Fully differential decay rate of a SM Higgs boson into a b-pair at NNLO

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in collaboration with

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Higgs boson has been discovered

- m_H [GeV]=125.5±0.2$_{stat}$±0.6$_{syst}$ (ATLAS 2013)
  125.7±0.3$_{stat}$±0.3$_{syst}$ (CMS 2013)
- All measured properties are consistent with SM expectations within experimental uncertainties
  - spin zero
  - parity +
  - couples to masses of W and Z (with c_V=1 within experimental uncertainty)
- Yet it still could be the first element of an extended Higgs sector (e.g. SUSY neutral Higgs)

Distinction requires high-precision prediction for both production and decay

Example: pp → H + X → b$\bar{\text{b}}$ + X in PT

- Γ_H [MeV]=4.07±0.16$_{\text{teo}}$

⇒ can use the narrow width approximation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O_{b\bar{b}}} = \left[\sum_{n=0}^{\infty} \frac{\mathrm{d}d^2\sigma^{(n)}_{pp\to H+X}}{\mathrm{d}p_{\perp,H}\,\mathrm{d}\eta_H}\right] \times \left[\frac{\sum_{n=0}^{\infty} \mathrm{d}\Gamma^{(n)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}}}{\sum_{n=0}^{\infty} \Gamma^{(n)}_{H\to \mathrm{b}\bar{\mathrm{b}}}}\right] \times \mathrm{Br}(H \to \mathrm{b}\bar{\mathrm{b}})$$

known up to NNLO

this talk: up to NNLO

known with 1% accuracy

pp → H + X → b b + X in PT

Including up to NNLO corrections for production and decay:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}O_{b\bar{b}}} = \Bigg[\frac{\mathrm{d}^2\sigma^{(0)}_{pp\to H+X}}{\mathrm{d}p_{\perp,H}\mathrm{d}\eta_H} \; \frac{\mathrm{d}\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}} + \mathrm{d}\Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}} + \mathrm{d}\Gamma^{(2)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}}}{\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}} + \Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}} + \Gamma^{(2)}_{H\to \mathrm{b}\bar{\mathrm{b}}}}$$

$$+ \frac{\mathrm{d}^2\sigma^{(1)}_{pp\to H+X}}{\mathrm{d}p_{\perp,H}\mathrm{d}\eta_H} \; \frac{\mathrm{d}\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}} + \mathrm{d}\Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}}/\mathrm{d}O_{b\bar{b}}}{\Gamma^{(0)}_{H\to \mathrm{b}\bar{\mathrm{b}}} + \Gamma^{(1)}_{H\to \mathrm{b}\bar{\mathrm{b}}}}$$

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Method

Problem

$$\sigma_m^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}}$$

$$\equiv \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1} + \int_m \mathrm{d}\sigma_m^{\mathrm{VV}} J_m$$

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- the three contributions are separately divergent in d = 4 dimensions:
  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$ after integration over phase space, no explicit ϵ-poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
  - in σ^{VV} explicit ϵ-poles at $O\ (\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$

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CoLoRFulNNLO

Completely Local SubtRactions for Fully Differential Predictions@NNLO

Structure

of subtractions is governed by the jet functions

$$\sigma_m^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}} = \sigma_{m+2} + \sigma_{m+1} + \sigma_m$$

$$\mathrm{d}\sigma_{m+2} = \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m \right] \right\}_{\epsilon=0}$$

$$\mathrm{d}\sigma_{m+1} = \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \right\}_{\epsilon=0}$$

$$\mathrm{d}\sigma_m = \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\}_{\epsilon=0} J_m$$

- RR,A_2 regularizes doubly-unresolved limits
- RR,A_1 regularizes singly-unresolved limits
- RR,A_{12} removes overlapping subtractions
- RV,A_1 regularizes singly-unresolved limits

Use known ingredients

- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one- and two-loop amplitudes
  - soft and collinear factorization of QCD matrix elements

tree-level 3-parton splitting, double soft current:

J.M. Campbell, E.W.N. Glover 1997, S. Catani, M. Grazzini 1998
V. Del Duca, A. Frizzo, F. Maltoni, 1999, D. Kosower, 2002

one-loop 2-parton splitting, soft gluon current:

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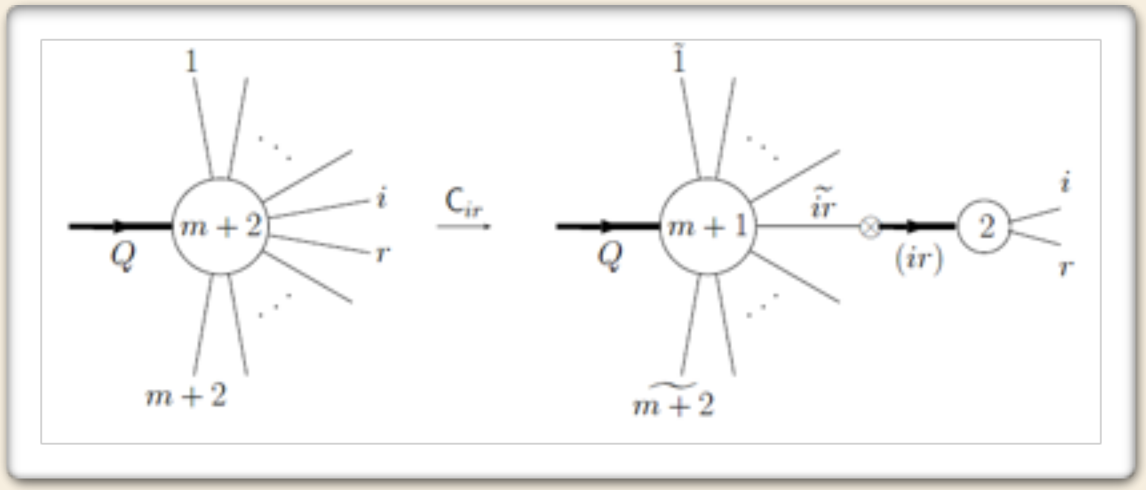
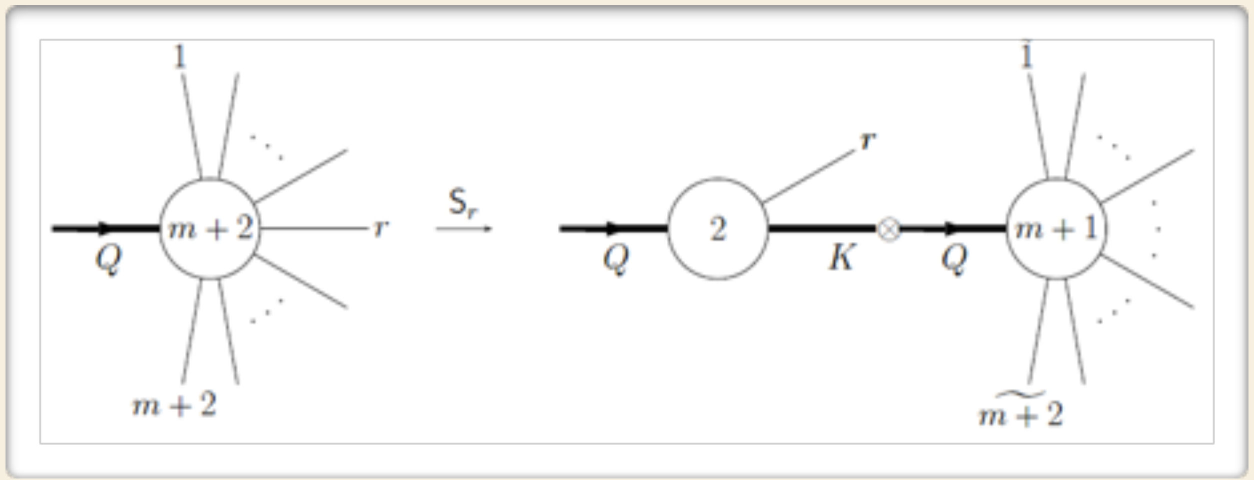
- Extension over whole phase space using momentum mappings (not unique): $\{p\}_{n+s} \to \{\tilde{p}\}_n$

Momentum mappings

$$\{p\}_{n+s} \to \{\tilde{p}\}_n$$

- implement exact momentum conservation
- recoil distributed democratically
- different mappings for collinear and soft limits





- lead to phase-space factorization
- can be generalized to any number s of unresolved patrons trivially

Momentum mappings

define subtractions

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

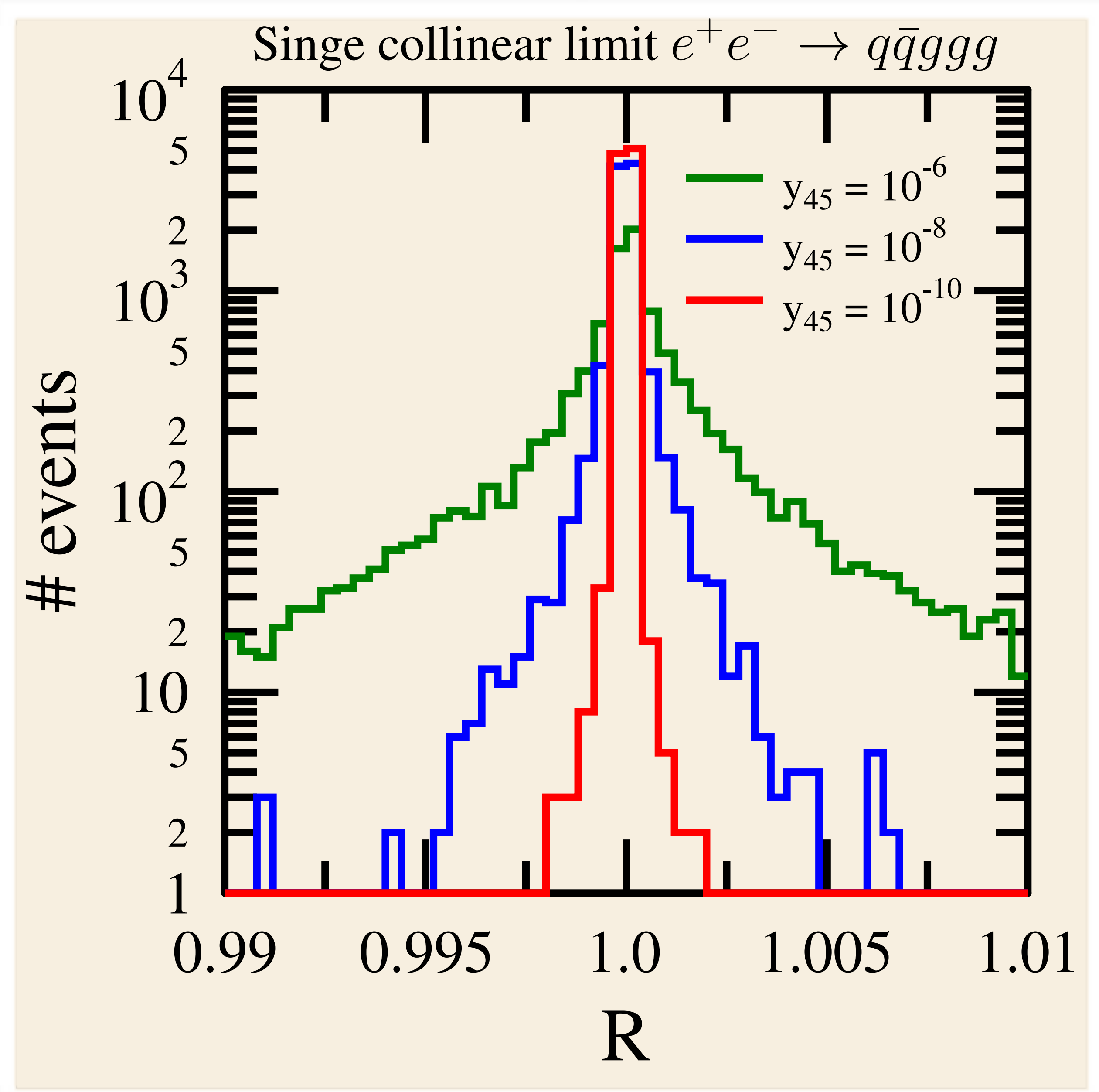
$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

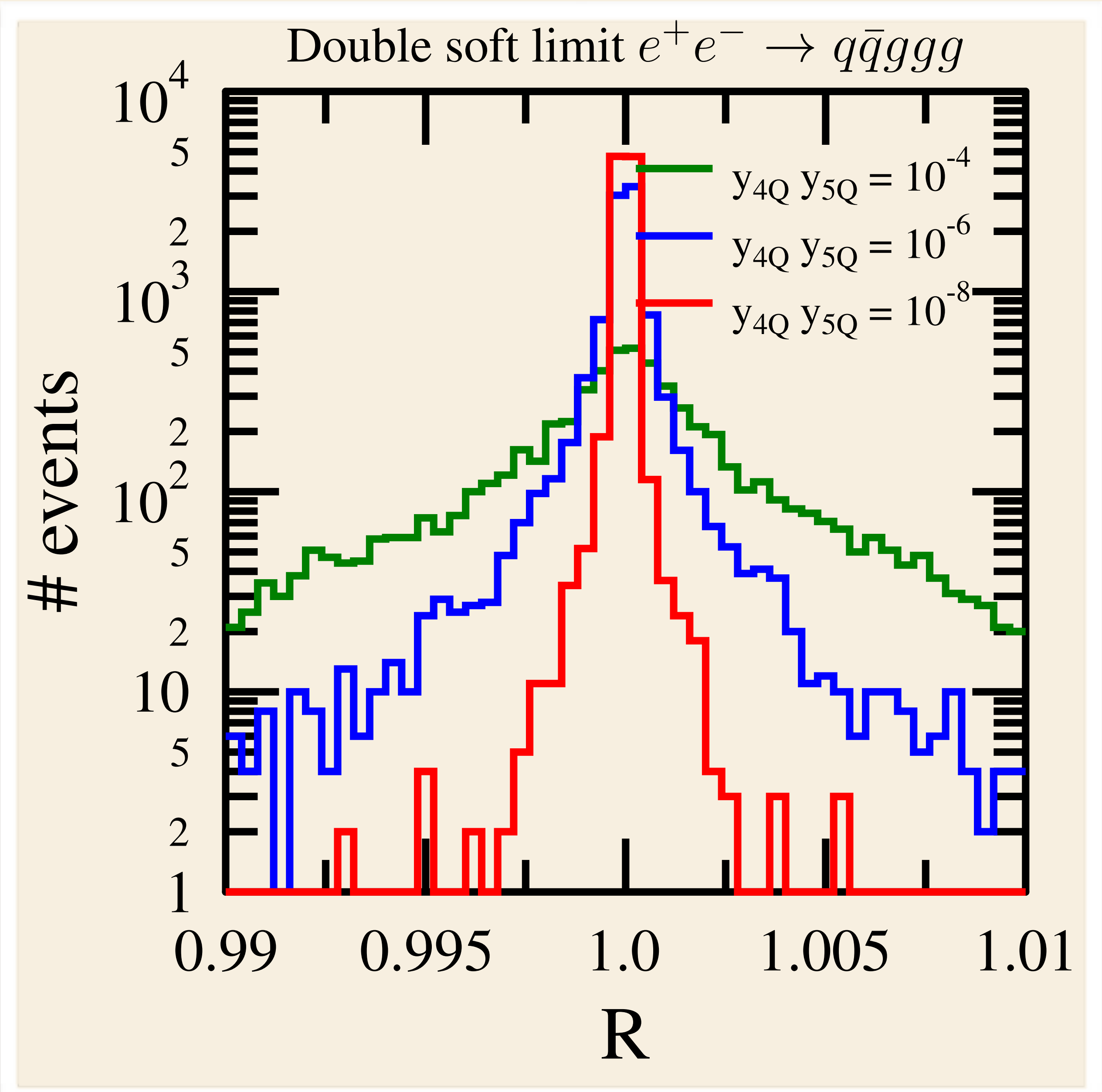
$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

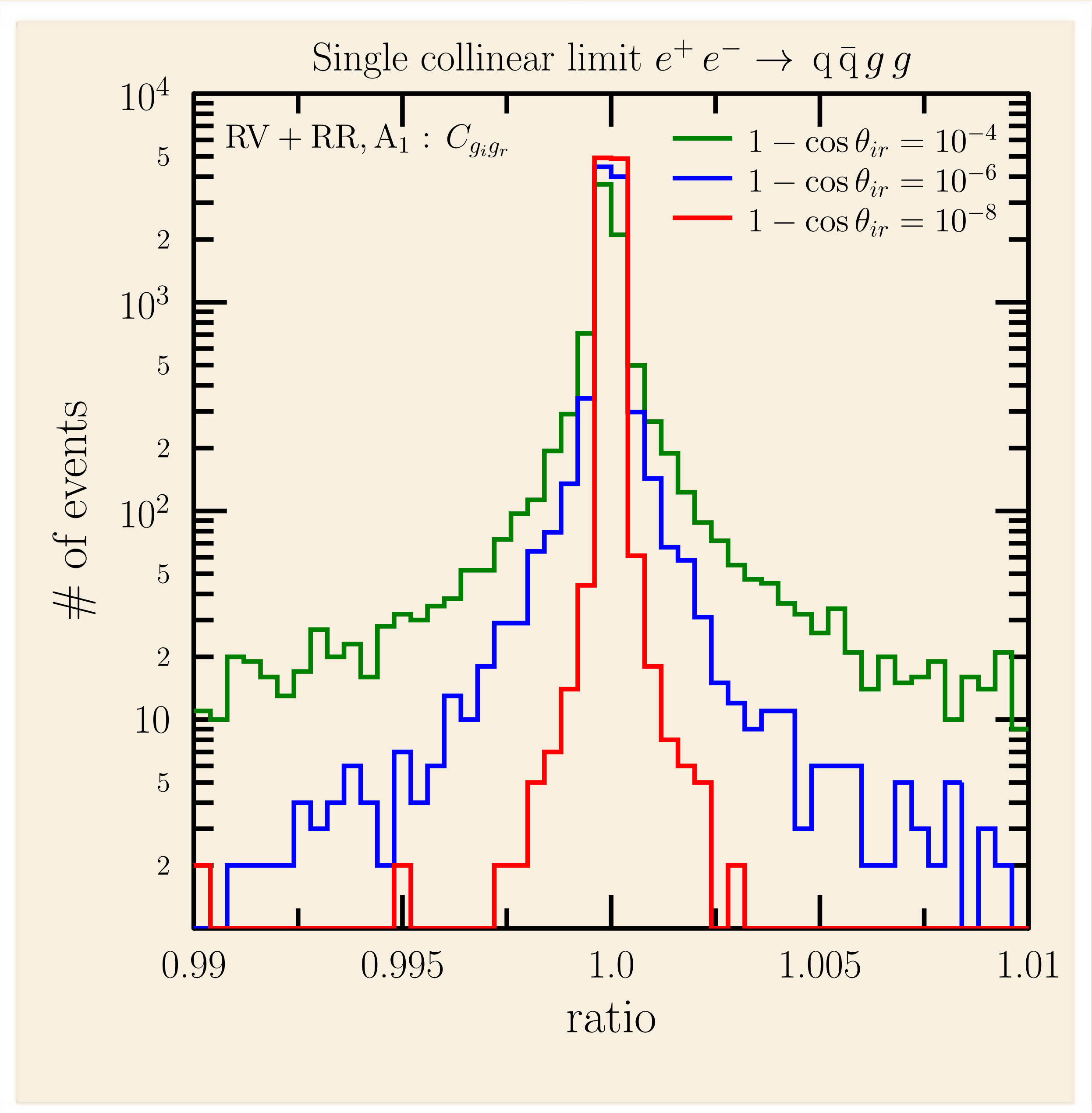
Kinematic singularities cancel in RR

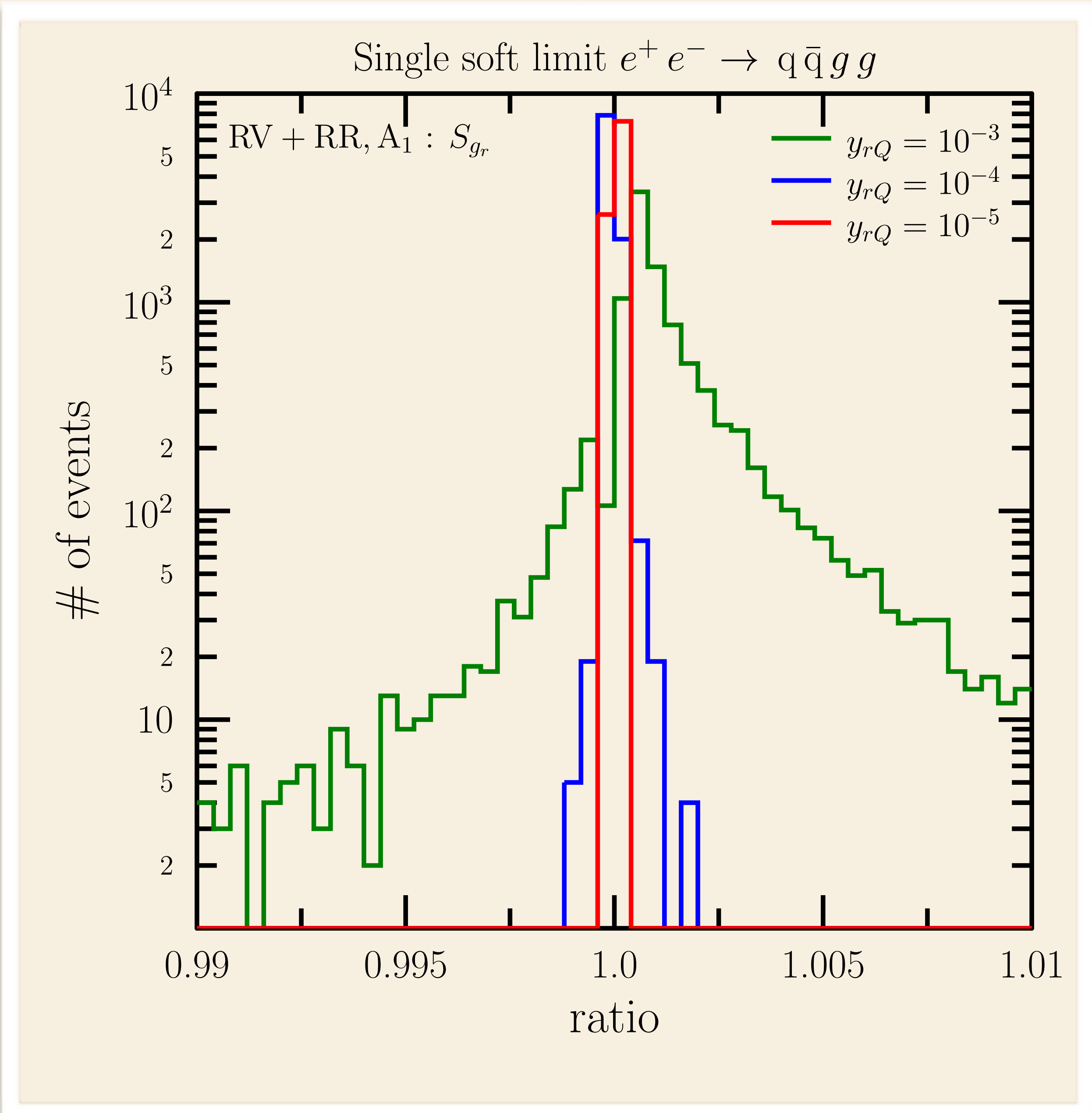




R = subtraction/RR

Kinematic singularities cancel in RV





R = subtraction/(RV+RR,A_1)

Regularized RR & RV contributions

can now be computed by numerical Monte Carlo integrations

(implementation for general m in progress)

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left({\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left({\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left({\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right) + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
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Difficulty

Integrated approximate xsections

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

After integrating over unresolved momenta & summing over unresolved colors and flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

$$\int_p \mathrm{d}\sigma^{\mathrm{RR,A}_p} = \boldsymbol{I}^{(0)}_p(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma^{\mathrm{B}}_n$$

Structure of insertion operators

- the color structures are independent of the precise definition of subtractions (momentum mappings), only subleading coefficients of ϵ-expansion in kinematic functions may depend
- we computed all insertion operators analytically (defined in our subtraction scheme) up to $O(\epsilon^{-2})$ for arbitrary m

Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2,
  - the insertion operators are independent of the kinematics (momenta are back-to-back, so MI's are needed at the endpoints only)
  - color algebra is trivial:

$$\boldsymbol{T}_1\boldsymbol{T}_2 = -\boldsymbol{T}_1^2 = -\boldsymbol{T}_2^2 = -C_{\mathrm{F}}$$

- so can demonstrate the cancellation of poles

Example: $H \to b\bar{b}$ at $\mu = m_H$

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

$$\mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{VV}} = \left(\frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{B}} \left\{ + \frac{2C_\mathrm{F}^2}{\epsilon^4} + \left(\frac{11 C_\mathrm{A} C_\mathrm{F}}{4} + 6C_\mathrm{F}^2 - \frac{C_\mathrm{F} n_\mathrm{f}}{2} \right) \frac{1}{\epsilon^3} \right.$$
$$+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_\mathrm{A} C_\mathrm{F} + \left(\frac{17}{2} - 2\pi^2 \right) C_\mathrm{F}^2 - \frac{2 C_\mathrm{F} n_\mathrm{f}}{9} \right] \frac{1}{\epsilon^2}$$
$$\left. + \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2} \right) C_\mathrm{A} C_\mathrm{F} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3 \right) C_\mathrm{F}^2 + \frac{65 C_\mathrm{F} n_\mathrm{f}}{108} \right] \frac{1}{\epsilon} \right\}$$

C. Anastasiou, F. Herzog, A. Lazopoulos, arXiv:0111.2368

$$\sum \int \mathrm{d}\sigma^{\mathrm{A}} = \left(\frac{\alpha_s(\mu^2)}{2\pi} \right)^2 \mathrm{d}\sigma_{H\to b\bar{b}}^{\mathrm{B}} \left\{ - \frac{2C_\mathrm{F}^2}{\epsilon^4} - \left(\frac{11 C_\mathrm{A} C_\mathrm{F}}{4} + 6C_\mathrm{F}^2 + \frac{C_\mathrm{F} n_\mathrm{f}}{2} \right) \frac{1}{\epsilon^3} \right.$$
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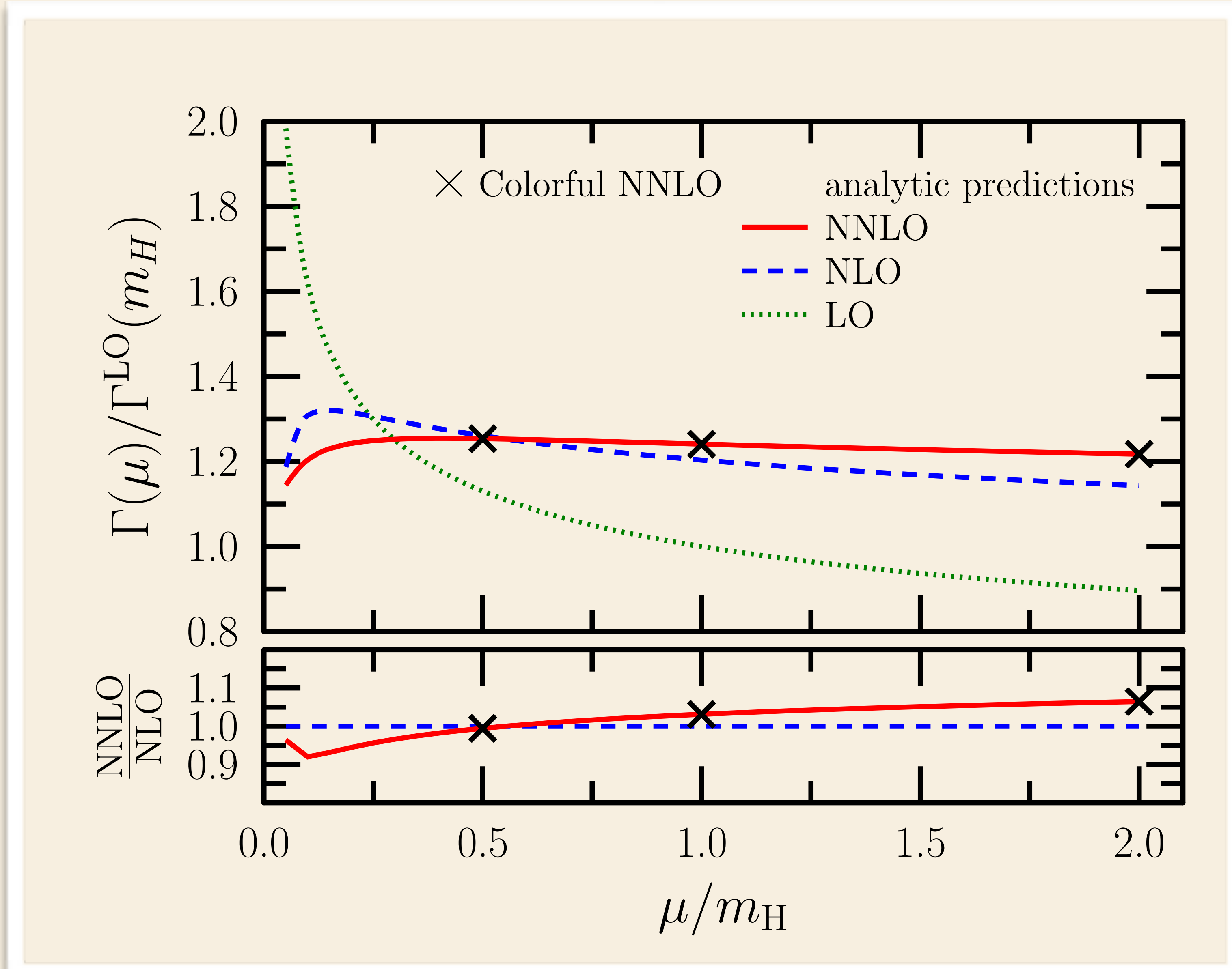
V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, Z. Trócsányi, arXiv:1501.07226

Application

Example: H→b$\bar{\text{b}}$

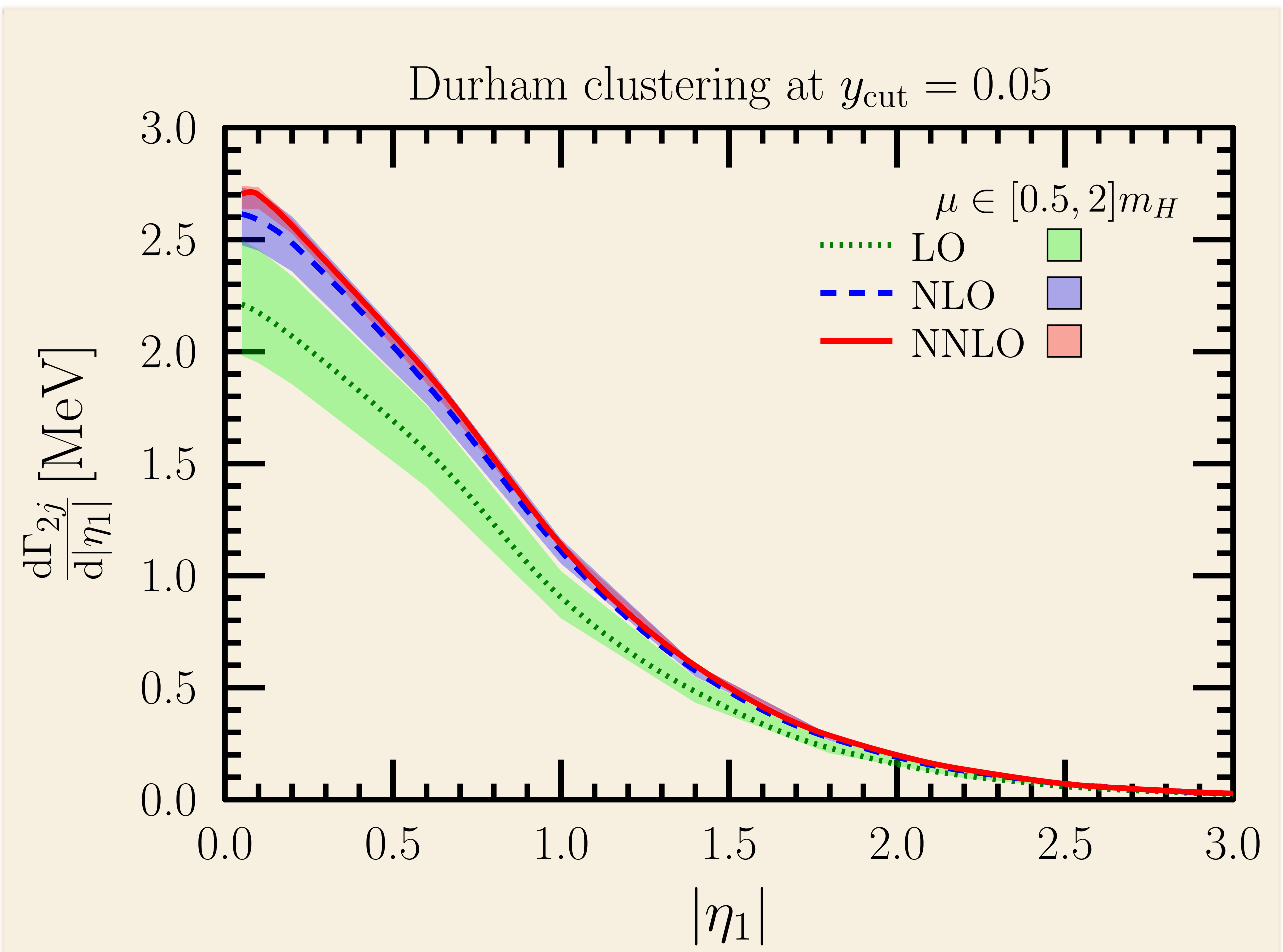
$$\Gamma^{\text{NNLO}}_{H\to b\bar{b}}(\mu = m_H) = \Gamma^{\text{LO}}_{H\to b\bar{b}}(\mu = m_H)\left[1 - \left(\frac{\alpha_s}{\pi}\right)5.666667 - \left(\frac{\alpha_s}{\pi}\right)^2 29.149 + \mathcal{O}(\alpha_s^3)\right]$$

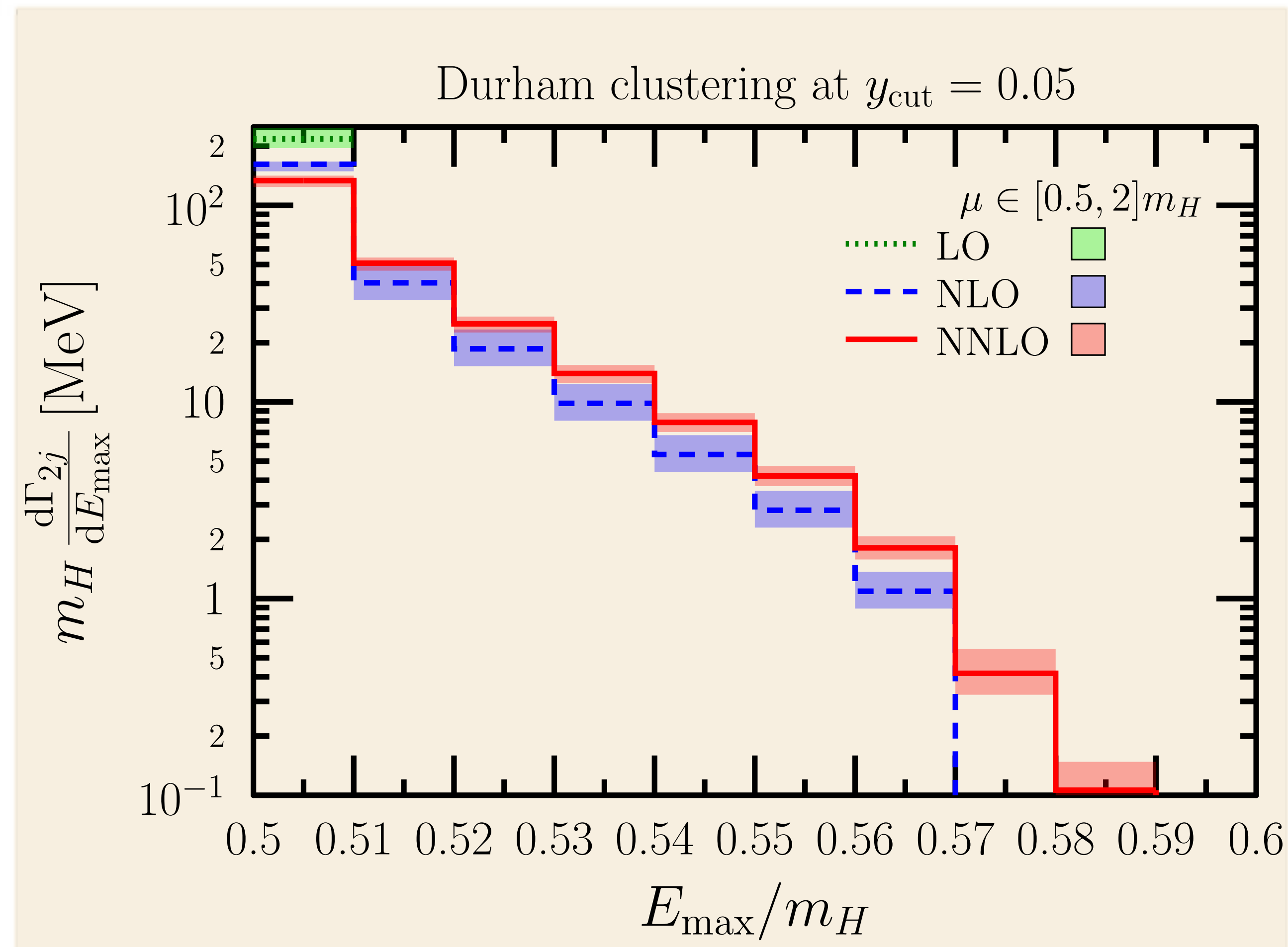


Scale dependence of the inclusive decay rate Γ(H -> b$\bar{\text{b}}$)

analytic: K.G. Chetyrkin hep-ph/9608318

Example: $H \to b\bar{b}$





rapidity distribution energy spectrum

of the leading jet in the rest frame of the Higgs boson.

jets are clustered using the Durham algorithm with $y_{cut} = 0.05$

Constrained subtractions

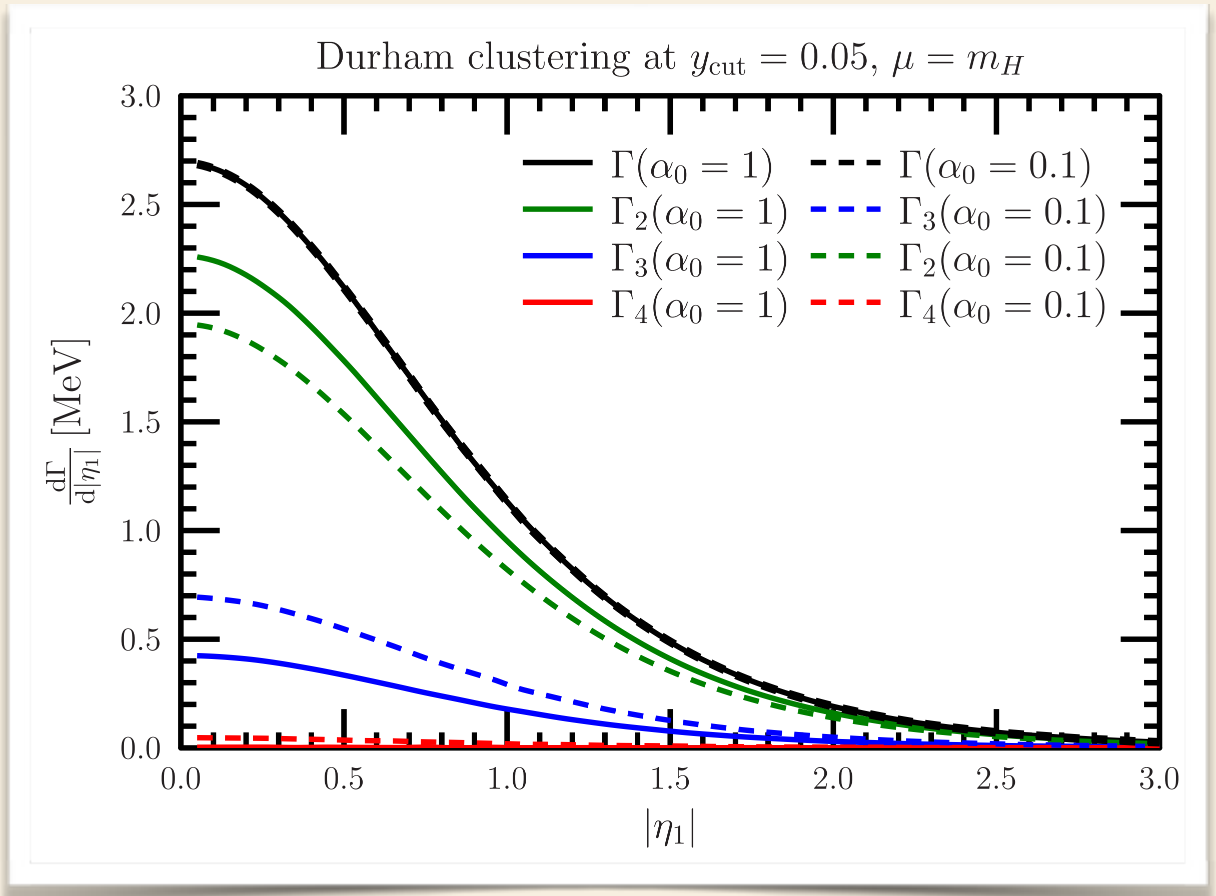
We can constrain subtractions near singular regions (α_0<1)

Poles cancel numerically (α_0 = 0.1)

$$\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} + \sum \int \mathrm{d}\sigma^{\mathrm{A}} = \frac{5.4\times10^{-8}}{\epsilon^4} + \frac{3.9\times10^{-5}}{\epsilon^3} + \frac{3.3\times10^{-3}}{\epsilon^2} + \frac{6.7\times10^{-3}}{\epsilon} + \mathcal{O}(1)$$

$$Err\left(\sum \int \mathrm{d}\sigma^{\mathrm{A}}\right) = \frac{3.1\times10^{-5}}{\epsilon^4} + \frac{5.0\times10^{-4}}{\epsilon^3} + \frac{8.1\times10^{-3}}{\epsilon^2} + \frac{7.7\times10^{-2}}{\epsilon} + \mathcal{O}(1)$$

Predictions remain the same:



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Conclusions

- ✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)
- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)
- ✓ Demonstrated the cancellation of ϵ-poles
- ✓ First application: Higgs-boson decay into a b-quark pair

Appendix

Example: $e^+e^- \to m(=3)$ jets at $\mu^2 = s$

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

$$\mathrm{d}\sigma_3^{\mathrm{VV}} = \mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{F}inite\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right)$$

$$\mathcal{P}oles\left(A_3^{(2\times 0)} + A_3^{(1\times 1)}\right) + \mathcal{P}oles \sum \int \mathrm{d}\sigma^{\mathrm{A}} =$$ 200k Mathematica lines

= zero numerically in any phase space point:

```
                0.           2    0. nf
        0. + --- + 0. Nc  + ----- + 0. Nc nf
                2               Nc
              Nc
Out[1]= ------------------------------------ +
                                   2
                                  e

                0.           2    0. nf
        0. + --- + 0. Nc  + ----- + 0. Nc nf
                2               Nc
              Nc                                      0
----------------------------------------------- + O[e]
                                  e
```

Example: $e^+e^- \to m(=3)$ jets at $\mu^2 = s$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ {\rm d}\sigma_m^{\rm VV} + \int_2 \left[{\rm d}\sigma_{m+2}^{\rm RR,A_2} - {\rm d}\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[{\rm d}\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 {\rm d}\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_3^{\rm VV} = \mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{F}inite\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right)$$

$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles \sum \int {\rm d}\sigma^{\rm A} =$ 200k Mathematica lines

= zero analytically according to C. Duhr

Message:

$$\sigma_3^{\rm NNLO} = \int_3 \left\{ {\rm d}\sigma_3^{\rm VV} + \sum \int {\rm d}\sigma^{\rm A} \right\}_{\epsilon=0} J_3$$

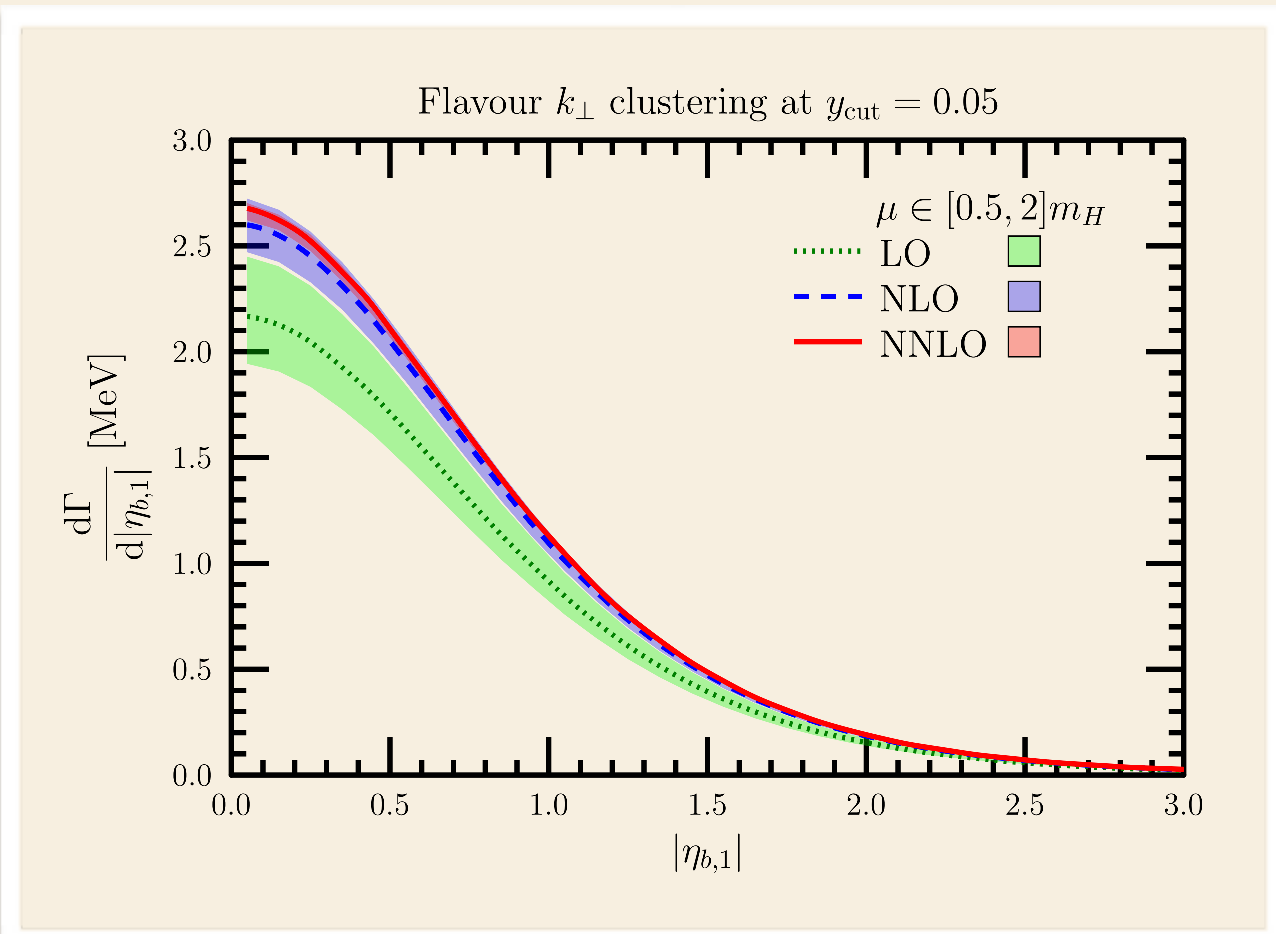
indeed finite in d=4 dimensions

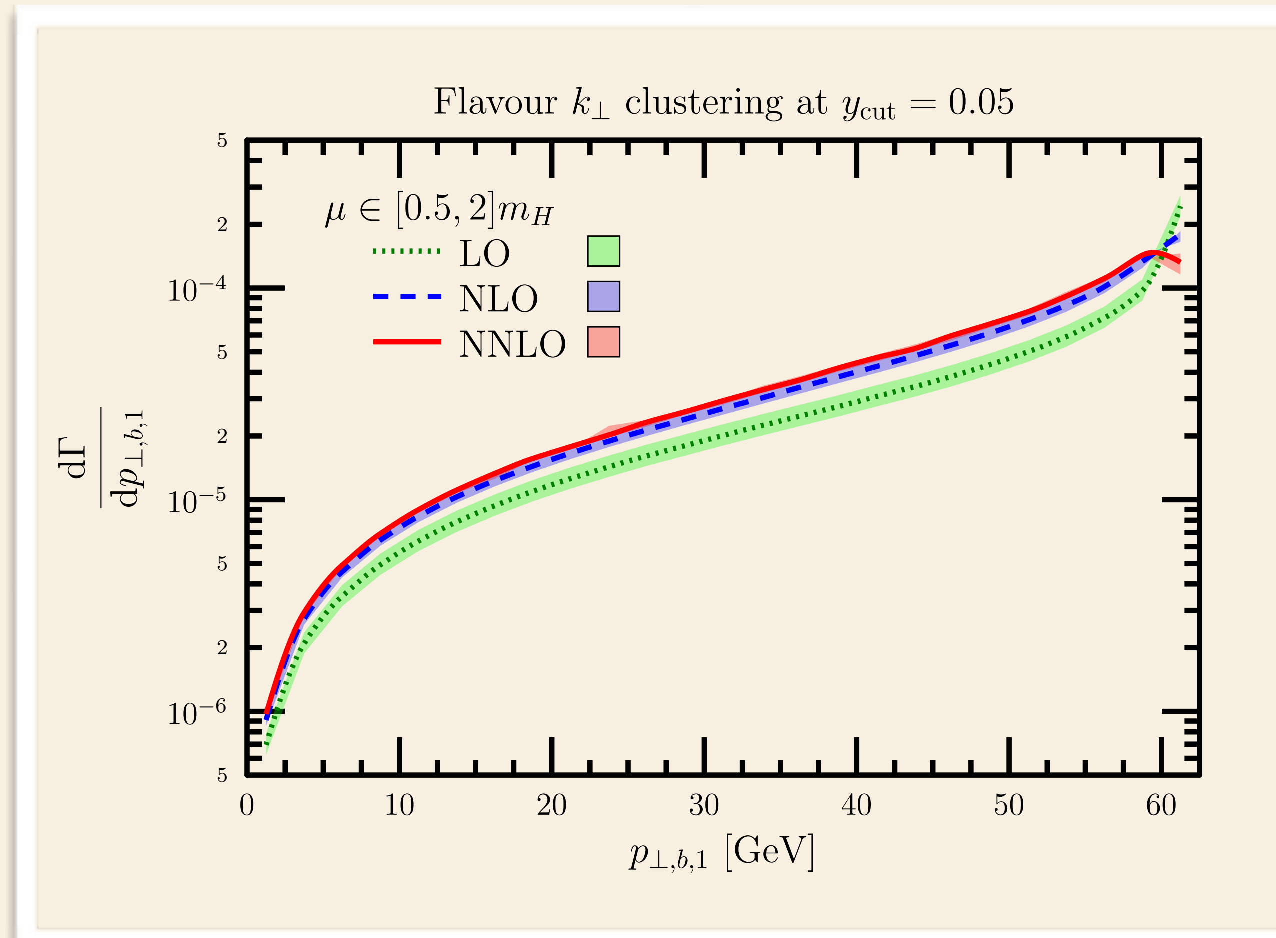
IR safe predictions w flavour-$k_\perp$

rapidity distribution / p_T spectrum

of the leading b-jet in the rest frame of the Higgs boson.

jets are clustered using the flavour-$k_\perp$ algorithm with $y_{cut} = 0.05$





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