Fully differential decay rate of a SM Higgs boson into a b-pair at NNLO

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Higgs boson has been discovered

\[ m_H \, [\text{GeV}] = 125.5^{+0.2}_{-0.2} \text{stat}^{+0.6}_{-0.6} \text{syst} \, (\text{ATLAS} \, 2013) \]

\[ 125.7^{+0.3}_{-0.3} \text{stat}^{+0.3}_{-0.3} \text{syst} \, (\text{CMS} \, 2013) \]

All measured properties are consistent with SM expectations within experimental uncertainties

- spin zero
- parity +
- couples to masses of W and Z (with \( c_V = 1 \) within experimental uncertainty)

Yet it still could be the first element of an extended Higgs sector (e.g. SUSY neutral Higgs)

Distinction requires high-precision prediction for both production and decay
Example: $pp \rightarrow H + X \rightarrow b\bar{b} + X$ in PT

- $\Gamma_H \text{[MeV]} = 4.07 \pm 0.16_{\text{teo}}$

  $\Rightarrow$ can use the narrow width approximation

$$\frac{d\sigma}{dO_{b\bar{b}}} = \left[ \sum_{n=0}^{\infty} \frac{dd^2\sigma_{pp \rightarrow H + X}^{(n)}}{dp_{\perp}, H d\eta_H} \right] \times \left[ \frac{\sum_{n=0}^{\infty} \frac{d\Gamma_{H \rightarrow b\bar{b}}^{(n)}}{\Gamma_{H \rightarrow b\bar{b}}}}{\sum_{n=0}^{\infty} \Gamma_{H \rightarrow b\bar{b}}^{(n)}} \right] \times \text{Br}(H \rightarrow b\bar{b})$$

- Known up to NNLO
- This talk: up to NNLO
- Known with 1% accuracy
Including up to NNLO corrections for production and decay:

\[
\frac{d\sigma}{dO_{b\bar{b}}} = \left[ \frac{d^2\sigma_{pp\rightarrow H+X}^{(0)}}{dp_{\perp},Hd\eta_H} \frac{d\Gamma_{H\rightarrow b\bar{b}}^{(0)}}{dO_{b\bar{b}}} + \frac{d\Gamma_{H\rightarrow b\bar{b}}^{(1)}}{dO_{b\bar{b}}} + \frac{d\Gamma_{H\rightarrow b\bar{b}}^{(2)}}{dO_{b\bar{b}}} \right] \times \text{Br}(H \rightarrow b\bar{b})
\]
Method
\[ \sigma_{NNLO}^{m} = \sigma_{m+2}^{RR} + \sigma_{m+1}^{RV} + \sigma_{m}^{VV} \]

\[ \equiv \int_{m+2} d\sigma_{m+2}^{RR} J_{m+2} + \int_{m+1} d\sigma_{m+1}^{RV} J_{m+1} + \int_{m} d\sigma_{m}^{VV} J_{m} \]
\[ \sigma_{\text{NNLO}}^m = \sigma_{m+2}^{\text{RR}} + \sigma_{m+1}^{\text{RV}} + \sigma_{m}^{\text{VV}} = \int_{m+2} \text{d}\sigma_{m+2}^{\text{RR}} J_{m+2} + \int_{m+1} \text{d}\sigma_{m+1}^{\text{RV}} J_{m+1} + \int_{m} \text{d}\sigma_{m}^{\text{VV}} J_{m} \]

- the three contributions are separately divergent in \( d = 4 \) dimensions:
  - in \( \sigma^{\text{RR}} \) kinematical singularities as one or two partons become unresolved yielding \( \epsilon \)-poles at \( O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1}) \) after integration over phase space, no explicit \( \epsilon \)-poles
  - in \( \sigma^{\text{RV}} \) kinematical singularities as one parton becomes unresolved yielding \( \epsilon \)-poles at \( O(\epsilon^{-2}, \epsilon^{-1}) \) after integration over phase space + explicit \( \epsilon \)-poles at \( O(\epsilon^{-2}, \epsilon^{-1}) \)
  - in \( \sigma^{\text{VV}} \) explicit \( \epsilon \)-poles at \( O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1}) \)
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How to combine to obtain finite cross section?
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CoLoRFu1NNLO
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CoLoRFuL NNLO

Completely Local Subtractions for Fully Differential Predictions@NNLO
Structure

of subtractions is governed by the jet functions

\[
\sigma^{\text{NNLO}}_m = \sigma^{\text{RR}}_{m+2} + \sigma^{\text{RV}}_{m+1} + \sigma^{\text{VV}}_m = \sigma_{m+2} + \sigma_{m+1} + \sigma_m
\]

\[
d\sigma_{m+2} = \left\{ d\sigma^{\text{RR}}_{m+2} J_{m+2} - d\sigma^{\text{RR},A_2}_{m+2} J_m - \left[ d\sigma^{\text{RR},A_1}_{m+2} J_{m+1} - d\sigma^{\text{RR},A_{12}}_{m+2} J_m \right] \right\}_{\epsilon=0}
\]

\[
d\sigma_{m+1} = \left\{ d\sigma^{\text{RV}}_{m+1} + \int_1 \left[ d\sigma^{\text{RR},A_1}_{m+2} \right] J_{m+1} - \left[ d\sigma^{\text{RV},A_1}_{m+1} + \left( \int_1 d\sigma^{\text{RR},A_1}_{m+2} \right)^{A_1} \right] J_m \right\}_{\epsilon=0}
\]

\[
d\sigma_{m} = \left\{ d\sigma^{\text{VV}}_{m} + \int_2 \left[ d\sigma^{\text{RR},A_2}_{m+2} - d\sigma^{\text{RR},A_{12}}_{m+2} \right] + \int_1 \left[ d\sigma^{\text{RV},A_1}_{m+1} + \left( \int_1 d\sigma^{\text{RR},A_1}_{m+2} \right)^{A_1} \right] \right\}_{\epsilon=0} J_m
\]

- RR,A_2 regularizes doubly-unresolved limits
- RR,A_1 regularizes singly-unresolved limits
- RR,A_{12} removes overlapping subtractions
- RV,A_1 regularizes singly-unresolved limits
Use known ingredients

• Universal IR structure of QCD (squared) matrix elements
  
  – $\epsilon$-poles of one- and two-loop amplitudes
  – soft and collinear factorization of QCD matrix elements

  tree-level 3-parton splitting, double soft current:
    V. Del Duca, A. Frizzo, F. Maltoni, 1999, D. Kosower, 2002
  
  one-loop 2-parton splitting, soft gluon current:
    L.J. Dixon, D.C. Dunbar, D.A. Kosower 1994
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  \begin{itemize}
    \item J.M. Campbell, E.W.N. Glover 1997, S. Catani, M. Grazzini 1998
    \item V. Del Duca, A. Frizzo, F. Maltoni, 1999, D. Kosower, 2002
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- Extension over whole phase space using momentum mappings (not unique):
  \[ \left\{ p \right\}_{n+s} \rightarrow \left\{ \tilde{p} \right\}_n \]
Momentum mappings

\[ \{p\}_{n+s} \rightarrow \{\tilde{p}\}_n \]

- Implement exact momentum conservation
- Recoil distributed democratically
- Different mappings for collinear and soft limits

- Lead to phase-space factorization
- Can be generalized to any number \( s \) of unresolved patrons trivially
Momentum mappings

define subtractions

\[
\sigma_{\text{NNLO}} = \sigma_{m+2}^{\text{RR}} + \sigma_{m+1}^{\text{RV}} + \sigma_{m}^{\text{VV}} = \sigma_{m+2}\text{NNLO} + \sigma_{m+1}\text{NNLO} + \sigma_{m}\text{NNLO}
\]

\[
\sigma_{m+2}^{\text{NNLO}} = \int_{m+2} \left\{ d\sigma_{m+2}^{\text{RR}} J_{m+2} - d\sigma_{m+2}^{\text{RR},A_2} J_{m} - \left( d\sigma_{m+2}^{\text{RR},A_1} J_{m+1} - d\sigma_{m+2}^{\text{RR},A_{12}} J_{m} \right) \right\}
\]

\[
\sigma_{m+1}^{\text{NNLO}} = \int_{m+1} \left\{ \left( d\sigma_{m+1}^{\text{RV}} + \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) J_{m+1} - \left[ d\sigma_{m+1}^{\text{RV},A_1} + \left( \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) A_1 \right] J_{m} \right\}
\]

\[
\sigma_{m}^{\text{NNLO}} = \int_{m} \left\{ d\sigma_{m}^{\text{VV}} + \int_{2} \left( d\sigma_{m+2}^{\text{RR},A_2} - d\sigma_{m+2}^{\text{RR},A_{12}} \right) + \int_{1} \left[ d\sigma_{m+1}^{\text{RV},A_1} + \left( \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) A_1 \right] \right\} J_{m}
\]

Z. Nagy, G. Somogyi, ZT hep-ph/0702273
Kinematic singularities cancel in $RR$

$R = \text{subtraction/}RR$

Single collinear limit $e^+e^- \rightarrow \bar{q}qggg$

Double soft limit $e^+e^- \rightarrow \bar{q}qggg$

$y_{45} = 10^{-6}$

$y_{45} = 10^{-8}$

$y_{45} = 10^{-10}$
Kinematic singularities cancel in RV

\[ R = \frac{\text{subtraction}}{RV+RR,A_1} \]
Regularized RR & RV contributions can now be computed by numerical Monte Carlo integrations (implementation for general m in progress)

\[
\sigma_{\text{NNLO}} = \sigma_{m+2}^{\text{RR}} + \sigma_{m+1}^{\text{RV}} + \sigma_{m}^{\text{VV}} = \sigma_{m+2}^{\text{NNLO}} + \sigma_{m+1}^{\text{NNLO}} + \sigma_{m}^{\text{NNLO}}
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\[
\sigma_{m+2}^{\text{NNLO}} = \int_{m+2} \left\{ d\sigma_{m+2}^{\text{RR}} J_{m+2} - d\sigma_{m+2}^{\text{RR},A_2} J_{m} - \left( d\sigma_{m+2}^{\text{RR},A_1} J_{m+1} - d\sigma_{m+2}^{\text{RR},A_12} J_{m} \right) \right\}
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\[
\sigma_{m+1}^{\text{NNLO}} = \int_{m+1} \left\{ \left( d\sigma_{m+1}^{\text{RV}} + \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) J_{m+1} - \left[ d\sigma_{m+1}^{\text{RV},A_1} + \left( \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) A_1 \right] J_{m} \right\}
\]

\[
\sigma_{m}^{\text{NNLO}} = \int_{m} \left\{ d\sigma_{m}^{\text{VV}} + \int_{2} \left( d\sigma_{m+2}^{\text{RR},A_2} - d\sigma_{m+2}^{\text{RR},A_12} \right) + \int_{1} \left[ d\sigma_{m+1}^{\text{RV},A_1} + \left( \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) A_1 \right] \right\} J_{m}
\]

Z. Nagy, G. Somogyi, ZT hep-ph/0702273
Difficulty
Integrated approximate xsections

\[ \sigma_{\text{NNLO}} = \sigma_{m+2}^{\text{RR}} + \sigma_{m+1}^{\text{RV}} + \sigma_{m}^{\text{VV}} = \sigma_{m+2}^{\text{NNLO}} + \sigma_{m+1}^{\text{NNLO}} + \sigma_{m}^{\text{NNLO}} \]

\[ \sigma_{m+2}^{\text{NNLO}} = \int_{m+2} \left\{ \frac{d\sigma_{m+2}^{\text{RR}}}{d\sigma_{m+2}^{\text{RR}} J_{m+2}} - d\sigma_{m+2}^{\text{RR},A_2} J_{m} - \left( d\sigma_{m+2}^{\text{RR},A_1} J_{m+1} - d\sigma_{m+2}^{\text{RR},A_{12}} J_{m} \right) \right\} \]

\[ \sigma_{m+1}^{\text{NNLO}} = \int_{m+1} \left\{ \left( d\sigma_{m+1}^{\text{RV}} + \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) J_{m+1} - \left[ d\sigma_{m+1}^{\text{RV},A_1} + \left( \int_{1} d\sigma_{m+2}^{\text{RR},A_1} \right) A_1 \right] J_{m} \right\} \]

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After integrating over unresolved momenta & summing over unresolved colors and flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

\[ \int_{p} d\sigma_{r_{\text{RR}, A}} = I_{r_{p}}^{(0)} (\{p\}_{n}; \epsilon) \otimes d\sigma_{n}^{B} \]
Structure of insertion operators

- the color structures are independent of the precise definition of subtractions (momentum mappings), only subleading coefficients of $\epsilon$-expansion in kinematic functions may depend

- we computed all insertion operators analytically (defined in our subtraction scheme) up to $O(\epsilon^{-2})$ for arbitrary $m$
Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary $m$

- for $m=2$,
  - the insertion operators are independent of the kinematics (momenta are back-to-back, so MI's are needed at the endpoints only)
  - color algebra is trivial:
    $$T_1 T_2 = -T_1^2 = -T_2^2 = -C_F$$
  - so can demonstrate the cancellation of poles
Example: \( \mathcal{H} \rightarrow b \bar{b} \) at \( \mu = m_{\mathcal{H}} \)

\[
\sigma_{\text{NNLO}}^{m} = \int \left\{ d\sigma_{m}^{\text{VV}} + \int_{2} \left[ d\sigma_{m+2}^{\text{RR},A_{2}} - d\sigma_{m+2}^{\text{RR},A_{12}} \right] + \int_{1} \left[ d\sigma_{m+1}^{\text{RV},A_{1}} + \left( \int_{1} d\sigma_{m+2}^{\text{RR},A_{1}} \right)^{A_{1}} \right] \right\} J_{m}
\]

\[
d\sigma_{\mathcal{H} \rightarrow b \bar{b}}^{\text{VV}} = \left( \frac{\alpha_{s}(\mu^{2})}{2\pi} \right)^{2} d\sigma_{\mathcal{H} \rightarrow b \bar{b}}^{\text{B}} \left\{ 1 + \frac{2C_{F}^{2}}{\epsilon^{4}} + \left( \frac{11C_{A}C_{F}}{4} + 6C_{F}^{2} - \frac{C_{F}n_{f}}{2} \right) \frac{1}{\epsilon^{3}} \right. \\
+ \left[ \left( \frac{8}{9} + \frac{\pi^{2}}{12} \right) C_{A}C_{F} + \left( \frac{17}{2} - 2\pi^{2} \right) C_{F}^{2} - \frac{2C_{F}n_{f}}{9} \right] \frac{1}{\epsilon^{2}} \\
+ \left[ \left( - \frac{961}{216} + \frac{13\zeta_{3}}{2} \right) C_{A}C_{F} + \left( \frac{109}{8} - 2\pi^{2} - 14\zeta_{3} \right) C_{F}^{2} + \frac{65C_{F}n_{f}}{108} \right] \frac{1}{\epsilon} \right\}
\]

C. Anastasiou, F. Herzog, A. Lazopoulos, arXiv:0111.2368

\[
\sum_{A} \int d\sigma_{A} = \left( \frac{\alpha_{s}(\mu^{2})}{2\pi} \right)^{2} d\sigma_{\mathcal{H} \rightarrow b \bar{b}}^{\text{B}} \left\{ - \frac{2C_{F}^{2}}{\epsilon^{4}} - \left( \frac{11C_{A}C_{F}}{4} + 6C_{F}^{2} + \frac{C_{F}n_{f}}{2} \right) \frac{1}{\epsilon^{3}} \right. \\
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\]

Application
Example: $H \rightarrow b \bar{b}$

\[
\Gamma_{H \rightarrow b \bar{b}}^{\text{NNLO}}(\mu = m_H) = \Gamma_{H \rightarrow b \bar{b}}^{\text{LO}}(\mu = m_H) \left[ 1 - \left( \frac{\alpha_s}{\pi} \right)^2 5.666667 - \left( \frac{\alpha_s}{\pi} \right)^2 29.149 + \mathcal{O}(\alpha_s^3) \right]
\]

Scale dependence of the inclusive decay rate $\Gamma(H \rightarrow b \bar{b})$

Example: $H \rightarrow b\bar{b}$

- Rapidity distribution of the leading jet in the rest frame of the Higgs boson.
- Jets are clustered using the Durham algorithm with $y_{\text{cut}} = 0.05$. 

Durham clustering at $y_{\text{cut}} = 0.05$

- $\mu \in [0.5, 2]m_H$

| $|\eta_1|$ | $\frac{d^2\hat{\sigma}}{d|\eta_1|}$ [MeV] |
|---------|------------------|
| 0.0     | 3.0              |
| 0.5     | 2.5              |
| 1.0     | 2.0              |
| 1.5     | 1.5              |
| 2.0     | 1.0              |
| 2.5     | 0.5              |
| 3.0     | 0.0              |

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Durham clustering at $y_{\text{cut}} = 0.05$

- $\mu \in [0.5, 2]m_H$

- LO (dotted green line)
- NLO (dashed blue line)
- NNLO (solid red line)
Constrained subtractions

We can constrain subtractions near singular regions ($\alpha_0 < 1$).

Poles cancel numerically ($\alpha_0 = 0.1$).

$$d\sigma_{H\rightarrow bb}^{VV} + \sum \int d\sigma^A = \frac{5.4 \times 10^{-8}}{\epsilon^4} + \frac{3.9 \times 10^{-5}}{\epsilon^3} + \frac{3.3 \times 10^{-3}}{\epsilon^2} + \frac{6.7 \times 10^{-3}}{\epsilon} + O(1)$$

$$\text{Err}\left(\sum \int d\sigma^A\right) = \frac{3.1 \times 10^{-5}}{\epsilon^4} + \frac{5.0 \times 10^{-4}}{\epsilon^3} + \frac{8.1 \times 10^{-3}}{\epsilon^2} + \frac{7.7 \times 10^{-2}}{\epsilon} + O(1)$$

Predictions remain the same:
Conclusions
✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)
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✓ Subtractions are

✓ fully local

✓ exact and explicit in color (using color state formalism)

✓ Demonstrated the cancellation of $\epsilon$-poles

✓ First application: Higgs-boson decay into a b-quark pair
Appendix
Example: $e^+e^- \rightarrow m(=3) \text{ jets at } \mu^2 = s$

$$\sigma_{NNLO}^m = \int \left\{ d\sigma_{VV}^m + \int_2 \left[ d\sigma_{m+2}^{RR,A_2} - d\sigma_{m+2}^{RR,A_{12}} \right] + \int_1 \left[ d\sigma_{m+1}^{RV,A_1} + \left( \int d\sigma_{m+2}^{RR,A_1} \right)^{A_1} \right] \right\} J_m$$

$$d\sigma_{VV}^3 = \text{Poles}(A_3^{(2\times0)} + A_3^{(1\times1)}) + \text{Finite}(A_3^{(2\times0)} + A_3^{(1\times1)})$$

$$\text{Poles}(A_3^{(2\times0)} + A_3^{(1\times1)}) + \text{Poles} \sum \int d\sigma^A = 200k \text{ Mathematica lines}$$

= zero numerically in any phase space point:
Example: $e^+e^- \rightarrow m(=3) \text{ jets at } \mu^2 = s$

$$\sigma_{m}^{\text{NNLO}} = \int_m \left\{ \sigma_{m}^{\text{VV}} + \int_2 \left[ \sigma_{m+2}^{\text{RR},A_2} - \sigma_{m+2}^{\text{RR},A_{12}} \right] + \int_1 \left[ \sigma_{m+1}^{\text{RV},A_1} + \left( \int_1 \sigma_{m+2}^{\text{RR},A_1} \right)^{A_1} \right] \right\} J_m$$

$$\sigma_{3}^{\text{VV}} = \text{Poles}(A_3^{(2\times0)} + A_3^{(1\times1)}) + \text{Finite}(A_3^{(2\times0)} + A_3^{(1\times1)})$$

$$\text{Poles}(A_3^{(2\times0)} + A_3^{(1\times1)}) + \text{Poles} \sum \int \sigma^A = 200k \text{ Mathematica lines}$$

= zero analytically according to C. Duhr

Message:

$$\sigma_{3}^{\text{NNLO}} = \int_3 \{ \sigma_{3}^{\text{VV}} + \sum \int \sigma^A \}_{\epsilon=0} J_3$$

indeed finite in $d=4$ dimensions
IR safe predictions \( w \) flavour-\( k_\perp \)

rapidity distribution
of the leading b-jet in the rest frame of the Higgs boson.
jets are clustered using the flavour-\( k_\perp \) algorithm with \( y_{\text{cut}} = 0.05 \)

\[
\begin{align*}
\text{Flavour } k_\perp \text{ clustering at } y_{\text{cut}} = 0.05 \\
\frac{d \Gamma}{d |\eta|} \quad [\text{MeV}] \\
|\eta|, |l| \\
\mu \in [0.5, 2]m_H \\
& \text{LO} \quad \text{NLO} \quad \text{NNLO} \\
\end{align*}
\]

\[
\begin{align*}
\text{Flavour } k_\perp \text{ clustering at } y_{\text{cut}} = 0.05 \\
\frac{d \Gamma}{d p_{\perp,b,1}} \quad [\text{GeV}] \\
p_{\perp,b,1} \\
\mu \in [0.5, 2]m_H \\
& \text{LO} \quad \text{NLO} \quad \text{NNLO} \\
\end{align*}
\]