Non-perturbative renormalization of the energy-momentum tensor in SU(3) Yang-Mills theory

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PLAN OF THE TALK

- Introduction
- Non-perturbative renormalization of $T_{\mu\nu}$ on the lattice
- A physical application: Equation of State
- Conclusions

Introduction

- The energy-momentum tensor is a fundamental quantity for a Quantum Field Theory: it contains the currents of Poincare' symmetry and of dilatations.

- For SU(N) Yang-Mills theory in the continuum in D dimensions it is given by

$$T_{\mu\nu}(x) = \frac{1}{g_0^2}\left[F^a_{\mu\rho}(x)F^a_{\nu\rho}(x) - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}(x)F^a_{\rho\sigma}(x)\right] = \tau_{\mu\nu} + \delta_{\mu\nu}\tau$$

$$\tau_{\mu\nu} = \frac{1}{g_0^2}\left[F^a_{\mu\alpha}F^a_{\nu\alpha} - \frac{1}{D}\delta_{\mu\nu}F^a_{\alpha\beta}F^a_{\alpha\beta}\right]$$

traceless part: two-index symmetric tensor of SO(D)

$$\tau = \frac{\epsilon}{2Dg_0^2}F^a_{\alpha\beta}F^a_{\alpha\beta}$$

anomalous part: singlet of SO(D)

- $T_{\mu\nu}$ is multiplicatively renormalizable and properly generates translations by Ward Identities

$$\int d^Dx \,\langle\partial_\mu T_{\mu\nu}(x)\mathcal{O}\rangle = -\langle\partial_\nu\mathcal{O}\rangle$$

- The energy-momentum tensor is a physical quantity: directly related to thermodynamics quantities like pressure, entropy and energy density.

The energy-momentum tensor on the lattice

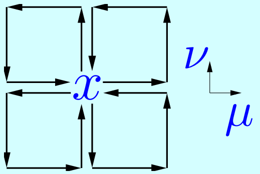
S. Caracciolo et al., NPB 375 (1992) 195; Ann. Phys. 197 (1990) 119

- Lattice: preferred framework for non-perturbative study from first principles

  Explicit breaking of space-time symmetries that must be recovered in the continuum limit; troubles with the energy-momentum tensor

- $T_{\mu\nu}$ on the lattice must generate the correct conserved currents in the cont. limit

$$F_{\mu\nu}(x) = \frac{1}{8}\left[Q_{\mu\nu}(x) - Q_{\nu\mu}(x)\right]_{traceless} \qquad Q_{\mu\nu}(x) = \sum$$



$$T_{\mu\nu} = -\frac{2}{g_0^2}\left[\mathrm{Tr}\left(F_{\mu\alpha}(x)F_{\nu\alpha}(x)\right) - \frac{1}{4}\delta_{\mu\nu}\mathrm{Tr}\left(F_{\alpha\beta}(x)F_{\alpha\beta}(x)\right)\right]$$

- $T_{\mu\nu}$ has dimension 4; on the lattice SO(4) breaks to SW_4, mixing with

$$T^{[1]}_{\mu\nu} = (1-\delta_{\mu\nu})\frac{1}{g_0^2}\left[F^a_{\mu\alpha}F^a_{\nu\alpha}\right] \qquad T^{[2]}_{\mu\nu} = \delta_{\mu\nu}\frac{1}{4g_0^2}F^a_{\alpha\beta}F^a_{\alpha\beta} \qquad T^{[3]}_{\mu\nu} = \delta_{\mu\nu}\frac{1}{g_0^2}\left[F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}\right]$$

- The renormalized energy-momentum tensor correctly generates translations and the trace anomaly. It can be written as

$$T^R_{\mu\nu} = Z_T\left\{T^{[1]}_{\mu\nu} + z_T T^{[3]}_{\mu\nu} + z_S[T^{[2]}_{\mu\nu} - \langle T^{[2]}_{\mu\nu}\rangle_0]\right\}$$

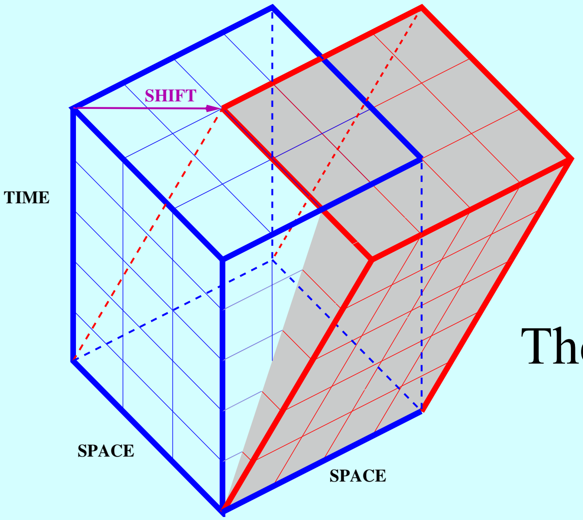
where Z_T, z_T and z_S are renormalizations constants and depend only on g_0^2.
Only a perturbative method to compute them; calculated at 1 loop.

A moving frame in Euclidean space: the shift

L. Giusti and H. Meyer,
PRL 2011, JHEP 2011 and 2013

Implement translations in Euclidean space by setting a thermal quantum field theory in a moving reference frame.

$$Z(L_0,\xi) = \mathrm{Tr}\left[e^{-L_0(H-\xi_k T_{0k})}\right]$$



That corresponds to introducing a shift $\vec{\xi}$ when closing the periodic boundary conditions along the temporal direction:

$$A_\mu(L_0,\vec{x}) = A_\mu(0,\vec{x}-L_0\vec{\xi})$$

The Lorentz invariance implies that the free-energy is given by

$$f\left(L_0\sqrt{1+\xi^2}\right) = -\lim_{V\to\infty}\frac{1}{L_0 V}\log Z(L_0,V,\vec{\xi})$$

i.e. the temperature of the system is given by $T = \dfrac{1}{L_0\sqrt{1+\xi^2}}$

By applying the machinery of quantum fields, one produces new Ward Identities generated by Lorentz invariance. Particularly interesting ones are

$$\langle T_{0k}\rangle_\xi = -\frac{1}{L_0 V}\frac{\partial}{\partial\xi_k}\log Z(L_0,\vec{\xi}) \qquad \langle T_{0k}\rangle_\xi = \frac{\xi_k}{1-\xi^2}\left[\langle T_{00}\rangle_\xi - \langle T_{kk}\rangle_\xi\right]$$

$$\frac{\partial}{\partial\xi_k}\langle T_{\mu\mu}\rangle_\xi = \frac{1}{(1+\xi^2)^2}\frac{\partial}{\partial\xi_k}\left[\frac{(1+\xi^2)^3}{\xi_k}\langle T_{0k}\rangle_\xi\right]$$

L. Giusti and M.P.,
PRD 91 (2015) 11, 114504

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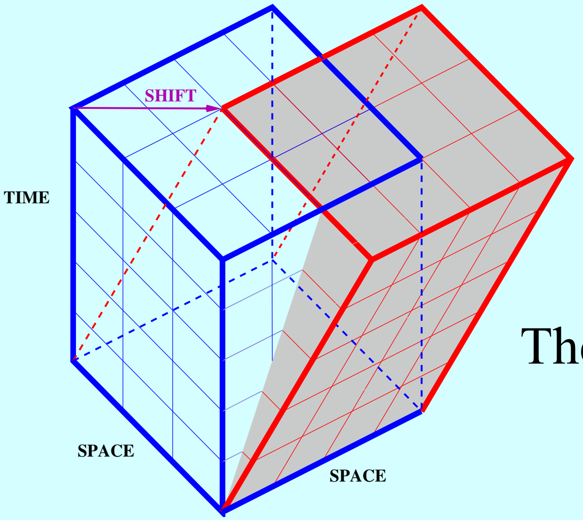

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By applying the machinery of quantum fields, one produces new Ward Identities generated by Lorentz invariance. Particularly interesting ones are

$$Z_T\langle T_{0k}\rangle_\xi = -\frac{1}{L_0 V}\frac{\partial}{\partial\xi_k}\log Z(L_0,\vec{\xi}) \qquad \langle T_{0k}\rangle_\xi = \frac{z_T\xi_k}{1-\xi^2}\left[\langle T_{00}\rangle_\xi - \langle T_{kk}\rangle_\xi\right]$$

$$z_S\frac{\partial}{\partial\xi_k}\langle T_{\mu\mu}\rangle_\xi = \frac{1}{(1+\xi^2)^2}\frac{\partial}{\partial\xi_k}\left[\frac{(1+\xi^2)^3}{\xi_k}\langle T_{0k}\rangle_\xi\right]$$

on the lattice

L. Giusti and M.P., PRD 91 (2015) 11, 114504

Non-perturbative method to calculate the renormalization constants

The calculation of $Z_T(g_0{}^2)$

The renormalization constant of the off-diagonal components can be obtained from

$$Z_T(g_0^2) = -\frac{1}{\langle T_{0k}\rangle_\xi L_0 V}\frac{\partial}{\partial \xi_k}\log Z(L_0,\vec{\xi})$$

on the lattice

$$Z_T(g_0^2) = \frac{1}{2V}\frac{1}{\langle T_{0k}\rangle_\xi}\log\frac{Z(L_0,\vec{\xi}+\frac{a}{\beta}\hat{k})}{Z(L_0,\vec{\xi}-\frac{a}{\beta}\hat{k})} = \frac{1}{\langle T_{0k}\rangle_\xi}\frac{\Delta f}{\Delta \xi_k}$$

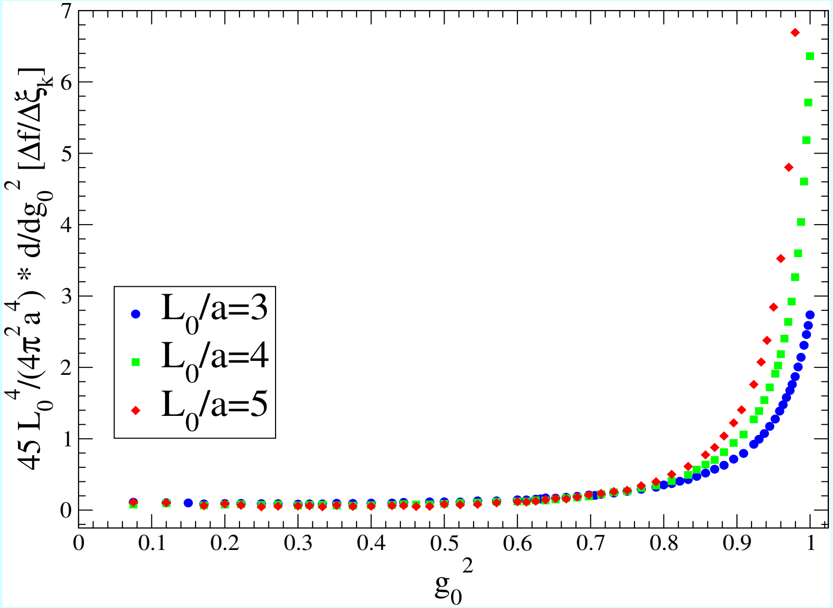
We need to measure $\langle T_{0k}\rangle_\xi$ and the ratio $\frac{\Delta f}{\Delta \xi_k}$ at fixed value of the bare coupling g_0.

Then, by taking the limits $V \longrightarrow \infty$ and $L_0 \longrightarrow \infty$, we can calculate Z_T.

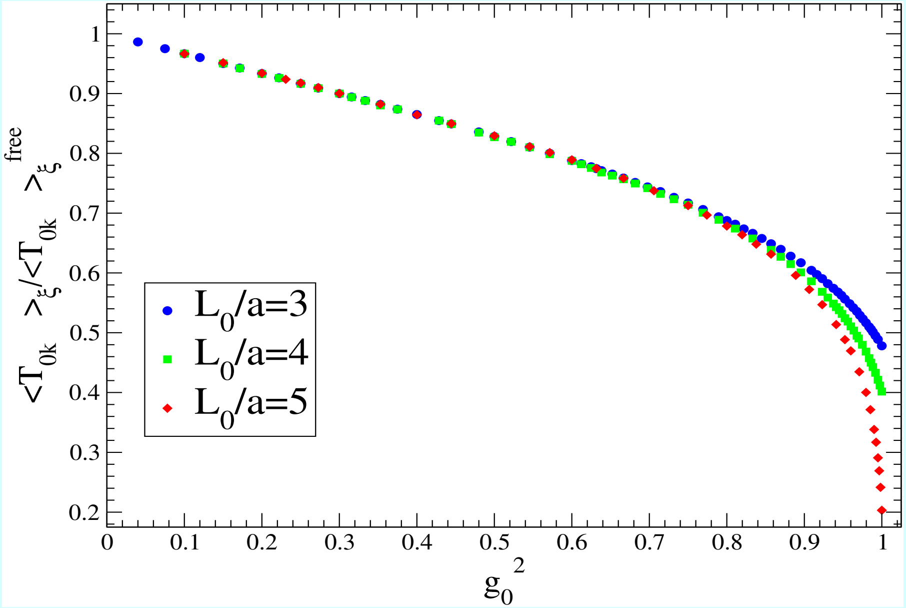
Calculation of $\frac{\Delta f}{\Delta \xi_k}$

$\frac{\Delta f}{\Delta \xi_k}$ is a smooth function of g_0

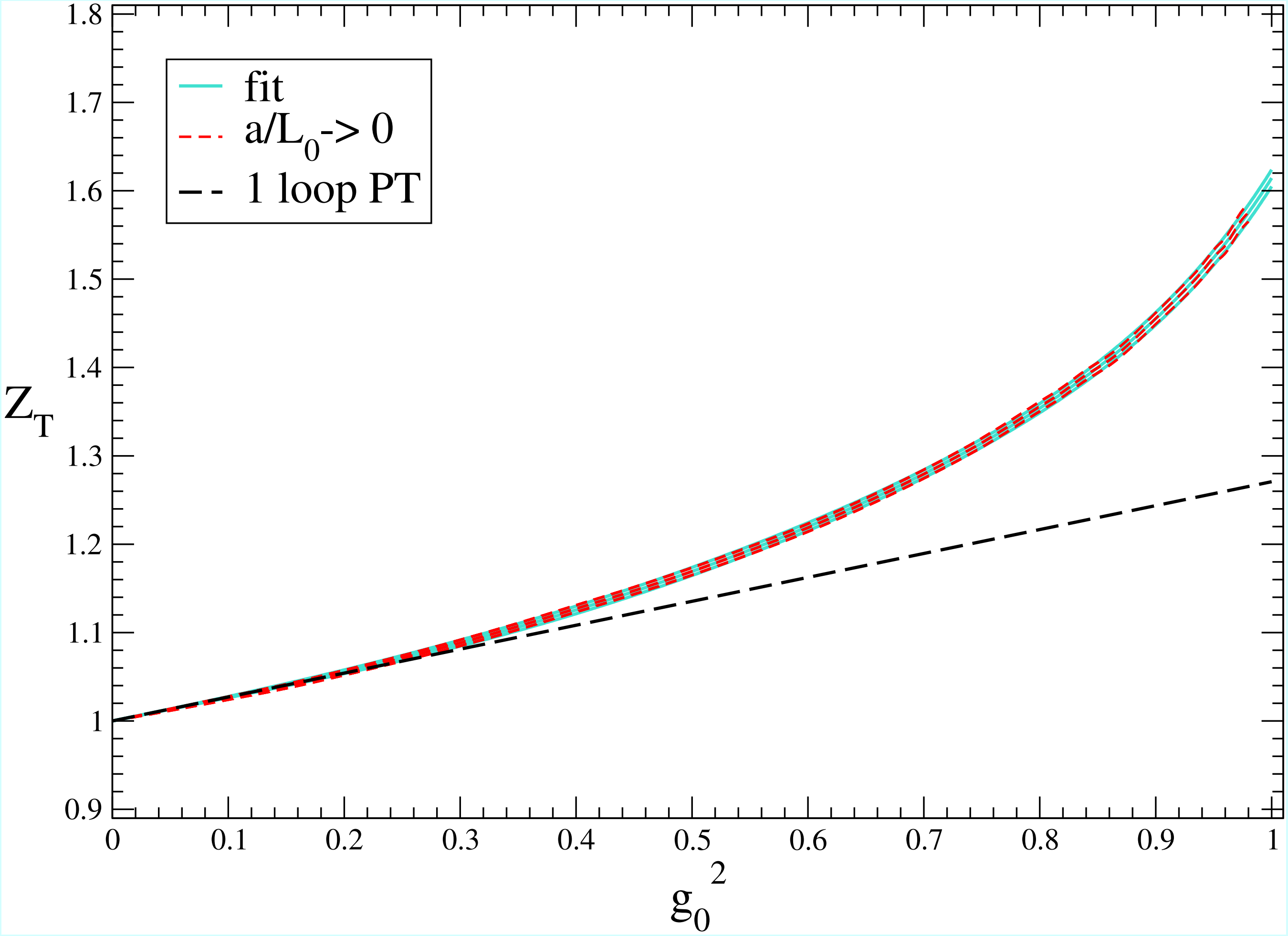
$$\frac{\Delta f}{\Delta \xi_k} = c_0 + \int_0^{g_0^2} d\bar{g}_0^2 \frac{d}{d\bar{g}_0^2}\frac{\Delta f}{\Delta \xi_k}$$



Calculation of $\langle T_{0k}\rangle_\xi$



The calculation of $Z_T(g_0^2)$



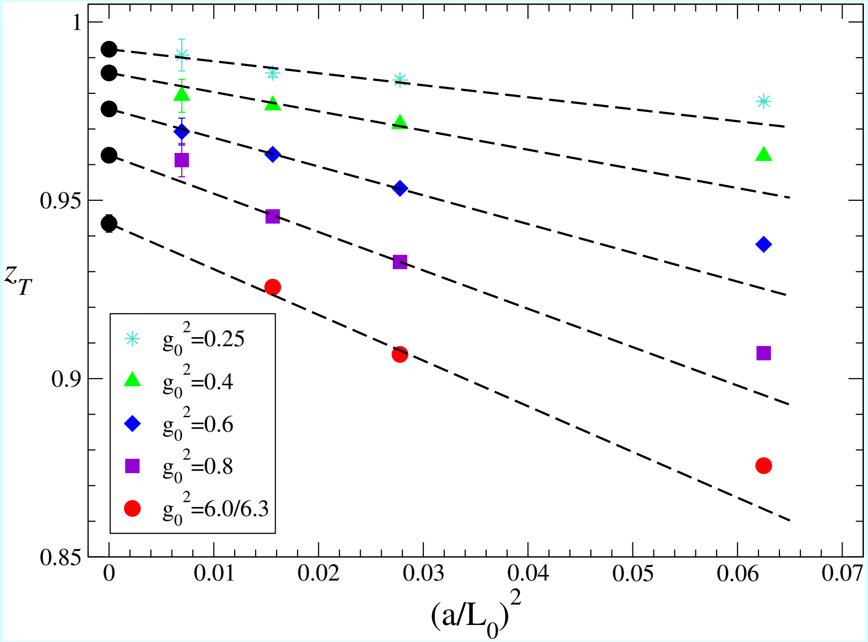
$$Z_T(g_0^2) = \frac{1 - 0.4457 g_0^2}{1 - 0.7165 g_0^2} - 0.2543 g_0^4 + 0.4357 g_0^6 - 0.521 g_0^8$$

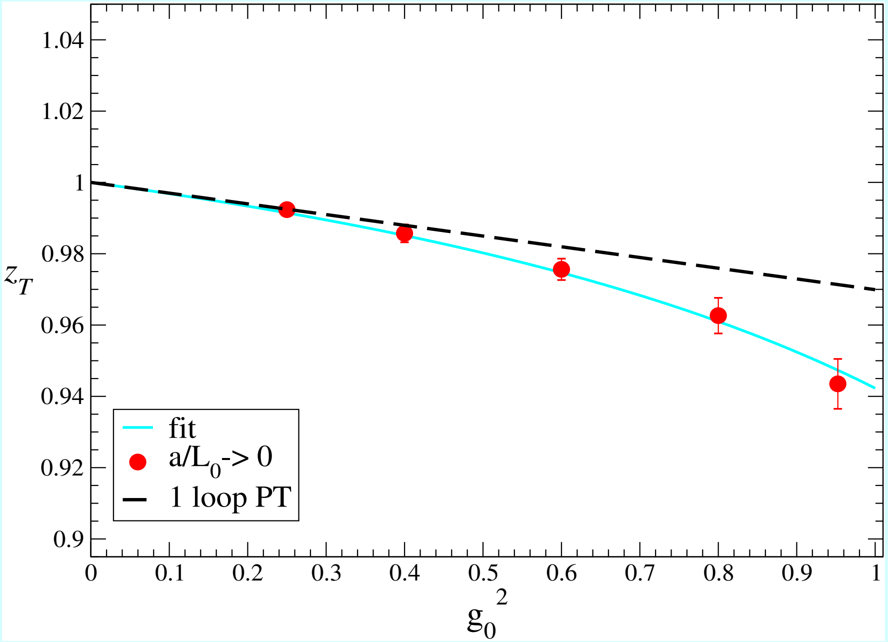
The accuracy is about 0.5% or smaller

The calculation of $z_T(g_0^2)$

$$z_T(g_0^2) = \frac{1-\xi_k^2}{\xi_k} \frac{\langle T_{0k}\rangle_\xi}{\langle T_{00}\rangle_\xi - \langle T_{kk}\rangle_\xi}$$

We need to measure $\langle T_{0k}\rangle_\xi$, $\langle T_{00}\rangle_\xi$ and $\langle T_{kk}\rangle_\xi$ at fixed value of the bare coupling g_0 in a single Monte Carlo simulation.





The calculation of $z_S(g_0^2)$

$$z_S(g_0^2) = \frac{1}{(1+\xi^2)^2} \frac{\partial}{\partial \xi_k} \left[\frac{(1+\xi^2)^3}{\xi_k} \langle T_{0k}\rangle_\xi \right] / \frac{\partial}{\partial \xi_k} \langle T_{\mu\mu}\rangle_\xi$$

work in progress
(simple measurement
of the trace anomaly)

A physical application: the calculation of Equation of State

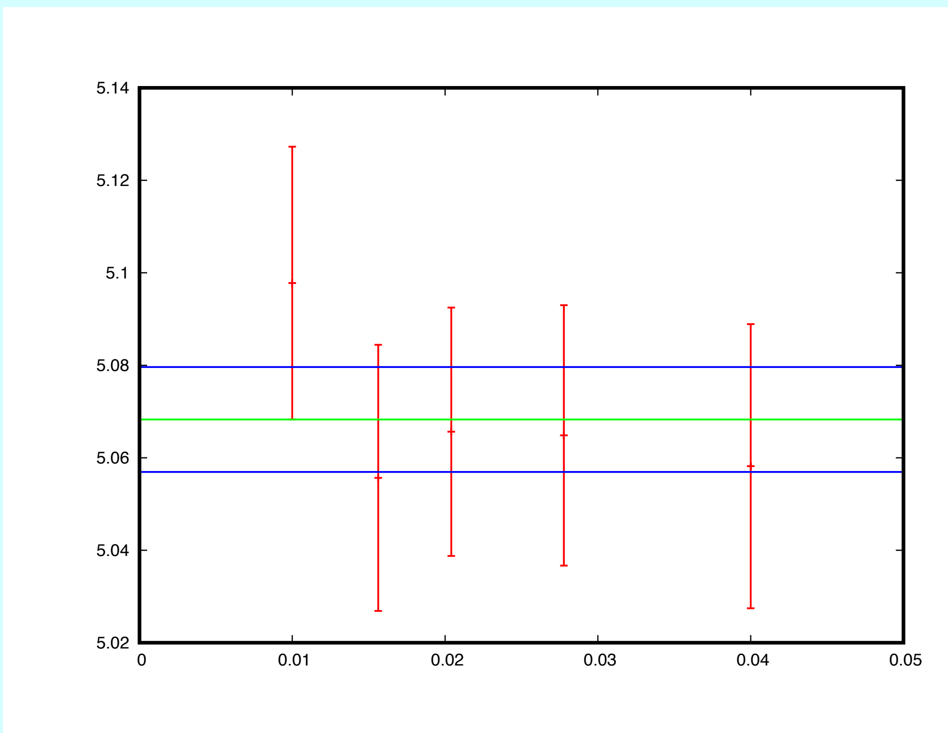
The energy-momentum tensor is a physical operator and it is related to the thermodynamic properties of a thermal quantum field theory.

The entropy density s can be obtained as follows

$$\frac{s}{T^3} = -\frac{L_0^4(1+\xi^2)^3}{\xi_k} \langle T_{0k} \rangle_\xi Z_T$$

L_0 lattice size in the temporal direction

$\frac{s}{T^3}$

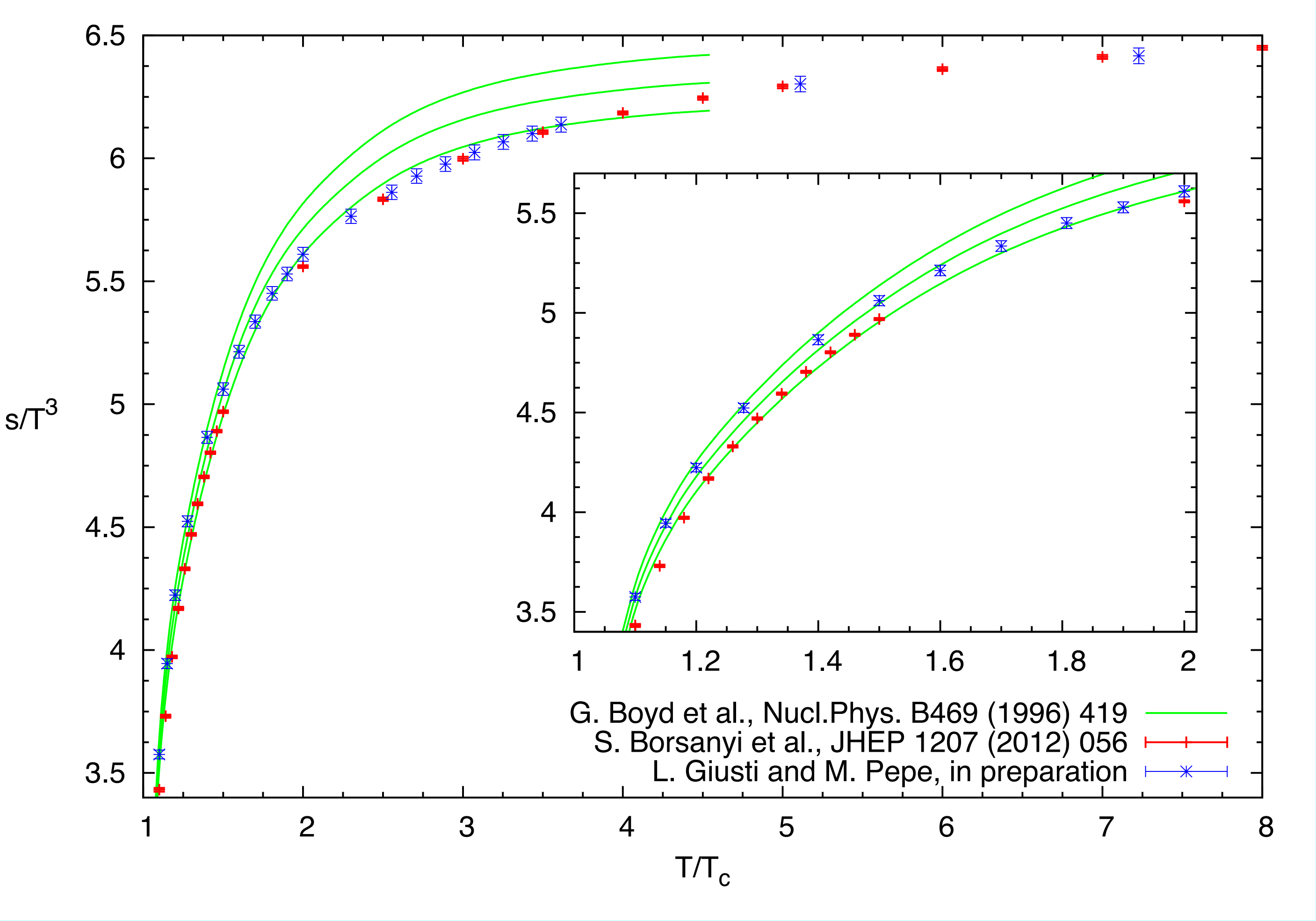


$\left(\frac{a}{L_0}\right)^2$

T=1.5 T_c

The Equation of State

We compare our preliminary results in the range 1-8 T_c with the data available in the literature



Numerical simulations are in progress to reach temperatures about 250 T_c. These runs are not computationally demanding and they will be over in a few months.

Conclusions

- The non-perturbative renormalization of the energy-momentum tensor on the lattice has been computed in the range [0,1] for the bare coupling g_0^2.

- Measurements of Z_T and z_T with an accuracy of about 0.5% or smaller have been shown.

- An equation for the renormalization of the trace anomaly is shown and the calculation of z_S is in progress.

- The renormalization factor Z_T of the space-time components of the energy-momentum tensor provide a new method to compute the Equation of State of SU(3).

- Our preliminary results are not in agreement with data in the literature. The discrepancy with the data by Borsanyi et al. below 2 T_c is significative, especially around 1.1-1.2 T_c where it is above 5σ. Our data are systematically above.