

# Entanglement Entropy and far-from-Equilibrium Energy Flow

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# Issues

## 1 Motivation

## 2 Energy flow in strongly correlated systems

- A universal regime of thermal transport
- Holographic Model
- Linearized solution

## 3 Information Flow

- Entanglement Entropy
- Mutual Information

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# Far from equilibrium dynamics

- Compute real time dynamics.
- Transition to hydrodynamics at late times.
- In strongly correlated systems:
  - Organizing principles out of equilibrium?
  - Universality classes?

## Examples:

- Thermalization of quark-gluon plasma.
- Quenches in condensed matter systems.
- Fluctuations in the fractional Hall effect.

- **Much simpler class of problems** → (non)equilibrium steady state.
- **Concrete setup:** Two heat reservoirs at different temperatures are put in contact at time  $t = 0$ .

$$\text{Energy density} \equiv \varepsilon(x, t=0) = \frac{C\pi}{12} [T_L^2 \Theta(-x) + T_R^2 \Theta(x)]$$



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# A universal regime of thermal transport

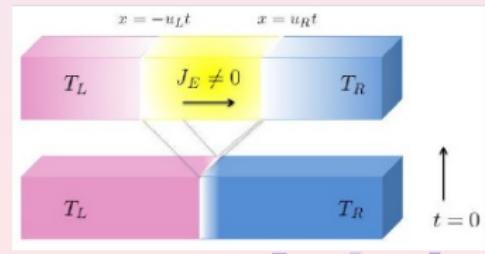
[Bernard, Doyon '12], [Bhaseen, Doyon, Lucas, Schalm '13]

- Two systems initially at equilibrium.
- Independently thermalized.
- Conservation equations and tracelessness:

$$\partial_x \langle T^{xx} \rangle = -\partial_t \langle T^{tx} \rangle = 0, \quad \langle T^{xx} \rangle = \langle T^{tt} \rangle$$

→ "shockwaves" emanating from interface:  $\begin{cases} \langle T^{tx} \rangle = F(x-t) - F(x+t) \\ \langle T^{tt} \rangle = F(x-t) + F(x+t) \end{cases}$

- In  $d+1$  dim:  $\langle T^{\mu\nu} \rangle = a_d T^{d+1} (\eta^{\mu\nu} + (d+1) u^\mu u^\nu)$



# Hydrodynamics

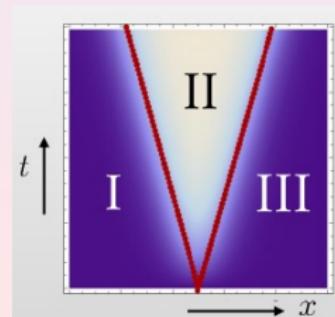
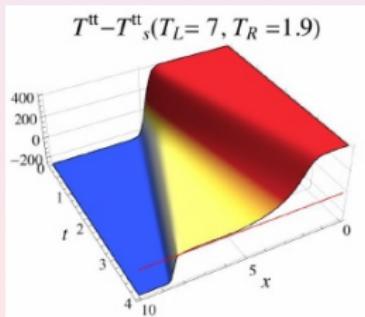
[Bernard, Doyon '12], [Bhaseen, Doyon, Lucas, Schalm '13]

Prediction for the **steady-state**:

- Lorentz-boosted energy density with  $u^\mu = (\cosh \theta, \sinh \theta, \vec{0})$  and temperature

$$T_L = T \cdot e^{\theta}, \quad T_R = T \cdot e^{-\theta} \quad \equiv \text{Doppler shift}$$

- Heat current:  $J_E \equiv \langle T^{tx} \rangle_{st} = a_d \frac{T_L^{d+1} - T_R^{d+1}}{u_L + u_R}$  with  $a_d \sim \frac{L^d}{G_N}$ .
- Temperature:  $T_{st} = \sqrt{T_L T_R}$
- Shockwave velocities:  $c_s^2 = \frac{1}{d} = u_L u_R$ .



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# Gravitational Dual

- Energy transport: Lorentz-boosted thermal distribution  
→ Gravity dual: **Boosted black brane**

$$S = \frac{1}{16\pi G} \int d^{d+1}x \sqrt{-g} \{R + 2\Lambda\} .$$

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= \frac{L^2}{z^2} \left[ \frac{dz^2}{f(z)} - f(z) (\cosh \theta dt - \sinh \theta dx)^2 + (\cosh \theta dx - \sinh \theta dt)^2 + dx_\perp^2 \right] \end{aligned}$$

where  $f(z) = 1 - \left(\frac{z}{z_h}\right)^d$  and  $z_h = \frac{d}{4\pi T}$ .

- $g_{\mu\nu}$  is a solution of e.o.m. as long as  $z_h \neq z_h(t, x)$  and  $\theta \neq \theta(t, x)$ .
- $(t, x)$ -dependent solutions → linearization

$$z_h(t, x) = z_h^{(0)} + \epsilon z_h^{(1)}(t, x) + \dots, \quad \theta(t, x) = \epsilon \theta^{(1)}(t, x) + \dots .$$

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# Linearization

$$z_h(t, x) = z_h^{(0)} + \epsilon z_h^{(1)}(t, x) + \dots, \quad \theta(t, x) = \epsilon \theta^{(1)}(t, x) + \dots$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \epsilon \begin{pmatrix} \delta g_{tt}(t, x, z) & \delta g_{tx}(t, x, z) & 0 & 0 \\ \delta g_{tx}(t, x, z) & \delta g_{xx}(t, x, z) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta g_{zz}(t, x, z) \end{pmatrix}$$

e.o.m.  $\rightarrow$

$$\begin{cases} 0 = 2\partial_t^2 z_h^{(1)}(t, x) - \partial_x^2 z_h^{(1)}(t, x) \\ 0 = -z_h^{(0)} \partial_x \theta^{(1)}(t, x) + 2\partial_t z_h^{(1)}(t, x) \end{cases}$$

**Solution:** Assuming some initial profile  $T_{ini}(x) \equiv T(t=0, x)$   $\rightarrow$

$$T(t, x) = \frac{3}{4\pi z_h(t, x)} = \frac{1}{2} \left[ T_{ini}\left(x + \frac{t}{\sqrt{2}}\right) + T_{ini}\left(x - \frac{t}{\sqrt{2}}\right) \right]$$

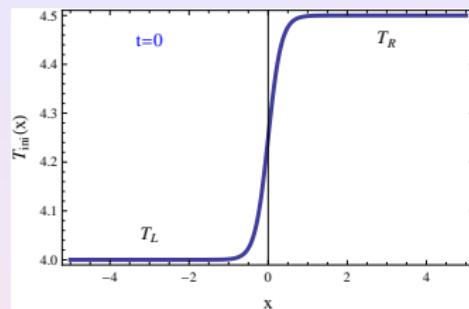
$$\theta(t, x) = -\epsilon \frac{2\sqrt{2}}{3} \pi z_h^{(0)} \left[ T_{ini}\left(x + \frac{t}{\sqrt{2}}\right) - T_{ini}\left(x - \frac{t}{\sqrt{2}}\right) \right]$$

with  $z_h^{(0)} = \frac{3}{2\pi(T_L + T_R)}$ . Note  $C_s = \frac{1}{\sqrt{2}}$ .

# Linearization

- Let us consider the initial profile:

$$T_{ini}(x) = \frac{T_R + T_L}{2} + \frac{T_R - T_L}{2} \tanh(\alpha x)$$



- Then one gets the  $t$ -dependent solution:

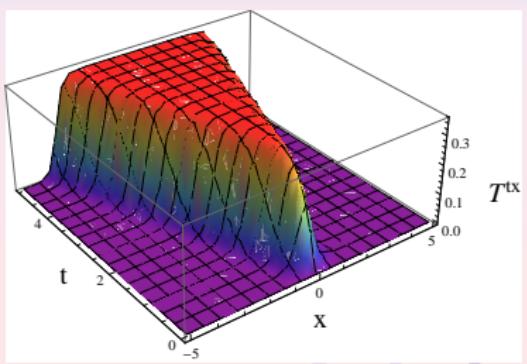
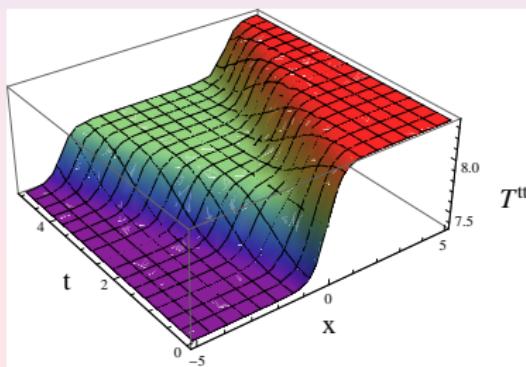
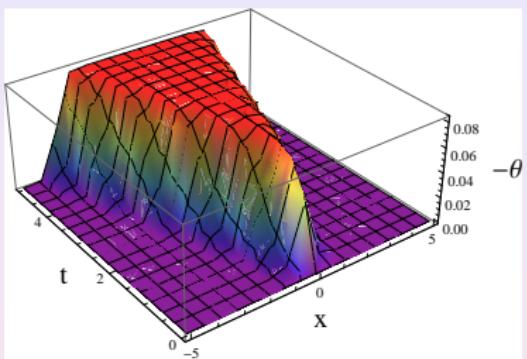
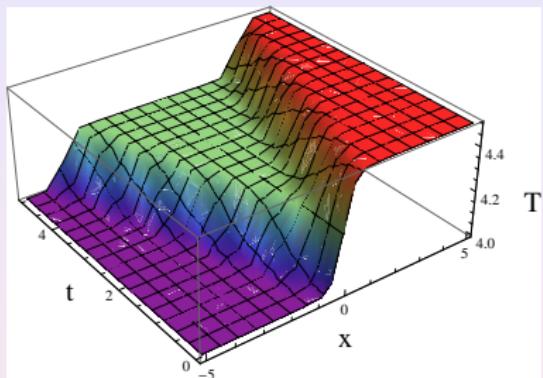
$$T(t, x) = \frac{T_R + T_L}{2} \left[ 1 + \epsilon \frac{T_R - T_L}{T_R + T_L} \left( \tanh(\alpha(x + t/\sqrt{2})) + \tanh(\alpha(x - t/\sqrt{2})) \right) \right]$$

$$\theta(t, x) = -\epsilon \frac{1}{\sqrt{2}} \frac{T_R - T_L}{T_R + T_L} \left[ \tanh(\alpha(x + t/\sqrt{2})) - \tanh(\alpha(x - t/\sqrt{2})) \right]$$

→  $\mathcal{O}(\epsilon)$  are corrections in  $\frac{|T_R - T_L|}{T_R + T_L} \ll 1$  → Small gradients in  $T(t, x)$ .

Example:  $T_{\text{steady state}} \equiv T(t \rightarrow +\infty, x) = \sqrt{T_L T_R} = \frac{T_L + T_R}{2} + \mathcal{O}(\epsilon^2)$

# Linearization



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# Entanglement Entropy

How does information get exchanged between the systems which are isolated at  $t = 0$ ?

- Entanglement entropy: Holographic measure  $\Rightarrow$  Generalization of Bekenstein-Hawking entropy formula  
[Ryu, Takayanagi '06], [Hubeny, Rangamani, Takayanagi '07]

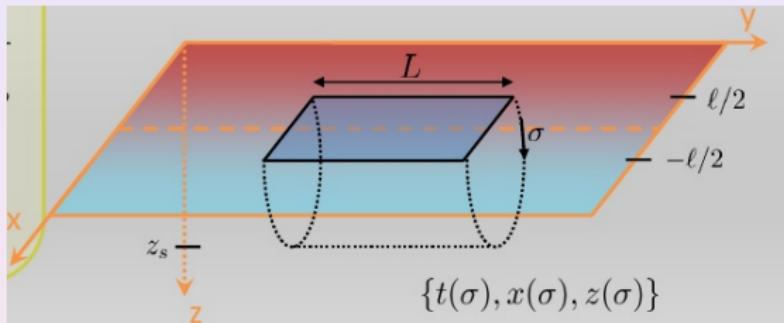
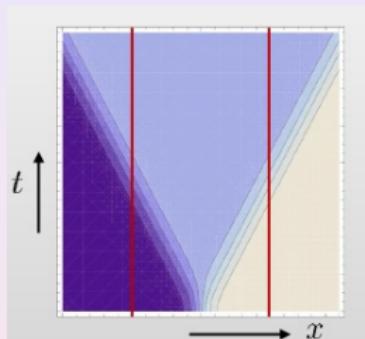
$$\mathcal{S}_A = \frac{\text{Area}(\gamma_A)}{4G_N^{(d+2)}}, \quad \text{Area with boundary } \partial A$$

- Regularization:  $\text{Area}(\gamma_A^{\text{div}}) \sim \frac{1}{z_{uv}}$ .
- Spatial induced metric on surface:  $ds^2 = h_{\sigma\sigma} d\sigma^2 + g_{yy} dy^2$ .
- Action:  $\mathcal{S} = L \int ds \sqrt{\frac{\partial x^a}{\partial \sigma} \frac{\partial x^b}{\partial \sigma} \tilde{g}_{ab}}$
- Minimal surface  $\rightarrow$  Geodesic equations for metric  $\tilde{g}_{ab} = g_{yy} g_{ab}$ :

$$\frac{d^2 x^a}{ds^2} + \tilde{\Gamma}_{bc}^a \frac{dx^b}{ds} \frac{dx^c}{ds} = 0, \quad a = t, x, z$$

$$s \equiv \text{affine parameter} \rightarrow \frac{\partial x^a}{\partial s} \frac{\partial x^b}{\partial s} \tilde{g}_{ab} = 1.$$

# Entanglement Entropy



$$S_A^{reg} = \frac{1}{4G_N^{d+2}} \left( \text{Area}(\gamma_A) - \text{Area}(\gamma_A^{div}) \right)$$

# Entanglement Entropy

$$\mathcal{S} = L \int_{-s_{uv}}^{s_{uv}} ds = 2L \cdot s_{uv}$$

$$\mathcal{S}^{\text{reg}} = \mathcal{S} - \mathcal{S}^{\text{div}} = 2L \cdot \left( s_{uv} - \frac{1}{z(s_{uv})} \right)$$

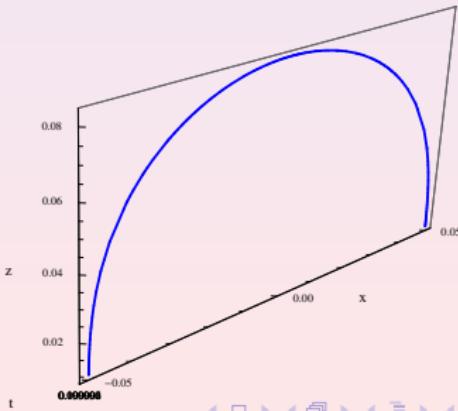
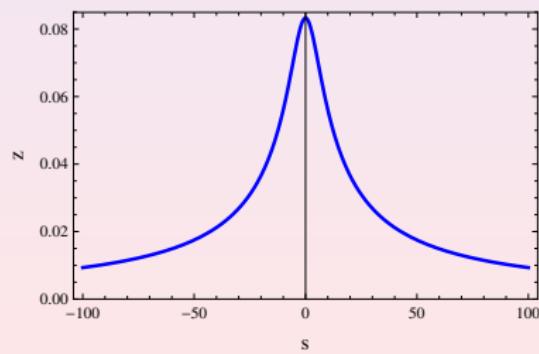
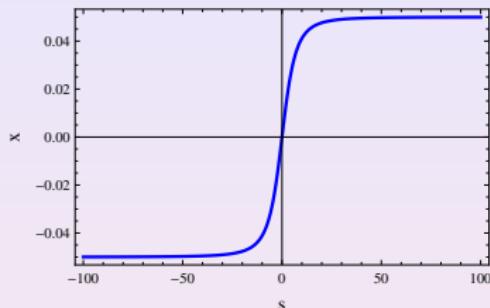
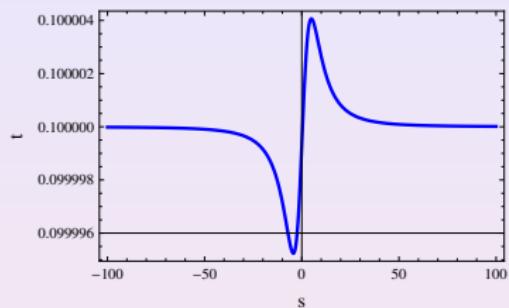
- Geodesic equations are 3 coupled equations of 2nd order  $\Rightarrow$  solutions:

$$t = t(s), \quad x = x(s), \quad z = z(s)$$

- 6 boundary conditions:

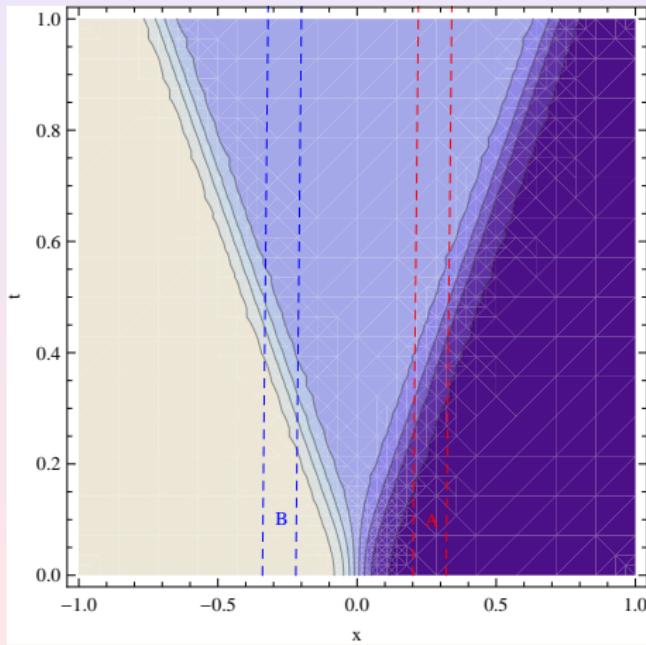
$$\begin{cases} t(s_{uv}) = t(-s_{uv}) = t_0 \\ x(s_{uv}) = -x(-s_{uv}) = \ell/2 \\ z(s_{uv}) = z(-s_{uv}) = z_{uv} \end{cases}$$

# Geodesics

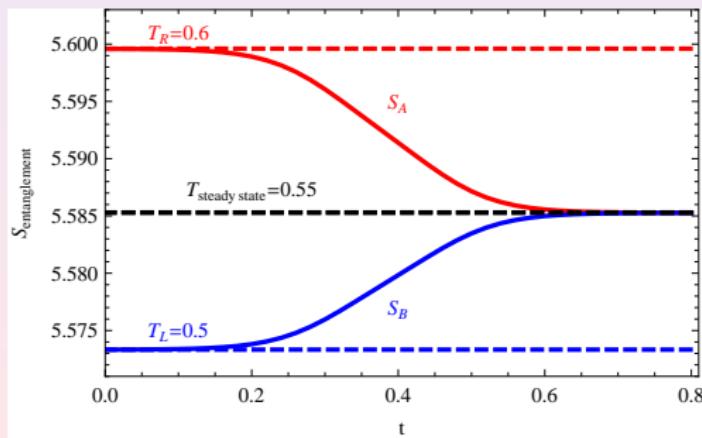
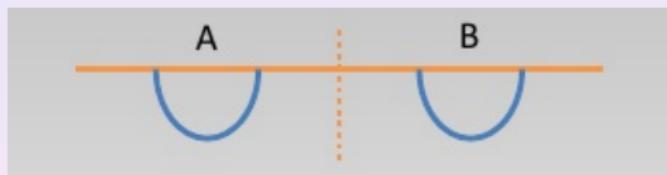


# Entanglement Entropy

**Contour plot of  $T(t, x)$  with  $T_L = 0.5$  and  $T_R = 0.6$ .**



# Entanglement Entropy



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# Mutual Information

What do we learn about A by looking at B?

→ Mutual information:  $I(A, B) = S(A) + S(B) - S(A \cup B)$



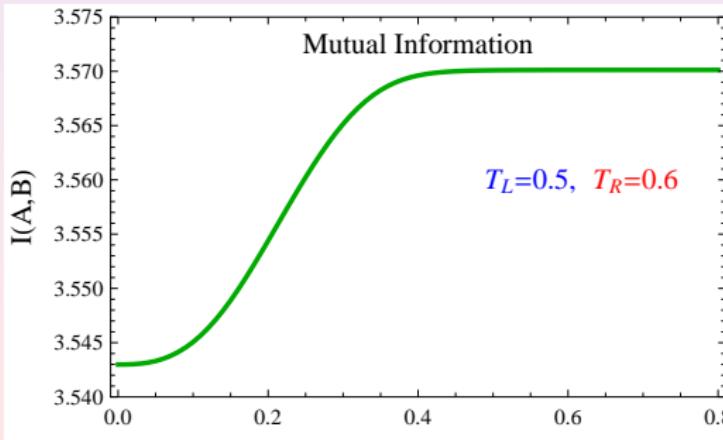
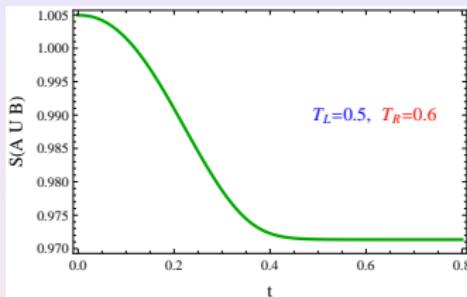
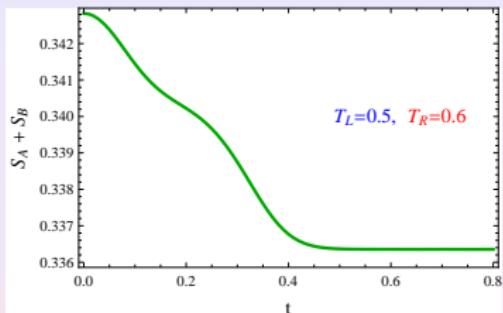
## Basic features:

- Shockwaves transport information.
- When does system A find out about the existence of system B?
- $I(A, B) = \text{finite} \rightarrow$  no need to regularize!!!

We find:

$$\partial_t I(A, B) \geq 0$$

# Mutual Information



# Conclusions and future directions

- We have studied a **holographic model for time-dep energy flow**.
- **Linearization** → analytical solution valid for  $|T_L - T_R| \ll T_L + T_R$  (*linear response regime*).
- We have computed the time evolution of **Entanglement Entropies**  
→ Mutual information grows with time.

What next?

- Complete analysis beyond linear response regime:  
 $0 < T_L/T_R < 1$  → **Full numerical solution of Einstein e.o.m.**  
**[Chesler, Yaffe '09], [Amado, Yarom '15], [Ecker, Grumiller, Stricker '15], [Erdmenger, Fernandez, Flory, EM, Straub '15]**.
- **Other possible solutions?** **[Chang, Karch, Yarom '14]**.
- **Higher dimensions:**  $(2+1)\text{dim} \rightarrow (3+1)\text{dim}$ . What changes?  
Preliminary analysis confirms the change in shockwave velocities:  $c_s^2 = 1/d$ .

# Thank You!