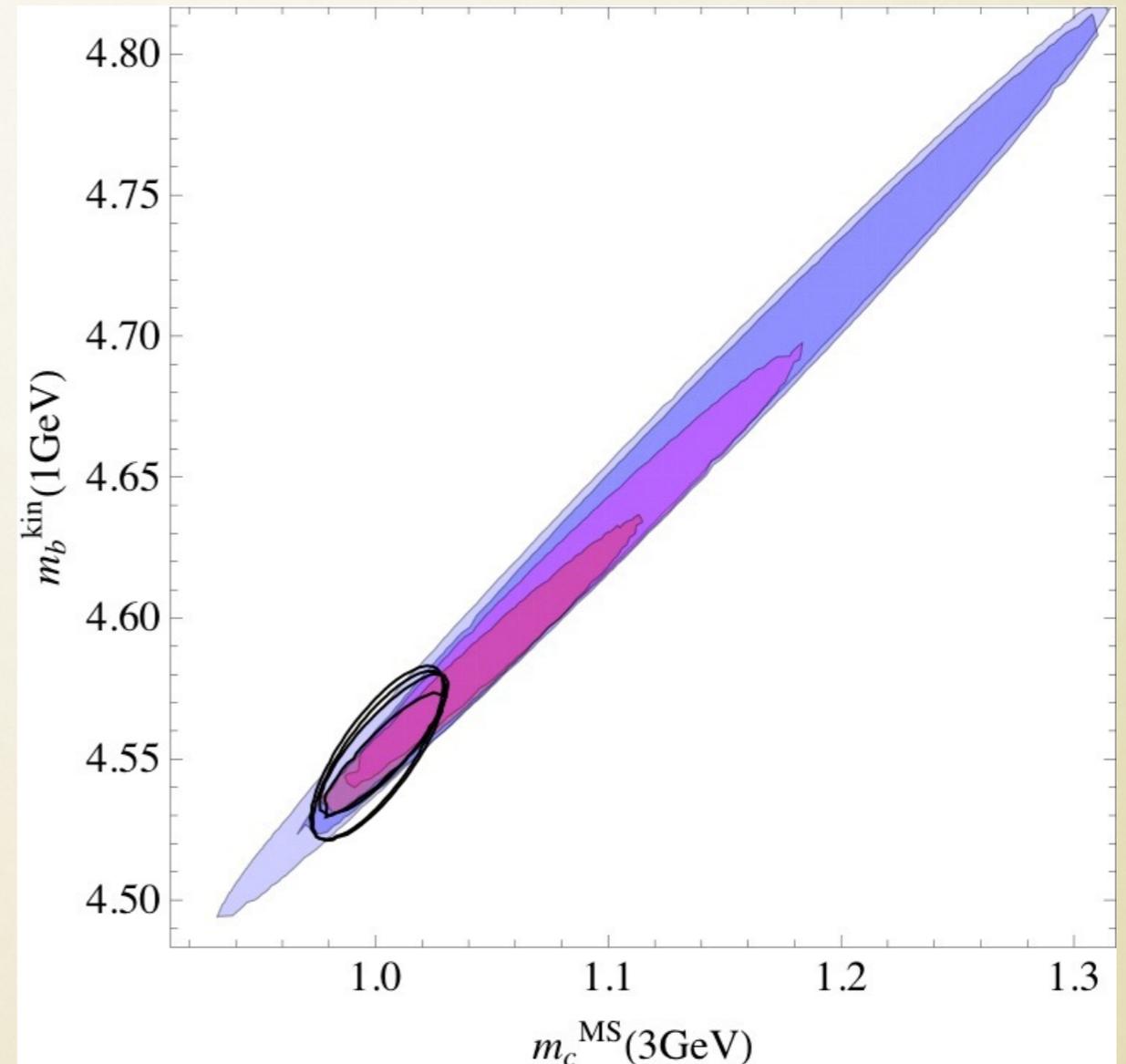
Previous fits: Buchmuller, Flaecher hep-ph/0507253,
Bauer et al, hep-ph/0408002 (1S scheme)

CHARM MASS DETERMINATIONS



sum rules studies of $\sigma(e^+e^- \rightarrow \text{hadrons})$
almost all at NNNLO

our default
choice



Remarkable improvement in recent years.

m_c can be used as precise input to fix m_b instead of radiative moments

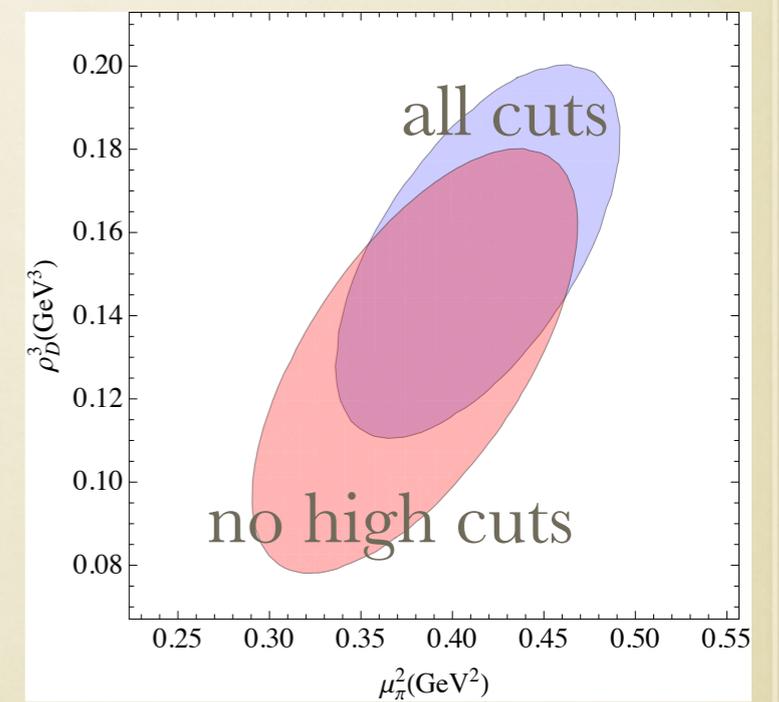
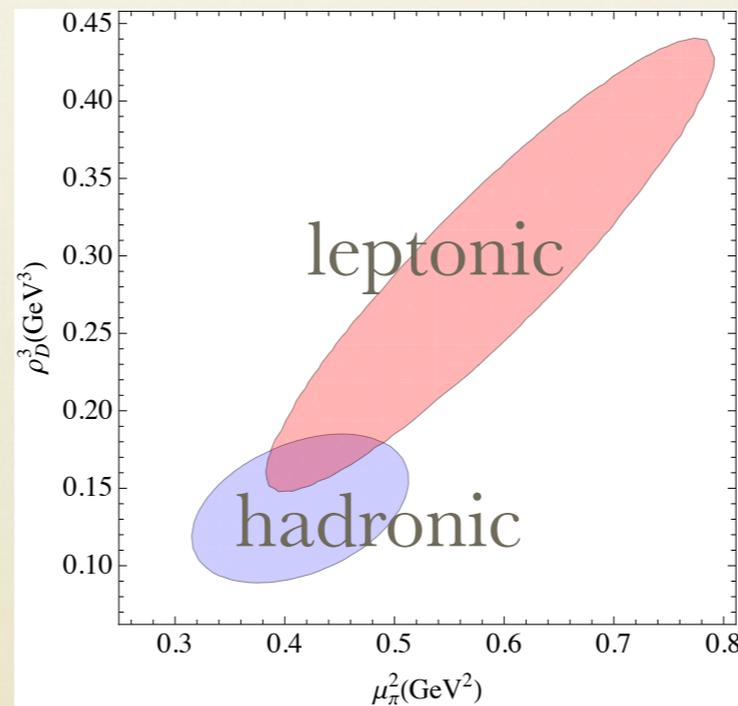
FIT RESULTS

m_b^{kin}	$\overline{m}_c(3\text{ GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$BR_{cl\nu}$	$10^3 V_{cb} $
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

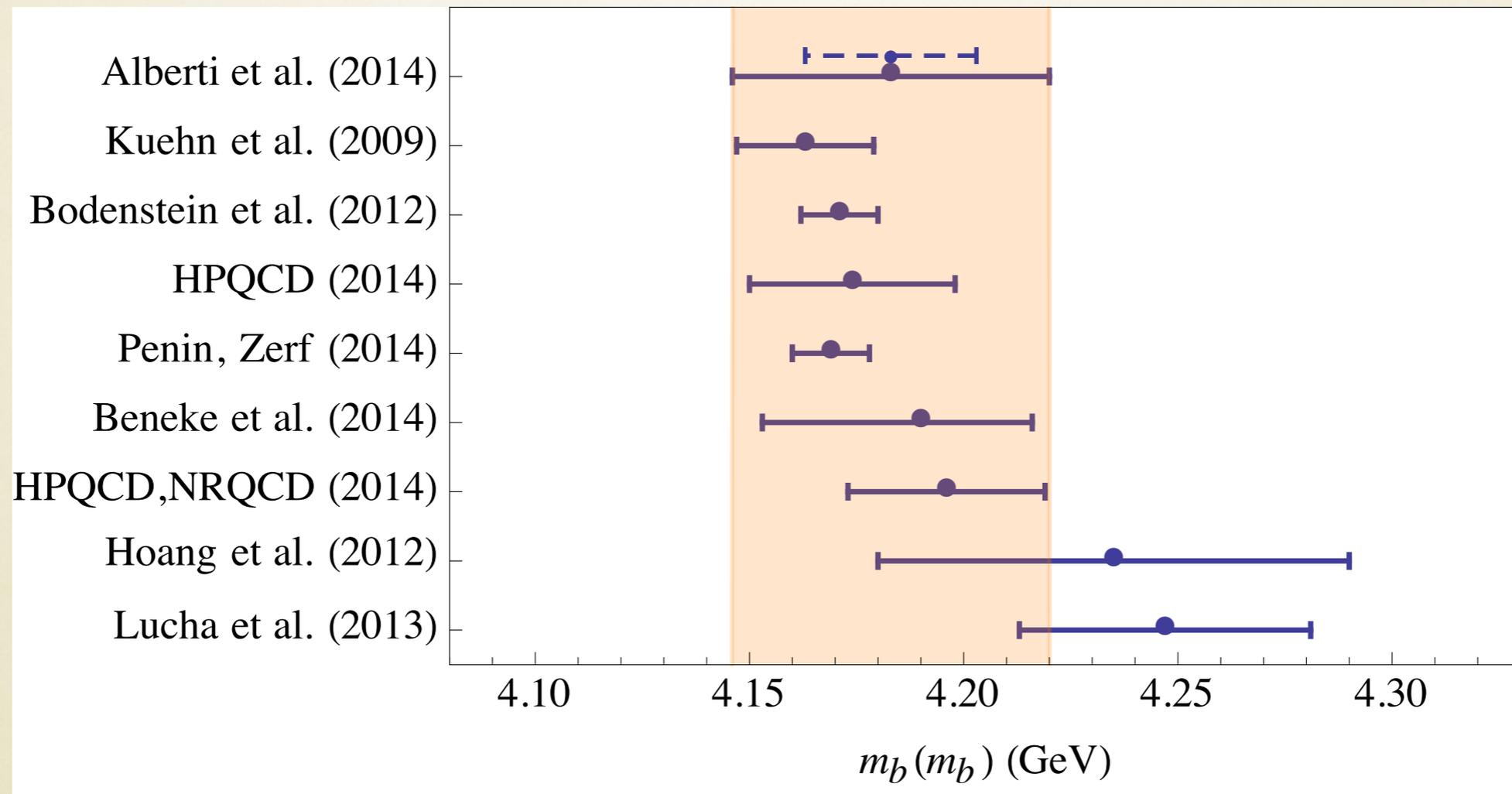
WITHOUT MASS CONSTRAINTS

$$m_b^{kin}(1\text{ GeV}) - 0.85 \overline{m}_c(3\text{ GeV}) = 3.714 \pm 0.018 \text{ GeV}$$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters



RESULTS: BOTTOM MASS



The fit gives $m_b^{kin}(1\text{GeV})=4.553(20)\text{GeV}$

scheme translation error $m_b^{kin}(1\text{GeV})=m_b(m_b)+0.37(3)\text{GeV}$

$\bar{m}_b(\bar{m}_b)=4.183(37)\text{GeV}$

HIGHER ORDER EFFECTS

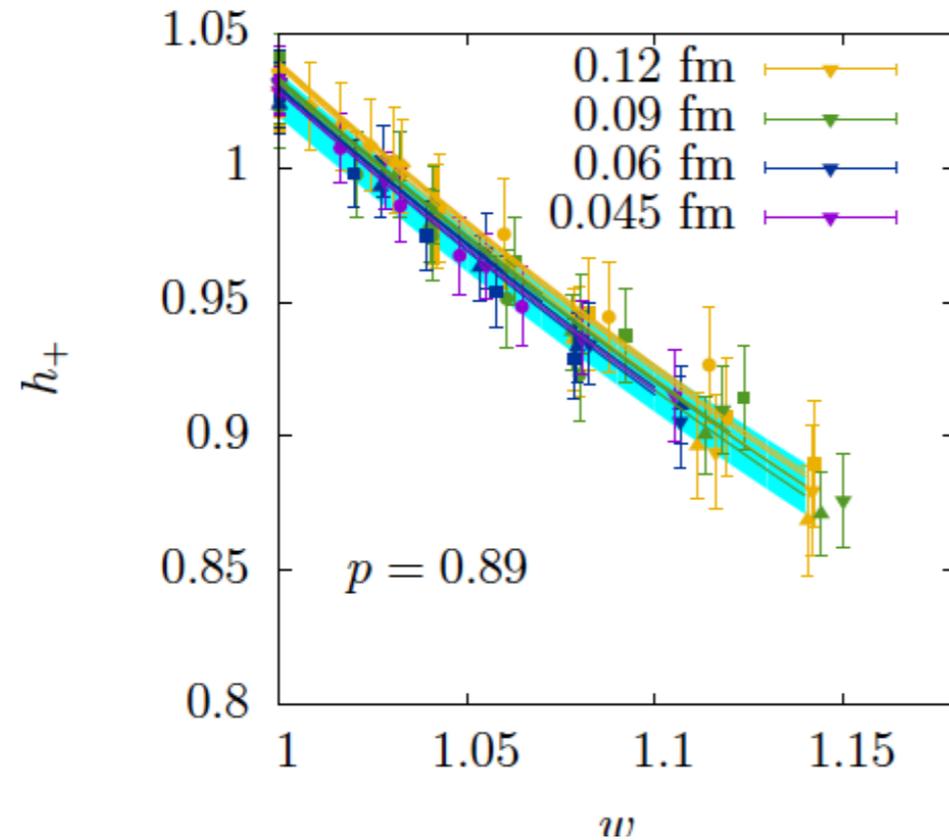
- Reliability of the method depends on our ability to control higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.
- **Purely perturbative corrections** complete at NNLO, small residual error (kin scheme)_{Melnikov,Biswas,Czarnecki,Pak,PG}
- **Higher power corrections** $O(1/m_Q^{4,5})$ known
Mannel,Turczyk,Uraltsev 2010 not included in fit
- **Mixed corrections** perturbative corrections to power suppressed coefficients completed at $O(\alpha_s/m_b^2)$
Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG, Mannel,Pivovarov, Rosenthal

PROSPECTS FOR THEORY

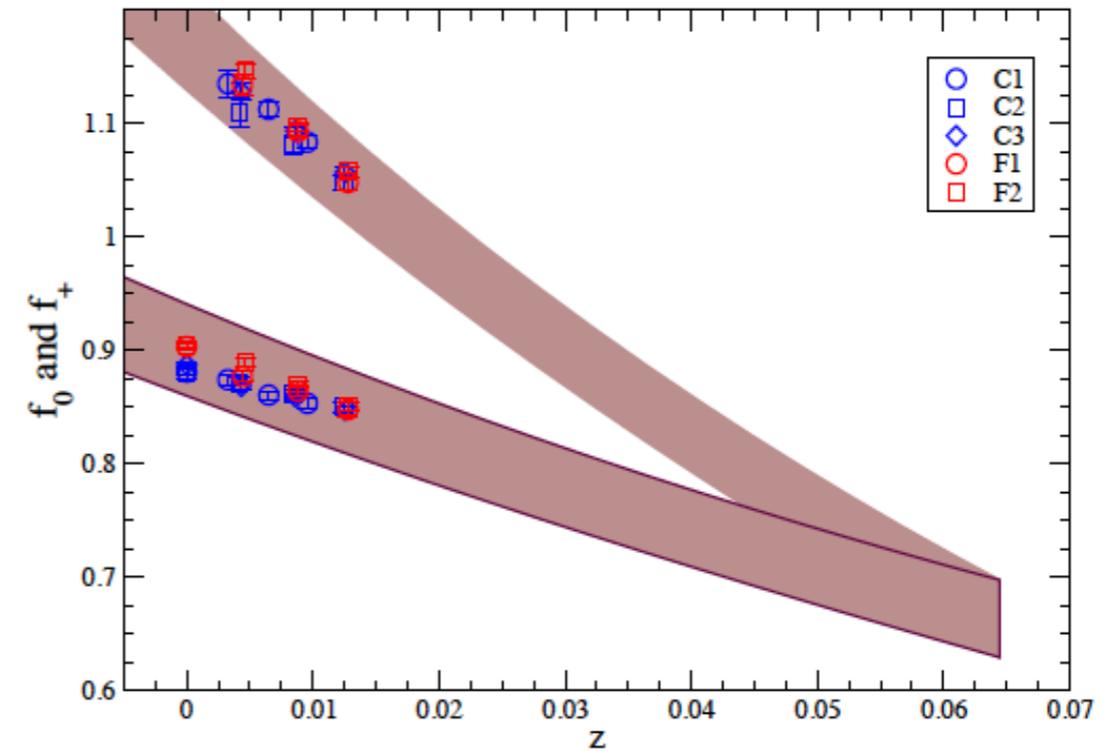
- Theoretical uncertainties dominate already
- $O(a_s/m_b^3)$ calculation under way
- $O(1/m_Q^{4,5})$ effects need further investigation: estimates based on vacuum saturation approx suggest small impact Turczyk, PG, preliminary
- NNNLO corrections to total width feasible
- Lattice QCD information on local matrix elements is the next frontier
- Electroweak corrections

NEW RESULTS FOR $B \rightarrow D l \nu$ F.F.

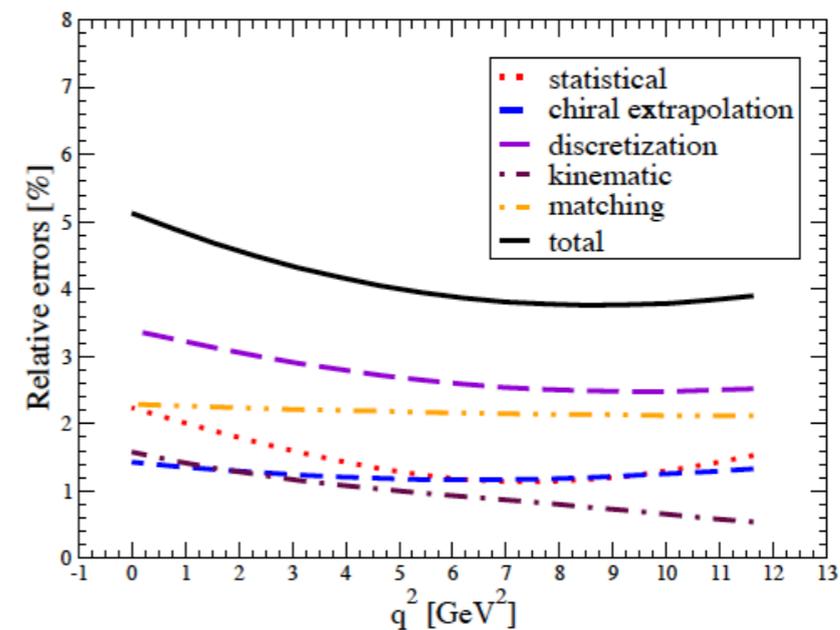
FNAL/MILC 1503.07237



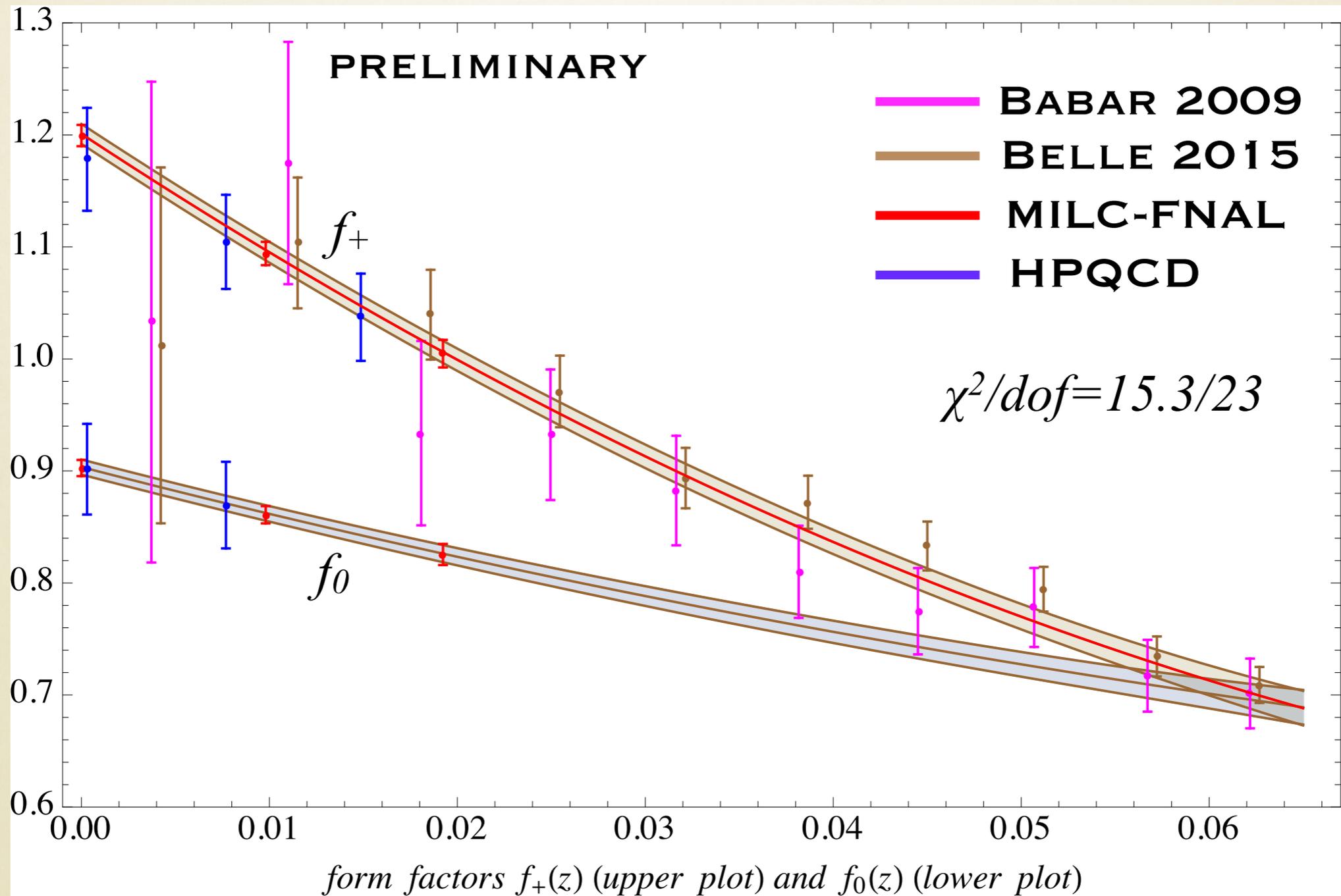
HPQCD 1505.03925



Source	f_+ (%)
Statistics+matching+ χ PT cont. extrap.	1.2
(Statistics)	(0.7)
(Matching)	(0.7)
(χ PT/cont. extrap.)	(0.6)
Heavy-quark discretization	0.4
Lattice scale r_1	0.2
Total error	1.2



Global fit to $B \rightarrow D\ell\nu$



A Global fit to $B \rightarrow D l \nu$

- $|V_{cb}| = 41.08(0.95) 10^{-3}$ preliminary
- based on z -expansion with unitarity constraints (BGL)
- assumes no correlation between FNAL and HPQCD, 3% syst error on Babar data, correct treatment of last bin, no finite size bin effect
- little dependence on parameterization if lattice data at non-zero recoil are included (CLN gives $40.99(97) 10^{-3}$)
- Non-zero recoil data are crucial: only zero recoil leads to $|V_{cb}| = 39.80(1.1) 10^{-3}$
- Babar prefers lower V_{cb}
- $R(D) = 0.302(9)$ (preliminary) 1.8σ from HFAG average

EXCLUSIVE $B \rightarrow D^* \ell \nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + \dots \right]$$

Recent progress in measurement of slopes and shape parameters,
exp error only ~1.3%

The ff $F(1)$ cannot be experimentally determined. Lattice QCD is the best hope to compute it. Only one unquenched Lattice calculation:

$$F(1) = 0.906(13) \implies |V_{cb}| = 39.04(49)_{\text{exp}}(53)_{\text{lat}}(19)_{\text{QED}} 10^{-3}$$

Bailey et al 1403.0635 (FNAL/MILC)

1.9% error (adding in quadrature)

$\sim 2.9\sigma$ or $\sim 8\%$ from inclusive determination

COMMENTS ON D^*

- **Heavy quark sum rules** (with BPS arguments) favor smaller $F(1)=0.86(2)$ leading to agreement with inclusive. Difficult to improve, how good is BPS limit?
- **Extrapolations to zero recoil** by experiments use CLN parameterization, based on NLO HQET, and **do not include a 1%** uncertainty. CLN has only 2 parameters, fits well exp data but makes assumptions. Lattice simulations at non zero recoil under way.
- Matching at $1/m_Q^3$ for **lattice discretization** effects under study by FNAL/MILC. Other collaborations working on $B \rightarrow D^*$ ff.
- **Indirect $|V_{cb}|$ determinations** assuming SM+unitarity CKM:
UTFit $42.05(65) \cdot 10^{-3}$ CKMFitter $41.4^{+2.4}_{-1.4} \cdot 10^{-3}$

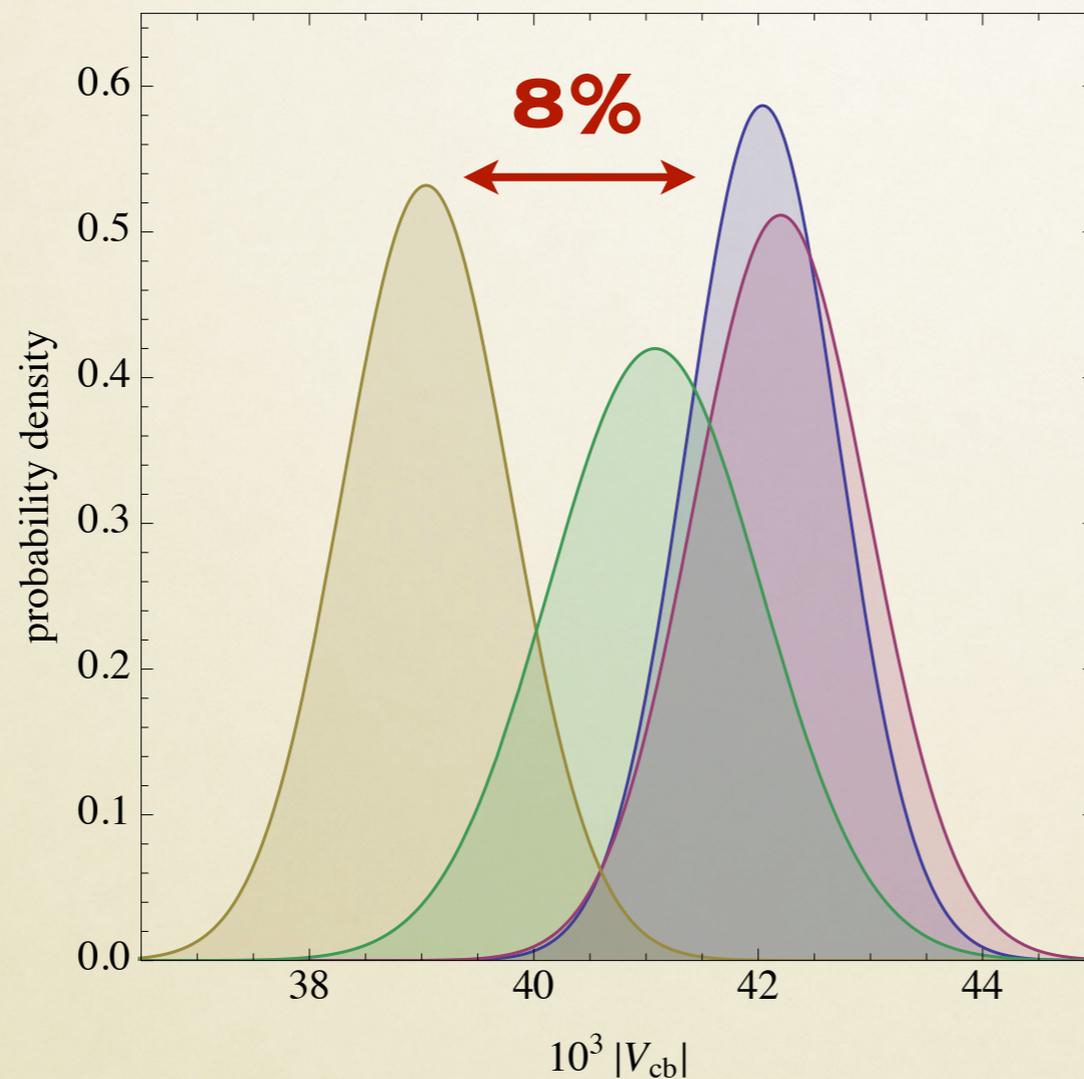
V_{cb} VISUAL SUMMARY

EXCLUSIVE $B \rightarrow D$

EXCLUSIVE $B \rightarrow D^*$

INCLUSIVE

UTFIT SM PREDICTION:
 $(42.04 \pm 0.68) 10^{-3}$



Latest lattice results
(FNAL/MILC+HPQCD)

form factors from
HQSR, HQE, LCSR
for exclusives also
available, less precise
Mannel, Uraltsev, PG, Khodjamirian et al

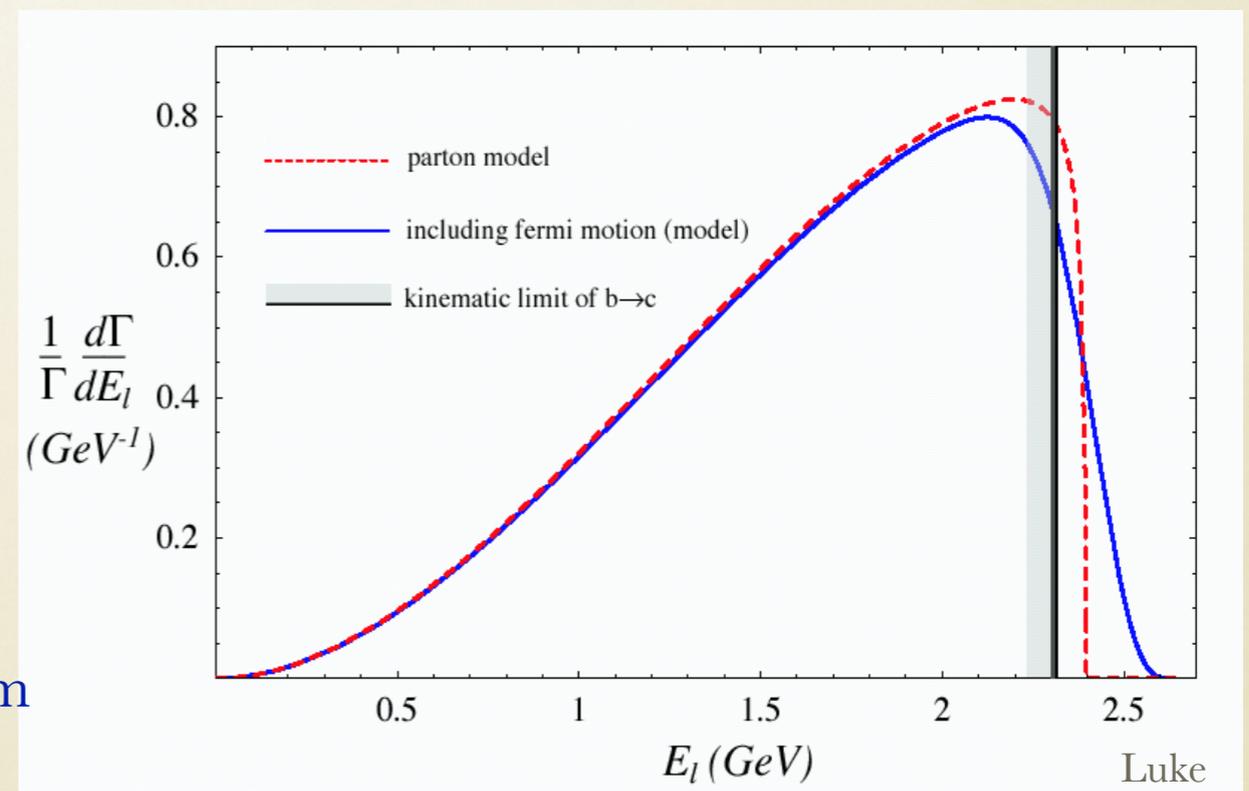
$B \rightarrow X_{ul} \nu$ AND CUTS

Experiments often use kinematic cuts to avoid the $\sim 100x$ larger $b \rightarrow c l \nu$ background:

$$m_X < M_D \quad E_l > (M_B^2 - M_D^2) / 2M_B \quad q^2 > (M_B - M_D)^2 \dots$$

The cuts destroy convergence of the OPE that works so well in $b \rightarrow c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** $f(k_+)$. Equivalently the SF is seen to emerge from soft gluon resummation



HOW TO ACCESS THE SF?

$$\frac{d^3\Gamma}{dp_+ dp_- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_+, p_-, k) F(k) + O\left(\frac{\Lambda}{m_b}\right)$$

Subleading SFs

Prediction *based on*
resummed pQCD

DGE, ADFR

OPE constraints +
parameterization
without/with resummation

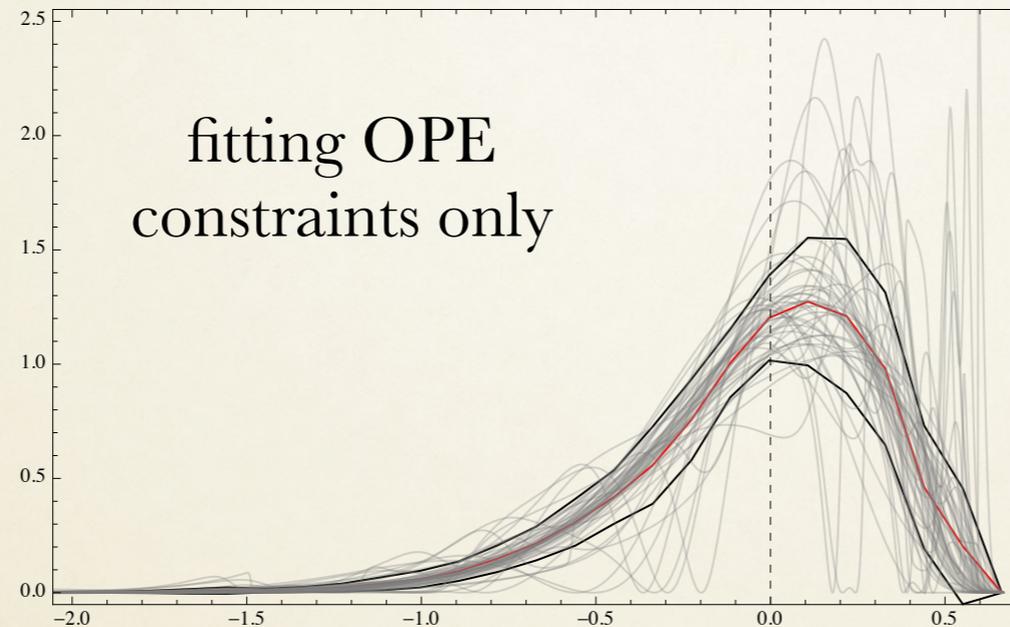
GGOU, BLNP

Fit semileptonic (and radiative) data

SIMBA, NN V_{ub}

The NNVub Project

K.Healey, C. Mondino, PG, in progress



- Use **Artificial Neural Networks** to fit shape functions to theoretical constraints and data without bias, extracting V_{ub} and HQE parameters in a model independent way (without assumptions on functional form).
- Belle-II will be able to measure some kinematic distributions, thus constraining directly the shape function. NNVub will provide a flexible tool to analyse data.

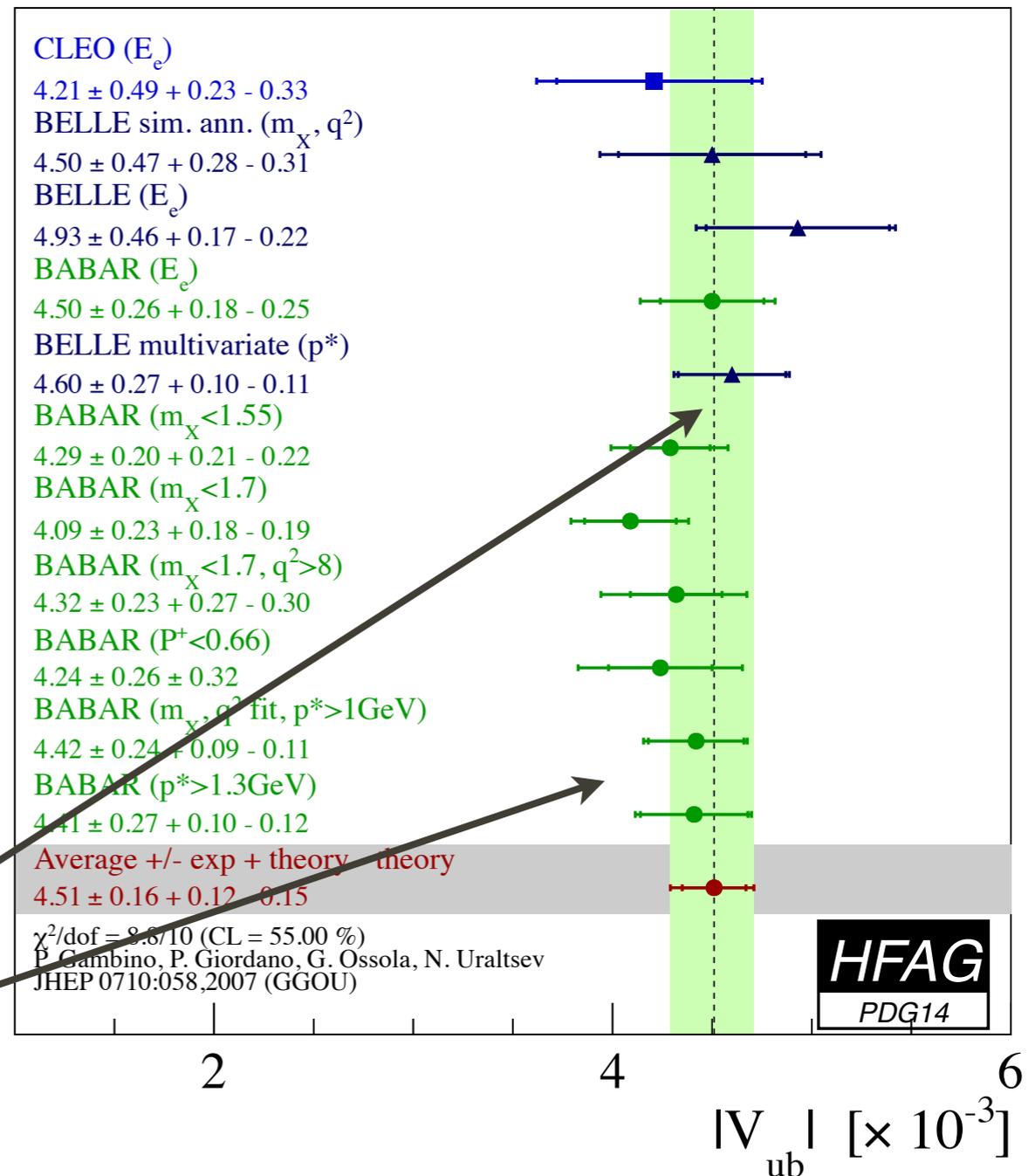
$|V_{ub}|$ DETERMINATIONS

Inclusive: 5% total error

HFAG 2014	Average $ V_{ub} $ 1000
DGE	4.52(16)(16)
BLNP	4.45(16)(22)
GGOU	4.51(16)(15)

UT fit (without direct V_{ub}):
 $V_{ub} = 3.62(12) \cdot 10^{-3}$

Recent experimental results are theoretically cleanest (2%) but based on background modelling. Signal simulation also relies on theoretical models...



LQCD calculations for $|V_{ub}|$: recent progress

➤ Disclaimer: the list is not meant to be inclusive. I am focusing on the publicized results.

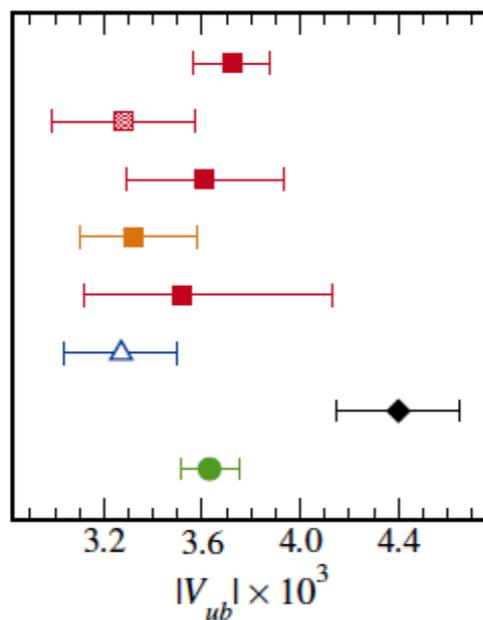
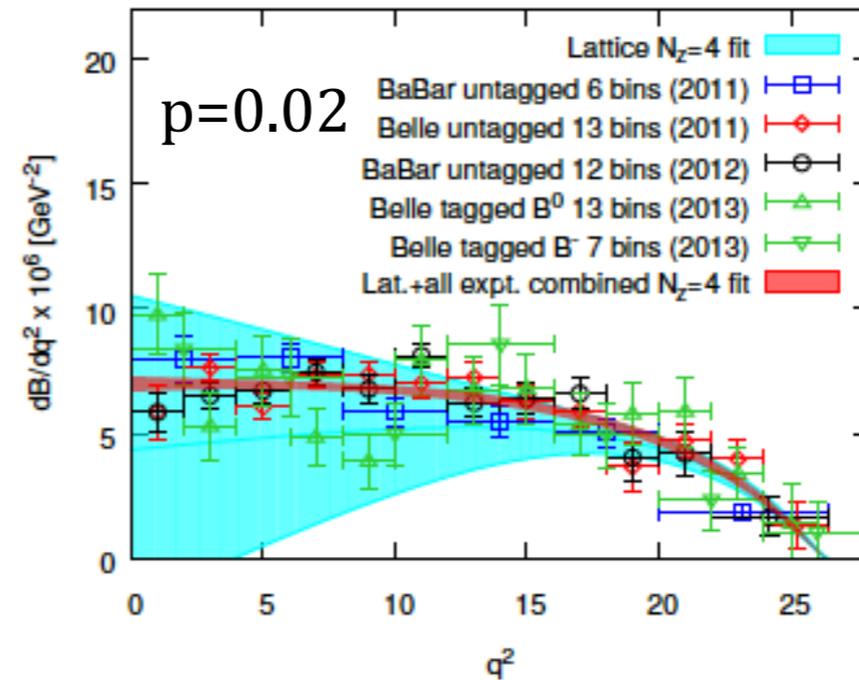
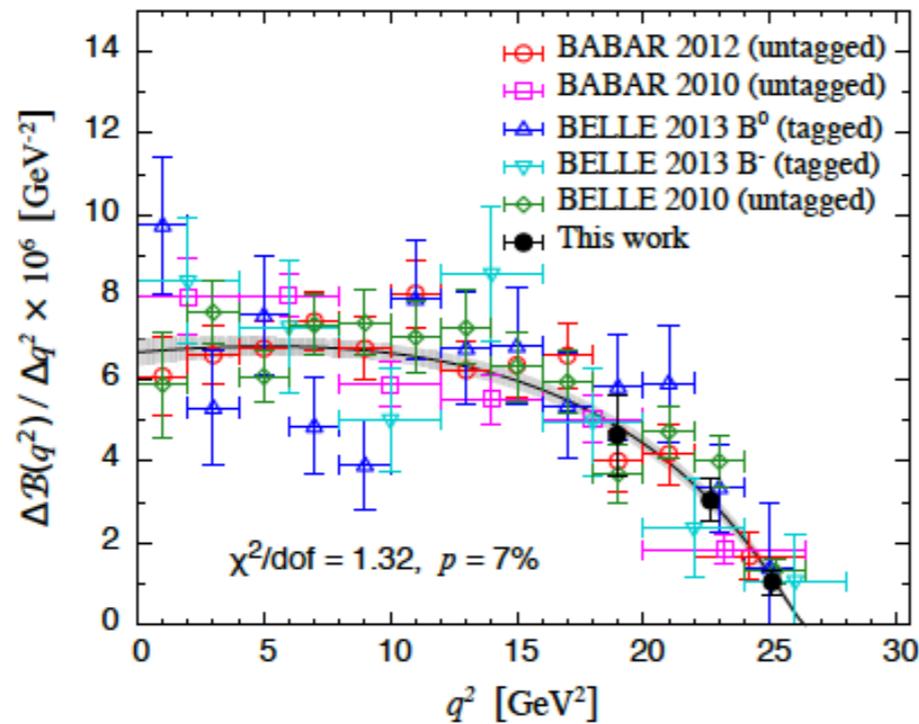
Lattice Group	Fermilab/MILC	HPQCD	RBC/UKQCD	Alpha	Detmold et al.
Process	$B \rightarrow \pi l \nu$ ($B_s \rightarrow K l \nu$)	$B_s \rightarrow K l \nu$ ($B \rightarrow \pi l \nu$)	$B \rightarrow \pi l \nu$ $B_s \rightarrow K l \nu$	($B_s \rightarrow K l \nu$)	$\Lambda_b \rightarrow p l \nu$
Gauge ensembles	MILC asqtad	MILC asqtad	Domain-Wall	CLS	Domain-Wall
Sea flavors	2+1	2+1	2+1	2	2+1
a (fm)	0.045–0.12	0.09–12	0.086–0.11	0.049–0.076	0.086–0.11
M_π	≥ 177 MeV	≥ 354 MeV	≥ 289 MeV	≥ 310 MeV	≥ 295 MeV
l -quark action	asqtad	HISQ	Domain-Wall	Imprv. Wilson	Domain-Wall
b -quark action	Fermilab Clover	NRQCD	RHQ	Lat. HQET	RHQ
χ PT	NNLO, SU(2), hard- π	HP χ PT+	NLO, SU(2), hard- π		
q^2 -extrapolation	functional BCL	modified z	synthetic BCL		modified- z
Ref.	arXiv:1503.07839 arXiv:1312.3197	arXiv:1406.2279	arXiv:1501.05373v2	arXiv:1411.3916	arXiv:1306.0446 arXiv:1503.01421v2 arXiv:1504.01568

• (): work in progress

NEW LATTICE RESULTS

RBC/UKQCD 1501.05373

FNAL/MILC 1503.07839



This work + BaBar + Belle, $B \rightarrow \pi l \nu$

Fermilab/MILC 2008 + HFAG 2014, $B \rightarrow \pi l \nu$

RBC/UKQCD 2015 + BaBar + Belle, $B \rightarrow \pi l \nu$

Imsong *et al.* 2014 + BaBar12 + Belle13, $B \rightarrow \pi l \nu$

HPQCD 2006 + HFAG 2014, $B \rightarrow \pi l \nu$

Detmold *et al.* 2015 + LHCb 2015, $\Lambda_b \rightarrow p l \nu$

BLNP 2004 + HFAG 2014, $B \rightarrow X_u l \nu$

UTFit 2014, CKM unitarity

FNAL $3.72(16) \cdot 10^{-3}$
only 4.3% error

2.2 σ from inclusive

RBC/UKQCD $3.61(32) \cdot 10^{-3}$

1.9 σ from inclusive

LCSR $3.32(26) \cdot 10^{-3}$

2.9 σ from inclusive

LHCb depends
on V_{cb} employed

NEW LATTICE RESULTS

FROM 1503.07839

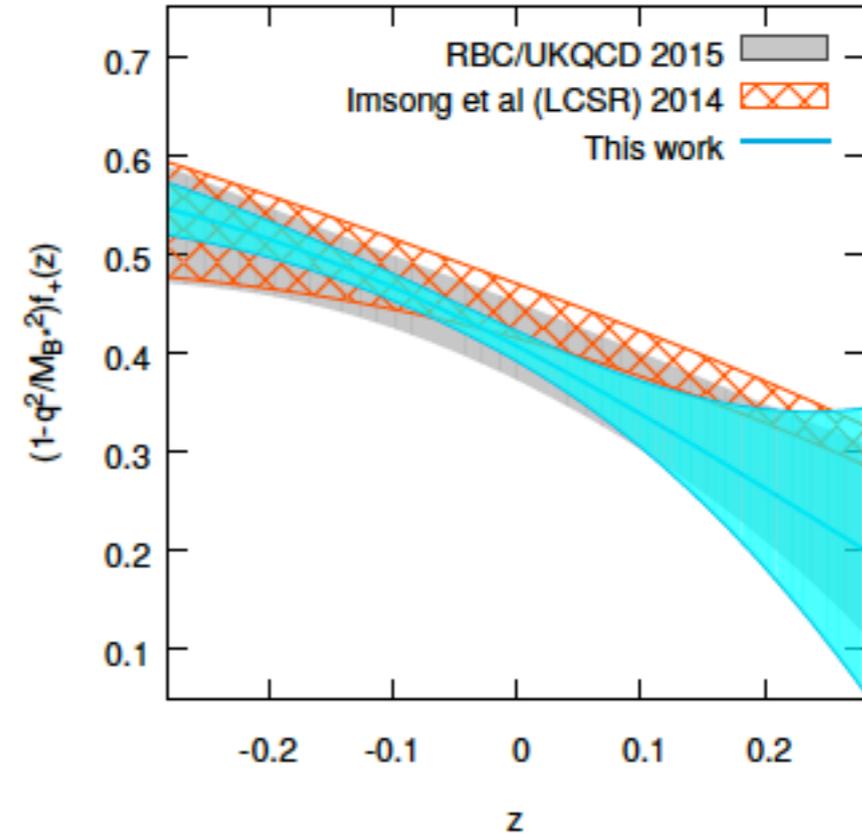
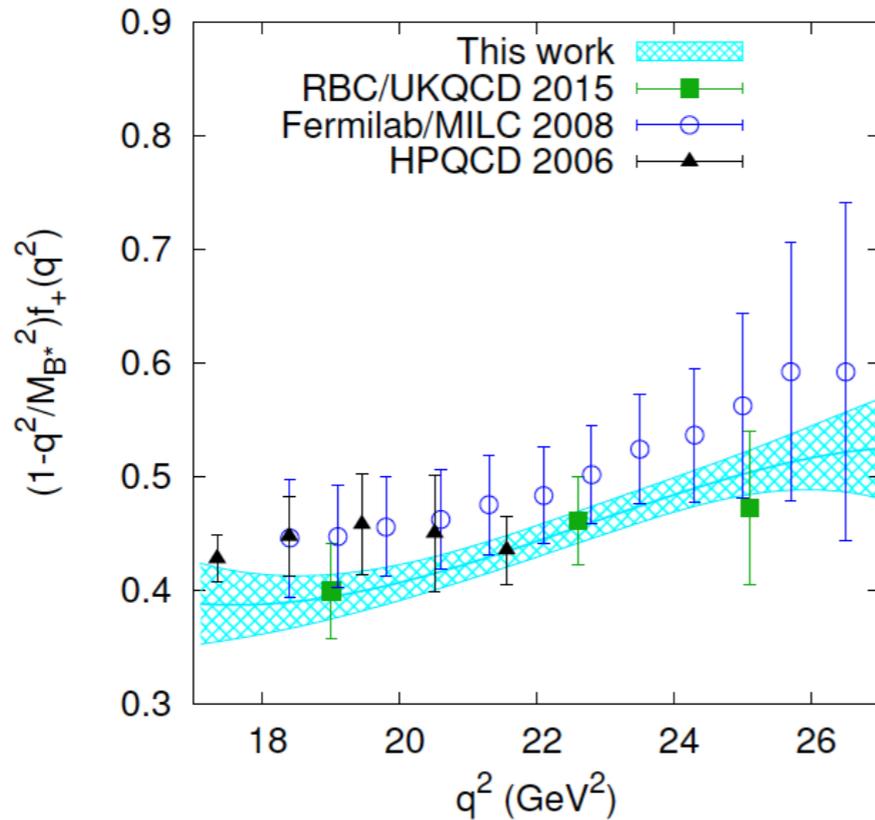


Table XVI. Results of the combined lattice+experiment fits with $N_z = 4$;

Fit	χ^2/dof	dof	p value	b_0^+	b_1^+	b_2^+	b_3^+	$ V_{ub} (\times 10^3)$
Lattice+exp.(all)	1.4	54	0.02	0.419(13)	-0.495(55)	-0.43(14)	0.22(31)	3.72(16)
Lattice+BaBar11 [7]	1.1	9	0.38	0.414(14)	-0.488(73)	-0.24(22)	1.33(44)	3.36(21)
Lattice+BaBar12 [8]	1.1	15	0.34	0.415(14)	-0.551(72)	-0.45(18)	0.27(41)	3.97(22)
Lattice+Belle11 [9]	0.9	16	0.55	0.412(13)	-0.574(65)	-0.40(16)	0.38(36)	4.03(21)
Lattice+Belle13 [10]	1.0	23	0.42	0.406(14)	-0.623(73)	-0.13(22)	0.92(45)	3.81(25)

NEW PHYSICS?

The difference in V_{cb} incl vs excl D^* with FNAL/MILC form factor is **quite large**: 3σ or about 8%. The perturbative corrections to inclusive V_{cb} total 5%, the power corrections about 4%.

Right Handed currents now excluded since

$$|V_{cb}|_{incl} \simeq |V_{cb}| \left(1 + \frac{1}{2} |\delta|^2 \right)$$

$$|V_{cb}|_{B \rightarrow D^*} \simeq |V_{cb}| \left(1 - \delta \right)$$

$$|V_{cb}|_{B \rightarrow D} \simeq |V_{cb}| \left(1 + \delta \right)$$

Chen, Nam, Crivellin, Buras, Gemmler, Isidori, ...

$$\delta = \epsilon_R \frac{\tilde{V}_{cb}}{V_{cb}} \approx 0.08$$

Most general SU(2) invariant dim 6 NP (without RH neutrino) can explain results, but it is incompatible with $Z \rightarrow b\bar{b}$ data

Explaining V_{ub} tension is easier

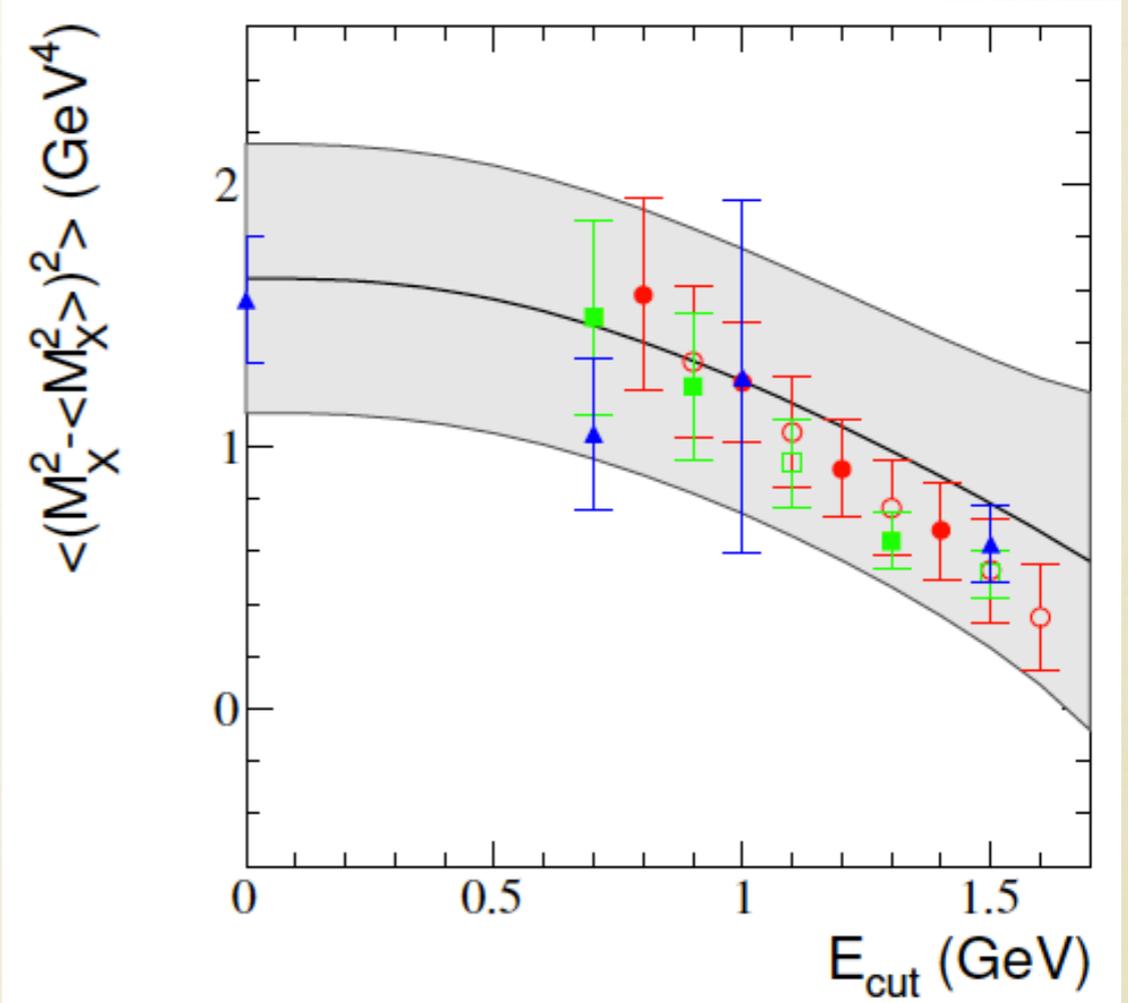
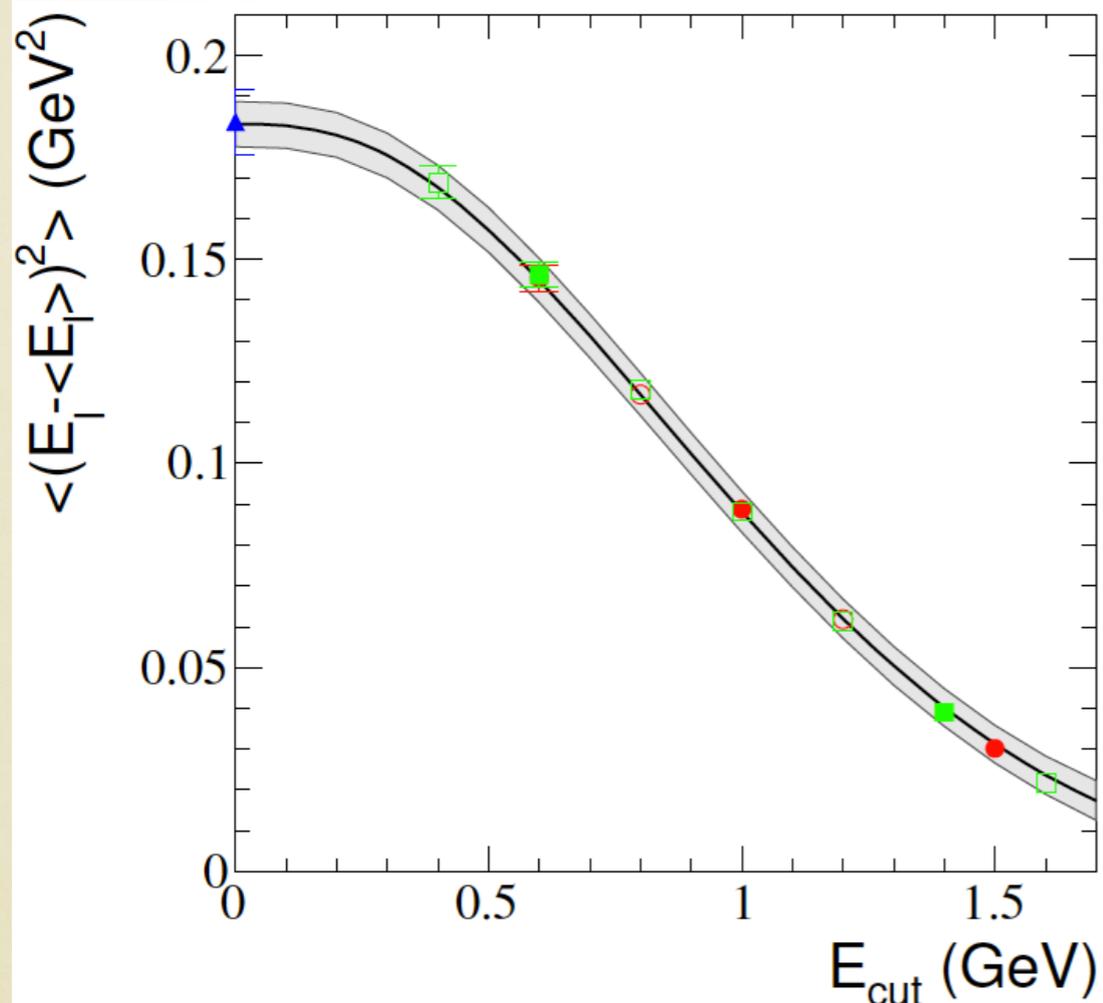
Crivellin, Pokorski 1407.1320

SUMMARY

- **New twists in the V_{ub}, V_{cb} saga!** Too early to draw conclusions though, we need more precise data and calculations.
- Improvements of OPE approach to semileptonic decays continue. All effects $O(\alpha_s \Lambda^2/m_b^2)$ implemented. **No sign of inconsistency in this approach so far, competitive m_b determination.**
- Exclusive/incl. tension in V_{cb} remains (3σ , 8%) only in the D^* channel. **The D channel is becoming competitive and agrees with inclusive.** The remaining tension calls for new lattice analyses and probably for model independent reanalyses of B factories data.
- Exclusive/incl tension in V_{ub} slightly receding because of new FNAL/MILC result. New physics explanations less constrained than for V_{cb} .
- Belle-II will improve precision and allow for consistency checks of our methods, especially for inclusive V_{ub} .

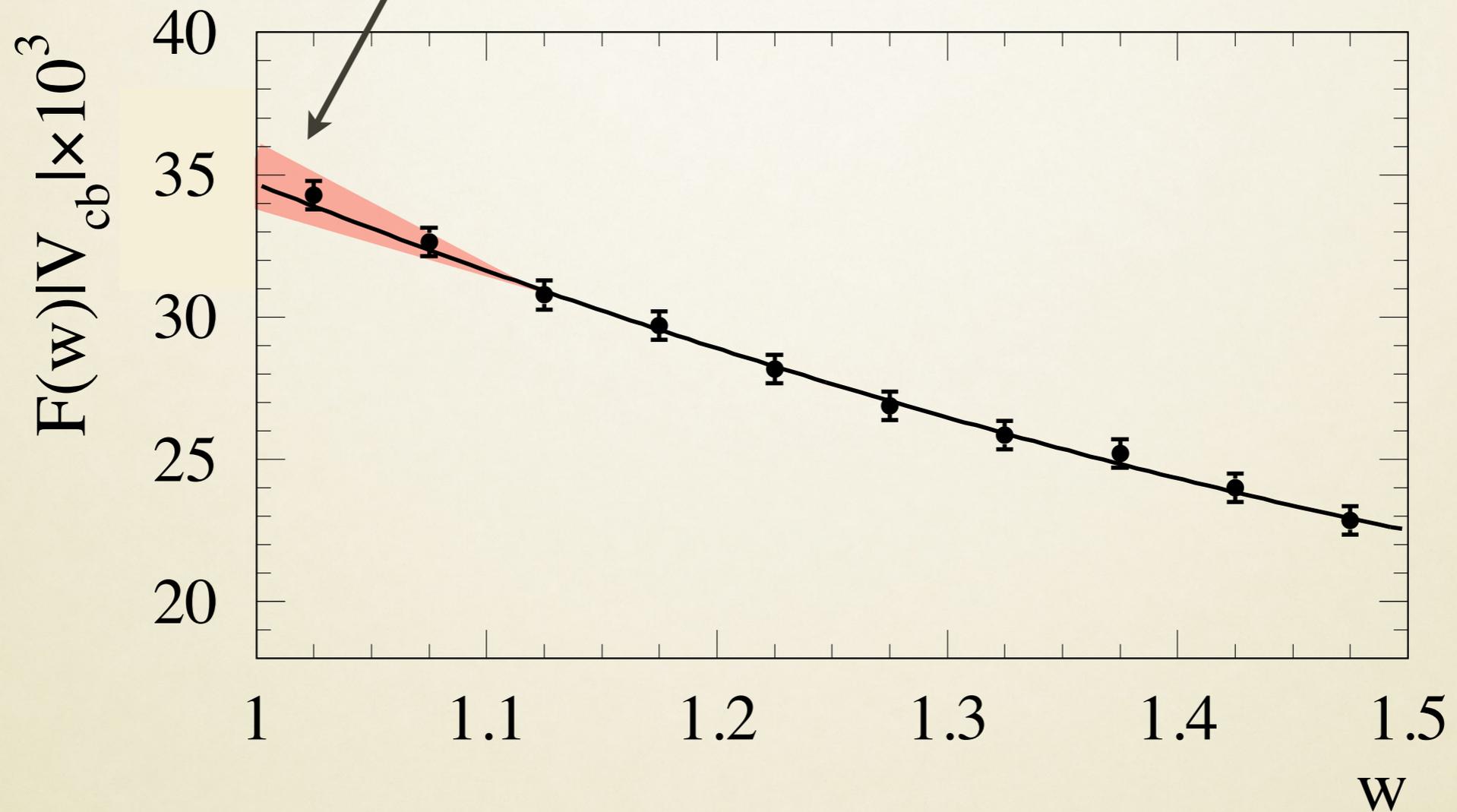
BACK-UP SLIDES

THEORETICAL ERRORS



Theoretical errors are generally the **dominant** ones in the fits. We estimate them in a **conservative** way, mimicking higher orders by varying the parameters by fixed amounts: $m_{c,b}$ 8MeV, $\alpha_s(m_b)$ 0.018, 7% in $1/m^2$ parameters, 30% in $1/m^3$ parameters. New corrections have been within theor. uncertainties so far.

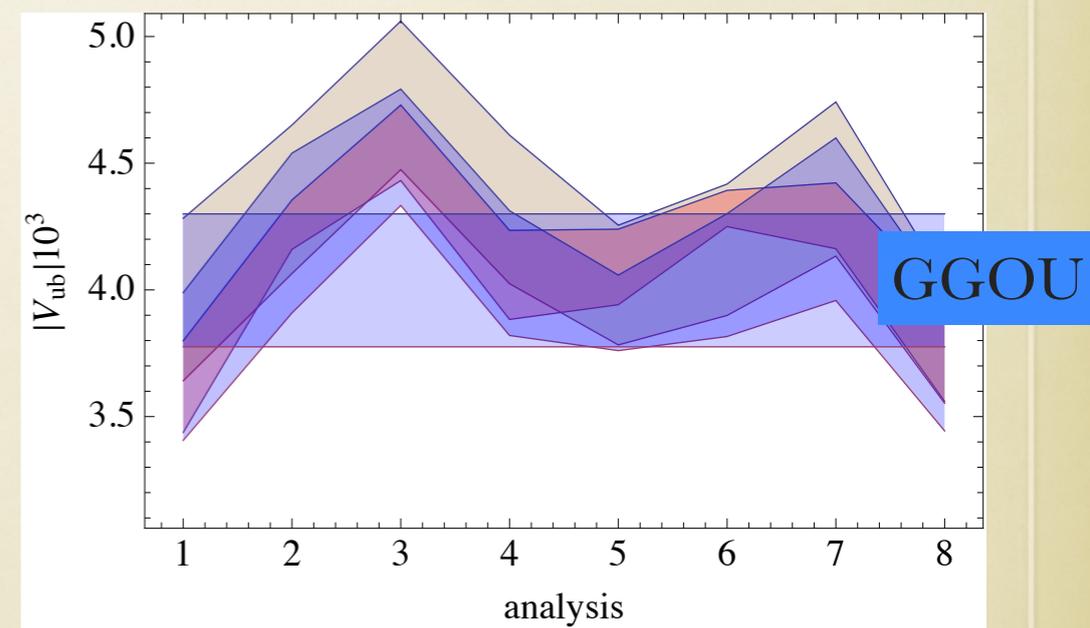
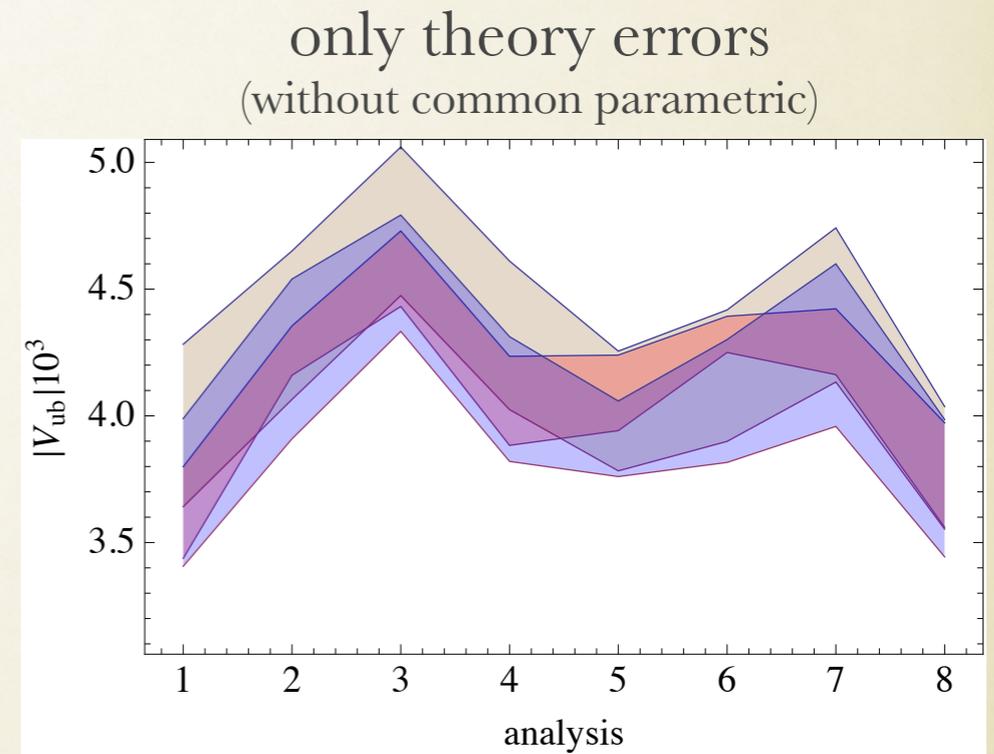
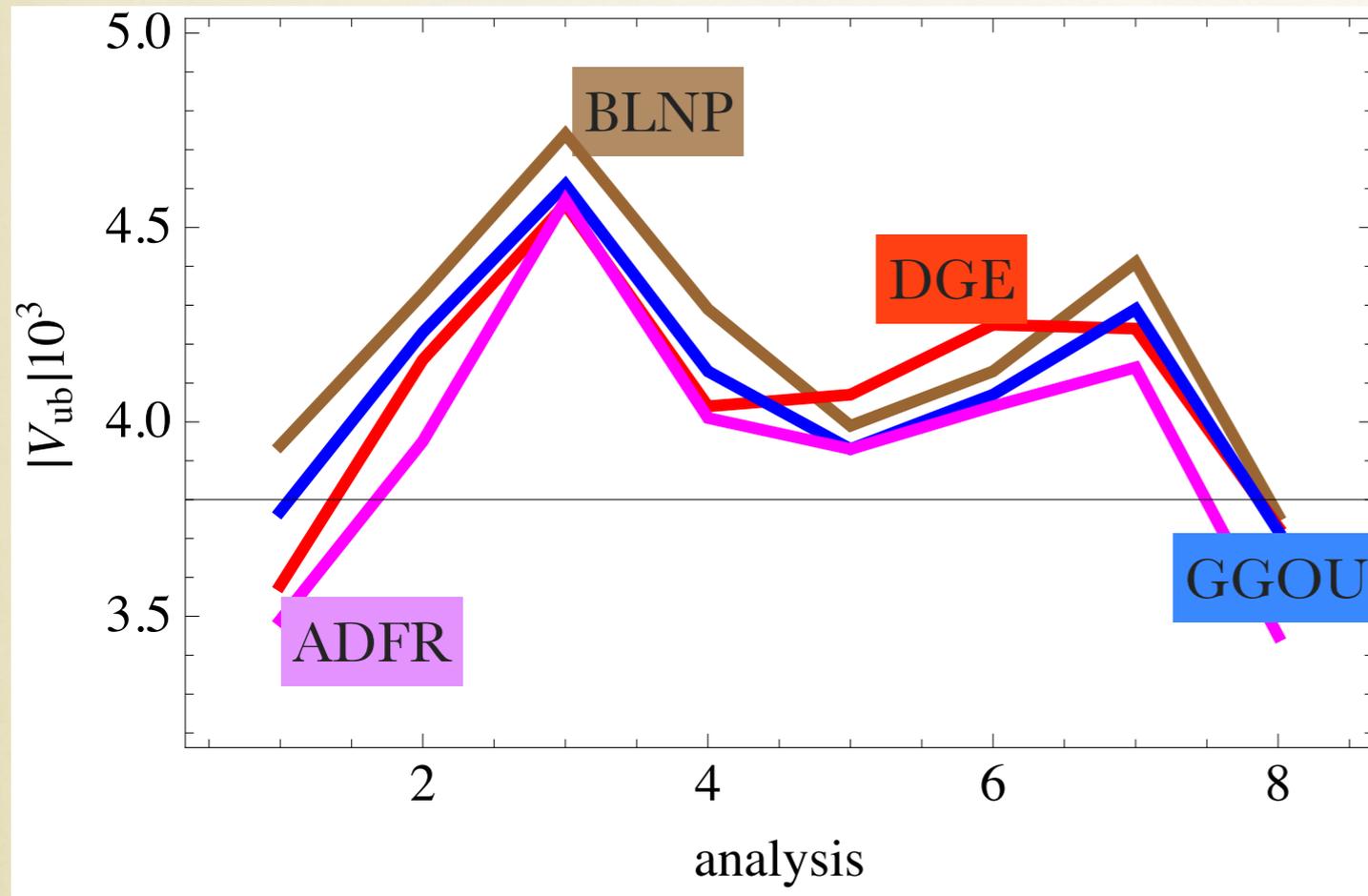
Extrapolation to zero recoil,
possible parameterization effect (qualitative picture)



Babar form factor shape from 0705.4008

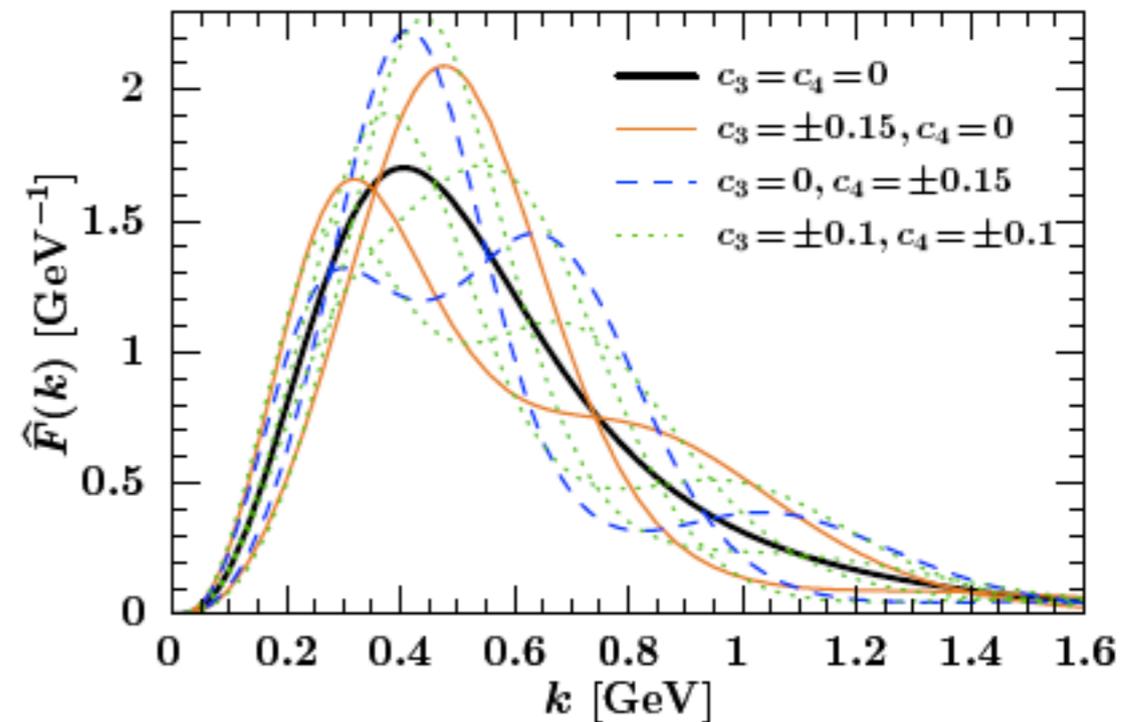
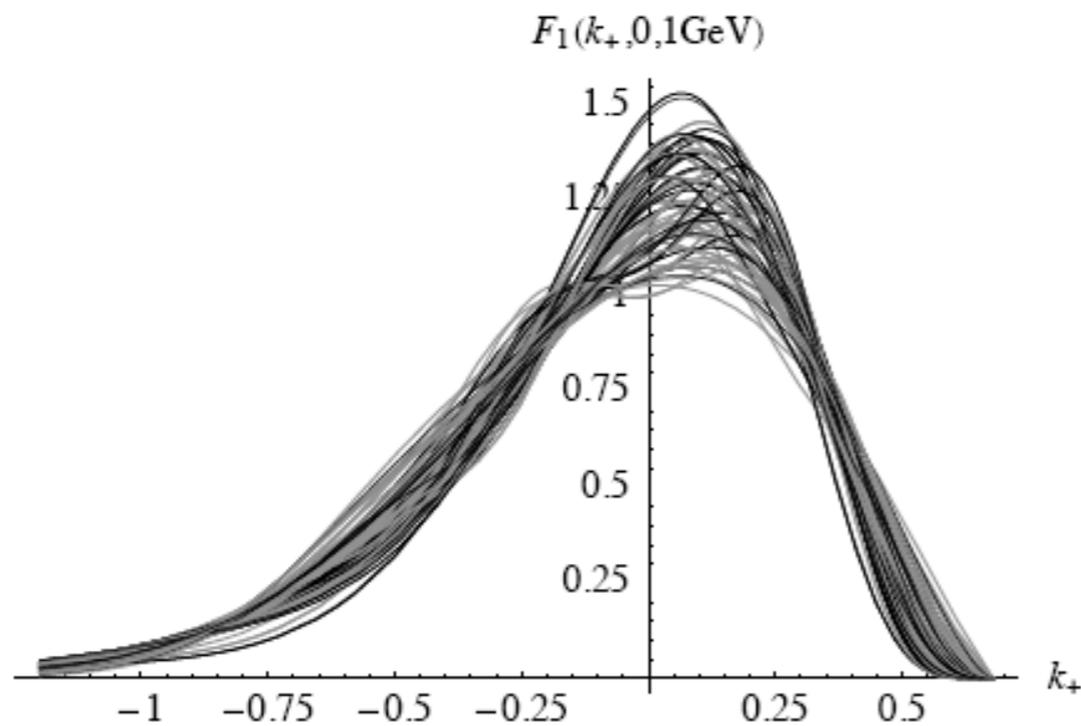
A GLOBAL COMPARISON

0907.5386, Phys Rept



- * common inputs (except ADFR)
- * Overall good agreement **SPREAD WITHIN THEORY ERRORS**
- * NNLO BLNP still missing: will push it up a bit
- * Systematic offset of central values: normalization? to be investigated

FUNCTIONAL FORMS



About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

A more systematic method by Ligeti et al. arXiv:0807.1926
Plot shows 9 SFs that satisfy all the first three moments

HIGHER POWER CORRECTIONS

Mannel, Turczyk, Uraltsev 1009.4622

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m^5$. At $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$

$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$

$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$

$$2M_B m_4 = g \langle \vec{p} \cdot \text{rot } \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$

$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$

$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$

$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$

$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by **Lowest Lying State Saturation** approx by truncating

$$\langle B | O_1 O_2 | B \rangle = \sum_n \langle B | O_1 | n \rangle \langle n | O_2 | B \rangle$$

In LLSA *good convergence* of the HQE. First fit with $1/m^{4,5}$:

$$\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\% \quad \text{Turczyk, PG preliminary}$$

Heinonen, Mannel 1407.4384 have more systematic approach

LLSA might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values*. Large corrections to LLSA have been found.

Mannel, Uraltsev, PG, 2012

Allowing 80% gaussian deviations from LLSA seem to leave V_{cb} unaffected.