A Higgs at 125.1* GeV and baryon mass spectra derived from a common U(3) framework

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Abstract

Baryons are described by a Hamiltonian on an intrinsic U(3) Lie group configuration space with electroweak degrees of freedom originating in specific Bloch wave factors. By opening the Bloch degrees of freedom pairwise via a U(2) Higgs mechanism, the strong and electroweak energy scales become related to yield the Higgs mass and the usual gauge boson masses. From the same Hamiltonian we derive both the relative neutron to proton mass ratio and the N and Delta mass spectra. All compare rather well with the experimental values.

We predict neutral flavour baryon singlets to be sought for in negative pions scattering on protons or in photoproduction on neutrons and in invariant pion-proton mass in various decays. The fundamental predictions are based on just one length scale and the fine structure coupling. The interpretation is to consider baryons as entire entities kinematically excited from laboratory space by three impact momentum generators, three rotation generators and three Runge-Lenz generators to internalize as nine degrees of freedom covering colour, spin and flavour.

Quark and gluon fields come about when the intrinsic structure is projected back into laboratory space depending on which exterior derivative one is taking. With such derivatives on the measure-scaled wavefunction, we derived approximate parton distribution functions for the u and d valence quarks of the proton that compare well with established experimental analysis.

Intrinsic space for baryon mass states

We use a reinterpreted Kogut-Susskind Hamiltonian with a Manton potential. We interpret it as describing the intrinsic dynamics for baryon mass states. Thus the Lie group U(3) is treated as intrinsic configuration space

$$\frac{\hbar c}{a}[-\tfrac{1}{2}\Delta+\tfrac{1}{2}\mathrm{Tr}\,\chi^2]\Psi(u)=E\Psi(u) \qquad u=e^{i\alpha_k T_k}\equiv e^{i\chi} \qquad (1)$$

It is the hypothesis of the present work, that the eigenstates of the above Schrödinger equation describe the baryon mass spectrum with u being the configuration variable of an entire baryonic entity and a is a scale settled by the classical electron radius

$$\pi a = r_e \equiv \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \qquad \Lambda\equiv\frac{\hbar c}{a}=\frac{\pi}{\alpha}m_e c^2$$

The potential is half the squared geodetic distance from the 'point' u to the 'origo' e

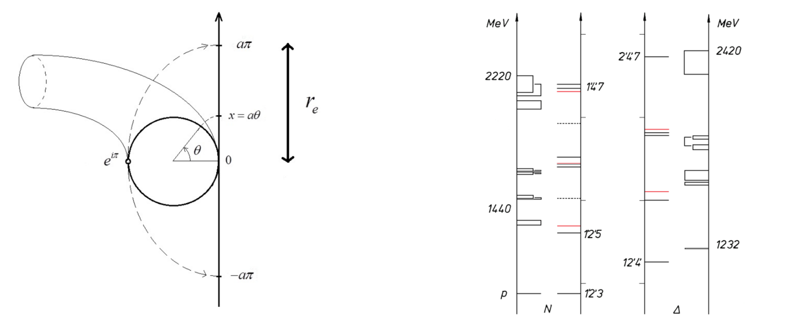
$$\frac{1}{2}\mathrm{Tr}\,\chi^2=w(\theta_1)+w(\theta_2)+w(\theta_3)=\frac{1}{2}d^2(e,u) \qquad (2)$$

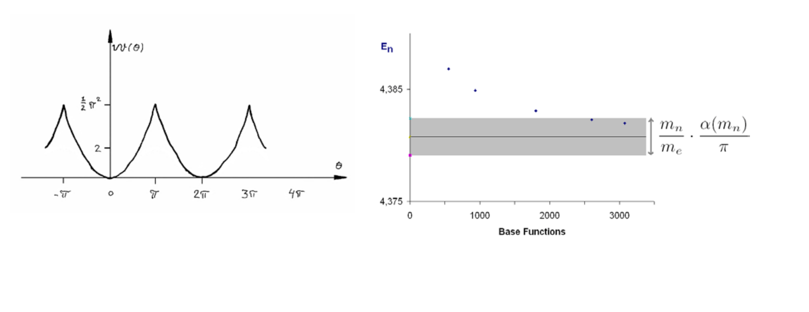
where $e^{i\theta_j}$ are the eigenvalues of u. The potential is periodic in parameter space

$$w(\theta)=\frac{1}{2}(\theta-q\cdot 2\pi)^2,\ \theta\in[(2q-1)\pi,(2q+1)\pi],\ q\in Z$$

We find exact solutions for alleged N-states and approximate solutions for both alleged N-states and Δ-states. From the ground state eigenvalue $\mathrm{E_n}=E_n/\Lambda$ we get

$$\frac{m_e}{m_n}=\frac{\alpha}{\pi}\frac{1}{\mathrm{E_n}}=\frac{1}{1838.\cdots}$$





The theory unfolded

The Laplacian in (1) contains off-toroidal derivatives which are represented by the off-diagonal Gell-Mann matrices. We choose three of these to represent spin and group them into $\mathbf{K}=(K_1,K_2,K_3)$. This interpretation is supported by their commutation relations as body fixed angular momentum. The relation between space and intrinsic space is like the relation in nuclear physics between fixed coordinate systems and intrinsic body fixed coordinate systems for the description of rotational degrees of freedom. The remaining three off-toroidal derivatives are grouped into $\mathbf{M}=(M_1,M_2,M_3)$, which is related to hypercharge and isospin. The Laplacian in polar decomposition thus reads

$$\Delta=\sum_{j=1}^{3}\frac{1}{J^2}\frac{\partial}{\partial\theta_j}J^2\frac{\partial}{\partial\theta_j}-\sum_{\substack{i<j\\k\neq i,j}}^{3}\frac{K_k^2+M_k^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}$$

$$J=\prod_{i<j}^{3}2\sin\tfrac{1}{2}(\theta_i-\theta_j) \qquad [M_i,M_j]=[K_i,K_j]=-i\hbar\varepsilon_{ijk}K_k$$

$$p_j=-i\hbar\frac{1}{a}\frac{\partial}{\partial\theta_j}=\frac{\hbar}{a}T_j \qquad K_1=a\theta_2 p_3-a\theta_3 p_2=\hbar\lambda_7$$

$$[p_j,a\theta_i]=-i\hbar\delta_{ij} \qquad M_1/\hbar=\theta_2\theta_3+\frac{a^2}{\hbar^2}p_2p_3=\lambda_6$$

The off-torus term is analogous to the centrifugal term in the usual treatment of the radial wave function for the hydrogen atom

$$-\frac{\hbar}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}-\frac{1}{r^2}\mathbf{L}^2\right]\psi(r,\theta,\varphi)+V(r)\psi(r,\theta,\varphi)=E\psi(r,\theta,\varphi) \qquad \psi(r,\theta,\varphi)=R(r)Y(\theta,\varphi)$$

With the periodic potential in (2) the complete Schrödinger equation reads with $\mathrm{E}=E/\Lambda$ and $\Lambda\equiv\hbar c/a=214.27\,\mathrm{MeV}$

$$\left[-\frac{1}{2}\left(\sum_{j=1}^{3}\frac{1}{J}\frac{\partial^2}{\partial\theta_j^2}J+2-\sum_{\substack{i<j\\k\neq i,j}}^{3}\frac{K_k^2+M_k^2}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}\right)+w(\theta_1)+w(\theta_2)+w(\theta_3)\right]\Psi(u)=\mathrm{E}\Psi(u). \qquad (3)$$

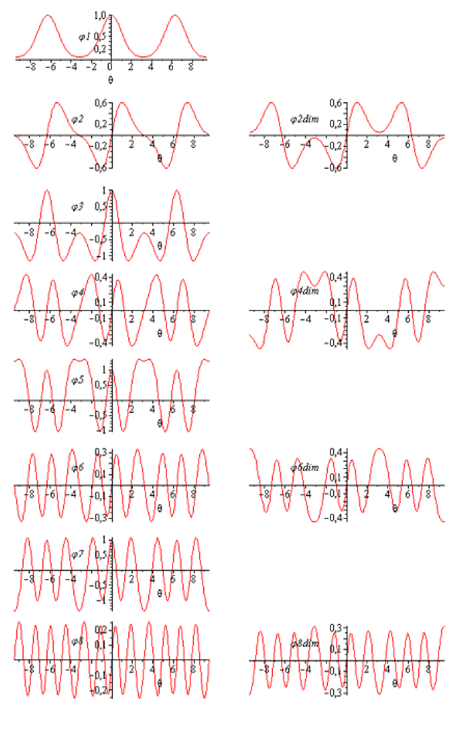
The constant term in the Laplacian is interpreted as a global curvature potential from differentiating through J.
A factorization of $\Psi(u)=\tau(\theta_1,\theta_2,\theta_3)\cdot\Upsilon(\alpha_4,\alpha_5,\alpha_6,\alpha_7,\alpha_8,\alpha_9)$ gives for $\Phi(u)=R(\boldsymbol{\theta})\cdot\Upsilon$ with $R(\boldsymbol{\theta})=J(\boldsymbol{\theta})\cdot\tau(\boldsymbol{\theta})$

$$[-\Delta_e+V]R(\theta_1,\theta_2,\theta_3)=2\mathrm{E}\,R(\theta_1,\theta_2,\theta_3)$$

where $\Delta_e=\sum_{j=1}^{3}\frac{\partial^2}{\partial\theta_j^2}$ and $V=-2+\frac{1}{3}(K(K+1)+M^2)\sum_{i<j}^{3}\frac{1}{8\sin^2\frac{1}{2}(\theta_i-\theta_j)}+2(w(\theta_1)+w(\theta_2)+w(\theta_3))$. Now R can be expanded on Slater determinants constructed from parametric eigenstates

$$R=\sum_{lmn}a_{lmn}R_{lmn}$$

$$R_{lmn}(\boldsymbol{\theta})=\begin{vmatrix}\varphi_l(\theta_1) & \varphi_l(\theta_2) & \varphi_l(\theta_3)\\ \varphi_m(\theta_1) & \varphi_m(\theta_2) & \varphi_m(\theta_3)\\ \varphi_n(\theta_1) & \varphi_n(\theta_2) & \varphi_n(\theta_3)\end{vmatrix}$$



The figure shows parametric eigenstates with periodicity 2π to the left and periodicity 4π for diminished states in the right column.
We can couple a diminishing period doubling in level two with an augmenting period doubling in level one. We interpret these coupled period doublings as representing the transformation from a neutral state (e.g. the neutron) to a charged state (e.g. the proton).

$$n\rightarrow p \qquad R_{\bar{1}'\bar{2}'3}(\boldsymbol{\theta})=\begin{vmatrix}e^{-i\frac{1}{2}\theta_1}g_1(\theta_1) & e^{-i\frac{1}{2}\theta_2}g_1(\theta_2) & e^{-i\frac{1}{2}\theta_3}g_1(\theta_3)\\ e^{i\frac{1}{2}\theta_1}g_2(\theta_1) & e^{i\frac{1}{2}\theta_2}g_2(\theta_2) & e^{i\frac{1}{2}\theta_3}g_2(\theta_3)\\ \varphi_3(\theta_1) & \varphi_3(\theta_2) & \varphi_3(\theta_3)\end{vmatrix}$$

The resulting shift in ground state eigenvalue is

$$\frac{E-E''}{E''}=0.13847\,\%\approx 0.13784\,\%=\frac{m_n-m_p}{m_p}$$

Spin and flavour inherent in the Laplacian

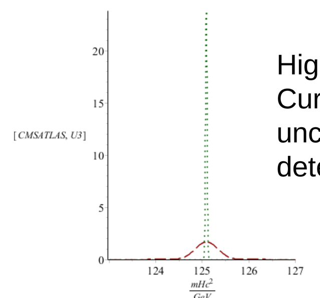
$$K(K+1)+M^2=\frac{4}{3}\left(n+\frac{3}{2}\right)^2-3-\frac{1}{3}y^2-4i_3^2,\ n=0,1,2,\cdots$$

Conclusions

The Hamiltonian in (1) or (3) may be seen as an effective model or interpreted more radically in an intrinsic conception:

When Resonances - the impact momentum act as introtangling operators to generate the maximal torus of U(3).
When Decay, fragmentation - the momentum form induces quark and gluon fields as projections.

Key predictions:
*accurate Higgs mass and Neutral flavour baryon singlets



Higgs mass gaussians. Curve widths represent uncertainties in peak determination

$$v_{SM}=\sqrt{2}\varphi_0\sqrt{V_{ud}} \qquad \varphi_0=\frac{2\pi}{\alpha}\Lambda$$

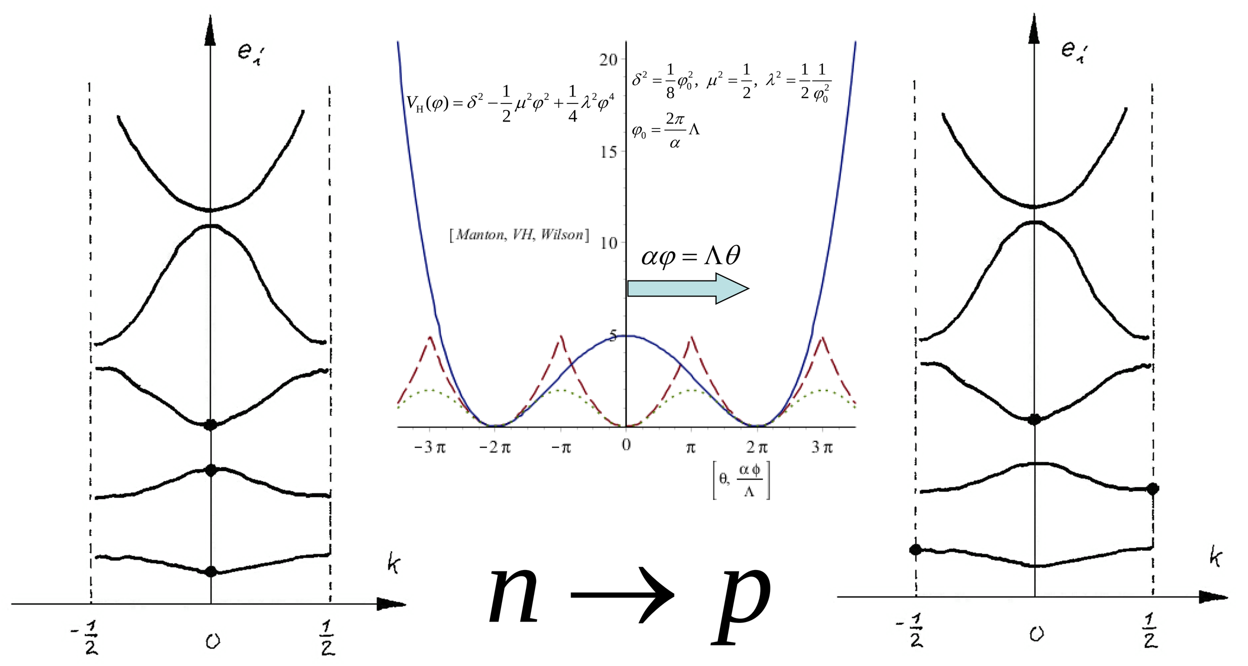
$$^{*}m_H c^2=\frac{1}{\sqrt{2}}\frac{2\pi}{\alpha(m_W)}\frac{\pi}{\alpha_0}m_e c^2=125.085\pm0.017\,\mathrm{GeV}$$

The Higgs mass, 125.085+/-0.017 GeV is in excellent agreement with the merged ATLAS and CMS result 125.09+/-0.24 GeV from LHC Run 1.

Singlet neutral flavour resonances N^0 to be sought at e.g. 2839, … 3206, 3707, 3859,…4228,…4723,… MeV.

Bloch degrees of freedom for electroweak scale and Higgs mass

Approximate energy levels for baryonic states are found by combinations of three parametric eigenstates of the three torus angles. These eigenstates originally have the same periodicity as the potential. However a coupled period doubling can decrease the total energy.

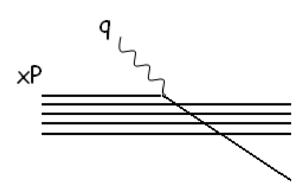


We interpret the period doublings as related by the Higgs mechanism to the creation of the proton charge in the neutron decay. Similar states all with one even label give the N resonances. Two even labels give possibilities of double charges which we interpret as Δ resonances.

For three even labels the complex phases factorize out and the states may contribute to neutral states.

The black dots in the figure show the Bloch wave number choices for the neutron (left) and the proton state (right).

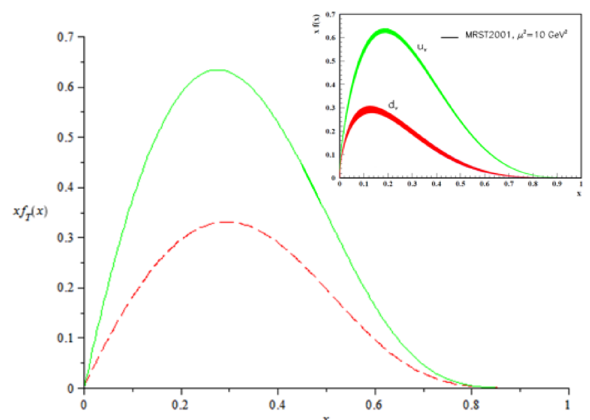
Laboratory space for parton distribution functions



We boost a proton from rest to energy E by impacting upon it a massless four-momentum q to hit a parton xP. After impact the parton carries a mass xE. Thus $(xP_\mu+q_\mu)\cdot(xP^\mu+q^\mu)=x^2E^2$ which yields for the parton fraction

$$x=\frac{2E_0}{E+E_0} \text{ and boost parameter}$$

$$\xi\equiv\frac{E-E_0}{E}=\frac{2-2x}{2-x}$$



$$f_T(x)dx=\left(\sum_{j=1}^{3}dR_{u=\exp(i\theta T)}(iT_j)\right)^2 d\theta,\quad \text{where } \theta=\pi\xi$$

$$xf_u(x)=x\left[D(\pi\frac{2-2x}{2-x}\frac{2}{3},\pi\frac{2-2x}{2-x}\cdot0,\pi\frac{2-2x}{2-x}\cdot(-1))\right]^2\cdot\frac{\pi\cdot2}{(2-x)^2}$$

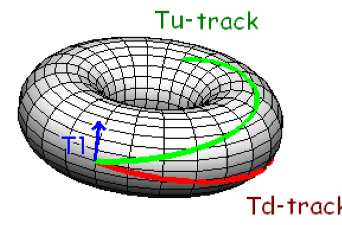
We project from a state constructed from trigonometric functions to mimic the period doublings in the proton state

$$R_{\bar{1}\bar{2}3}(\theta_1,\theta_2,\theta_3)=\frac{1}{N}\begin{vmatrix}1 & 1 & 1\\ \sin\frac{1}{2}\theta_1 & \sin\frac{1}{2}\theta_2 & \sin\frac{1}{2}\theta_3\\ \cos\theta_1 & \cos\theta_2 & \cos\theta_3\end{vmatrix}$$

The projection involves the exterior derivative dR summed over all three colours. The result we denote as D (directional derivative)

We track the boost with different toroidal generators

$$T_u=\begin{Bmatrix}2/3 & & \\ & 0 & \\ & & -1\end{Bmatrix} \text{ and } T_d=\begin{Bmatrix}-1/3 & & \\ & 0 & \\ & & -1\end{Bmatrix}$$



$$ND(\theta_1,\theta_2,\theta_3)=-\frac{1}{2}\cos\frac{\theta_1}{2}\cdot(\cos\theta_3-\cos\theta_2)-\sin\theta_1\cdot(\sin\frac{\theta_3}{2}-\sin\frac{\theta_2}{2})$$
$$+\frac{1}{2}\cos\frac{\theta_2}{2}\cdot(\cos\theta_3-\cos\theta_1)+\sin\theta_2\cdot(\sin\frac{\theta_3}{2}-\sin\frac{\theta_1}{2})$$
$$-\frac{1}{2}\cos\frac{\theta_3}{2}\cdot(\cos\theta_2-\cos\theta_1)-\sin\theta_3\cdot(\sin\frac{\theta_2}{2}-\sin\frac{\theta_1}{2})$$

No fitting parameters

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See also:
* O. L. Trinhammer, H. G. Bohr, M. S. Jensen, *A calculation of the exact Higgs mass from a U(2) mechanism on the Lie group U(3)*, ResearchGate , May 8, 2015.

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