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Neutrinoless double beta decay, nuclear environment and structure

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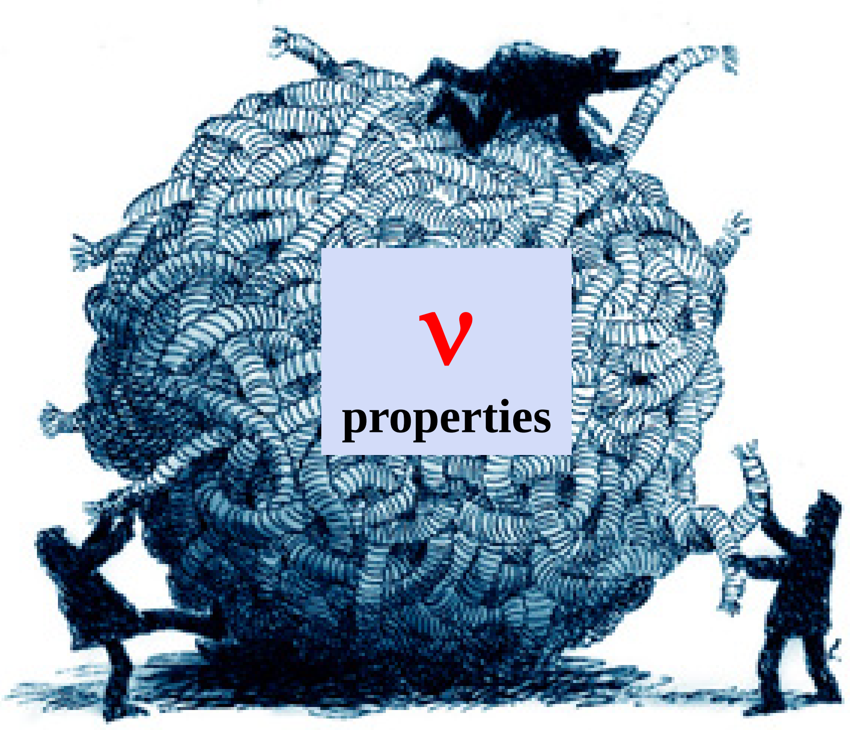
OUTLINE

- ***Introduction***
- ***Effective Majorana neutrino mass in vacuum and nuclear matter***
- ***$0\nu\beta\beta$-decay NMEs (uncertainty and quenching)***
- ***DBD NMEs and SU(4) symmetry***
- ***$0\nu\beta\beta$-decay with emission of $p_{1/2}$-electrons***
- ***Heavy/sterile neutrinos***
- ***$0\nu\beta\beta$-decay with right-handed currents revisited***
- ***Conclusions***

Amand Faesler, V. Rodin (U. Tuebingen), P. Vogel (Caltech), J. Engel (North Caroline U.), S. Kovalenko (Valparaiso U.), M. Krivoruchenko (ITEP Moscow), J. Vergados (U. Ioannina), S. Petcov (SISSA), D. Štefánik, R. Dvornický (Comenius U.), E. Lisi, G. Fogli (U. Bari) etc

After 59 years we know

- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)



No answer yet

- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?

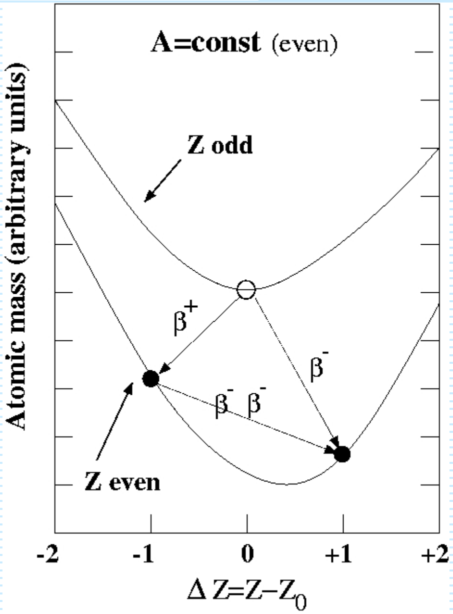
Currently main issue

$0\nu\beta\beta$-decay: Nature, Mass hierarchy, CP-properties, sterile ν

The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

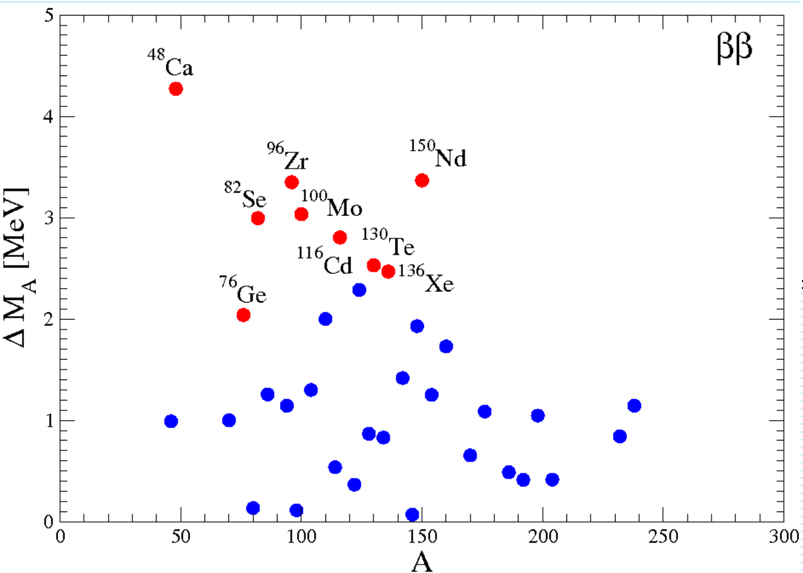
Neutrinoless Double-Beta Decay

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$



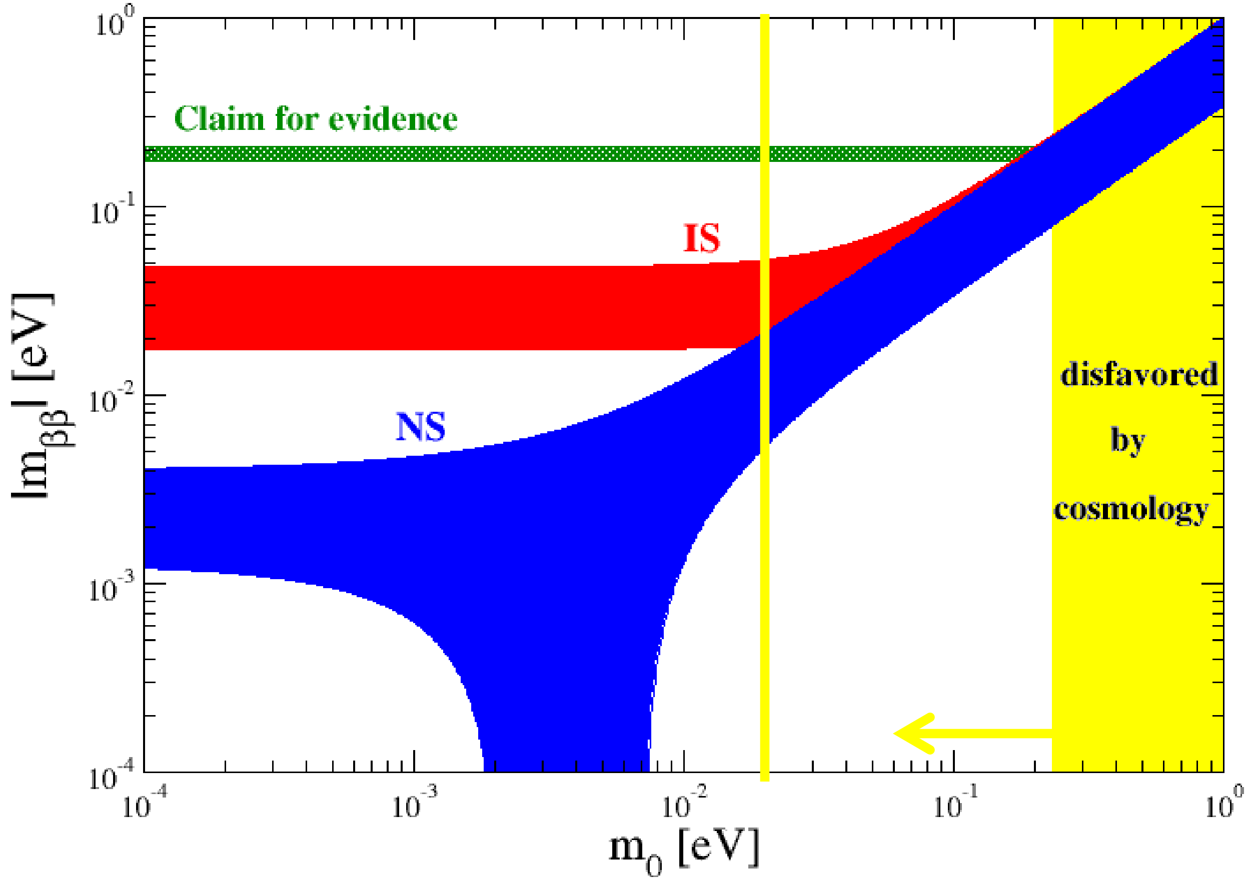
$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_\nu\right|^2 G^{0\nu}$$

transition	$G^{01}(E_0,Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\lvert M^{0\nu}\rvert^2$
$^{150}Nd \rightarrow {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \rightarrow {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \rightarrow {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \rightarrow {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \rightarrow {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \rightarrow {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \rightarrow {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \rightarrow {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \rightarrow {}^{76}Se$	0.79	2.040	8	?



The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

Issue: Lightest neutrino mass m_0



GUT's

Complementarity of 0νββ-decay, β-decay and cosmology

β-decay (Mainz, Troitsk)

$$m_\beta^2 = \sum_i |U_{ei}^L|^2 m_i^2 \leq (2.2\ \mathrm{eV})^2$$

KATRIN: $(0.2\ \mathrm{eV})^2$

Cosmology (Planck)

$$\sum_i m_i \leq 0.23\text{–}1.08\ \mathrm{eV}$$

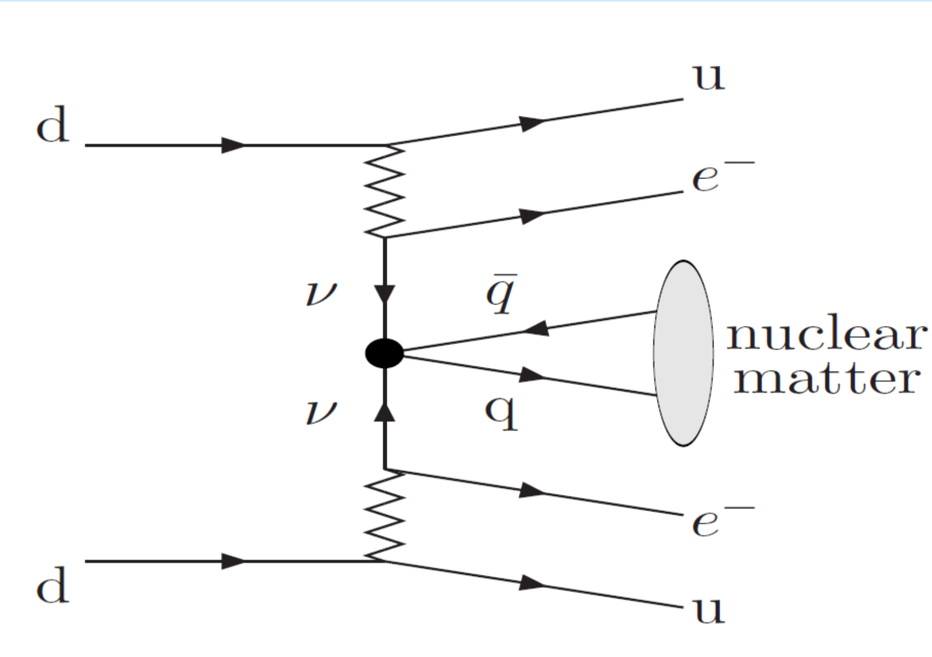
$$m_0 \leq 0.07\ \mathrm{eV}$$

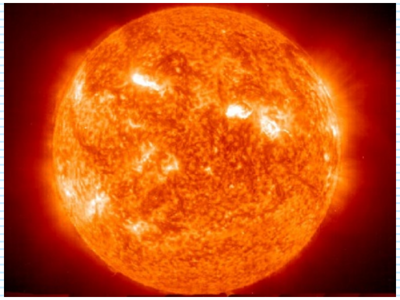
Nuclear medium effect on the light neutrino mass exchange mechanism of the $0\nu\beta\beta$ decay

S.G. Kovalenko, M.I. Krivoruchenko, F. Š., Phys. Rev. Lett. 112 (2014) 142503

A novel effect in $0\nu\beta\beta$ decay related with the fact, that its underlying mechanisms take place in the nuclear matter environment:

+ Low energy 4-fermion $\Delta L \neq 0$ Lagrangian
+ In-medium Majorana mass of neutrino
+ $0\nu\beta\beta$ constraints on the universal scalar couplings

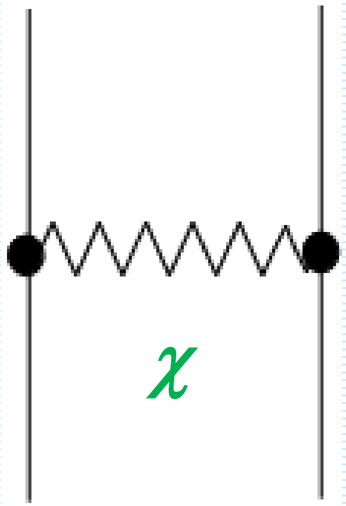




Non-standard ν–int. discussed e.g., in the context of ν–osc. at Sun

$\rho_{Sun} = 1.4\ g/cm^3$
$\rho_{Earth} = 5.5\ g/cm^3$
$\rho_{nucleus} = 2.3\ 10^{14}\ g/cm^3$

Non-standard interactions might be easily detected in nucleus rather than in vacuum



Low energy 4-fermion ΔL ≠ 0 Lagrangian

$$L_{\text{eff}} = \frac{g^2}{m_\chi^2} \sum_A (\bar{q} O_A q)(\bar{\nu} O'_A \nu),$$

$$m_\chi \gtrsim M_W .$$

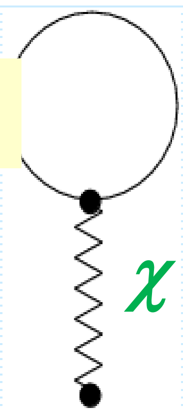
0νββ-decay

oscillation experiments
tritium β-decay, cosmology

density →

$$\Sigma_\nu^{\text{vac}} = \text{-}\times\text{-},$$

$$\Sigma_\nu^{\text{medium}} = \text{-}\times\text{-} +$$



Classification of the vertices gO_A and gO'_A

$$\mathcal{L}_{\mathrm{free},\nu} = \frac{1}{4}\sum_i \bar{\nu}_i i\gamma^\mu \overleftrightarrow{\partial}_\mu \nu_i - \frac{1}{2}\sum_i m_i \bar{\nu}_i \nu_i .$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{g_\chi}{m_\chi^2}\bar{q}q \sum_{a=1}^{6}\sum_{ij} g_{ij}^a J_{ij}^a$$

In nuclei, mean fields are created by scalar and vector currents (σ, ω).
Vector currents do not flip the spin of neutrinos
and do not contribute to the $0\nu\beta\beta$ decay.

Symmetric and antisymmetric scalar neutrino currents J^a_{ij}

a	S	a	S	a	A
1	$\bar{\nu}_i^c \nu_j$	3	$\partial_\mu(\bar{\nu}_i^c \gamma_5 \gamma^\mu \nu_j)$	5	$\partial_\mu(\bar{\nu}_i^c \gamma^\mu \nu_j)$
2	$\bar{\nu}_i^c i\gamma_5 \nu_j$	4	$\bar{\nu}_i^c \gamma^\mu i \overleftrightarrow{\partial}_\mu \nu_j$	6	$\bar{\nu}_i^c \gamma_5 \gamma^\mu i \overleftrightarrow{\partial}_\mu \nu_j$

g^a_{ij} are real symmetric for a = 1,2,3,4 and imaginary antisymmetric for a = 5,6. In the limit of $R = \infty$, the currents a = 3,5 vanish.

Mean field approximation

Mean field: $\bar{q}q \to \langle \bar{q}q \rangle$ **and** $\langle \bar{q}q \rangle \approx 0.5 \langle q^\dagger q \rangle \approx 0.25\,\mathrm{fm}^{-3}$

The effect depends on the product $\langle \chi \rangle = -\dfrac{g_\chi}{m_\chi^2} \langle \bar{q}q \rangle$

To compare with weak interaction: $\dfrac{g_\chi g_{ij}^a}{m_\chi^2} = \dfrac{G_F}{\sqrt{2}} \varepsilon_{ij}^a$

Typical scale:

$$\langle \chi \rangle g_{ij}^a = -\frac{G_F}{\sqrt{2}} \langle \bar{q}q \rangle \varepsilon_{ij}^a \approx -25\,\varepsilon_{ij}^a \text{ eV}$$

We expect:

$$25\,\varepsilon_{ij}^a < 1 \to m_\chi^2 > 25 \frac{g_\chi g_{ij}^a \sqrt{2}}{G_F} \sim 1\,\mathrm{TeV}^2$$

In-medium Lagrangian of Majorana ν

$$\Delta\mathcal{L} = \frac{1}{4}\sum_{ij}\bar{\nu}_i(Z_{ij}+\gamma_5 Z'_{ij})i\gamma^\mu \overleftrightarrow{\partial}_\mu \nu_j - \frac{1}{2}\sum_{ij}\bar{\nu}_i(M_{ij}+i\gamma_5 M'_{ij})\nu_j,$$

where

$$Z_{ij} = \delta_{ij} - \langle\chi\rangle g^4_{ij}, \qquad Z'_{ij} = -\langle\chi\rangle g^6_{ij}$$

$$M_{ij} = m_i\delta_{ij} + \langle\chi\rangle g^1_{ij}, \quad M'_{ij} = \langle\chi\rangle g^2_{ij}.$$

Universal scalar interaction

$$g^a_{ij} = \delta_{ij} g_a \quad \varepsilon^a_{ij} = \delta_{ij}\varepsilon_a$$

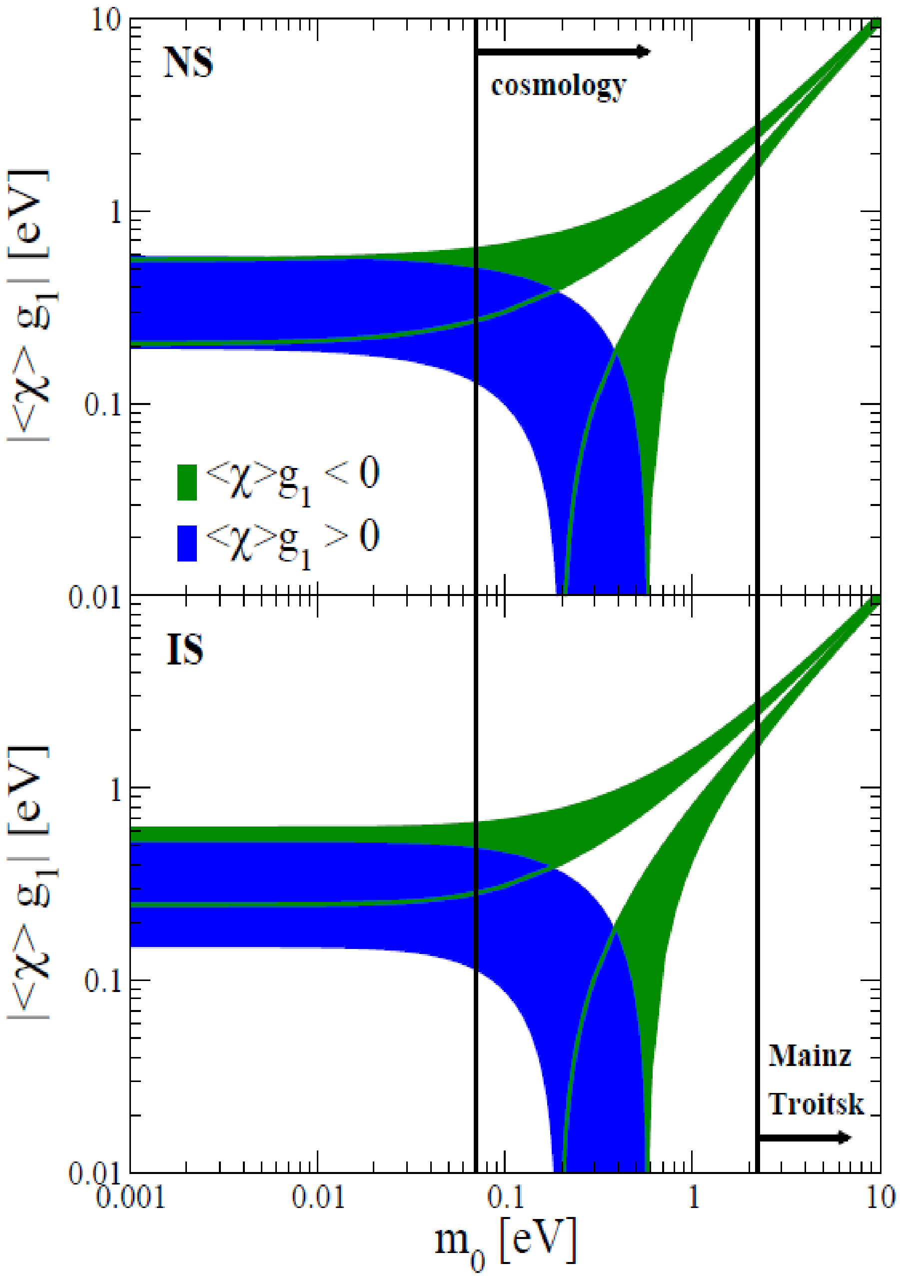
In medium ν mass

$$\mu_i = \frac{\sqrt{(m_i + \langle\chi\rangle g_1)^2 + (\langle\chi\rangle g_2)^2}}{\lambda_i}$$

In medium effective Majorana ν mass

$$m_{\beta\beta} = \sum_{i=1}^{n} U^2_{ei}\xi_i \frac{\sqrt{(m_i + \langle\chi\rangle g_1)^2 + (\langle\chi\rangle g_2)^2}}{(1-\langle\chi\rangle g_4)^2}.$$

Regions of admissible values of $\langle\chi\rangle g_1$ and m_0 ($m_{\beta\beta}$=0.2 eV)



$$\langle\chi\rangle = 0.17\ fm^{-3} = \frac{0.17}{(5.07)^3} GeV^3$$

$$\Lambda_{LNV} \geq 2.4\ \text{TeV (Planck)}$$

$$1.1\ \text{TeV (Tritium)}$$

$$\varepsilon_{ij} \leq 0.02\ \text{(Planck)},\ 0.1\ \text{(Tritium)}$$

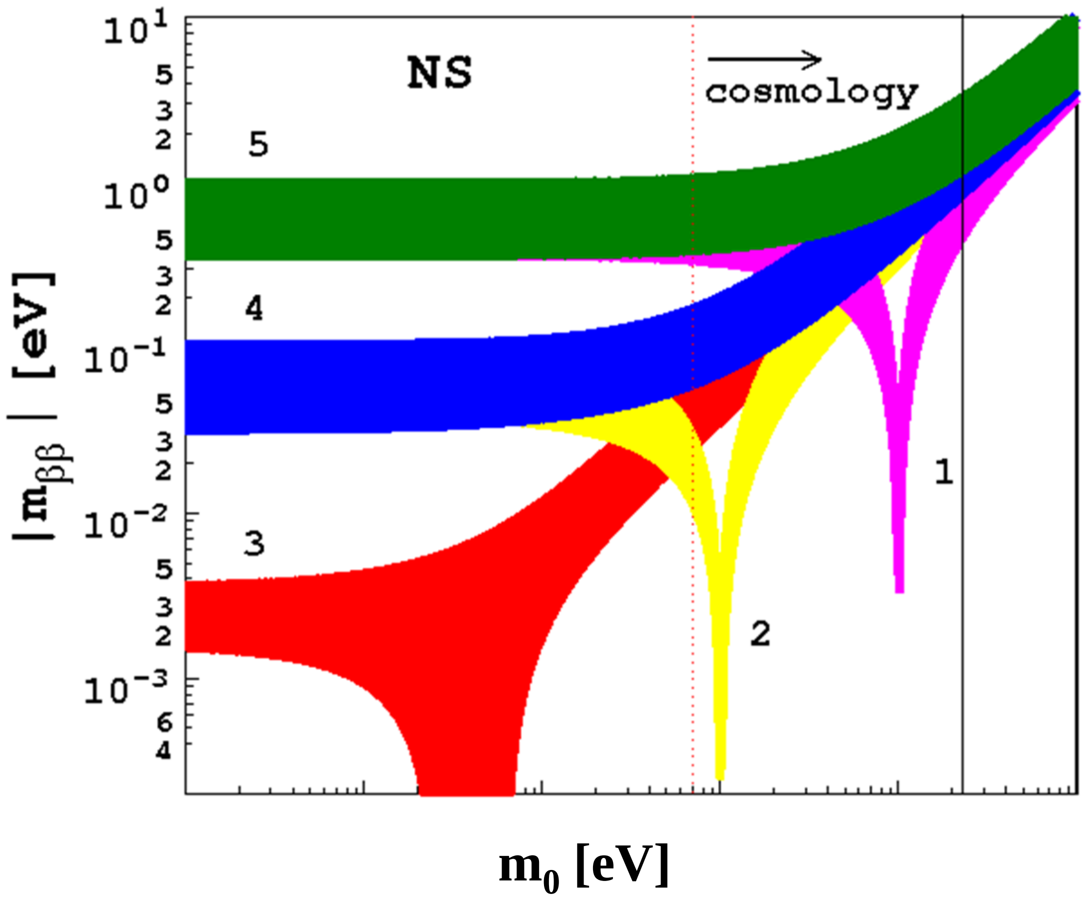
Using experimental data on the 0νββ decay in combination with β-decay and cosmological data we evaluated the characteristic scales of 4-fermion neutrino-quark operators, which is Λ_{LNV} > 2.4 TeV.

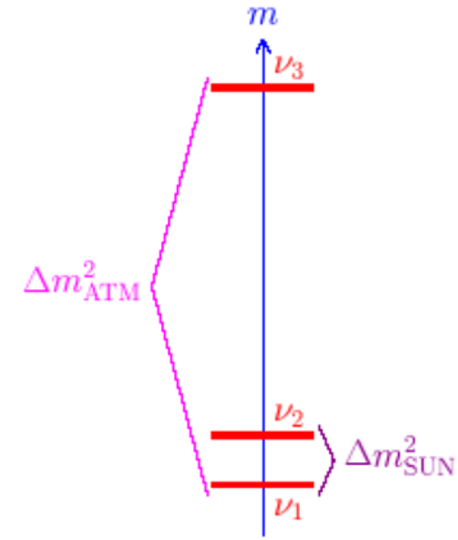
Pion decay: BR($\pi^0 \to \nu\nu$) ≤ 2.7 10^{-7}

$$\Lambda_{LNV} \geq 560\ \text{GeV}$$

The normal neutrino mass spectrum (NS)

$$m_1 < m_2 < m_3$$





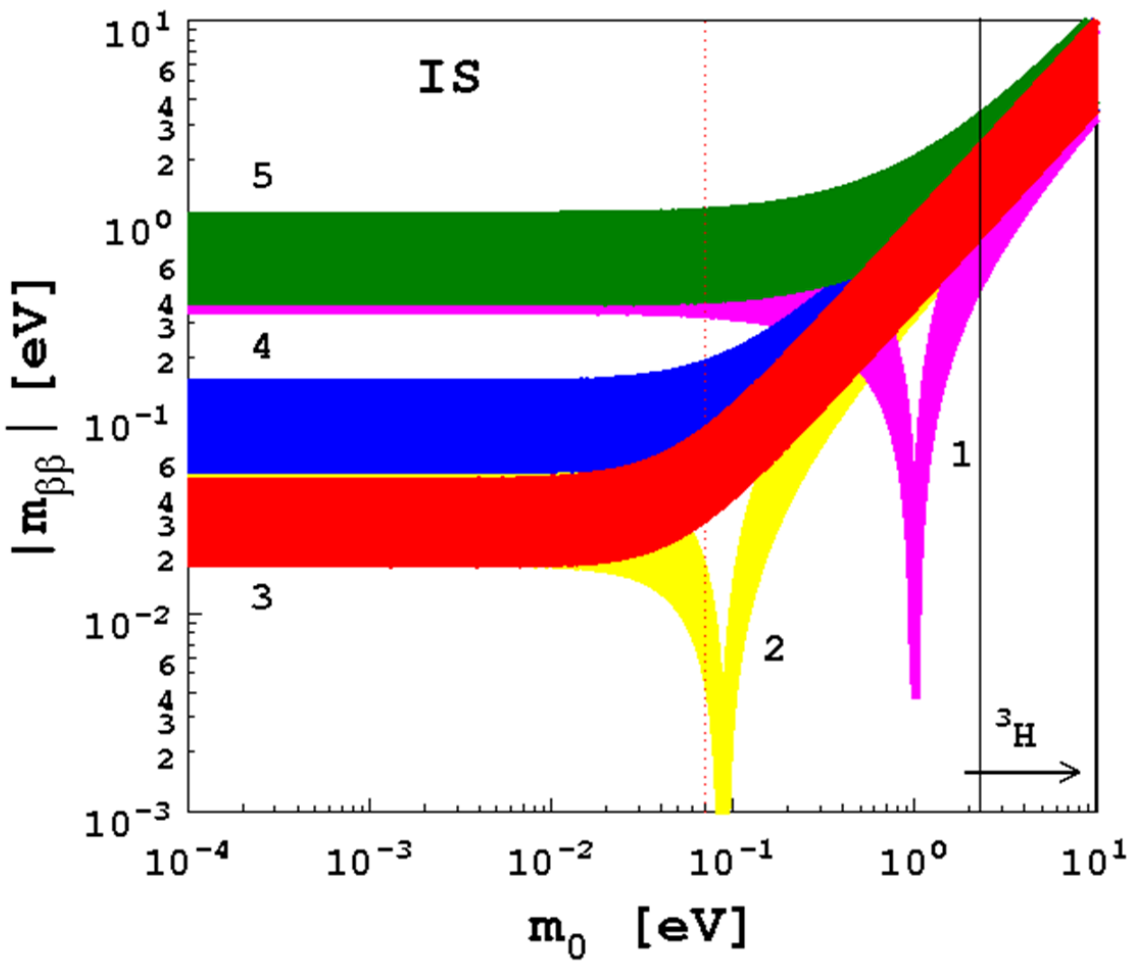
Area	$\langle\chi\rangle g_1$ [eV]
1	-1
2	-0.1
3	0
4	0.1
5	1

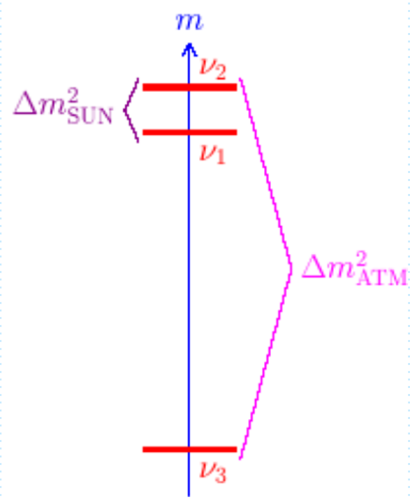
$$g_2 = g_4 = 0$$

No possibility to conclude about hierarchy

The inverted neutrino mass spectrum (IS)

$$m_3 < m_1 < m_2$$

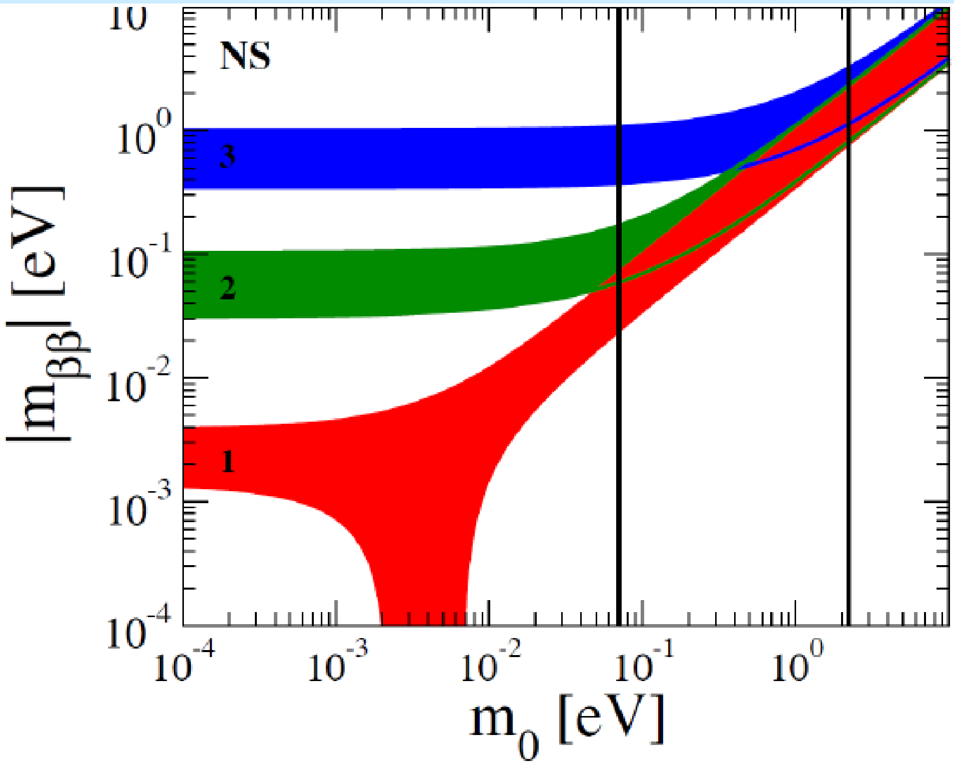




Area	$\langle\chi\rangle g_1$ [eV]
1	-1
2	-0.1
3	0
4	0.1
5	1

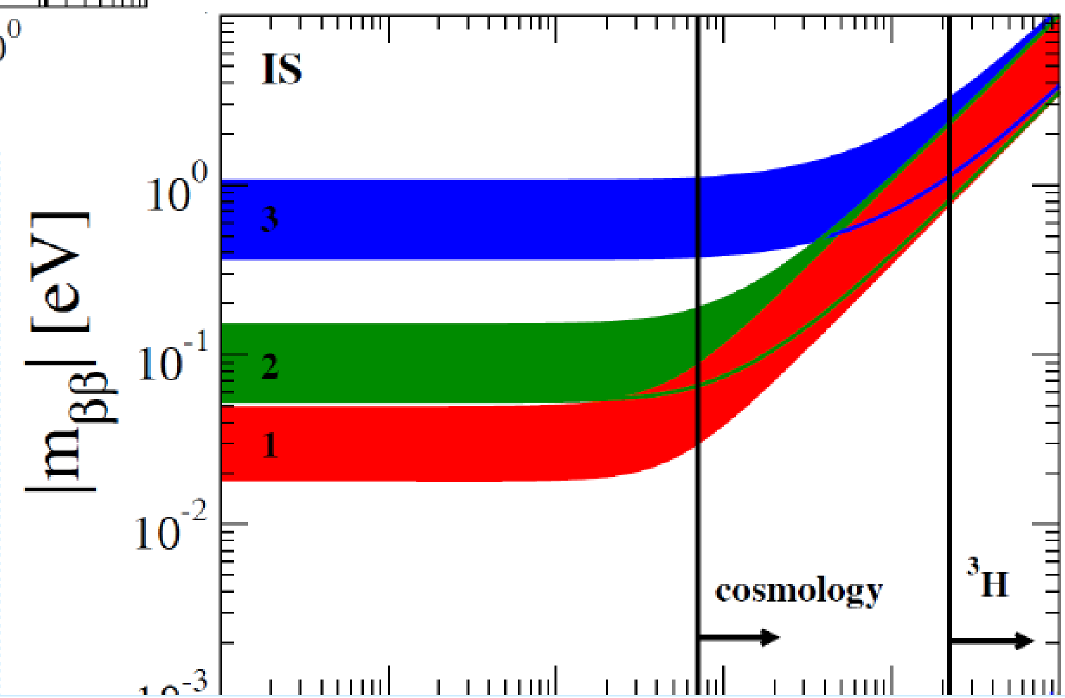
$$g_2 = g_4 = 0$$

No possibility to conclude about hierarchy



Difference between the NS and IS disappears

For positive LNV interaction effective Majorana neutrino mass becomes larger



The 0νββ-decay Nuclear Matrix Elements

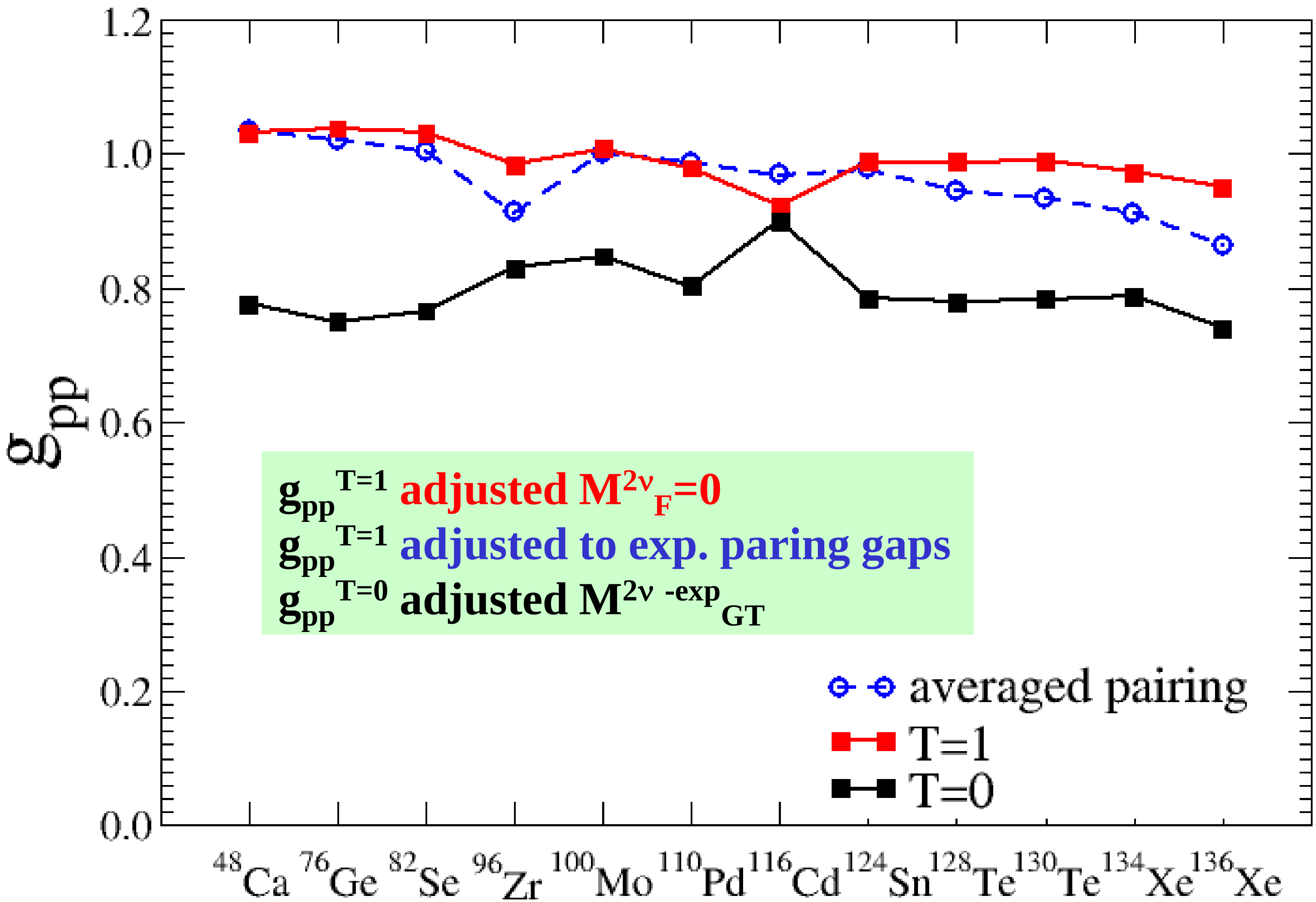
$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

QRPA and isospin symmetry restoration

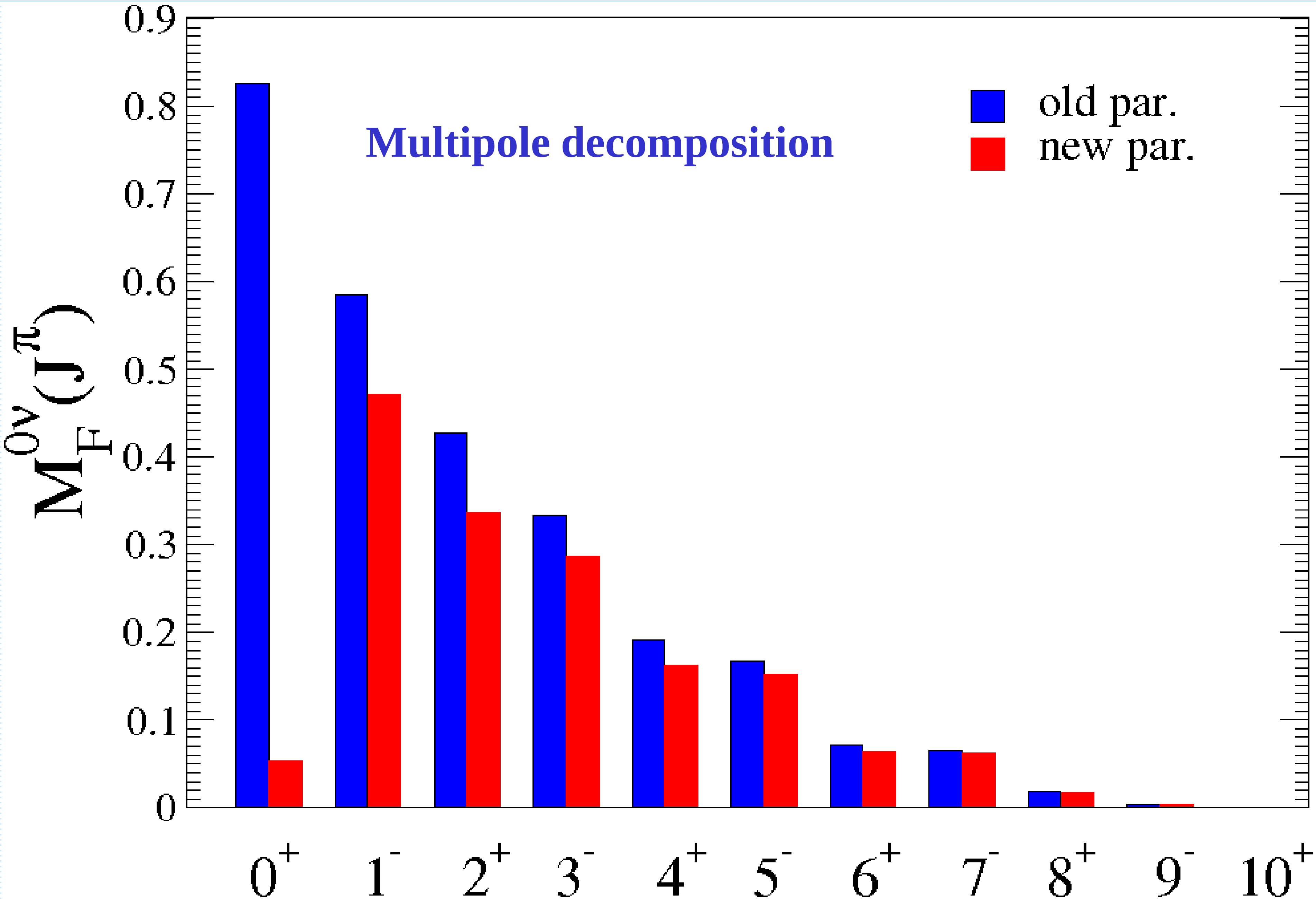
F.Š., V. Rodin, A. Faessler, and P. Vogel

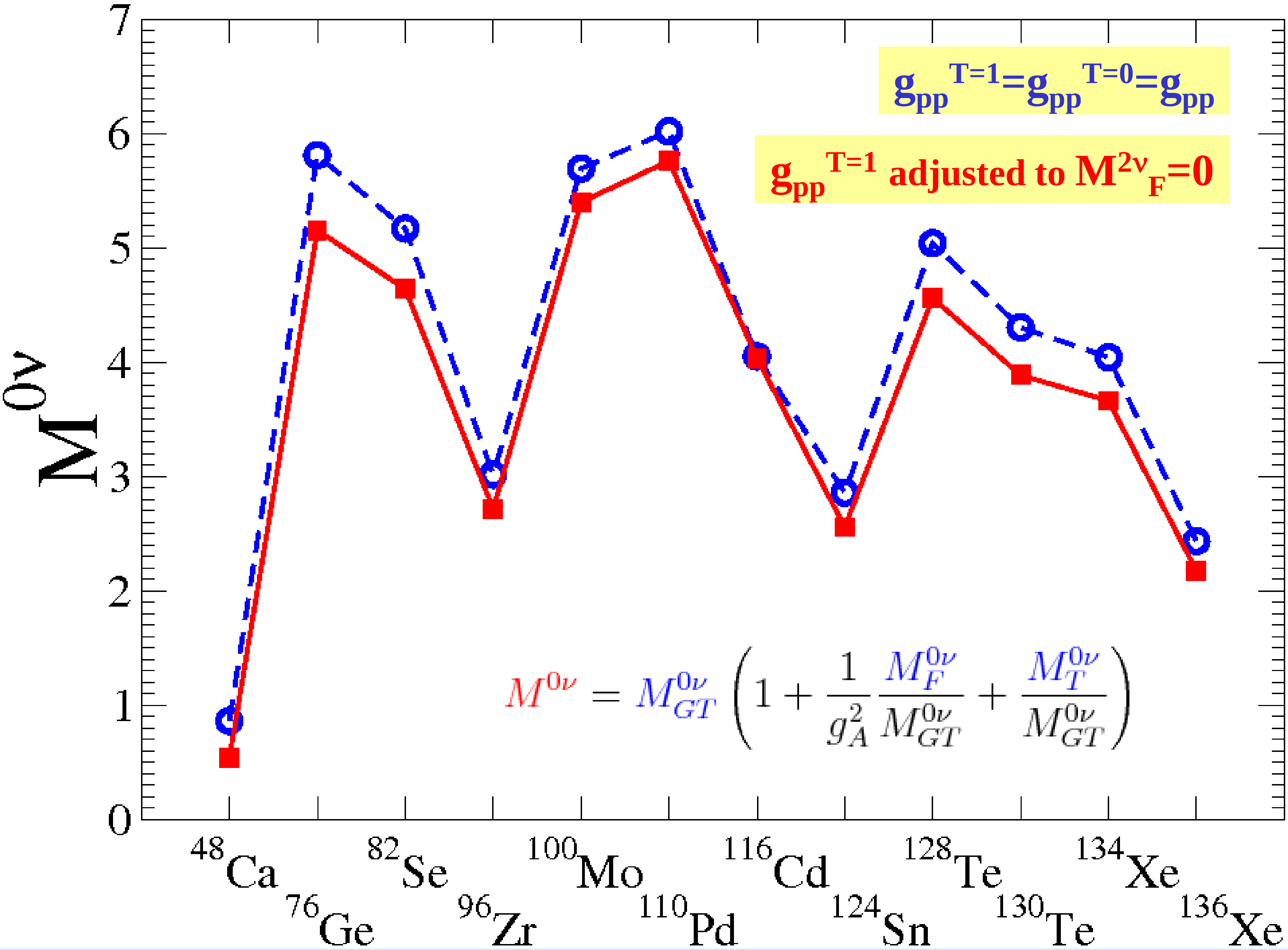
PRC 87, 045501 (2013)

Close values ■ and ■ => no new parameter



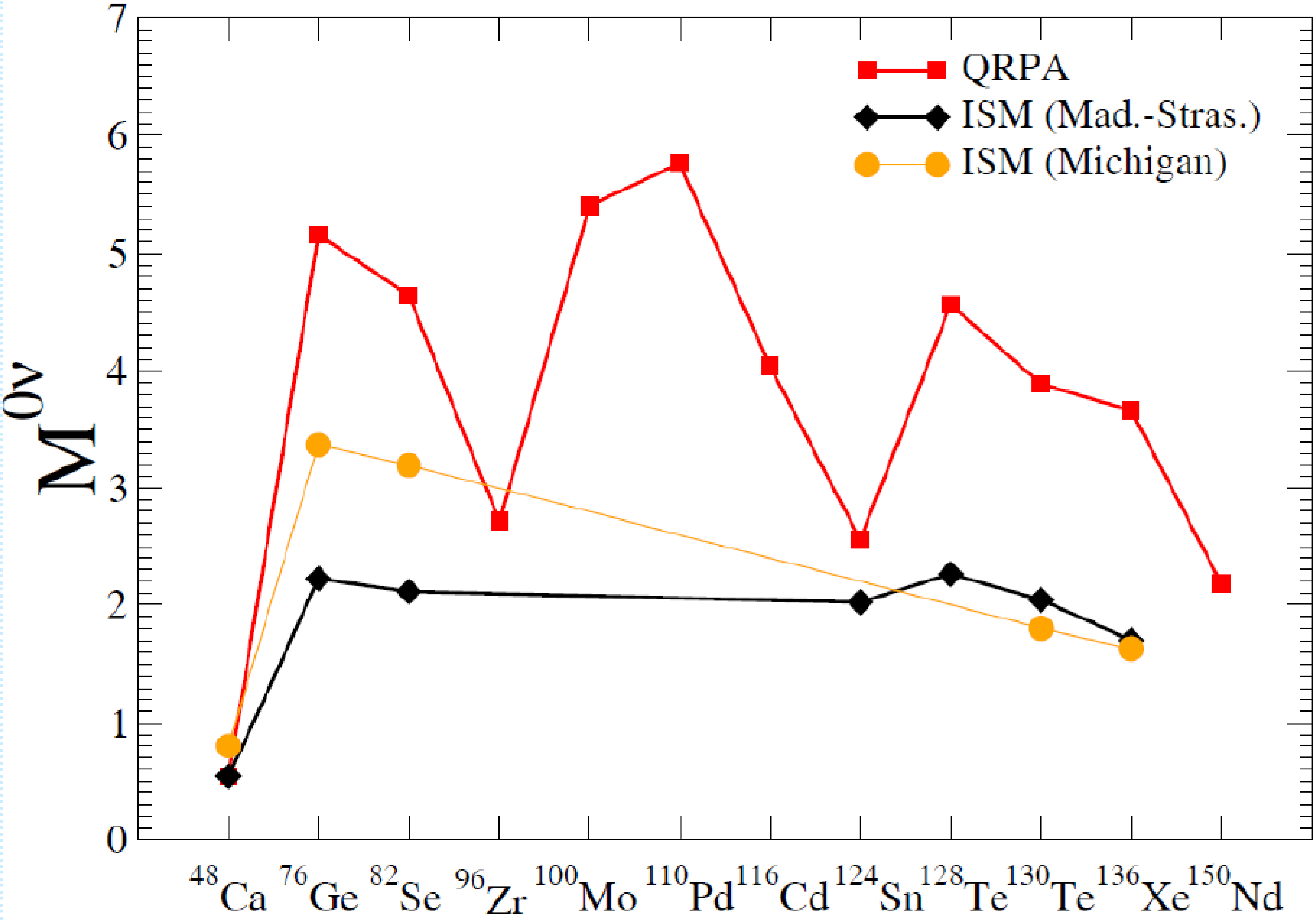
Separation of g_{pp} into $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$


Multipole decomposition
old par.
new par.
$M_F^{0\nu}(J^\pi)$
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0^+
1^-
2^+
3^-
4^+
5^-
6^+
7^-
8^+
9^-
10^+



$M^{0\nu}$
$g_{pp}^{T=1}=g_{pp}^{T=0}=g_{pp}$
$g_{pp}^{T=1}$ adjusted to $M^{2\nu}_F=0$
$M^{0\nu} = M^{0\nu}_{GT}\left(1+\frac{1}{g_A^2}\frac{M^{0\nu}_F}{M^{0\nu}_{GT}}+\frac{M^{0\nu}_T}{M^{0\nu}_{GT}}\right)$
0
1
2
3
4
5
6
7
^{48}Ca
^{76}Ge
^{82}Se
^{96}Zr
^{100}Mo
^{110}Pd
^{116}Cd
^{124}Sn
^{128}Te
^{130}Te
^{134}Xe
^{136}Xe

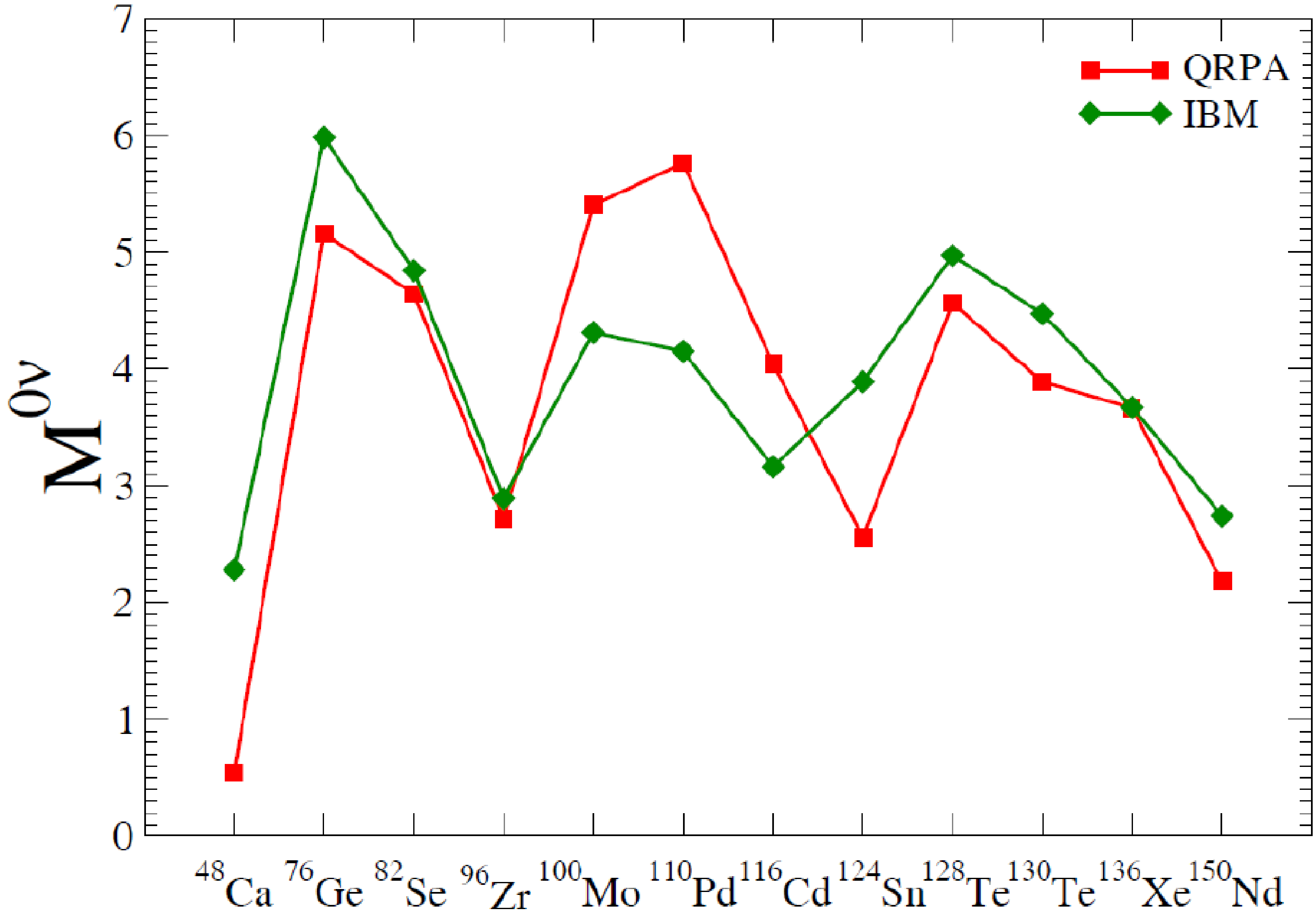
QRPA versus Interacting Shell Model



ISM: Menendez et al. NPA 818 (2009) 139

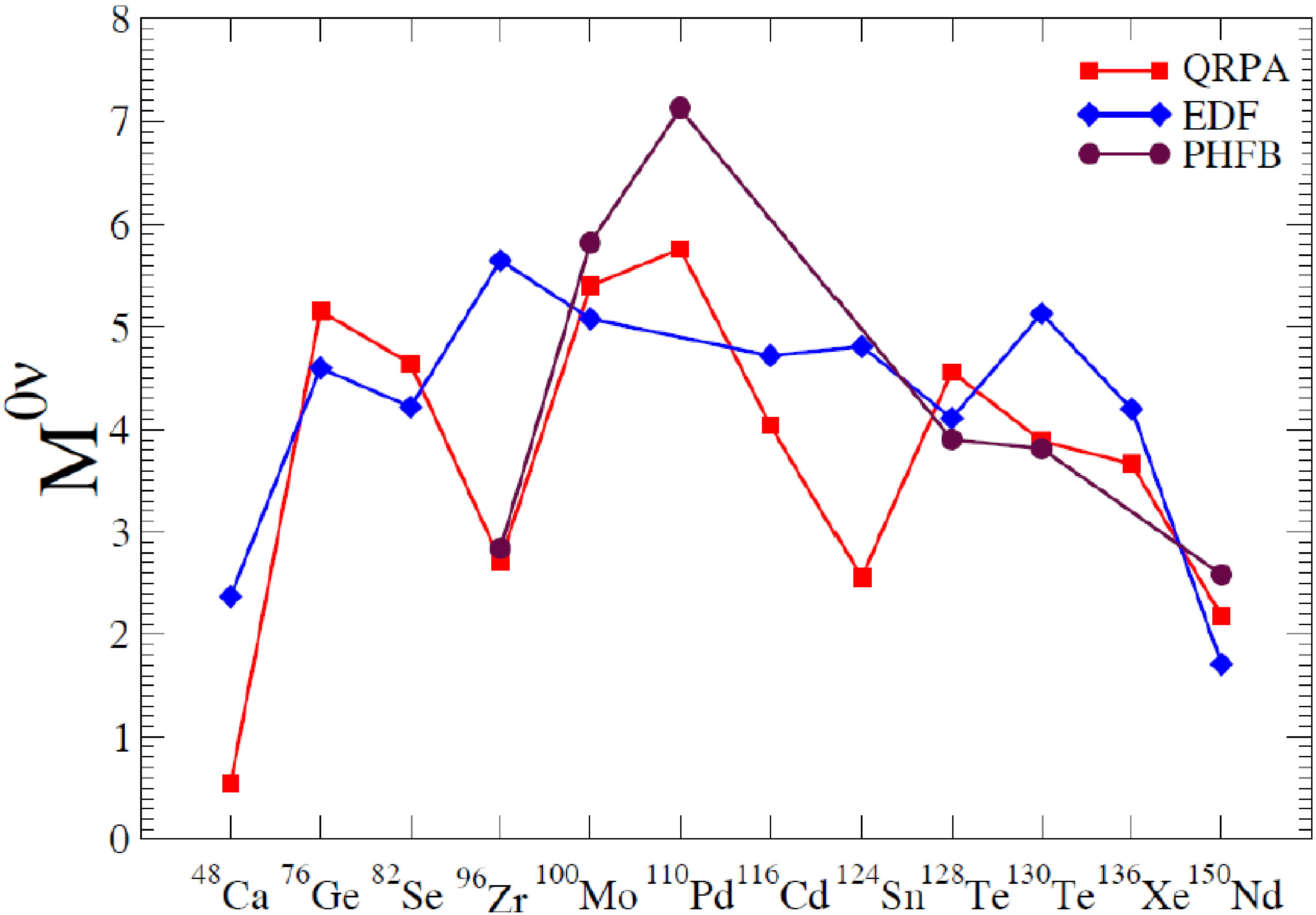
ISM: Brown, Horoi etc., Phys.Rev.Lett. 113 (2014) 262501 …

QRPA versus IBM



IBM: Barea, Kotila, Iachello, PRC (2013) 014315

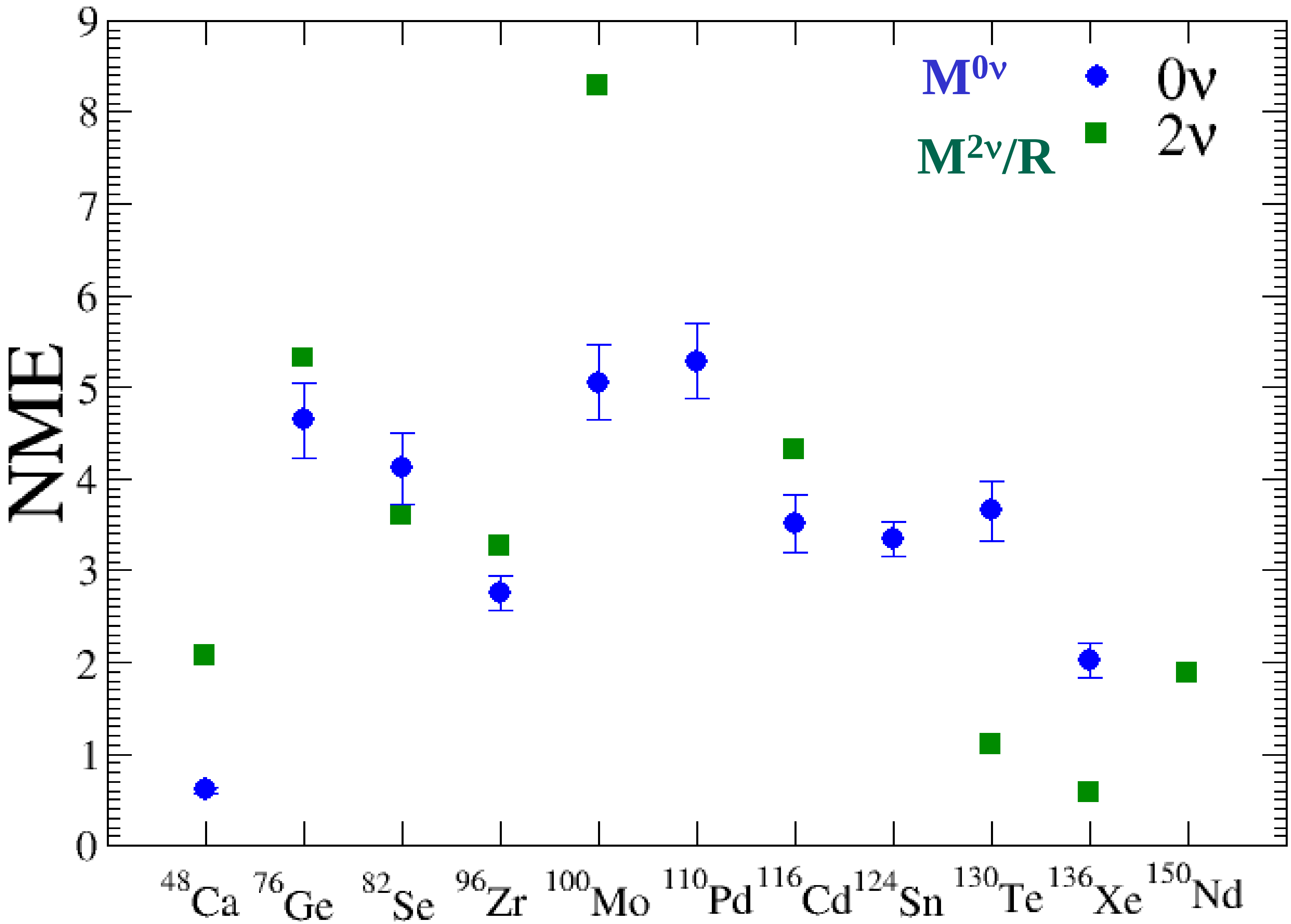
QRPA versus EDF/PHFB



EDF: Rodrigez, Martinez-Pinedo, PRL (2010) 105

PHFB: K. Rath et al., PRC 85 (2012) 014308

Is there a connection between 0νββ- and 2νββ-decay NME?



F. Š., Nucl. Phys. B, Proc. Suppl. (2014) – Proceeding of NOW14 conference

Quenching of g_A (systematic error)

$g_A{}^4 = (1.269)^4 = 2.6$

$(g^{eff}{}_A)^4 = 1.0$

Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment

$g_A = 1.269 \Rightarrow g^{eff}{}_A = 0.75\, g_A \approx 1$

$(g^{eff}{}_A)^4 = (0.8)^4 = 0.41$

$(g^{eff}{}_A)^4 = (0.7)^4 = 0.24$

In QRPA $g^{eff}{}_A$ and isoscalar force were fitted to reproduce the $2\nu\beta\beta$-decay half-life, β^- decay rate and β^+/EC rate $\Rightarrow g^{eff}{}_A$ is smaller than unity.

Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$g_A{}^{eff\text{-}ISM} = 0.57\text{-}0.90$

$g_A{}^{eff\text{-}IBM} = 0.35\text{-}0.71$

$g_A{}^{eff}$ is highly dependent on the model calculations and assumptions made

Barea, Kotila, Iachello, PRC 87, 014315 (2013)

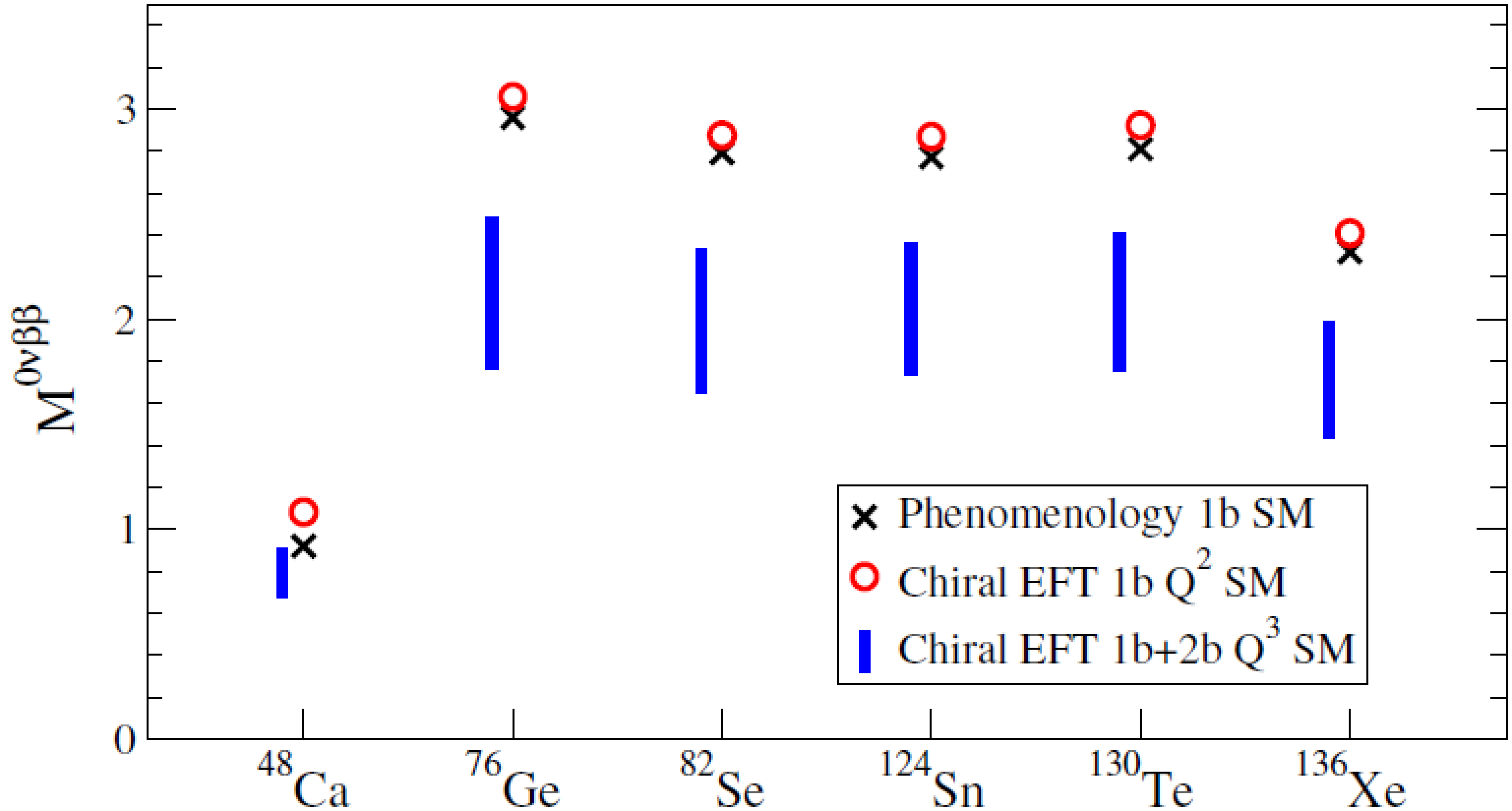
Is $g_A{}^{eff}$ different for different (A,Z) and different spin-dependent transition operators?

Quenching of g_A and two-body currents

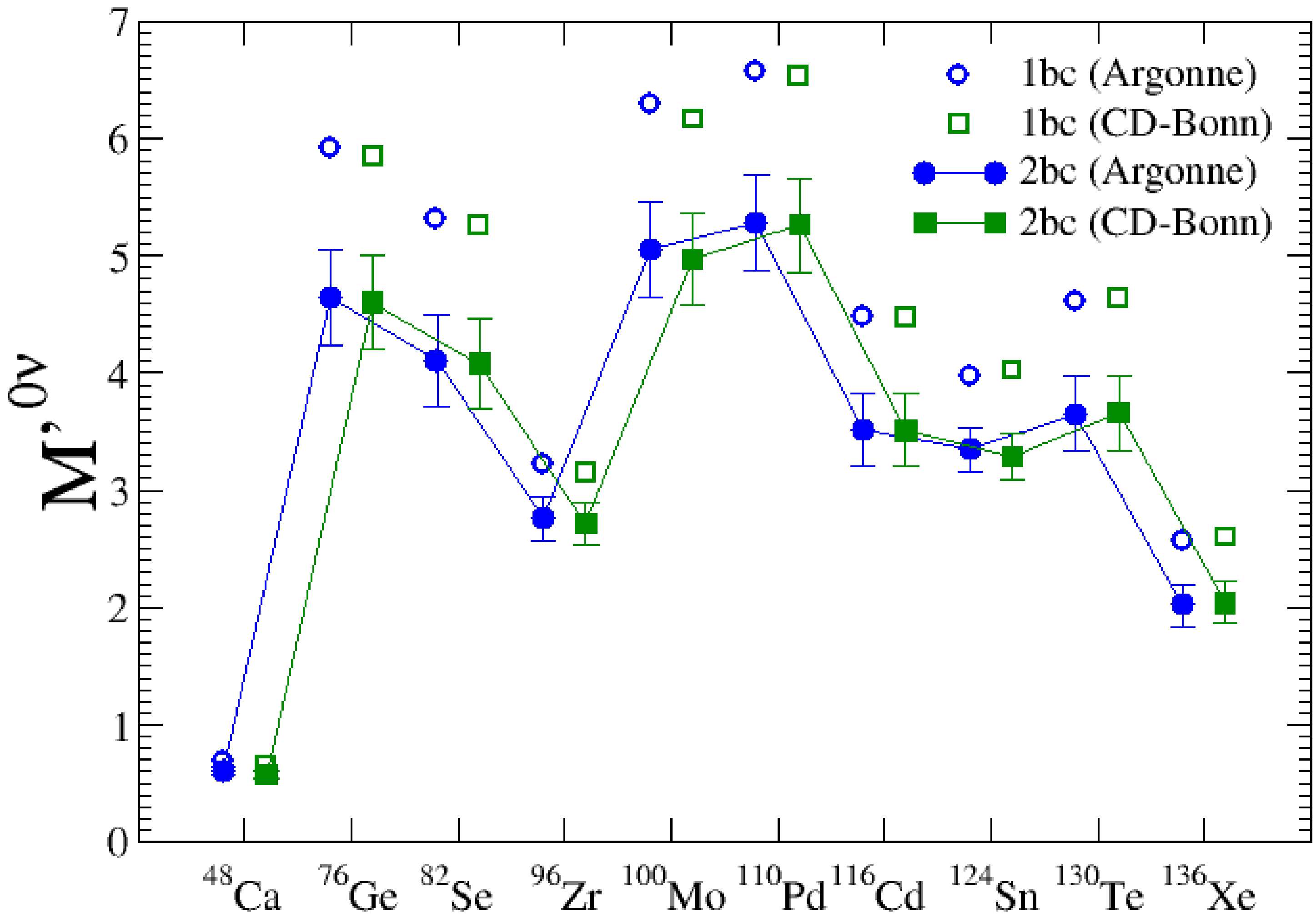
Menendez, Gazit, Schwenk, PRL 107 (2011) 062501; MEDEX13 contribution

$$\mathbf{J}^{\mathrm{eff}}_{i,2b} = -g_A \boldsymbol{\sigma}_i \tau_i^- \frac{\rho}{F_\pi^2}\left[\frac{2}{3}c_3\frac{p^2}{4m_\pi^2+p^2} + I(\rho,P)\left(\frac{1}{3}(2c_4-c_3)+\frac{1}{6m}\right)\right] = -g_A \delta(p) \boldsymbol{\sigma}_i \tau_i^-$$

The $0\nu\beta\beta$ operator calculated within effective field theory. Corrections appear as 2-body current predicted by EFT. The 2-body current contributions are related to the quenching of Gamow-Teller transitions found in nuclear structure calc.



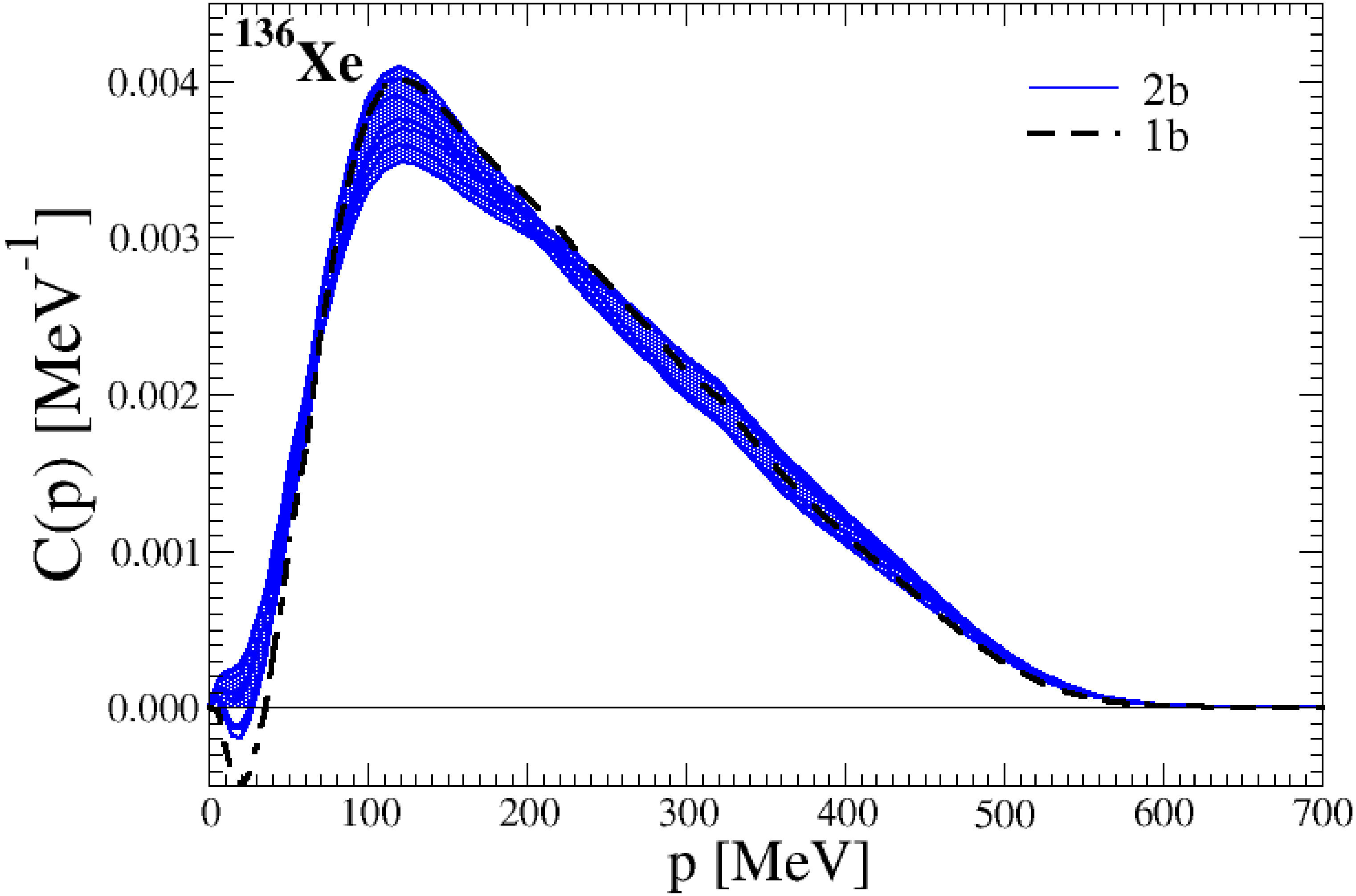
Quenching of g_A, two-body currents and QRPA
(Suppression of about 20%)



Engel, Vogel, Faessler, F.Š., PRC 89 (2014) 064308

Momentum distribution of NME normalized to unity

<p> ≈ 230 MeV, sqrt(<p²>) ≈ 250 MeV



The DBD Nuclear Matrix Elements and the SU(4) symmetry

D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

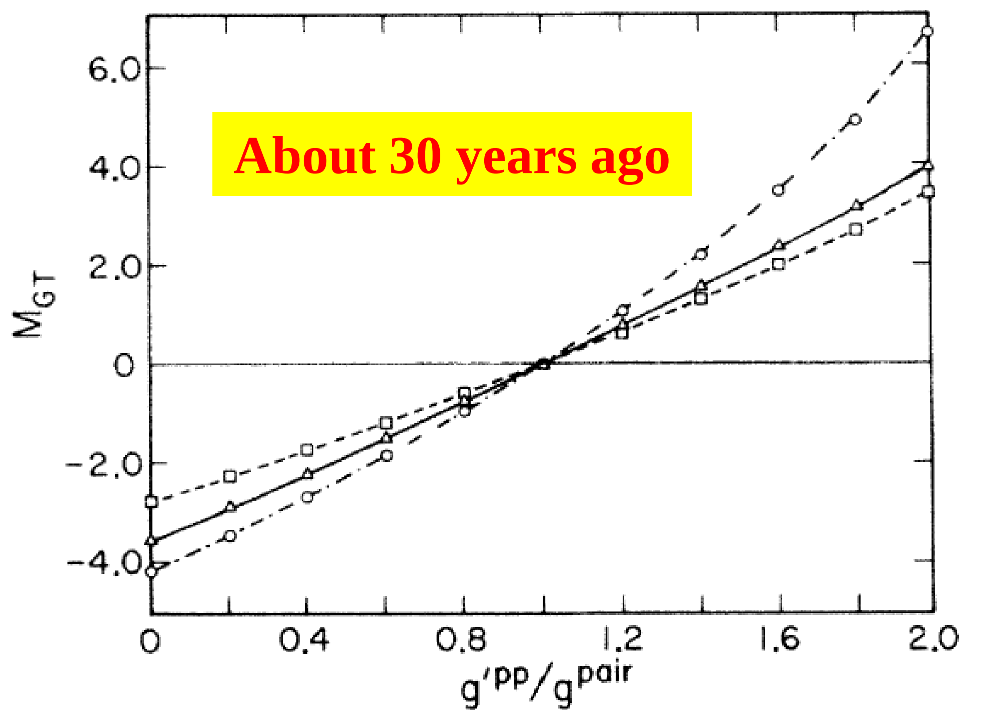
Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11

E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53

…



The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior? Is it an artifact of the QRPA?

2νββ-decay rate

$$\left[T^{2\nu\beta\beta}_{1/2}(0^+)\right]^{-1} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 I^{2\nu}(0^+),$$

$$I^{2\nu}(0^+) = \frac{1}{m_e^9}\int_{m_e}^{E_i-E_f-m_e} F_0(Z_f,E_{e_1})p_{e_1}E_{e_1}dE_{e_1} \times \int_{m_e}^{E_i-E_f-E_{e_1}} F_0(Z_f,E_{e_2})p_{e_2}E_{e_2}dE_{e_2} \times \int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathcal{A}^{2\nu} dE_{\nu_1}.$$

$$\mathcal{A}^{2\nu} = g_V^4\left[\frac{1}{4}|M_F^K + M_F^L|^2 + \frac{3}{4}|M_F^K - M_F^L|^2\right] - g_V^2 g_A^2 \mathrm{Re}\left\{M_F^{K*}M_{GT}^L + M_{GT}^{K*}M_F^L\right\} + \frac{g_A^4}{3}\left[\frac{3}{4}|M_{GT}^K + M_{GT}^L|^2 + \frac{1}{4}|M_{GT}^K - M_{GT}^L|^2\right]$$

$$M_F^K = \sum_n \frac{K(0_n^+)}{2}F_n, \quad M_F^L = \sum_n \frac{L(0_n^+)}{2}F_n,$$
$$M_{GT}^K = \sum_n \frac{K(1_n^+)}{2}G_n, \quad M_{GT}^L = \sum_n \frac{L(1_n^+)}{2}G_n,$$

$$F_n = \langle 0_f^+ \| \sum_m \tau_m^- \| 0_n^+\rangle\langle 0_n^+ \| \sum_m \tau_m^- \| 0_i^+\rangle,$$
$$G_n = \langle 0_f^+ \| \sum_m \tau_m^-\sigma_m \| 1_n^+\rangle\langle 1_n^+ \| \sum_m \tau_m^-\sigma_m \| 0_i^+\rangle$$

$$K_n(J^+) = \frac{2}{(2E_n(J^+)-E_i-E_f)+\epsilon_K} + \frac{2}{(2E_n(J^+)-E_i-E_f)-\epsilon_K}$$
$$L_n(J^+) = \frac{2}{(2E_n(J^+)-E_i-E_f)+\epsilon_L} + \frac{2}{(2E_n(J^+)-E_i-E_f)-\epsilon_L}$$

$$\epsilon_K = E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}$$
$$\epsilon_L = E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}$$

In the limit $2E_n - E_i - E_f = 0$ $\Rightarrow$ $\mathcal{A}^{2\nu} = 0$

s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^\dagger_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g_{pp}^{T=1}) A^\dagger_{0,1}(0,0)A_{0,1}(0,0)\,.}_{H_I}$$

H_I violates SU(4) symmetry

g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)

$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0

$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)

g_{ph}- strength of particle-hole force

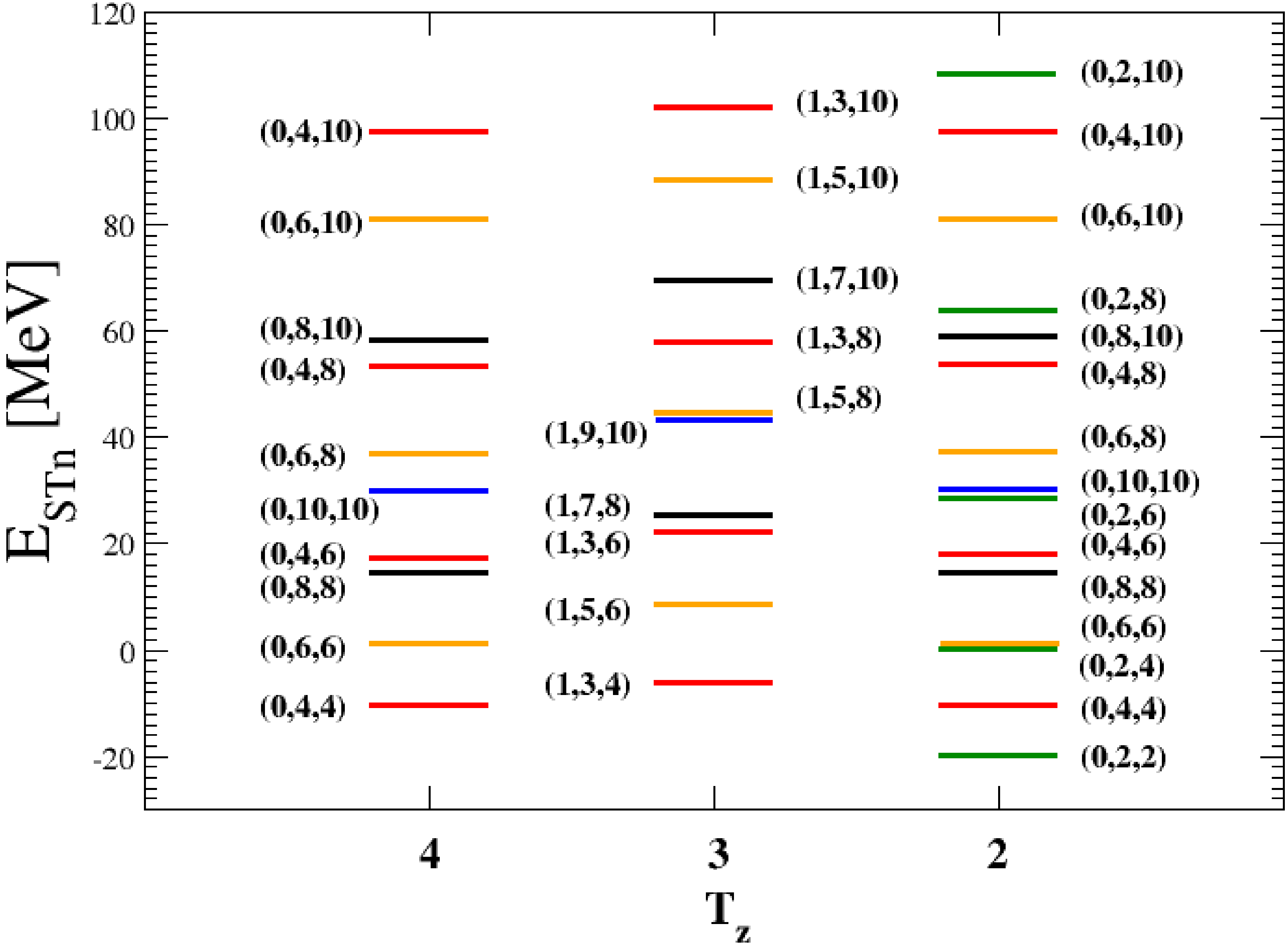
M_F and M_{GT} do not depend on the mean-field part of H and are governed by a weak violation of the **SU(4)** symmetry by the particle-particle interaction of H

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\,(g_{pair} - g_{pp}^{T=1})}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}$$

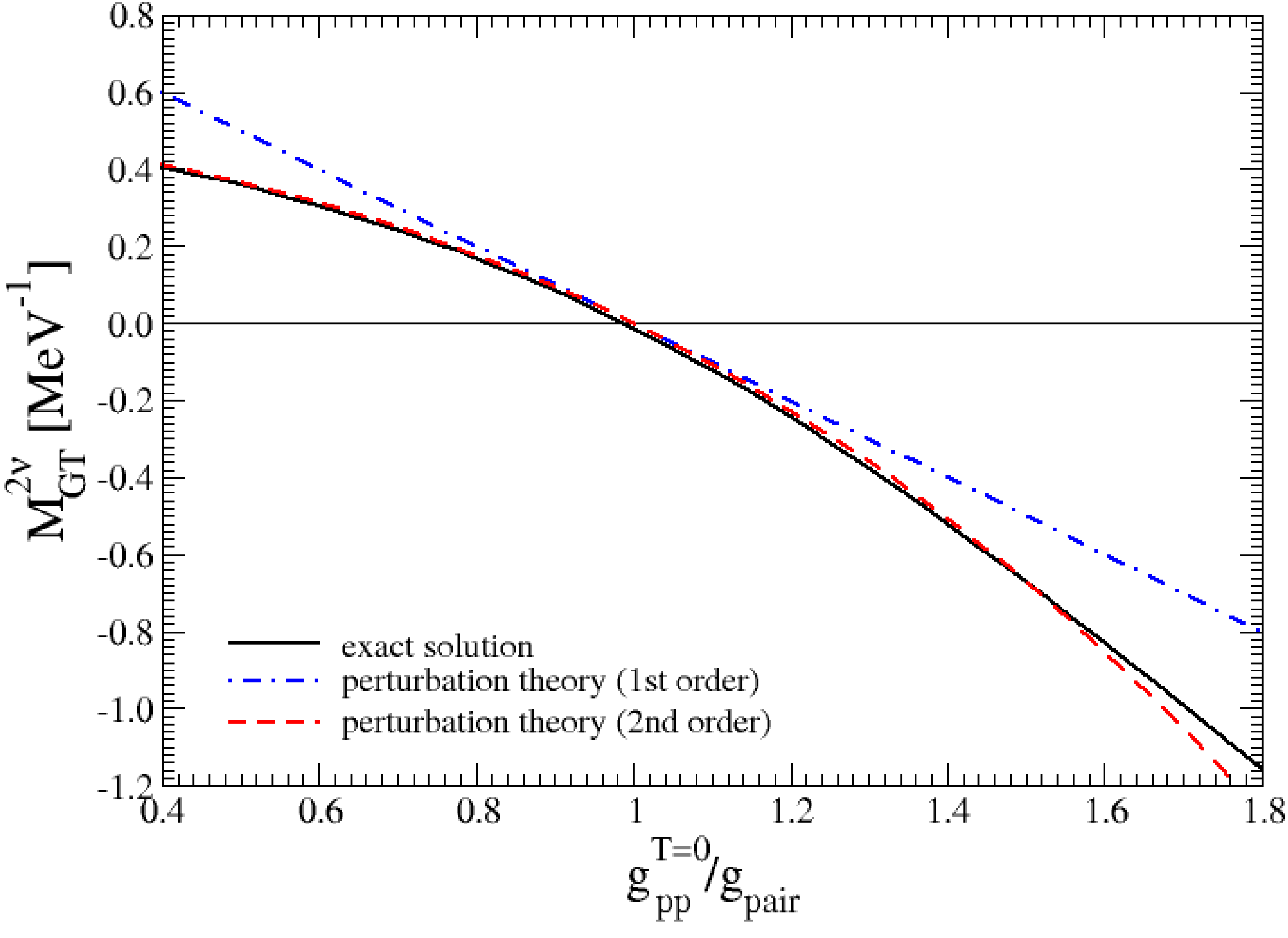
$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 9g_{ph}} \left\{ \frac{(g_{pair} - g_{pp}^{T=0})}{(10g_{pair} + 20g_{ph})} + \frac{2g_{ph}(g_{pair} - g_{pp}^{T=1})}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})} \right\}$$

Energies of excited states for the case of conserved SU(4) symmetry

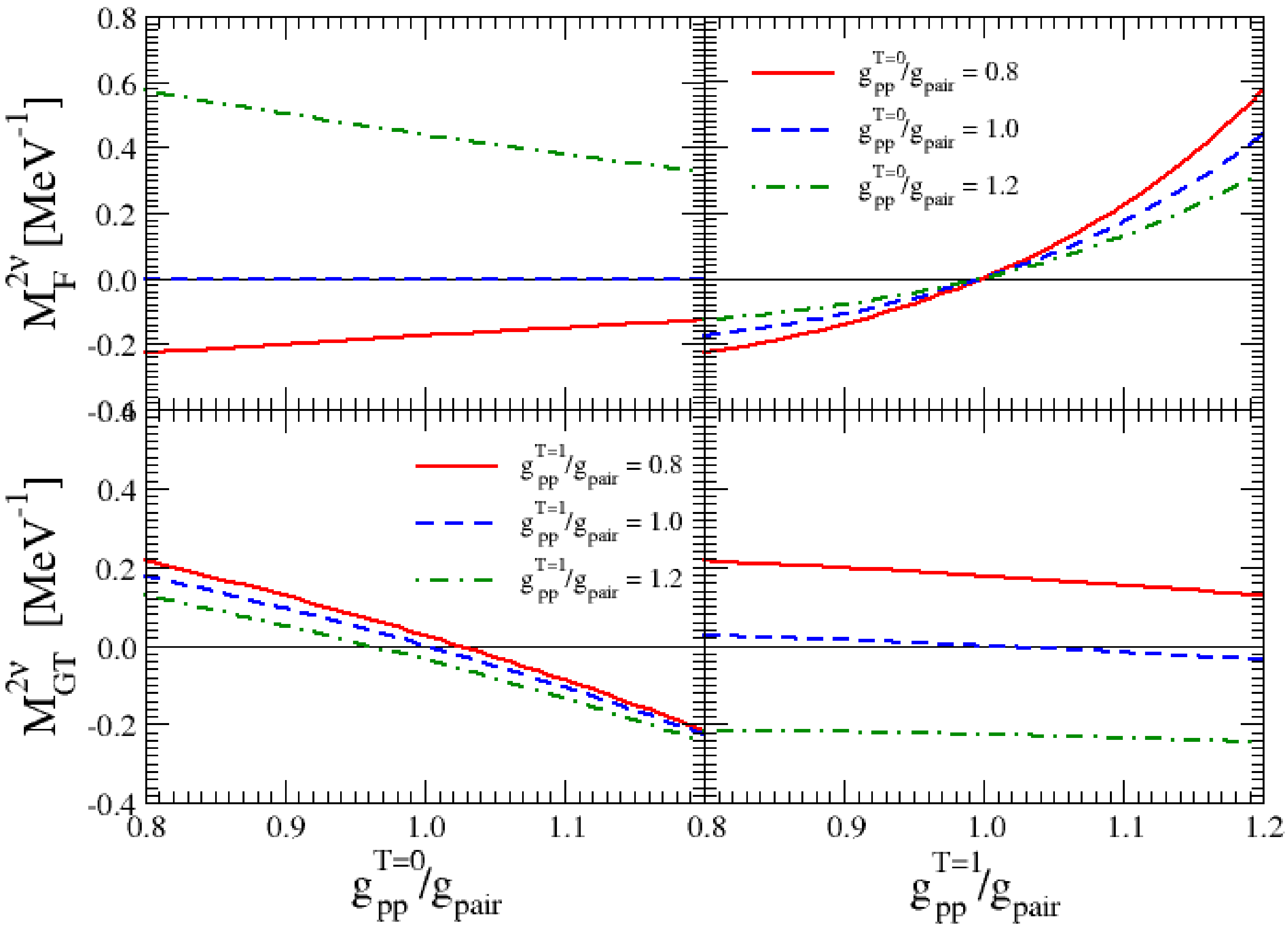
$M_F=0$, $M_{GT}=0$ (see SU(4) multiplets)



M_{GT} up to the second order of perturbation theory due to violation of the SU(4) symmetry by the particle-particle interaction of H



Results confirm dependence of M_F and M_{GT} on $g_{pp}^{T=0}$ and $g_{pp}^{T=1}$ by the QRPA



The 0νββ-decay with emission of electrons in $p_{1/2}$ wave state

D. Štefánik, R. Dvornický, F.Š., Nuclear Theory 33 (2014) 115

$$\psi(\mathbf{r},p,s) \simeq \psi_{s_{1/2}}(\mathbf{r},p,s) + \psi_{p_{1/2}}(\mathbf{r},p,s) =$$

$$\begin{pmatrix} g_{-1}(\varepsilon,r)\chi_s \\ f_{+1}(\varepsilon,r)\,(\vec{\sigma}\cdot\hat{\mathbf{p}})\,\chi_s \end{pmatrix} + \begin{pmatrix} i g_{+1}(\varepsilon,r)\,(\vec{\sigma}\cdot\hat{\mathbf{r}})\,(\vec{\sigma}\cdot\hat{\mathbf{p}})\,\chi_s \\ -i f_{-1}(\varepsilon,r)\,(\vec{\sigma}\cdot\hat{\mathbf{r}})\,\chi_s \end{pmatrix}$$

Exact relativ. electron w.f.

$$J^\rho(\mathbf{x}) = \sum_n \tau_n^+ \delta(\mathbf{x}-\mathbf{r}_n)\left[(g_V - g_A C_n)\, g^{\rho 0} + g^{\rho k}\right.$$

$$\left.\times\left(g_A\sigma_n^k - g_V D_n^k - g_P\left(p_n^k - p_n^{\prime k}\right)\frac{\vec{\sigma}_n\cdot\left(\mathbf{p}_n - \mathbf{p}_n^{\prime}\right)}{2m_N}\right)\right]$$

Higher order terms of nucleon current with nucleon recoil

$$C_n = \frac{\vec{\sigma}_n\cdot\left(\mathbf{p}_n + \mathbf{p}_n^{\prime}\right)}{2m_N} - \frac{g_P}{g_A}\left(E_n - E_n^{\prime}\right)\frac{\vec{\sigma}_n\cdot\left(\mathbf{p}_n - \mathbf{p}_n^{\prime}\right)}{2m_N}$$

$$\mathbf{D}_n = \frac{\left(\mathbf{p}_n + \mathbf{p}_n^{\prime}\right)}{2m_N} - i\left(1+\frac{g_M}{g_V}\right)\frac{\vec{\sigma}_n\times\left(\mathbf{p}_n - \mathbf{p}_n^{\prime}\right)}{2m_N}.$$

0νββ-decay rate with $p_{1/2}$ electrons (2 additional NMEs and 5 phase-space factors)

$$\left[T_{1/2}^{0\nu\beta\beta}\right]^{-1} = \frac{|m_{\beta\beta}|^2}{m_e^2} g_A^4 \left(2Re\left\{M_s M_r^*\right\} G_{sr} + 2Re\left\{M_s M_p^*\right\} G_{sp} + 2Re\left\{M_r M_p^*\right\} G_{rp} + G_{ss}|M_s|^2 + G_{rr}|M_r|^2 + G_{pp}|M_p|^2\right),$$

$$M_s = -\frac{M_F}{g_A^2} + M_{GT} + M_T$$

$$M_{F,GT,T} = \sum_{r,s} \langle 0| h_{F,GT,T}(r_-) \mathcal{O}_{\mathcal{F},\mathcal{GT},\mathcal{T}} |0\rangle$$

$$M_p = -\frac{M'_F}{g_A^2} + M'_{GT} + M'_T + M_V + M_A + M'_A$$

$$M_V = i\sum_{r,s} \langle 0| \frac{h_{AV}(r_-) + h_{VP}(r_-)}{2R^2} \tau_r^+ \tau_s^+ (\mathbf{r}_- \times \mathbf{r}_+) \cdot \vec{\sigma}_r |0\rangle$$

$$M'_{F,GT,T} = \sum_{r,s} \langle 0| h_{F,GT,T}(r_-) \mathcal{O}_{\mathcal{F},\mathcal{GT},\mathcal{T}} \left(\frac{|\mathbf{r}_-|^2 - |\mathbf{r}_+|^2}{4R^2}\right)$$

$$M_A = \sum_{r,s} \langle 0| \frac{h_{AP}(r_-) + h_{AA}(r_-) + h_{MM}(r_-)}{2R^2} \times \tau_r^+ \tau_s^+ (\vec{\sigma}_r \cdot \mathbf{r}_-)(\vec{\sigma}_s \cdot \mathbf{r}_+) |0\rangle$$

$$M'_A = \sum_{r,s} \langle 0| h_A(r_-) \frac{(\mathcal{O}_{\mathcal{T}} + \mathcal{O}_{\mathcal{GT}}) \mathbf{r}_- \cdot \mathbf{r}_+}{2R^2} |0\rangle$$

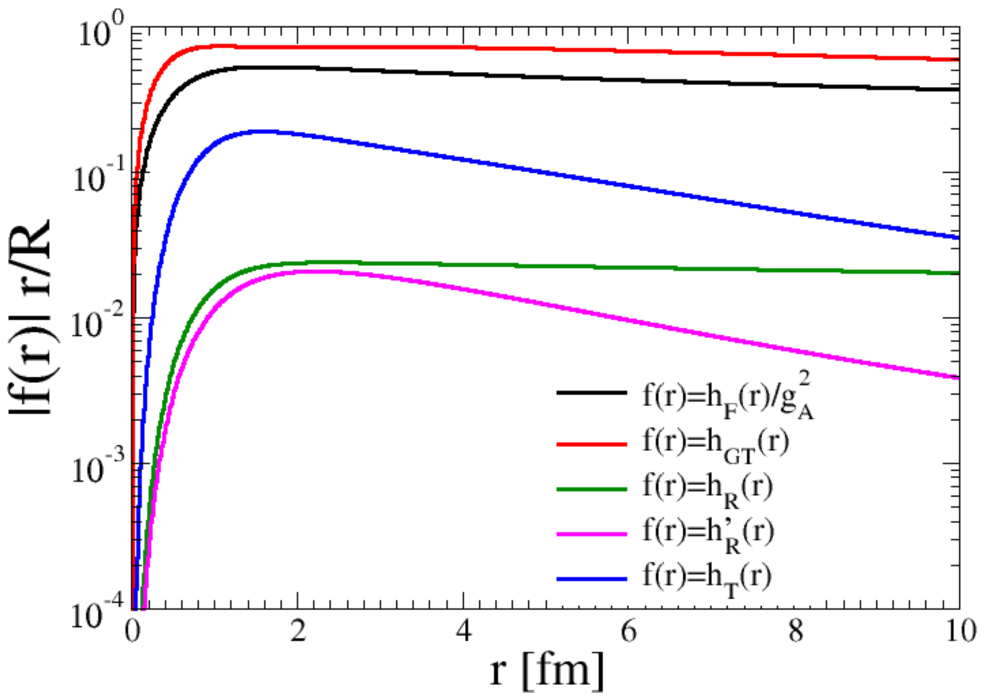
$$M_r = \sum_{r,s} \langle 0| \left(h_R(r_-) + h'_R(r_-)\right) \mathcal{O}_{\mathcal{T}} - 2h_R(r_-) \mathcal{O}_{\mathcal{GT}} |0\rangle$$

	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn	^{130}Te	^{136}Xe	^{150}Nd
$Q_{\beta\beta}$ [MeV]	4.27226	2.03904	2.99512	3.35037	3.03440	2.01785	2.8135	2.28697	2.52697	2.45783	3.37138
G_{ss} $[10^{-18} yr^{-1}]$	24 834.	2 368.1	10 176.	20 621.	15 953.	4 828.5	16 734.	9 063.5	14 255.	14 619.	63 163.
G_{sr} $[10^{-18} yr^{-1}]$	−4 138.3	−529.26	−2 499.4	−5 929.3	−4 738.2	−1 504.8	−5 569.5	−3 082.8	−5 071.1	−5 385.7	−26 409.
G_{rr} $[10^{-18} yr^{-1}]$	690.26	118.37	614.25	1 705.7	1 407.9	469.16	1 854.5	1 049.0	1 804.7	1 984.9	11 045.
G_{sp} $[10^{-18} yr^{-1}]$	−171.01	−29.513	−152.98	−424.86	−350.88	−117.07	−462.44	−261.74	−450.22	−495.23	−2 754.1
G_{rp} $[10^{-18} yr^{-1}]$	28.553	6.6047	37.619	122.29	104.31	36.518	154.05	89.101	160.29	182.59	1 152.3
G_{pp} $[10^{-18} yr^{-1}]$	1.1824	0.36878	2.3055	8.7718	7.7325	2.8437	12.802	7.5711	14.242	16.803	120.25

Calculated phase-space factor for $0\nu\beta\beta$-decay with emission of $s_{1/2}$ and $p_{1/2}$ electrons

($m_{\beta\beta}$ mechanism)

	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn	^{130}Te	^{136}Xe	^{150}Nd
r = 1 fm											
$2h_R(r)/h_{GT}(r)$	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.03
$\Delta[\%]$	0.31	0.55	0.93	1.81	2.03	2.44	4.22	3.24	3.70	4.10	5.74
r = 2 fm											
$2h_R(r)/h_{GT}(r)$	0.71	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05
$\Delta[\%]$	1.19	0.46	0.20	0.60	0.82	1.21	3.60	1.95	2.37	2.75	4.27



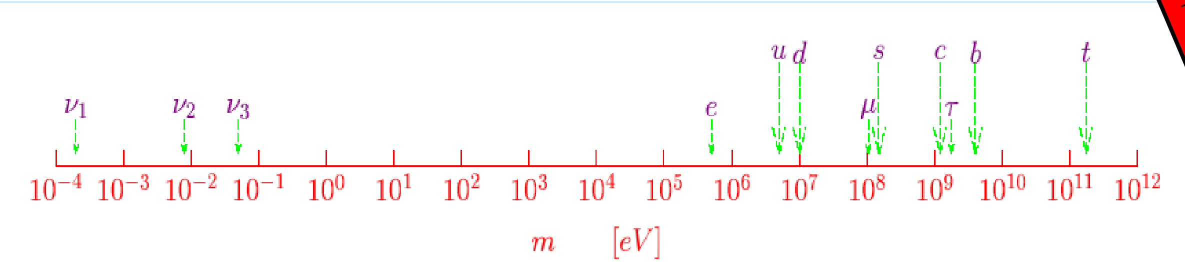
Effect of $p_{1/2}$ wave is below 10%.

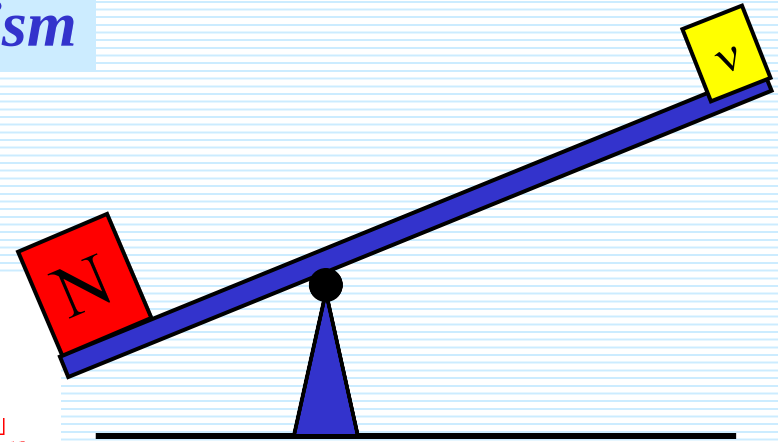
The 0$\nu\beta\beta$-decay mechanisms with light and heavy neutrinos

Assumption $M_R \gg m_D$

See-Saw mechanism

$$\begin{pmatrix} \overline{\nu}_L & \overline{(\nu_R)^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix}$$





Left-right symmetric models SO(10)

$$\nu_{eL} = \sum_{i=1}^{light} U_{ei}\chi_{iL} + \sum_{i=1}^{heavy} U_{ei} N_{iL}$$

large small

$$(\nu_{eR})^c = \sum_{i=1}^{light} V_{ei}\chi_{iL} + \sum_{i=1}^{heavy} V_{ei} N_{iL}$$

small large

Probability of Neutrino Oscillations

As N increases, the formalism gets rapidly more complicated!

N	Δm_{ij}^2	θ_{ij}	CP
2	1	1	0+1
3	2	3	1+2
6	5	15	10+9

Why TeV Seesaws?

Is the seesaw scale very close to a fundamental physics scale?

How heavy are the heavy Majorana neutrinos or the Higgs triplet?

10^{19} GeV — **Planck**

10^{16} GeV — **GUT** — to unify strong, weak & electromagnetic forces?

Conventional (Type-one) Seesaw Picture: close to the GUT scale

10^{3} GeV — **TeV** — **TeV Seesaw Idea:** driven by testability at LHC

to solve the unnatural gauge hierarchy problem?

GeV/5 — **Fermi**

10^{-6} GeV — **kev** — Hot dark mattter

Left-handed neutrinos: Majorana neutrino mass eigenstate N with arbitrary mass m_N

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha}\, \nu_\alpha$$

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N \left(U^2_{eN} m_N \right) m_p\, M'^{0\nu}(m_N, g_A^{\rm eff}) \right|^2$$

General case

$$M'^{0\nu}(m_N, g_A^{\rm eff}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p \times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0^+_F | J^{\mu\dagger}(\mathbf{x}) | n\rangle \langle n | J^\dagger_\mu(\mathbf{y}) | 0^+_I\rangle}{\sqrt{p^2 + m_N{}^2}\left(\sqrt{p^2 + m_N{}^2} + E_n - \frac{E_I - E_F}{2}\right)}$$

$$M'^{0\nu}(m_N \to 0, g_A^{\rm eff}) = \frac{1}{m_p m_e} M'^{0\nu}_\nu(g_A^{\rm eff})$$

$$M'^{0\nu}(m_N \to \infty, g_A^{\rm eff}) = \frac{1}{m_N^2} M'^{0\nu}_N(g_A^{\rm eff})$$

Particular cases

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_A^4 \times \times \begin{cases} \left|\frac{\langle m_\nu\rangle}{m_e}\right|^2 \left| M'^{0\nu}_\nu(g_A^{\rm eff})\right|^2 & \text{for } m_N \ll p_F \\ \left|\langle\frac{1}{m_N}\rangle m_p\right|^2 \left| M'^{0\nu}_N(g_A^{\rm eff})\right|^2 & \text{for } m_N \gg p_F \end{cases}$$

$$\langle m_\nu\rangle = \sum_N U^2_{eN} m_N$$

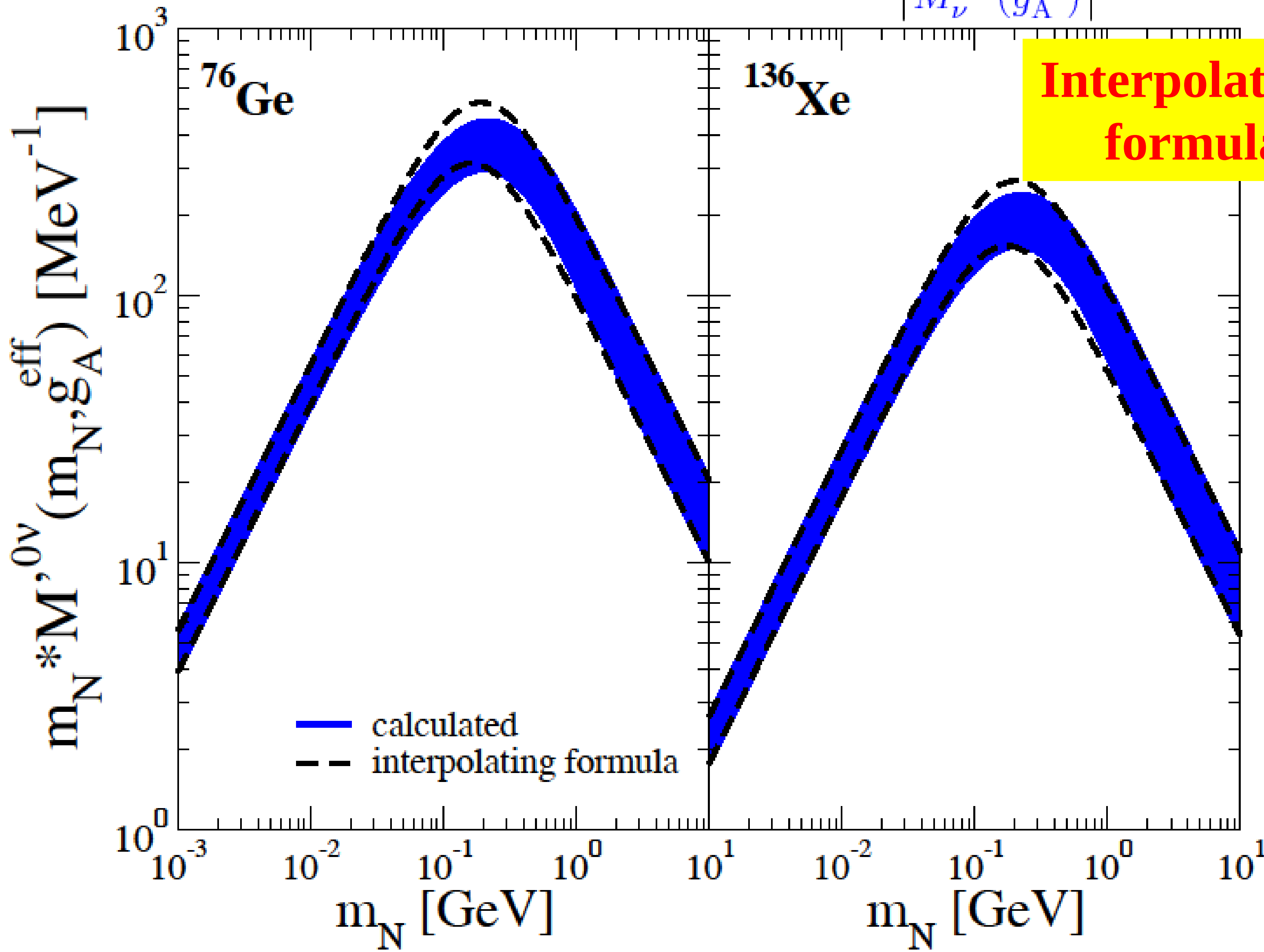
$$\left\langle \frac{1}{m_N} \right\rangle = \sum_N \frac{U^2_{eN}}{m_N}$$

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A} \cdot \left| m_{\rm p} \sum_{\rm N} U^2_{e{\rm N}} \frac{m_{\rm N}}{\langle p^2 \rangle + m_{\rm N}^2} \right|^2,$$

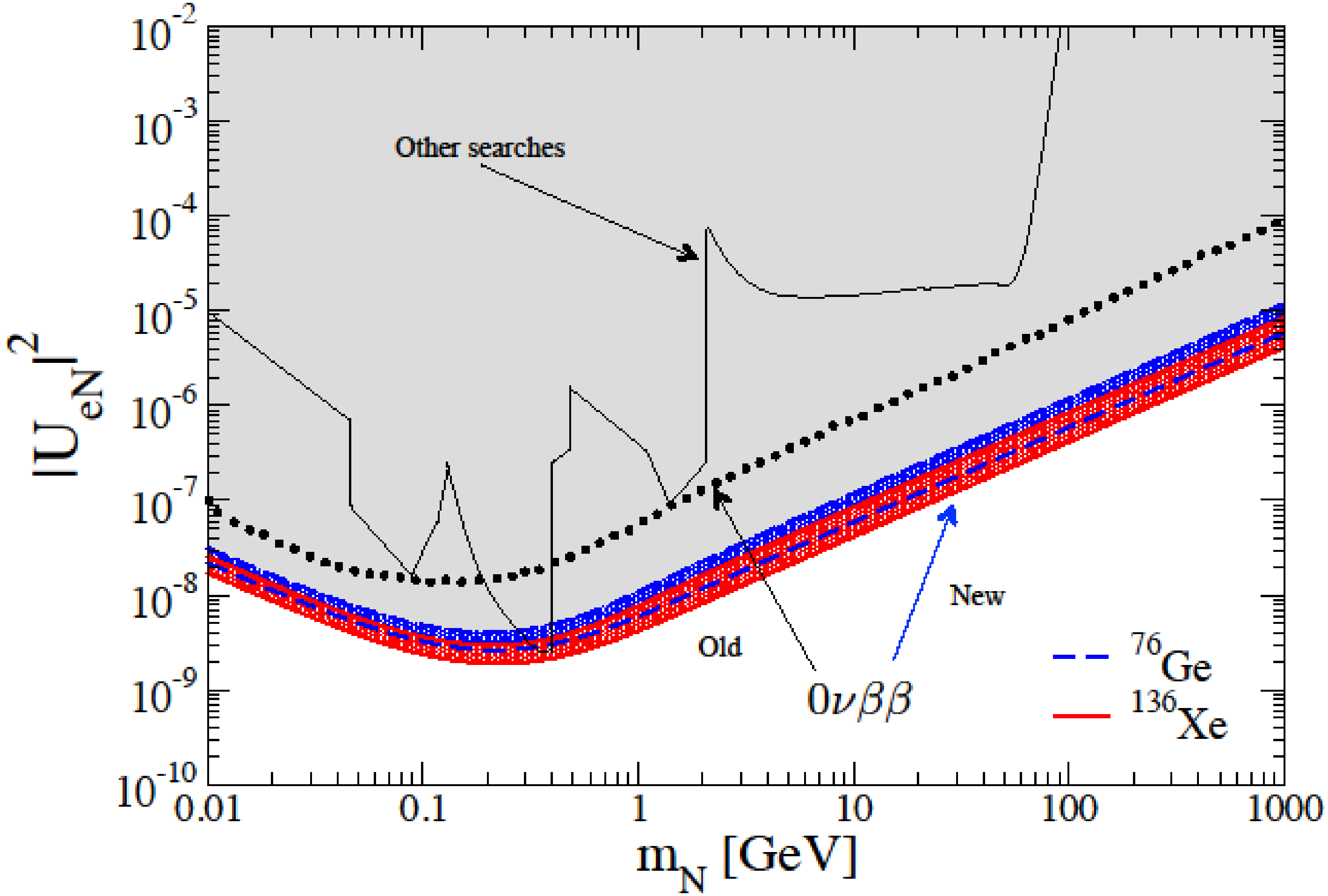
$$\mathcal{A} = G^{0\nu} g_{\rm A}^4 \left| M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff}) \right|^2,$$

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \left| \frac{M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff})}{M'^{0\nu}_{\nu}(g_{\rm A}^{\rm eff})} \right|^{\cancel{2}} \approx 200 \;\; MeV$$

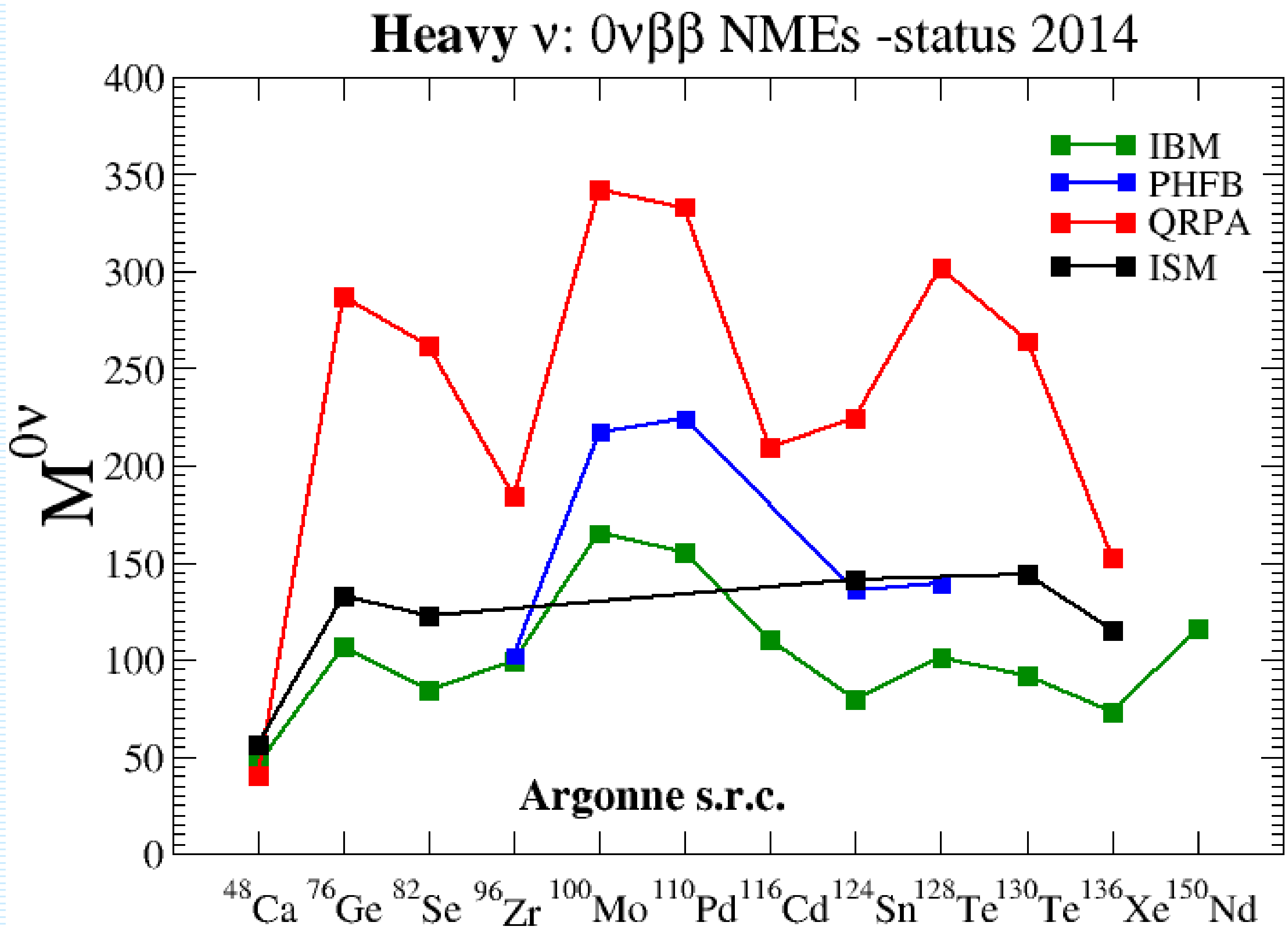
Interpolating formula



Exclusion plot in $|U_{eN}|^2 - m_N$ plane



Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry …), ii) More stringent limits on the 0νββ half-life

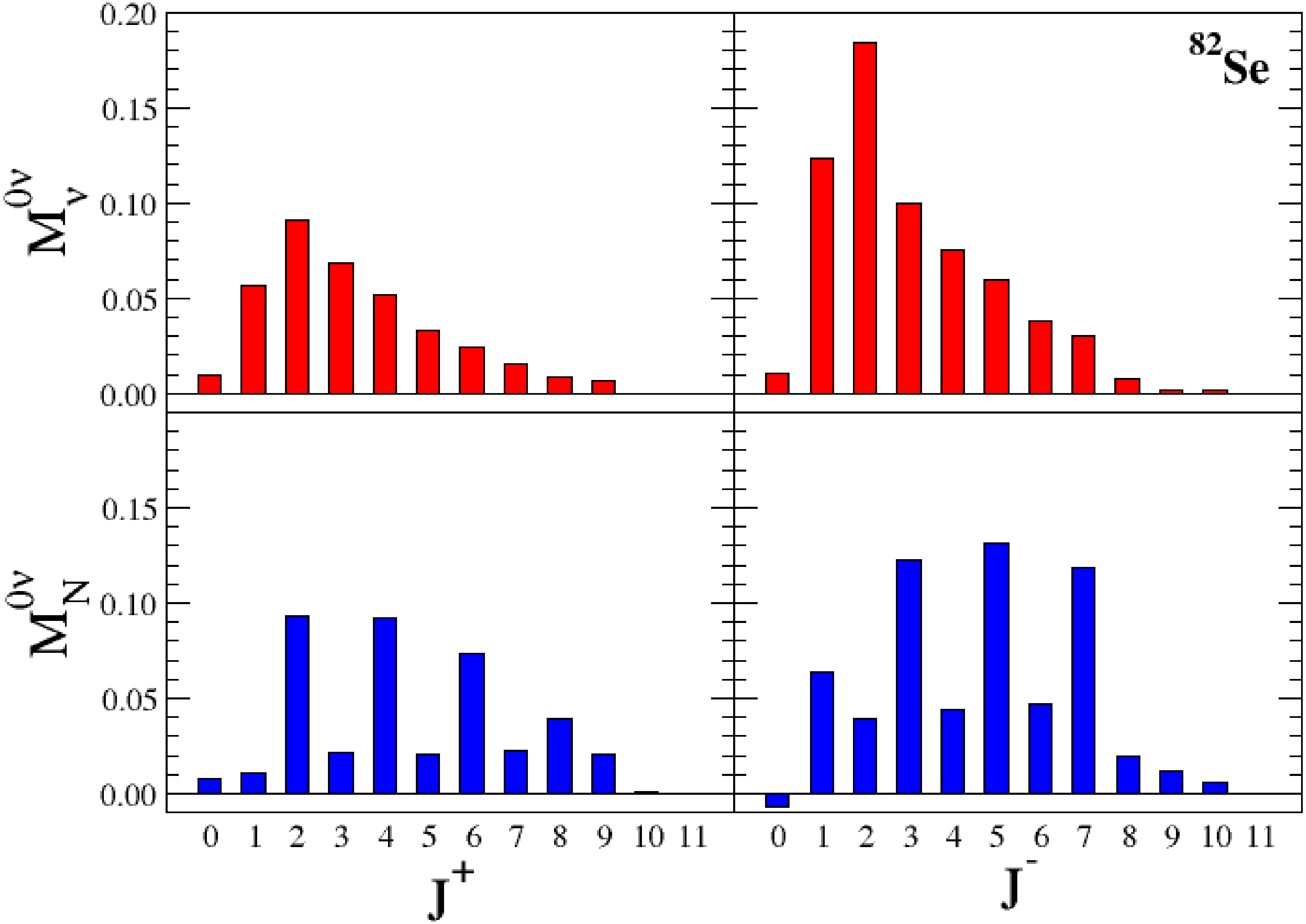


PHFB: K. Rath et al., PRC 85 (2012) 014308
IBM: Barea, Kotila, Iachello, PRC (2013) 014315

Fedo

QRPA: Faessler, Gonzales, , F. Š., Kovalenko, PRD 90 (2014) 096010 ``Vergados, Ejiri, F. Š., RPP 75 (2012) 106301
ISM: Menendez, privite communications

Multipole decomposition of NMEs normalized to unity



Co-existence of light and heavy neutrino mass Mechanisms of the $0\nu\beta\beta$-decay

It may happen that in year 201? (or 2???) the $0\nu\beta\beta$-decay will be detected for 2-3 or more isotopes …

Co-existence of 2, 3 or more interferring mechanisms of 0νββ-decay

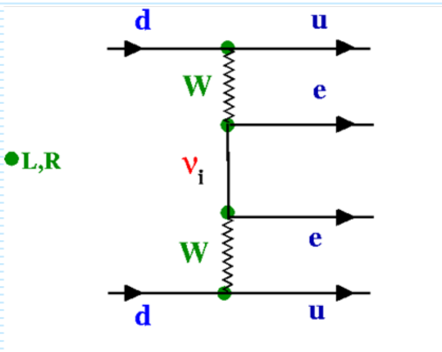
It is well-known that there exist many mechanisms that may contribute to the 0νββ. Let consider 3 mechanisms: i) light ν-mass mechanism, ii) heavy ν-mass mechanism, iii) R-parity breaking SUSY mechanism with gluino exchange and CP conservation

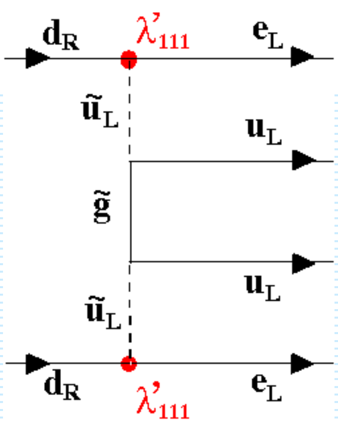
$$\frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(E_0, Z)\left|\frac{m_{\beta\beta}}{m_e} M^{0\nu}_{\nu} + \eta^L_N M^{0\nu}_N + \eta_{\lambda'_{111}} M^{0\nu}_{\lambda'_{111}} \ldots\right|^2$$

$$m_{\beta\beta} = \sum_k \left(U^L_{ek}\right)^2 \xi_k m_k$$

$$\eta^L_N = \sum_k^{heavy} U^2_{ek} \frac{m_p}{M_k}$$

$$\eta_{\lambda'_{111}} = \frac{\pi\alpha_s}{6} \frac{\lambda'^2_{111}}{G^2_F m^4_{\tilde{d}_R}} \frac{m_p}{m_{\tilde{g}}} \left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\right]^2 .$$





Claim of evidence:

Klapdor-Kleingrothaus, Krivosheina, Mod. Phys. A 21, 1547 (2009)

$$T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{100}\mathrm{Mo}) \geq 5.8 \times 10^{23}\ \mathrm{y}$$

$$T^{0\nu}_{1/2}(^{130}\mathrm{Te}) \geq 3.0 \times 10^{24}\ \mathrm{y}$$

$\xi_{Te} < 1.2$

$\xi_{Mo} < 2.6$

We introduce

$$\xi = \frac{|M^{\nu}_1|\sqrt{T_1\ G_1}}{|M^{\nu}_2|\sqrt{T_2\ G_2}}$$

r S

ξ=0, non-observation ($T_2 \to \infty$)
ξ=1, solution for single active mech. is reproduced

4 sets of two linear eq. (sign problem)

$$\frac{\pm 1}{\sqrt{T_1\ G_1}} = \frac{m_{\beta\beta}}{m_e} M_1^\nu + \eta M_1^\eta$$
$$\frac{\pm 1}{\sqrt{T_2\ G_2}} = \frac{m_{\beta\beta}}{m_e} M_2^\nu + \eta M_2^\eta$$

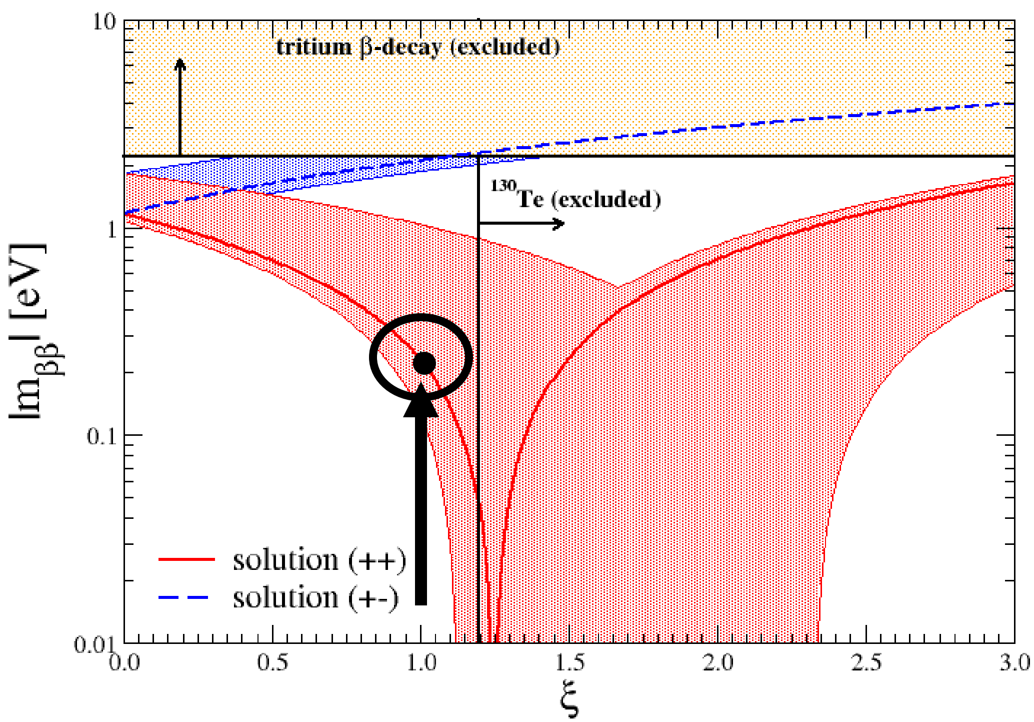
2 different solutions

CP-conservation assumed

$$|m_{\beta\beta}| = \left| \frac{m_e}{M_1^\nu \sqrt{T_1\ G_1}} \frac{M_1^\nu\ M_2^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \pm \frac{m_e}{M_2^\nu \sqrt{T_2\ G_2}} \frac{M_2^\nu\ M_1^\eta}{(M_1^\nu M_2^\eta - M_2^\nu M_1^\eta)} \right|$$

2 active mechanisms of the 0νββ-decay: Light and heavy ν-mass mechanism

F.Š., J.D. Vergados, A. Faessler, PRD 82, 113015 (2010)

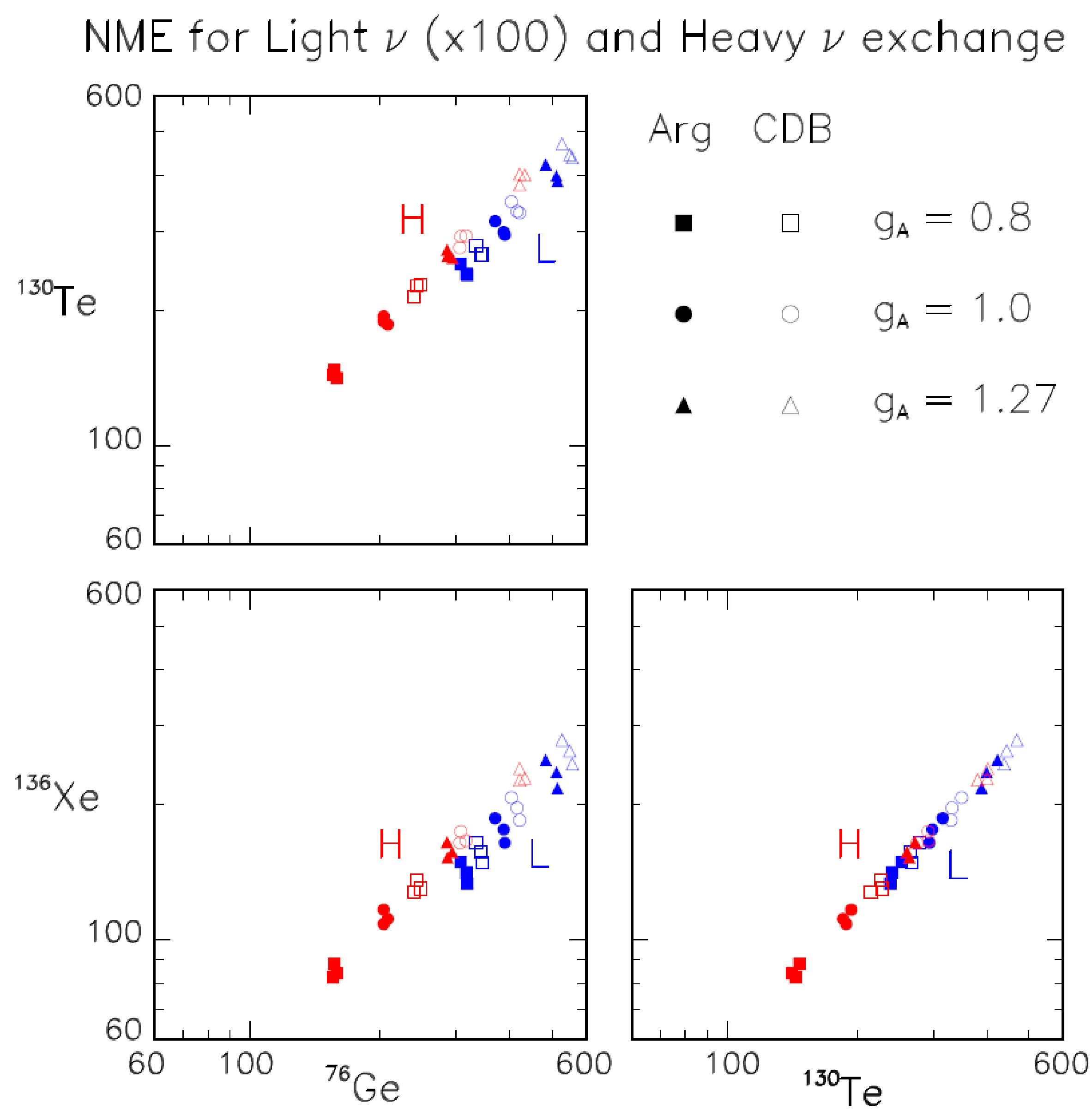


Non-observation of the 0νββ-decay for some isotopes might be in agreement with non- zero $m_{\beta\beta}$

Non-observation for ^{130}Te

- *Single solution for light ν-mass mech.*

Only positive signs:
There is a correlation of errors;
It is practically not possible to distinguish both mechanisms even observating the 0νββ-decay for 3 nuclei



E. Lisi, A. Rotunno, F. Š., arXiv:1506.04058 [hep-ph]

Two non-interfering mechanisms of the $0\nu\beta\beta$-decay (light LH and heavy RH neutrino exchange)

$$T^{0\nu}_{1/2}(^{76}Ge) \geq 1.9 \times 10^{25} y, \qquad T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25} y$$

$$5.8 \times 10^{23} y \leq T^{0\nu}_{1/2}(^{100}Mo) \leq 5.8 \times 10^{24} y, \qquad 3.0 \times 10^{24} y \leq T^{0\nu}_{1/2}(^{130}Te) \leq 3.0 \times 10^{25} y$$

Half-life:

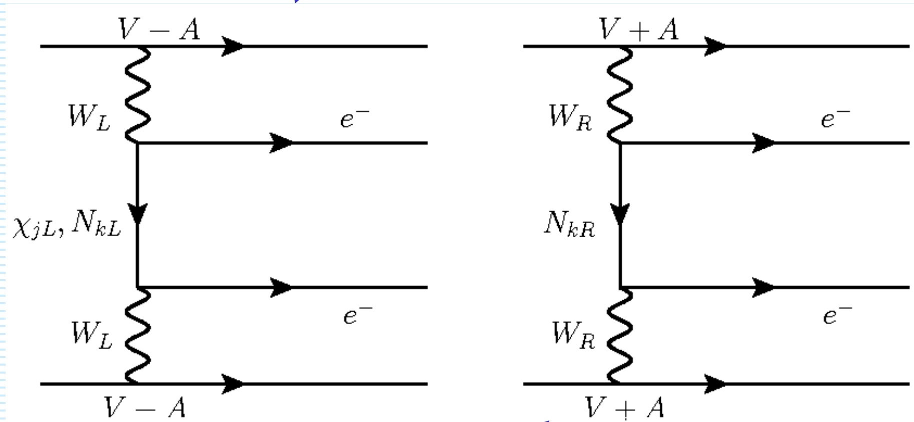
$$\frac{1}{T^{0\nu}_{1/2,i} G^{0\nu}_i(E,Z)} \cong |\eta_\nu|^2 |M'^{0\nu}_{i,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{i,N}|^2$$

$$\eta_\nu = \frac{m_{\beta\beta}}{m_e}$$

Set of equations:

$$\frac{1}{T_1 G_1} = |\eta_\nu|^2 |M'^{0\nu}_{1,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{1,N}|^2$$

$$\frac{1}{T_2 G_2} = |\eta_\nu|^2 |M'^{0\nu}_{2,\nu}|^2 + |\eta_R|^2 |M'^{0\nu}_{2,N}|^2$$



Solutions:

$$|\eta_\nu|^2 = \frac{|M'^{0\nu}_{2,N}|^2/T_1G_1 - |M'^{0\nu}_{1,N}|^2/T_2G_2}{|M'^{0\nu}_{1,\nu}|^2|M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2|M'^{0\nu}_{2,\nu}|^2}$$

$$|\eta_R|^2 = \frac{|M'^{0\nu}_{1,\nu}|^2/T_2G_2 - |M'^{0\nu}_{2,\nu}|^2/T_1G_1}{|M'^{0\nu}_{1,\nu}|^2|M'^{0\nu}_{2,N}|^2 - |M'^{0\nu}_{1,N}|^2|M'^{0\nu}_{2,\nu}|^2}$$

$$\eta^R_N = \left(\frac{M_W}{M_{WR}}\right)^4 \sum_k^{heavy} V^2_{ek} \frac{m_p}{M_k}$$

A. Faessler, A. Meroni, S.T. Petcov, F. Š., J.D. Vergados,
Phys. Rev. D 83, 113003 (2011); JHEP 1302, 025 (2013)

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Two non-interfering mechanisms of the $0\nu\beta\beta$-decay (light LH and heavy RH neutrino exchange)

The positivity condition:

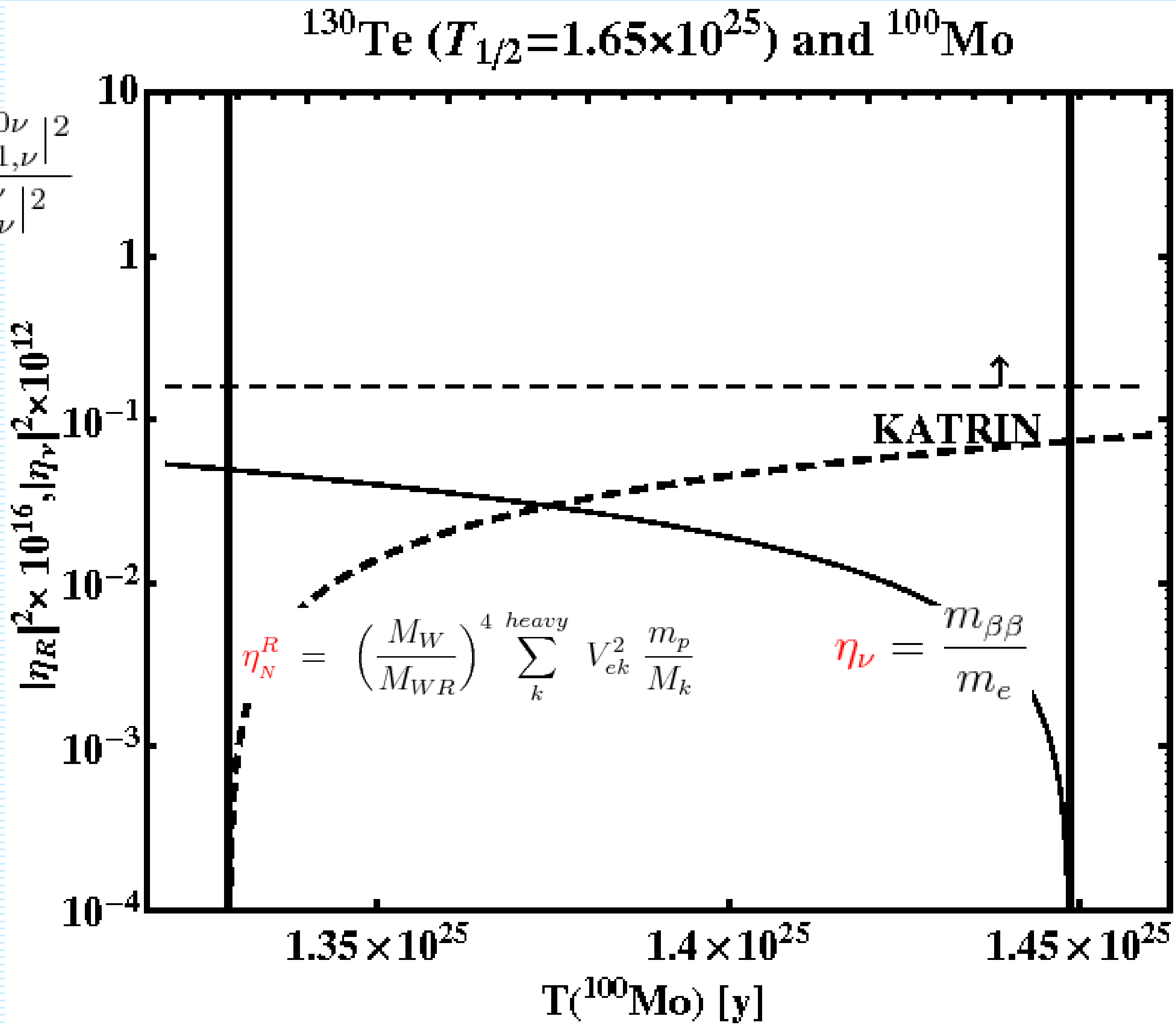
$$\frac{T_1 G_1 |M'^{0\nu}_{1,N}|^2}{G_2 |M'^{0\nu}_{2,N}|^2} \leq T_2 \leq \frac{T_1 G_1 |M'^{0\nu}_{1,\nu}|^2}{G_2 |M'^{0\nu}_{2,\nu}|^2}$$

Very narrow ranges!

$$0.15 \leq \frac{T^{0\nu}_{1/2}(^{100}Mo)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.18$$

$$0.17 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{76}Ge)} \leq 0.22$$

$$1.14 \leq \frac{T^{0\nu}_{1/2}(^{130}Te)}{T^{0\nu}_{1/2}(^{100}Mo)} \leq 1.24$$



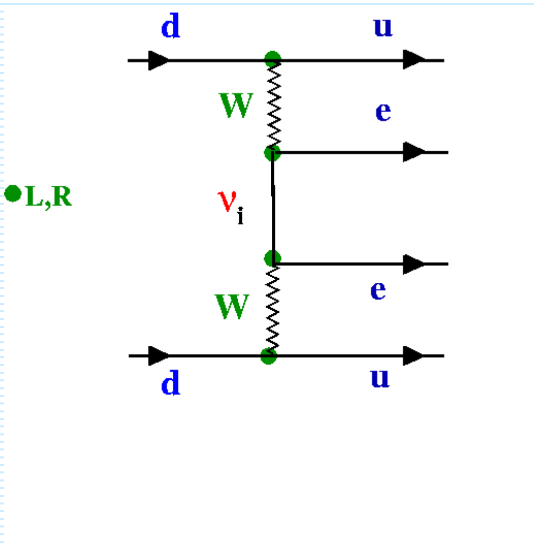
The $0\nu\beta\beta$-decay with right-handed currents revisited (exchange of light neutrinos)

D. Štefánik, R. Dvornický, F.Š., P. Vogel, arXiv:1506.07145 [hep-ph]

$$H^{\beta} = \frac{G_{\beta}}{\sqrt{2}} \left[j_L{}^{\rho} J^{\dagger}_{L\rho} + \chi j_L{}^{\rho} J^{\dagger}_{R\rho} + \eta j_R{}^{\rho} J^{\dagger}_{L\rho} + \lambda j_R{}^{\rho} J^{\dagger}_{R\rho} + h.c. \right]$$

$$\eta \simeq -\tan\zeta \qquad \chi = \eta$$

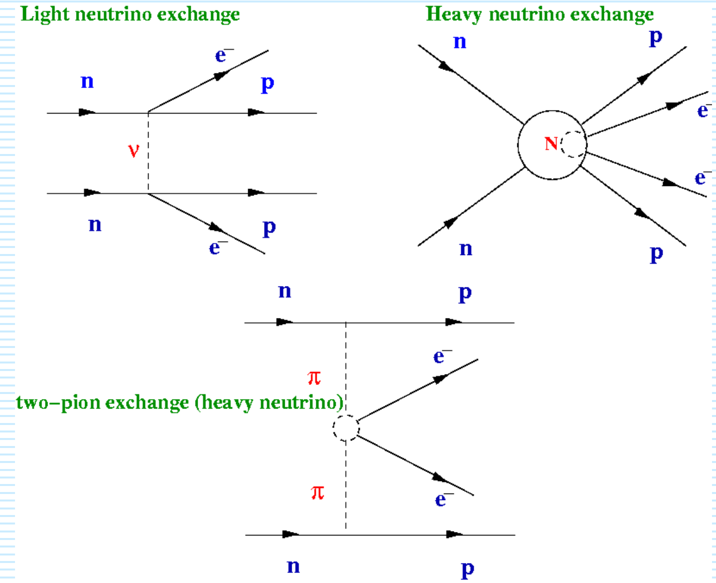
$$\lambda \simeq (M_{W_1}/M_{W_2})^2$$



$$\mathrm{P}_L \frac{\hat{q}+im}{q^2+m^2} P_L \Rightarrow \frac{im}{q^2}$$

$$\mathrm{P}_L \frac{\hat{q}+im}{q^2+m^2} P_R \Rightarrow \frac{i\hat{q}}{q^2}$$

$$\mathrm{P}_{L,R} \frac{\hat{q}+iM}{q^2+M^2} P_{L,R} \Rightarrow \frac{i}{M}$$



quark level **Mechanism** **nucleon level**

neutrino	*lept.v.*	*quarkv.*	*hadr.m.*	*supp.f.*	*LNV p.*	**parameters**
light	LL	LL	$2n$		$\sum^{light} UUm$	$m_{\beta\beta}$
	LR	LR	$2n$	$(M_1/M_2)^2$	$\sum^{light} UV$	$<\lambda>$
	LR	LL	$2n$	$\tan\zeta$	$\sum^{light} UV$	$<\eta>$
heavy	LL	LL	$2n$	$-$	$\sum^{heavy} U\ U\ m_p/M$	η_N
	RR	RR	$2n$	$(M_1/M_2)^4$	$\sum^{heavy} V\ V\ m_p/M$	
	RR	LL	$2n$	$(\tan\zeta)^4$	$\sum^{heavy} V\ V\ m_p/M$	
	RR	RL	2π	$\tan\zeta\ (M_1/M_2)^2$	$\sum^{heavy} V\ V\ m_p/M$	

3x3 block matrices U, S, T, V are generalization of PMNS matrix

Zhi-zhong Xing, Phys. Rev. D **85**, 013008 (2012)

Basis

6x6 neutrino mass matrix

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

$$(\nu_L, (N_R)^C)^T$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

Decomposition

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Type seesaw I

$$A \approx \mathbf{1}, B \approx \mathbf{1}, R \approx \frac{m_D}{m_{LNV}}\mathbf{1}, S \approx -\frac{m_D}{m_{LNV}}\mathbf{1}$$

Approximation

$$U_0 \simeq V_0$$

LNV parameters

$$\langle\lambda\rangle \approx (M_{W_1}/M_{W_2})^2 \frac{m_D}{m_{LNV}} |\xi|$$

$$\langle\eta\rangle \approx -\tan\zeta \frac{m_D}{m_{LNV}} |\xi| \,,$$

$$\begin{aligned} |\xi| &= |c_{23}c_{12}^2 c_{13} s_{13}^2 - c_{12}^3 c_{13}^3 - c_{13} c_{23} c_{12}^2 s_{13}^2 \\ &\quad - c_{12} c_{13} \left(c_{13}^2 s_{12}^2 + s_{13}^2\right)| \\ &\simeq 0.82 \end{aligned}$$

The 0νββ-decay rate with right-handed currents

$$\left[T^{0\nu}_{1/2}\right]^{-1} = \frac{\Gamma^{0\nu}}{\ln 2} = g_A^4 \left|M_{GT}\right|^2 \Bigg\{ C_{mm} \left(\frac{|m_{\beta\beta}|}{m_e}\right)^2 + C_{m\lambda}\frac{|m_{\beta\beta}|}{m_e}\langle\lambda\rangle\cos\psi_1 + C_{m\eta}\frac{|m_{\beta\beta}|}{m_e}\langle\eta\rangle\cos\psi_2 + C_{\lambda\lambda}\langle\lambda\rangle^2 + C_{\eta\eta}\langle\eta\rangle^2 + C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle\cos(\psi_1-\psi_2) \Bigg\}$$

Two additioanal phase-space factor G_{010} and G_{011} (For w.f. A $G_{010}=G_{03}$, $G_{011}=G_{04}$)

The induced pseudoscalar term included

$$\begin{aligned}
\langle\lambda\rangle &= \lambda\Big|\sum_j U_{ej}V_{ej}(g'_V/g_V)\Big|,\\
\langle\eta\rangle &= \eta\Big|\sum_j{}' U_{ej}V'_{ej}\Big|,\\
\psi_1 &= \arg[\{\sum_j{}' m_j U_{ej}^2\}\{\sum_j{}' U_{ej}V_{ej}(g'_V/g\\
\psi_2 &= \arg[\{\sum_i{}' m_j U_{ej}^2\}\{\sum_i{}' U_{ej}V'_{ej}\}^*].
\end{aligned}$$

$$\begin{aligned}
C_{mm} &= (1-\chi_F+\chi_T)^2 G_{01},\\
C_{m\lambda} &= -(1-\chi_F+\chi_T)\left[\chi_{2-}G_{03}-\chi_{1+}G_{04}\right],\\
C_{m\eta} &= (1-\chi_F+\chi_T)\\
&\times \left[\chi_{2+}G_{03}-\chi_{1-}G_{04}-\chi_P G_{05}+\chi_R G_{06}\right],\\
C_{\lambda\lambda} &= \chi_{2-}^2 G_{02}+\frac{1}{9}\chi_{1+}^2 G_{011}-\frac{2}{9}\chi_{1+}\chi_{2-}G_{010},\\
C_{\eta\eta} &= \chi_{2+}^2 G_{02}+\frac{1}{9}\chi_{1-}^2 G_{011}-\frac{2}{9}\chi_{1-}\chi_{2+}G_{010}+\chi_P^2 G_{08}\\
&- \chi_P\chi_R G_{07}+\chi_R^2 G_{09},\\
C_{\lambda\eta} &= -2\Big[\chi_{2-}\chi_{2+}G_{02}-\frac{1}{9}\left(\chi_{1+}\chi_{2+}+\chi_{2-}\chi_{1-}\right)G_{010}\\
&+ \frac{1}{9}\chi_{1+}\chi_{1-}G_{011}\Big].
\end{aligned} \qquad (37)$$

Different types of electron wave functions

$$\Psi(\varepsilon, \boldsymbol{r}) = \Psi^{(s_{1/2})}(\varepsilon, \boldsymbol{r}) + \Psi^{(p_{1/2})}(\varepsilon, \boldsymbol{r})$$

w.f. A (Doi et al.), uniform charge distribution, only the lowest term in expansion r/R

$$\begin{pmatrix} g_{-1}(\varepsilon, r) \\ f_{+1}(\varepsilon, r) \end{pmatrix} \approx \sqrt{F_0(Z_f, \varepsilon)} \begin{pmatrix} \sqrt{\frac{\varepsilon+m_e}{2\varepsilon}} \\ \sqrt{\frac{\varepsilon-m_e}{2\varepsilon}} \end{pmatrix} \qquad \begin{pmatrix} g_{+1}(\varepsilon, r) \\ f_{-1}(\varepsilon, r) \end{pmatrix} \approx \sqrt{F_0(Z_f, \varepsilon)} \begin{pmatrix} \sqrt{\frac{\varepsilon-m_e}{2\varepsilon}}[\alpha Z_f/2 + (\varepsilon+m_e)r/3] \\ -\sqrt{\frac{\varepsilon+m_e}{2\varepsilon}}[\alpha Z_f/2 + (\varepsilon-m_e)r/3] \end{pmatrix}$$

w.f. B, the analytical solution of the Dirac equation for a point-like nucleus

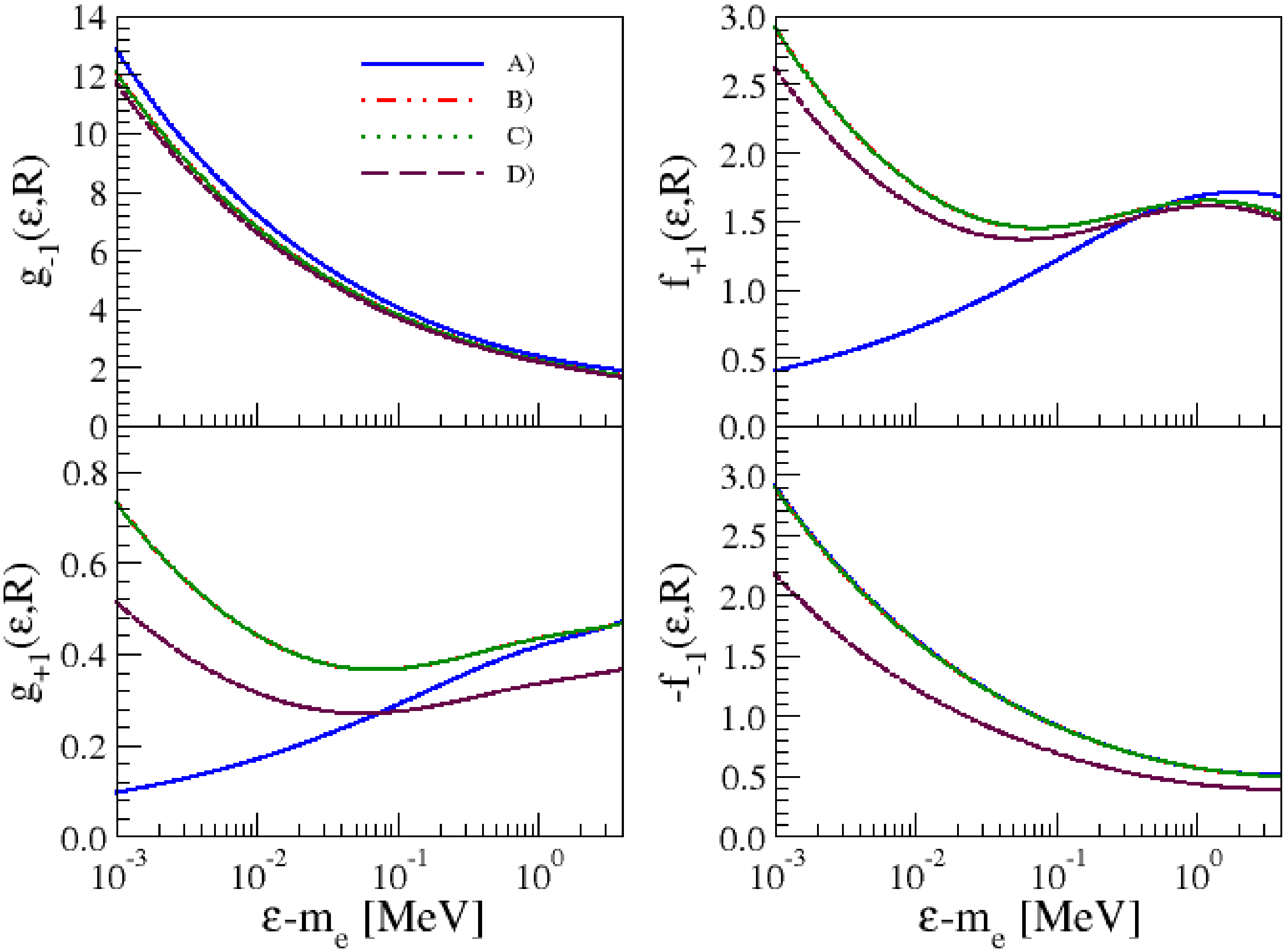
$$g_\kappa(\varepsilon, r) = \frac{1}{pr}\sqrt{\frac{\varepsilon+m_e}{2\varepsilon}} \frac{|\Gamma(1+\gamma_k+iy)|}{\Gamma(1+2\gamma_k)}(2pr)^{\gamma_k} \Im\left\{e^{i(pr+\xi)}\ {}_1F_1(\gamma_k - iy, 1+2\gamma_k, -2ipr)\right\}$$

$$f_\kappa(\varepsilon, r) = \frac{1}{pr}\sqrt{\frac{\varepsilon-m_e}{2\varepsilon}} \frac{|\Gamma(1+\gamma_k+iy)|}{\Gamma(1+2\gamma_k)}(2pr)^{\gamma_k} \Re\left\{e^{i(pr+\xi)}\ {}_1F_1(\gamma_k - iy, 1+2\gamma_k, -2ipr)\right\}$$

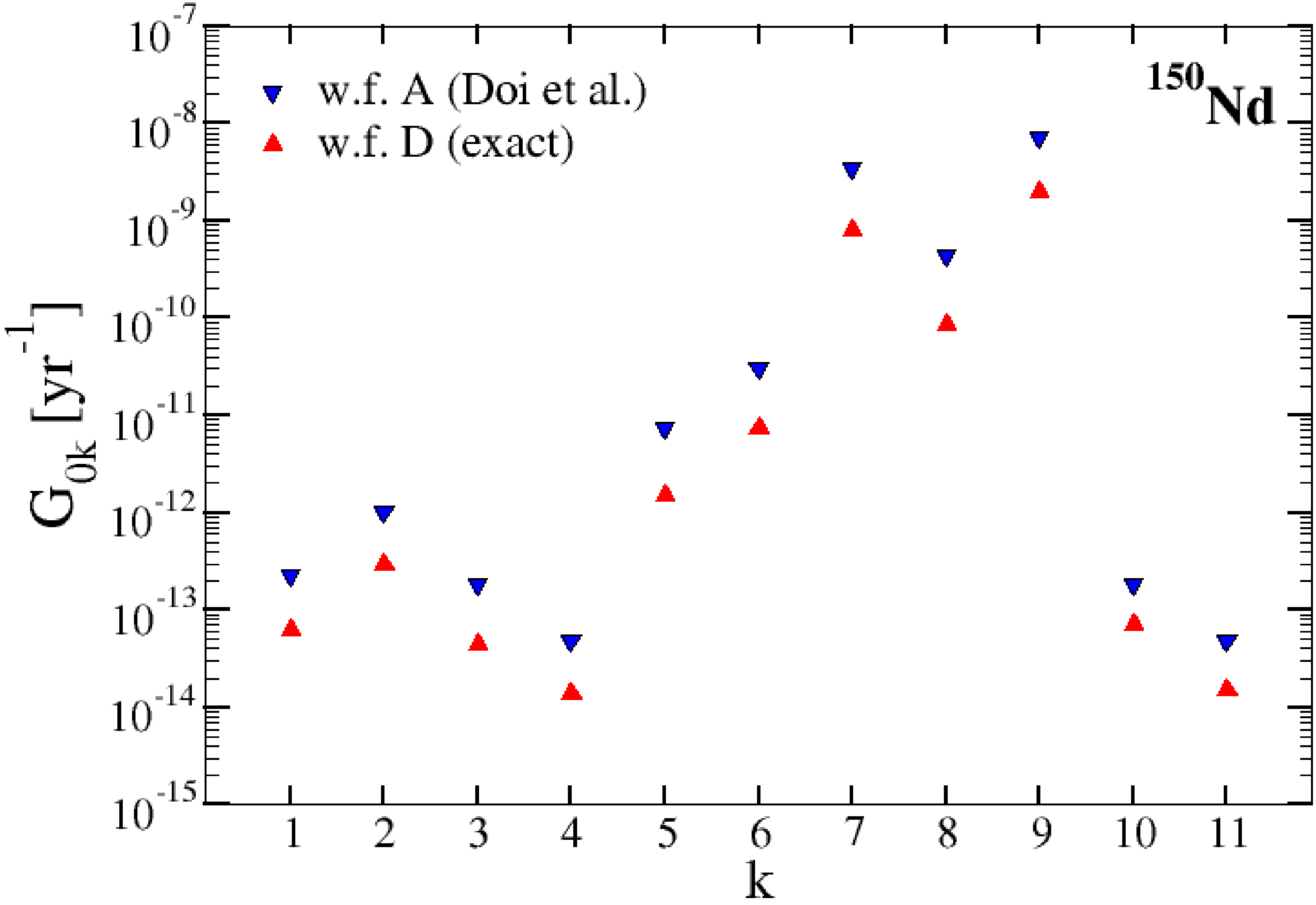
w.f. C, the exact Dirac wave functions with finite nuclear size corrections, which are taken into account in by a uniform charge distribution in a sphere of nucleus

w.f. D , the same as w.f. C but the screening of atomic electrons included

Radial components of electron wave functions at nuclear surface



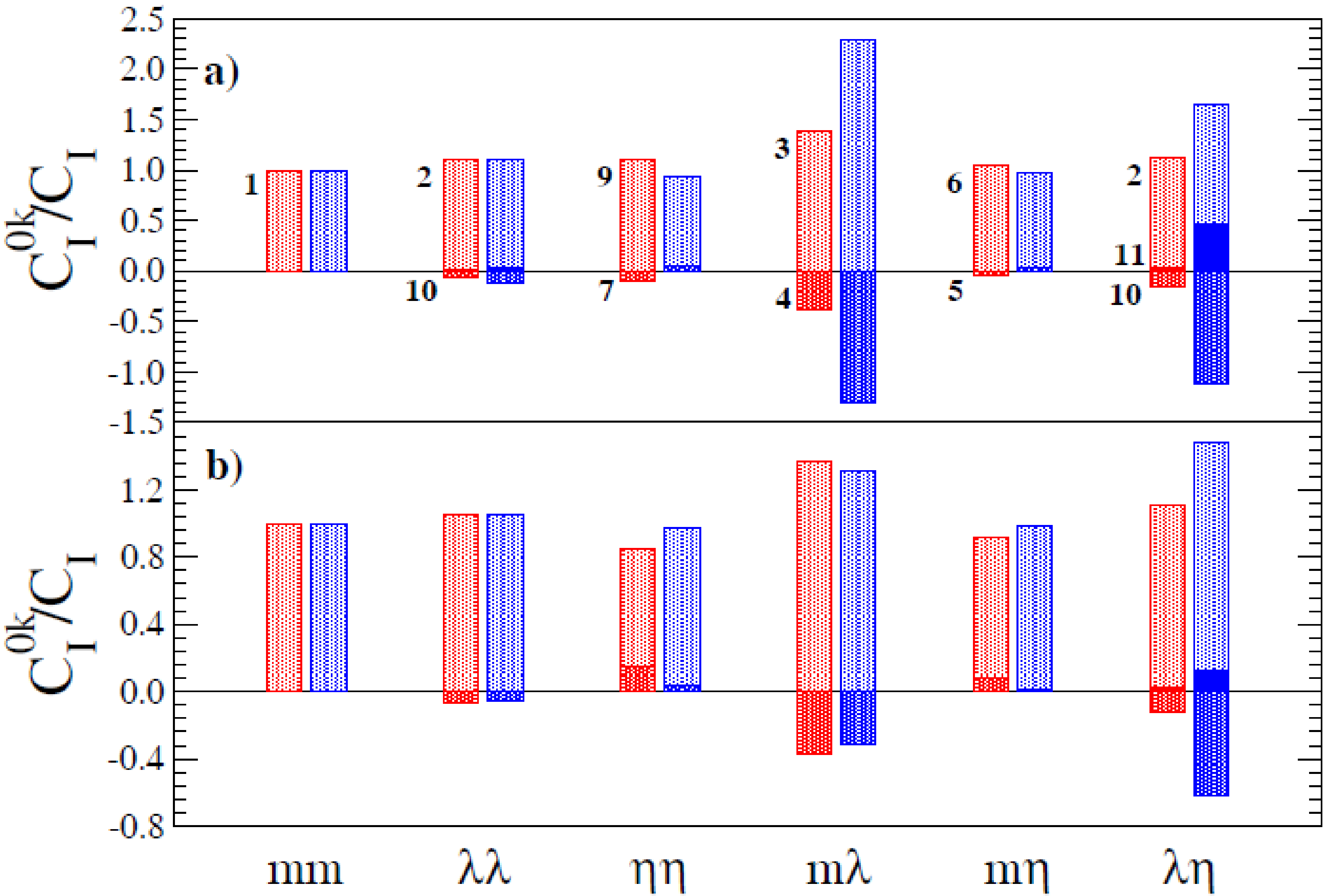
Phase-space factors for ^{150}Nd



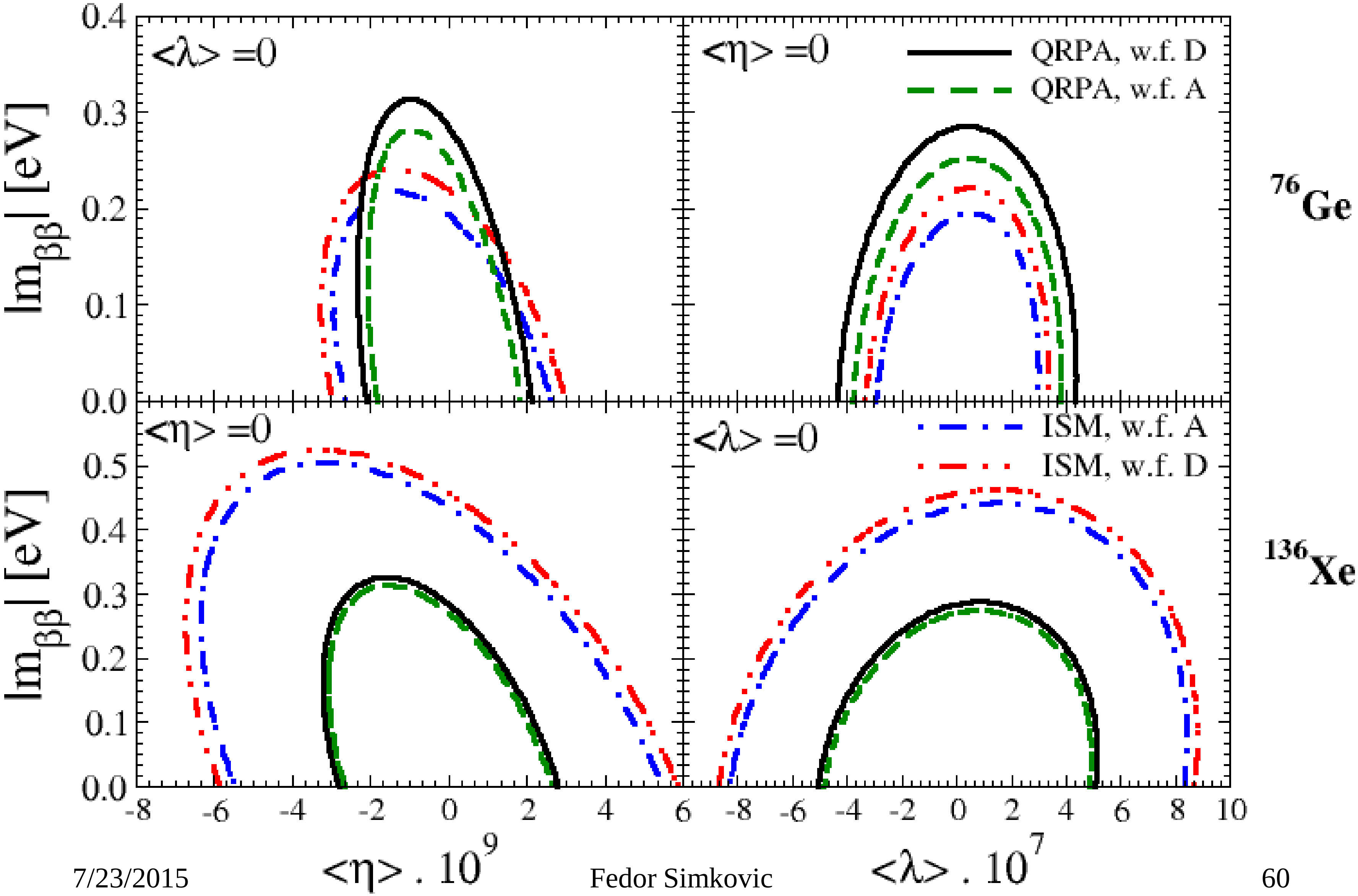
Phase-space factors for nuclei of experimental interest in the case of left and right-handed current mechanisms of the $0\nu\beta\beta$-decay (light neutrino exchange)

	^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn	^{130}Te	^{136}Xe	^{150}Nd
$Q_{\beta\beta}$ [MeV]	4.27226	2.03904	2.99512	3.35037	3.03440	2.01785	2.8135	2.28697	2.52697	2.45783	3.37138
$G_{01}.10^{14}$	2.483	0.237	1.018	2.062	1.595	0.483	1.673	0.906	1.425	1.462	6.316
$G_{02}.10^{14}$	16.229	0.391	3.529	8.959	5.787	0.814	5.349	1.967	3.761	3.679	29.187
$G_{03}.10^{15}$	18.907	1.305	6.913	14.777	10.974	2.672	11.128	5.403	8.967	9.047	45.130
$G_{04}.10^{15}$	5.327	0.470	2.141	4.429	3.400	0.978	3.569	1.886	3.021	3.099	14.066
$G_{05}.10^{13}$	3.007	0.566	2.004	4.120	3.484	1.400	4.060	2.517	3.790	4.015	14.873
$G_{06}.10^{12}$	3.984	0.531	1.733	3.043	2.478	0.934	2.563	1.543	2.227	2.275	7.497
$G_{07}.10^{10}$	2.682	0.270	1.163	2.459	1.927	0.599	2.062	1.113	1.755	1.812	8.085
$G_{08}.10^{11}$	1.109	0.149	0.708	1.755	1.420	0.462	1.703	0.939	1.549	1.657	8.405
$G_{09}.10^{10}$	16.246	1.223	4.779	8.619	6.540	1.939	6.243	3.301	4.972	4.956	19.454
$G_{010}.10^{14}$	2.116	0.141	0.801	1.855	1.359	0.309	1.418	0.660	1.146	1.165	7.115
$G_{011}.10^{15}$	5.376	0.476	2.183	4.557	3.502	1.010	3.704	1.955	3.148	3.238	15.055

Importance of contribution associated with G_{0k} for a given mechanism



Constraints on LNV mechanisms from the GERDA and EXO+KamLAND-Zen half-life limits



Current constraints on the effective neutrino mass and effective right-handed current parameters

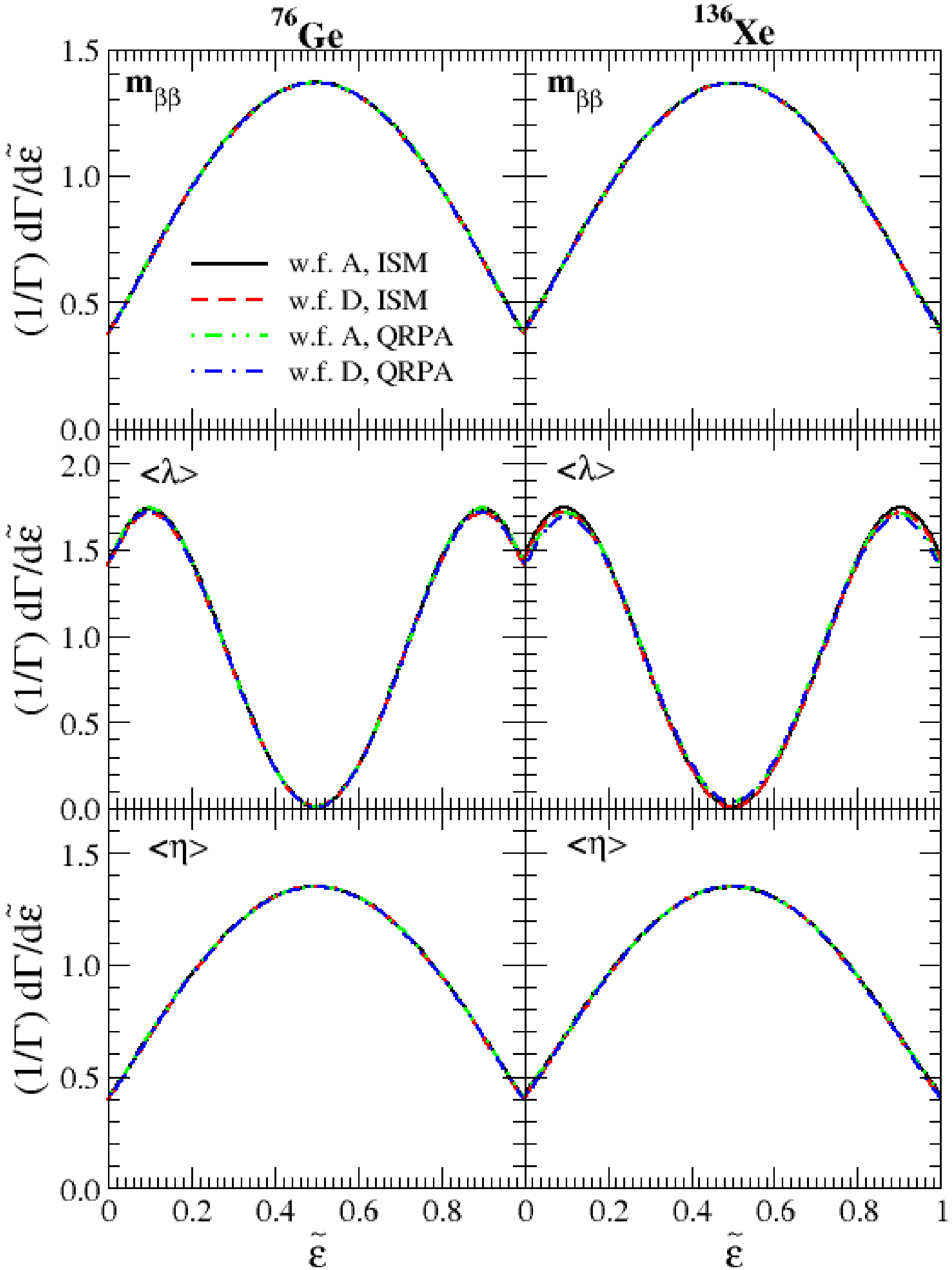
w.f.	^{76}Ge A	^{76}Ge D	^{136}Xe A	^{136}Xe D
		QRPA		
$\lvert m_{\beta\beta}\rvert$ [eV]	0.321	0.333	0.285	0.315
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\eta\rangle = 0$)	0.271	0.284	0.251	0.285
$\langle\eta\rangle \times 10^{-9}$	3.093	3.239	2.077	2.337
$\langle\lambda\rangle \times 10^{-7}$	4.943	5.163	3.822	4.370
		ISM		
$\lvert m_{\beta\beta}\rvert$ [eV]	0.515	0.535	0.222	0.245
$\lvert m_{\beta\beta}\rvert$ [eV] (for $\langle\eta\rangle = \langle\eta\rangle = 0$)	0.436	0.458	0.193	0.220
$\langle\eta\rangle \times 10^{-9}$	6.370	6.760	2.975	3.291
$\langle\lambda\rangle \times 10^{-7}$	8.462	8.841	3.000	3.378

$^{76}Ge\ T^{0\nu}_{1/2} \geq 3.0 \times 10^{25}$

ISM E. Caurier, F. Nowacki, A. Poves and J. Retamosa, Phys. Rev. Lett. **77**, 1954 (1996)

$^{136}Xe\ T^{0\nu}_{1/2} \geq 3.4 \times 10^{25}$

QRP K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A **334**, 187 (1989)



The single differential decay rate normalized to the total decay rate as function of electron energy for 3 limiting cases:

Results do not depend on isotope, NME and type of w.f.

i) Case $m_{\beta\beta} \neq 0$

$(\langle\lambda\rangle = 0$ and $\langle\eta\rangle = 0)$

ii) Case $\langle\lambda\rangle \neq 0$

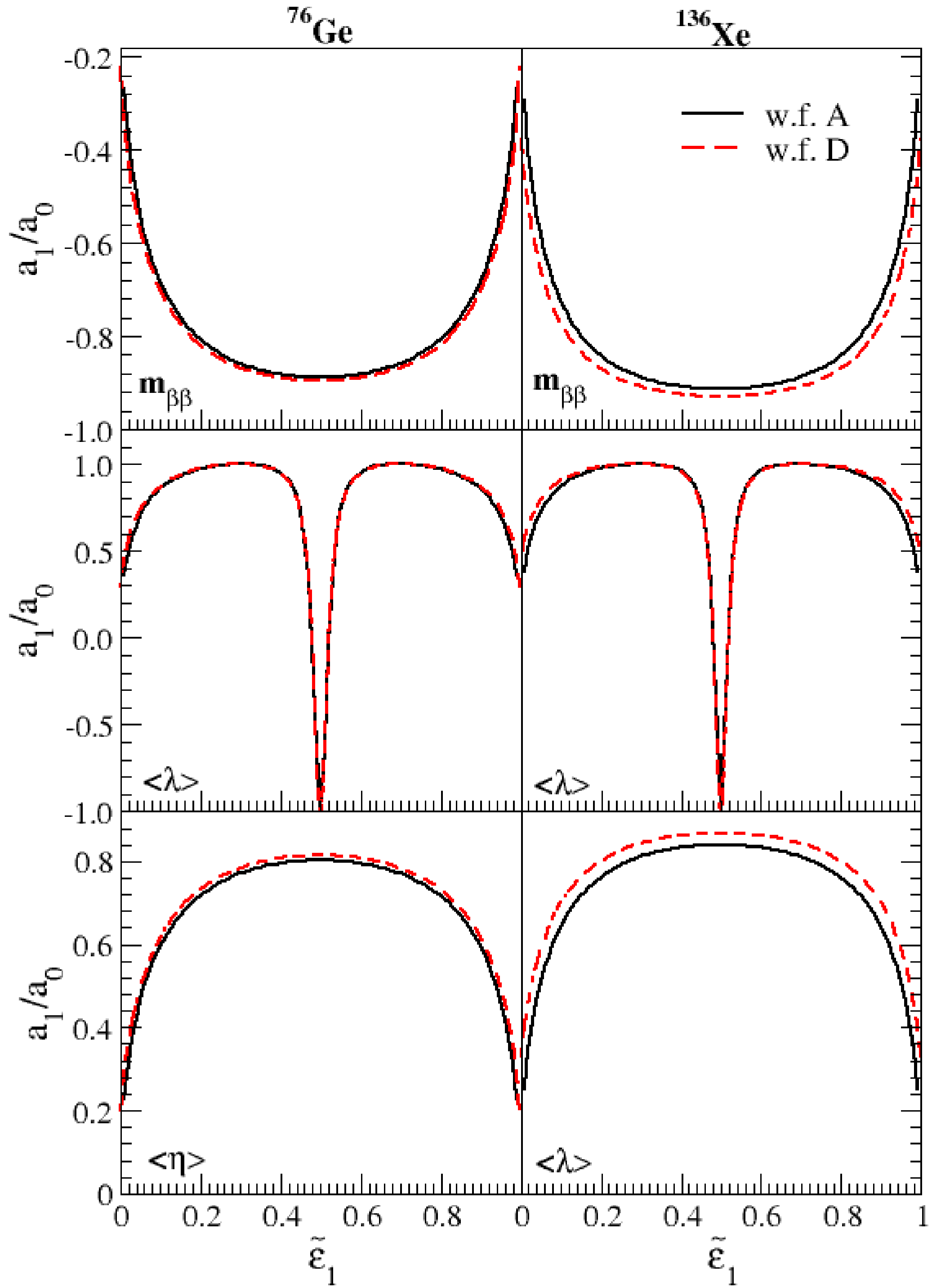
$(m_{\beta\beta} = 0$ and $\langle\eta\rangle = 0)$

iii) Case $\langle\eta\rangle \neq 0$

$(m_{\beta\beta} = 0$ and $\langle\lambda\rangle = 0)$

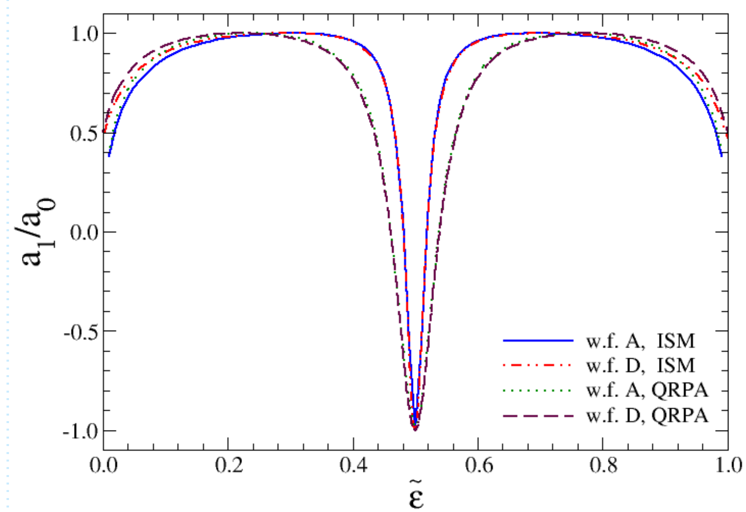
$$\varepsilon_1 = \tilde{\varepsilon}_1 Q_{\beta\beta} + m_e$$

$$\varepsilon_2 = Q_{\beta\beta} + 2m_e - \varepsilon_1)$$



Angular correlation factor as function of electron energy

$$\frac{d\Gamma}{d\cos\theta d\tilde{\varepsilon}_1} = a_0\left(1 + a_1/a_0 \cos\theta\right)$$



Instead of Conclusions

We are at the beginning of the Road…

The future of neutrino physics is bright.