

THERMALIZATION OF A BOOST-INVARIANT NON ABELIAN PLASMA WITH BOUNDARY SOURCING

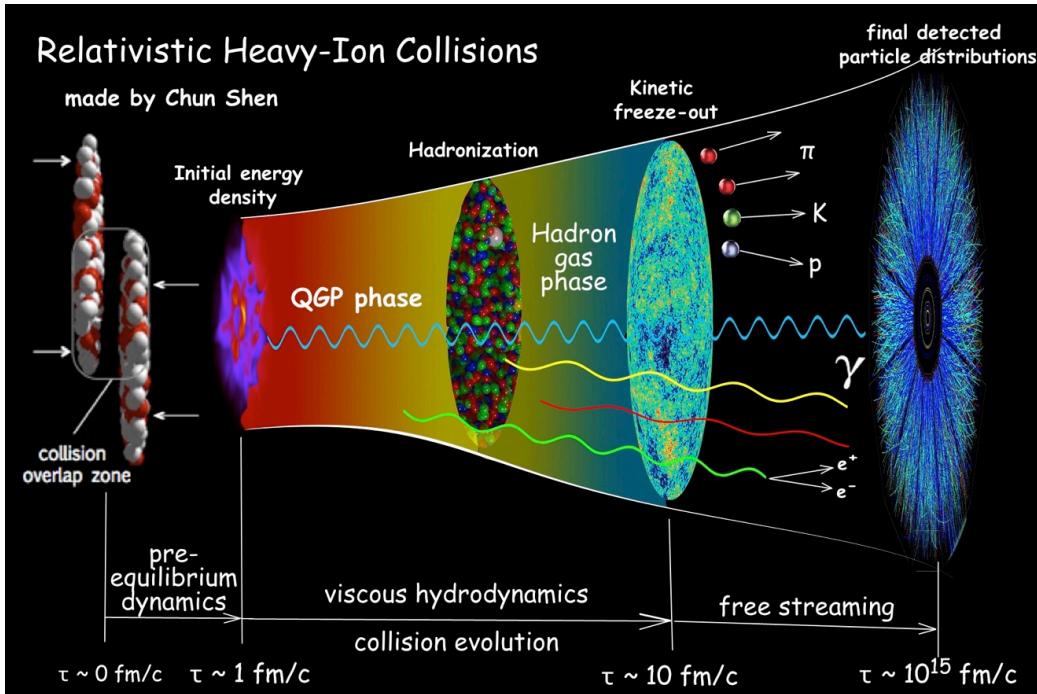
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Vienna

Based on JHEP 07 (2015) 053
in collaboration with L. Bellantuono, P. Colangelo, F. Giannuzzi

Quark-gluon Plasma (QGP)

- A new state of matter composed by deconfined quarks and gluons
- Believed to be produced by Heavy –Ion collision experiments
- Unknown the evolution from pre-equilibrium condition towards the final state



Indications from HI collision experiments (RHIC, LHC, ...)

- models based on near-ideal hydrodynamics suggest that after a pre equilibrium phase, hydrodynamics becomes applicable
- simulations that successfully reproduce the elliptic flow describing pressure anisotropy are consistent with an almost perfect fluid behaviour
- confirmed by the small viscosity/entropy ratio
- fast thermalization : $\tau \approx O(1 \text{ fm}/c)$

Questions

- what is the role of the underlying gauge theory, i.e. QCD?
- why QGP behaves like a perfect fluid ?
- can we understand thermalization and the typical times to reach it?

To be pointed out...

- perturbative QCD calculations give much larger values of viscosity/entropy ratio:
non perturbative QCD should be the relevant regime
- Lattice QCD can predict static properties of QGP but not the dynamics of HI collisions

Gauge/Gravity Duality

- original proposal (Maldacena 98)
Correspondence between a low energy supergravity approximation to a type IIB string theory on $\text{AdS}_5 \times S^5$ and a $N=4$ SYM theory living on its boundary
- subsequent development (Gubser, Polyakov, Klebanov, Witten)
duality between a gravity theory in $\text{AdS}_{d+1} \times C$ (C =compact manifold) and a CFT on the boundary of $\text{AdS}_{d+1}(M_d)$
- Holographic principle: two theories, one defined in a bulk space and the other on its boundary are equivalent (analogy with a hologram)

However...

- we do not know the dual theory of QCD (if any)
- QGP is a deconfined phase of QCD therefore duality could be applicable.



demonstrated by the successful calculation of the ratio η/s

Policastro, Son, Starinets PRD 2001

HI Collisions and Fluid/Gravity Duality

- Possible description: HI collisions give raise to a dense interacting medium
- relevant degrees of freedom are not individual partons
- a description of the system as a fluid might be appropriate

To be studied

- stress-energy tensor (energy density, pressures)
- entropy density



In gauge/gravity the hydrodynamic regime emerges as the late time behaviour of the expanding strongly coupled plasma

Perfect Fluid Hydrodynamics: Stress Energy Tensor under boost invariance assumption

Assumptions:

Bjorken PRD 1983

- boost invariance along the collision axis
- translational and rotational symmetries in the transverse plane

Coordinates & metrics

Proper Time τ , rapidity y , transverse coordinates x^\perp

$$x^0 = \tau \cosh y \quad x^1 = \tau \sinh y.$$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2.$$

Under the above assumptions, the stress-energy tensor $T_{\mu\nu}$ is diagonal and depends only on τ

Stress Energy Tensor in the perfect fluid hydrodynamics

$$\epsilon(\tau) = \frac{\text{const}}{\tau^{4/3}}$$

$$p_{\parallel}(\tau) = -\epsilon(\tau) - \tau \epsilon'(\tau)$$

$$p_{\perp}(\tau) = \epsilon(\tau) + \frac{\tau}{2} \epsilon'(\tau) .$$

Subleading corrections to the Stress Energy Tensor -> viscous hydrodynamics

$$\epsilon(\tau) = \frac{3\pi^4 \Lambda^4}{4(\Lambda\tau)^{4/3}} \left[1 - \frac{2c_1}{(\Lambda\tau)^{2/3}} + \frac{c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right]$$

$$p_{\parallel}(\tau) = \frac{\pi^4 \Lambda^4}{4(\Lambda\tau)^{4/3}} \left[1 - \frac{6c_1}{(\Lambda\tau)^{2/3}} + \frac{5c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right]$$

$$p_{\perp}(\tau) = \frac{\pi^4 \Lambda^4}{4(\Lambda\tau)^{4/3}} \left[1 - \frac{c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right] ,$$

Pressure ratio & anisotropy

$$\frac{p_{\parallel}}{p_{\perp}} = 1 - \frac{6c_1}{(\Lambda\tau)^{2/3}} + \frac{6c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right),$$

$$\frac{\Delta p}{\epsilon} = \frac{p_{\perp} - p_{\parallel}}{\epsilon} = 2 \left[\frac{c_1}{(\Lambda\tau)^{2/3}} + \frac{2c_1^2 - c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right]$$

$c_1 = \frac{1}{3\pi}$ and $c_2 = \frac{1 + 2\log 2}{18\pi^2}$

an effective temperature can be defined

$$\epsilon(\tau) = \frac{3}{4}\pi^4 T_{eff}(\tau)^4$$

Result

$$T_{eff}(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}} \left[1 - \frac{1}{6\pi(\Lambda\tau)^{2/3}} + \frac{-1 + \log 2}{36\pi^2(\Lambda\tau)^{4/3}} + \frac{-21 + 2\pi^2 + 51\log 2 - 24(\log 2)^2}{1944\pi^3(\Lambda\tau)^2} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^{8/3}}\right) \right]$$

Heller, Janik, Witaszczyk, PRD 85 (12) 126002

Heller, Janik, PRD 76 (07) 025027

Baier et al, JHEP 0804 (08) 100

How to mimic HI collisions: Introducing a perturbation (quench)



Duality



Operators in gauge theory

fields in $\text{AdS}_5 \times \text{S}^5$

$T_{\mu\nu}$

g_{MN}



modifications of the stress-energy tensor
are produced by modifications of the metric

the relation is provided by the procedure of
Holographic Renormalization

de Haro et al, Comm. Mat. Phys. 217 (01) 595
Kinoshita et al, Prog. Theor. Phys., 121 (09) 121

Holographic renormalization

Example: Fefferman-Graham coordinates:

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z)dx^\mu dx^\nu + dz^2}{z^2}.$$

The metric should be a solution of the Einstein's equations with negative cosmological constant

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 6g_{\mu\nu} = 0$$

Expansion near the boundary $z=0$

$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g_{\mu\nu}^{(4)}(x^\rho) + \dots .$$

Holographic renormalization:

$$\langle T_{\mu\nu}(x^\rho) \rangle = \frac{N_c^2}{2\pi^2} \cdot g_{\mu\nu}^{(4)}(x^\rho)$$

Two possibilities:

solution of the Einstein's equations \rightarrow corresponding $T_{\mu\nu}$ in gauge theory
 given $T_{\mu\nu}$ one can find the dual geometry solution of the Einstein's eqs



Criterion to select the right solution: absence of singularities

Distorting the boundary

procedure proposed by Chesler & Yaffe PRL 102 (09) 211601
PRD 82 (10) 026006

- a system described by a gauge theory in 4D is carried far from equilibrium
-> the 4D metric is deformed introducing a profile function $\gamma(\tau)$

$$ds^2 = -d\tau^2 + e^{\gamma(\tau)} dx_{\perp}^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$



- corresponding 5D metric of the dual theory in Eddington-Finkelstein coordinates

$$ds^2 = 2drd\tau - Ad\tau^2 + \Sigma^2 e^B dx_{\perp}^2 + \Sigma^2 e^{-2B} dy^2$$

- The functions A, B, Σ are obtained
- solving the Einstein eqs
with as boundary condition
for $r \rightarrow \infty$

$$\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0$$

$$\Sigma(\dot{B})' + \frac{3}{2} (\Sigma' \dot{B} + B' \dot{\Sigma}) = 0$$

$$A'' + 3B'\dot{B} - 12 \frac{\Sigma' \dot{\Sigma}}{\Sigma^2} + 4 = 0$$

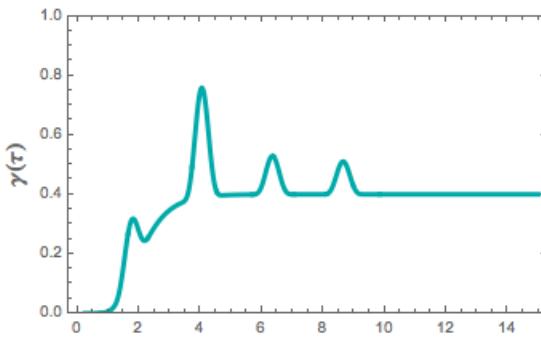
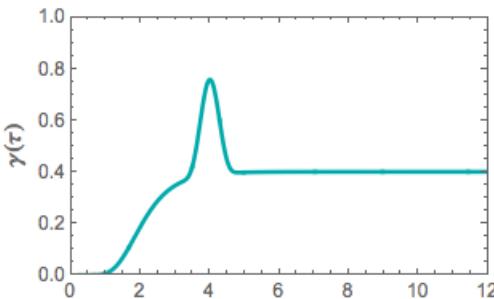
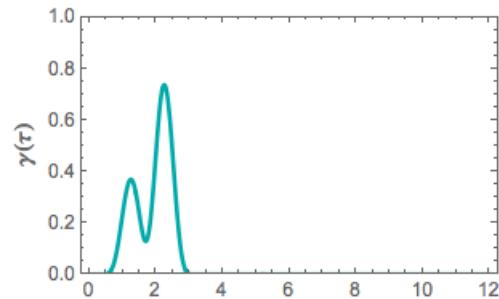
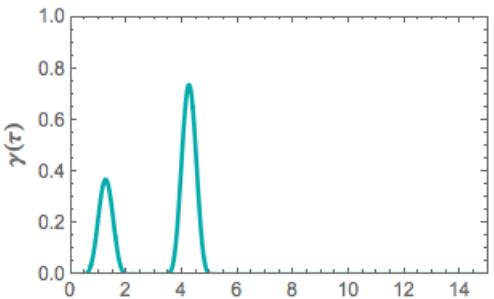
$$\ddot{\Sigma} + \frac{1}{2} (\dot{B}^2 \Sigma - A' \dot{\Sigma}) = 0$$

$$\Sigma'' + \frac{1}{2} B'^2 \Sigma = 0$$

- If the perturbation described by $\gamma(\tau)$ sets in at $\tau = \tau_i$ the condition that the metric is AdS_5 at τ_i is also imposed

Profiles

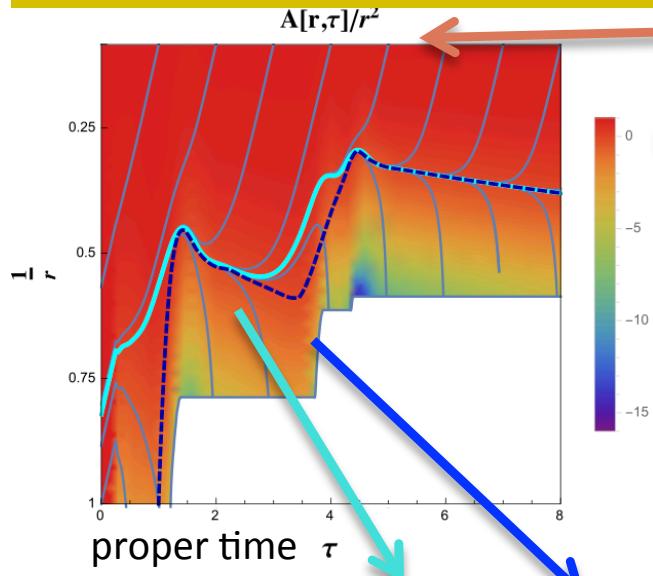
The effects of several profiles distorting the boundary can be considered.
Possible cases studied:



Results

Einstiens' eqs solutions for the metric vs asymptotic black brane solution

Model A (1)



$$ds^2 = 2drd\tau - Ad\tau^2 + \Sigma^2 e^B dx_1^2 + \Sigma^2 e^{-2B} dy^2$$

The red region corresponds to $A(r, \tau) = r^2$
 i.e. as for the black brane solution
 holding asymptotically for $r \rightarrow \infty$ and
 before the onset of the quench

two horizons

AH: apparent horizon= outermost null trapped surface

EH: event horizon=critical geodesics

separates outgoing radial null geodesics from infalling ones

Effective temperature from the horizon position:

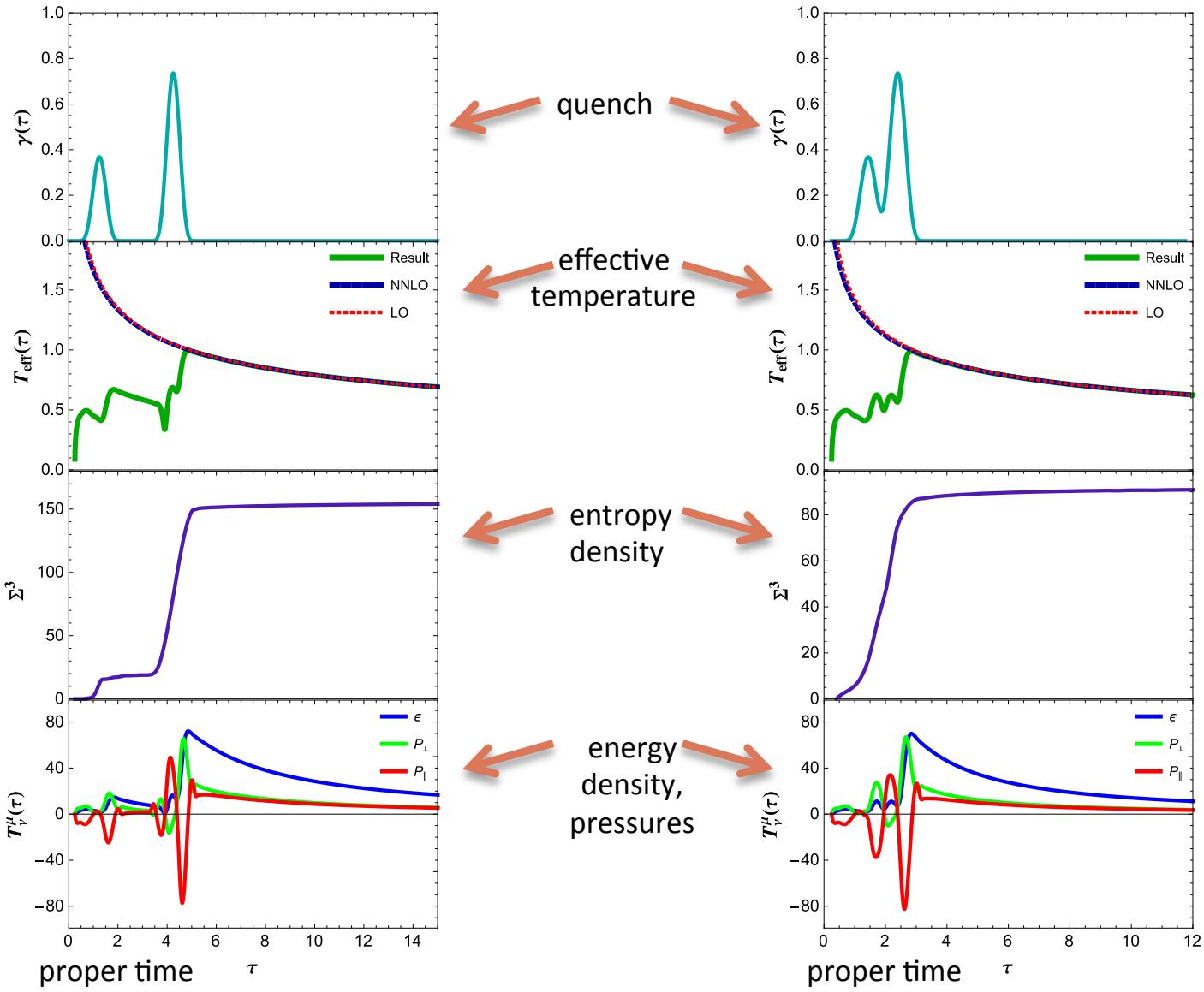
$$T_{\text{eff}}(\tau) = \frac{r_h(\tau)}{\pi}$$

The excision should in principle lie inside the horizon.

It is determined finding the region where the Einstein's equations can be solved

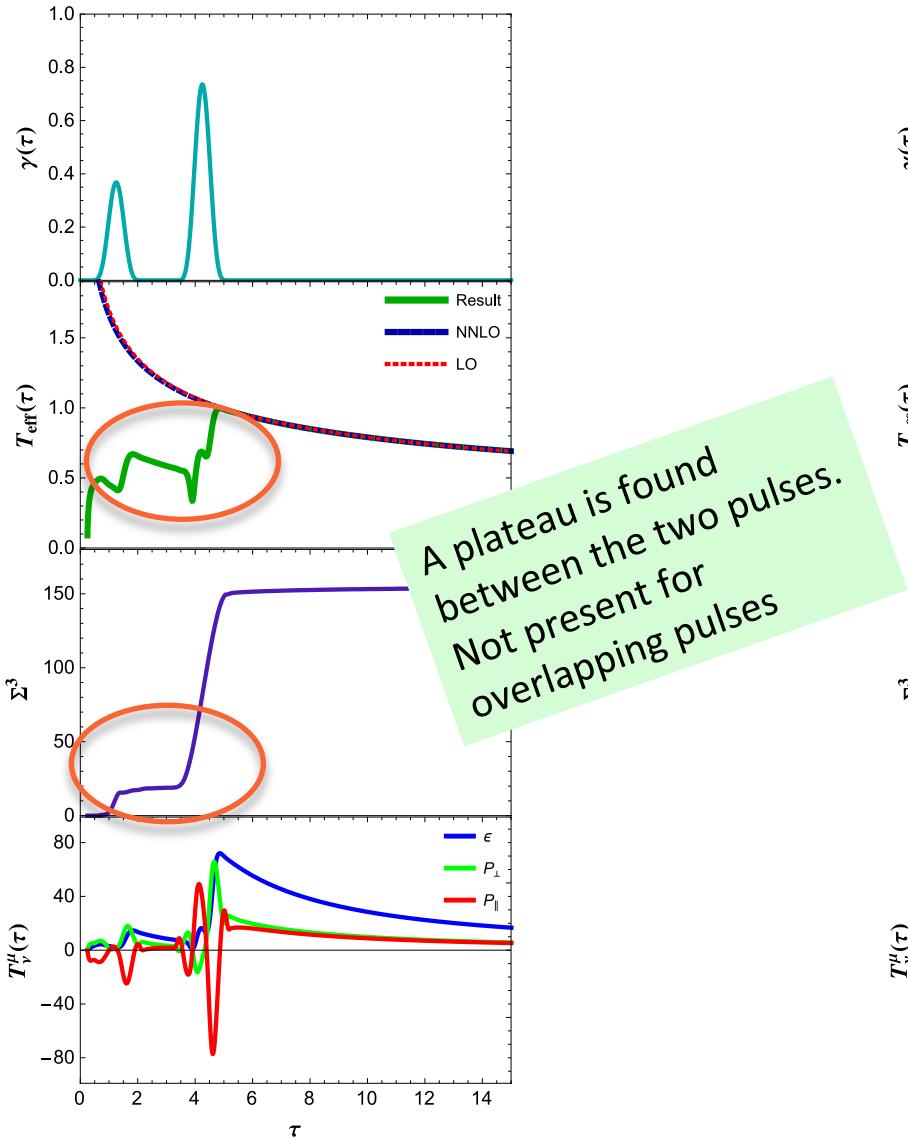
→ Outcome of the calculation!!

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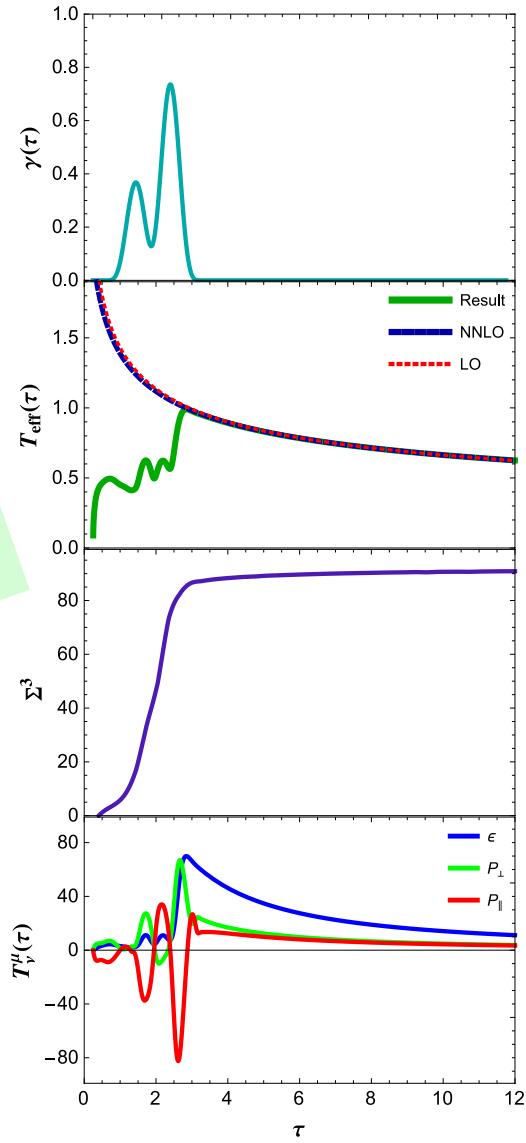


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Model A (1)

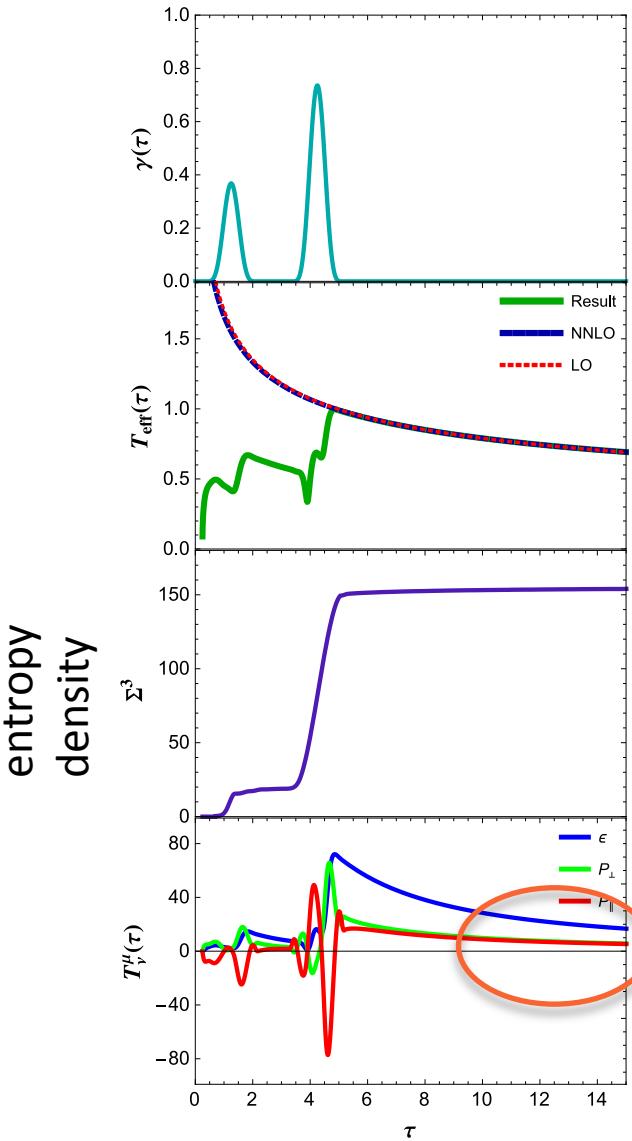


Model A (2)

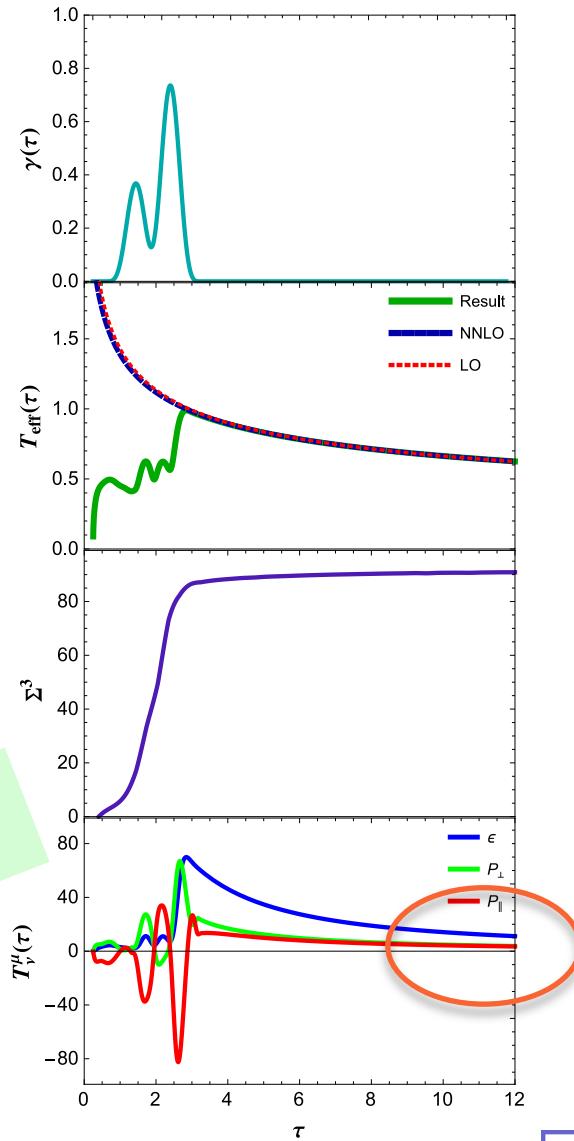


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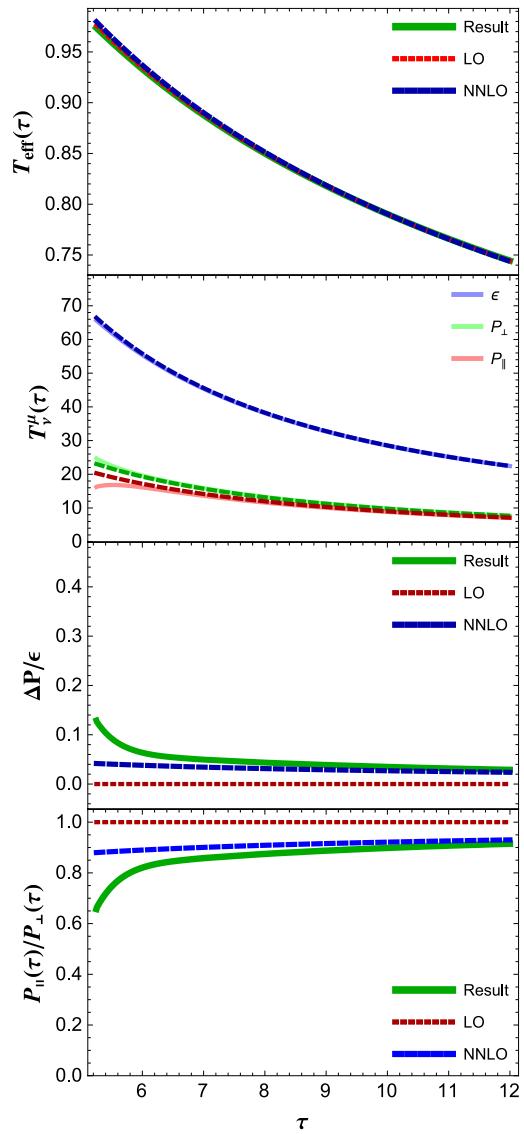
Model A (2)



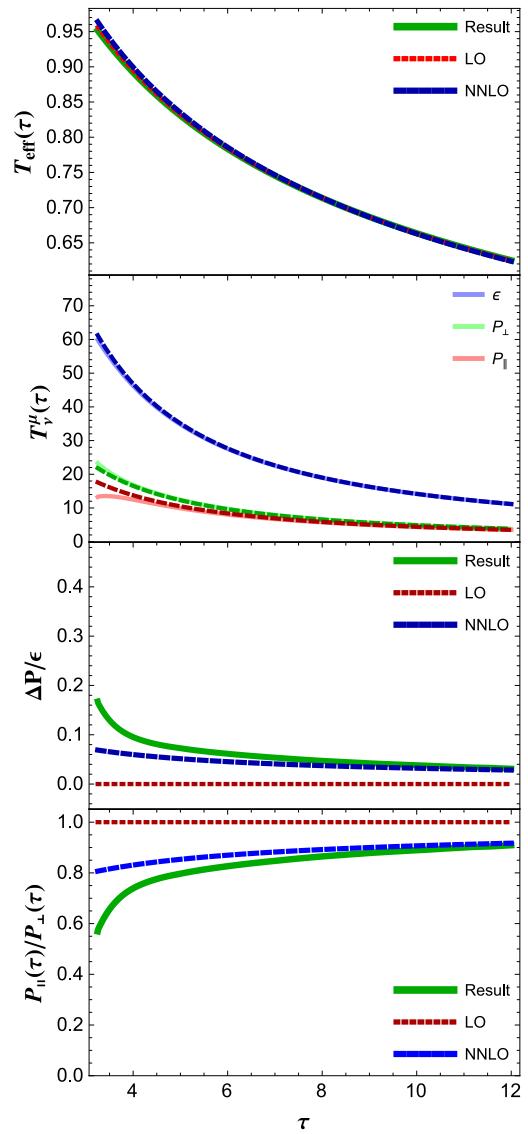
Late time dynamics
Thermalization?

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Model A (1)

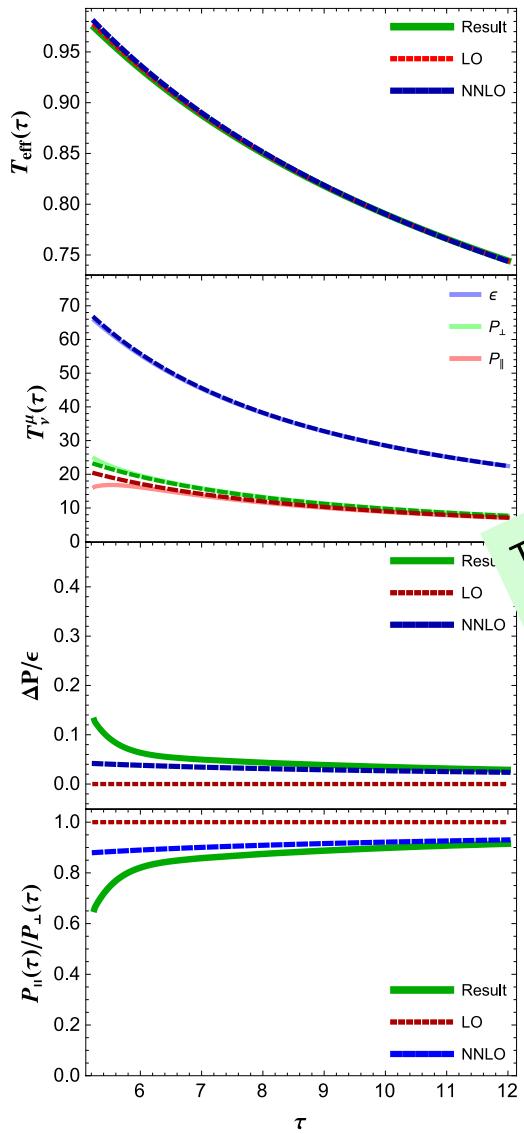


Model A (2)

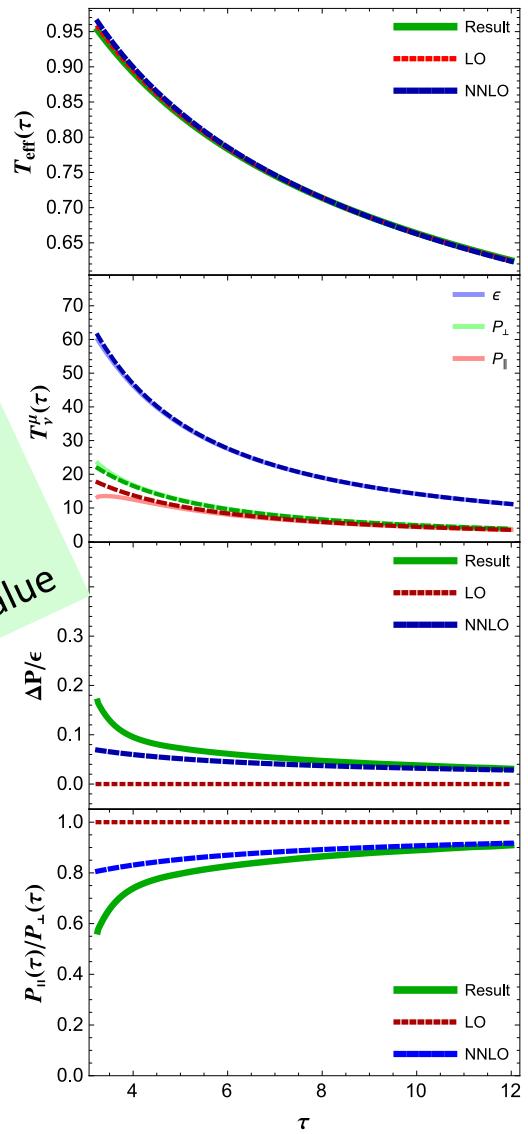


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Model A (1)



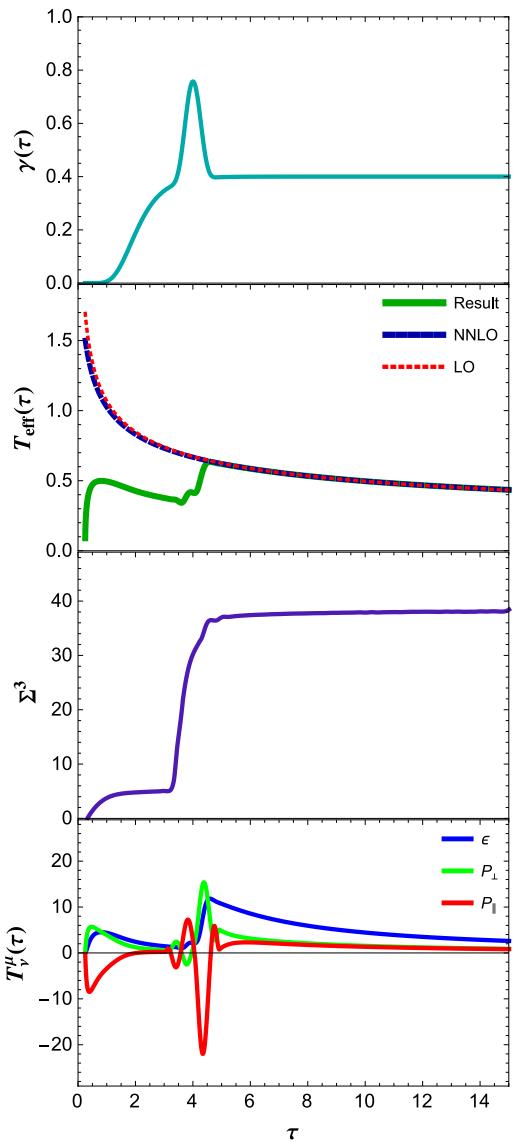
Model A (2)



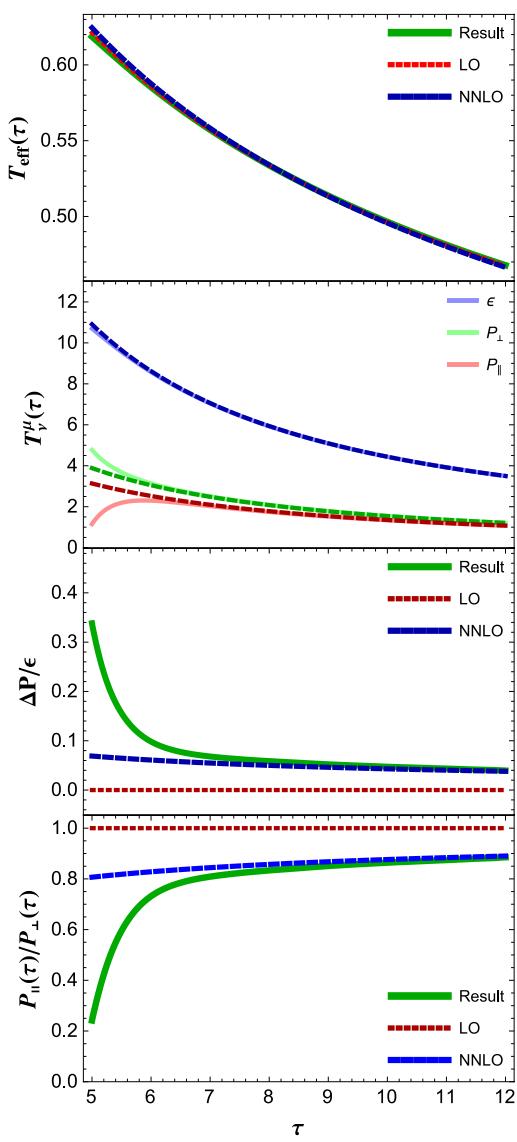
T_{eff} and ϵ reach their hydro value right after the quench (τ^*)
Pressures take a bit longer:
at τ_p they are within 5% of their hydro NNLO value

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Model B

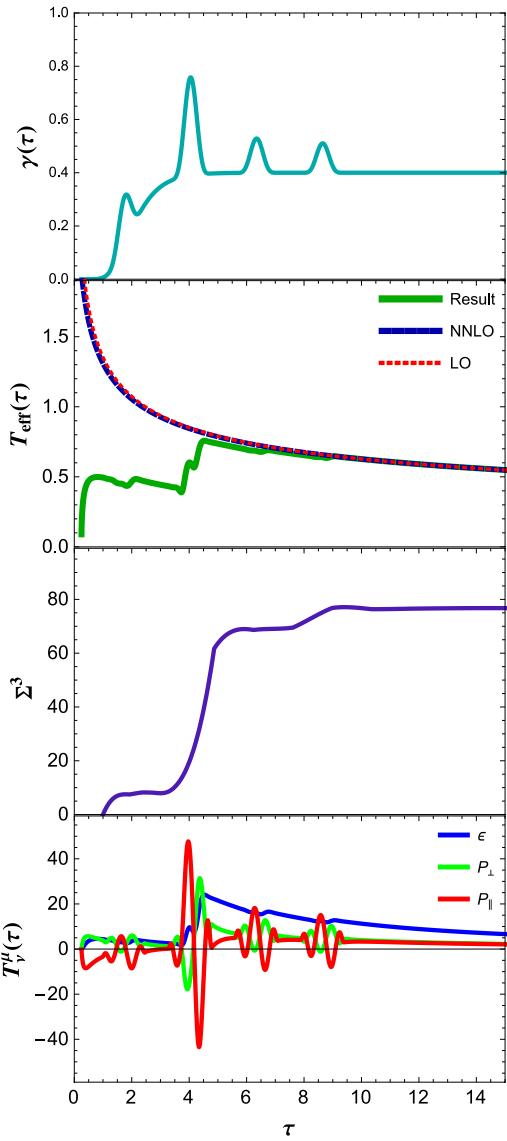


Model B

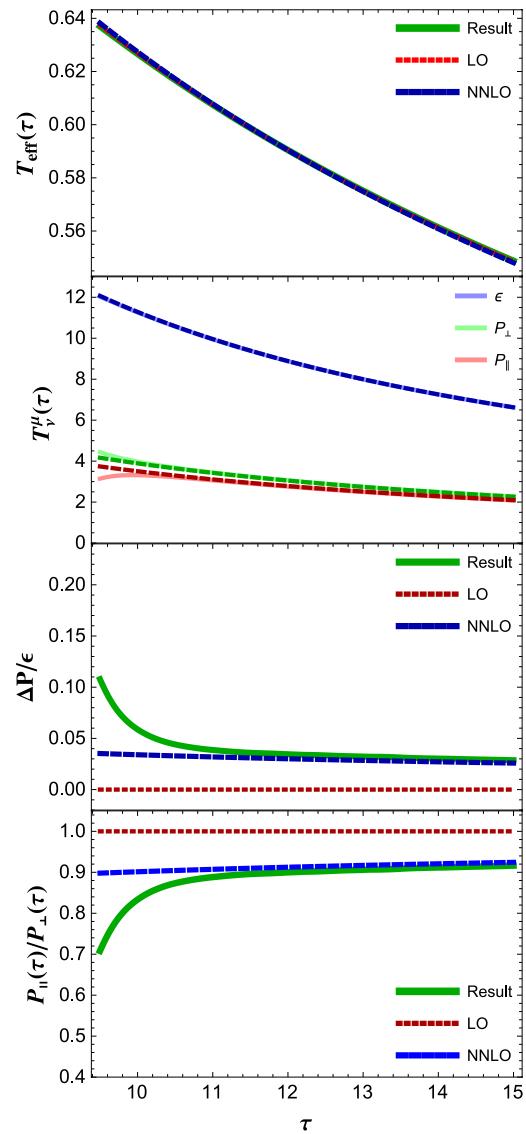


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Model C



Model C



Summary

A scale can be introduced fixing T_{eff} at the end of the quench to 500 MeV

model	τ^*	τ_p	Λ	$\Delta\tau = \tau_p - \tau^* \text{ (fm/c)}$
\mathcal{A} (1)	5.25	6.8	2.25	0.60
\mathcal{A} (2)	3.25	6.0	1.73	1.03
\mathcal{B}	5	6.74	1.12	0.42
\mathcal{C}	9.45	10.24	1.59	0.20



Thermalization times are a fraction of 1 fm

Conclusions

- present data on HI collisions indicate the occurrence of a collective flow
- regimes to be studied: fluid carried out of equilibrium / late time dynamics

highly non perturbative phenomenon

hydrodynamics seems
an appropriate description here

Gauge/Gravity duality may be used to mimic the situation

Main findings:

- the system evolves towards an equilibrium phase
- the position of the horizon can be computed -> effective temperature
- time scales for the hydrodynamic regime to set in : $\tau \approx O(1 \text{ fm})$