Higgs Evolution in Inflation

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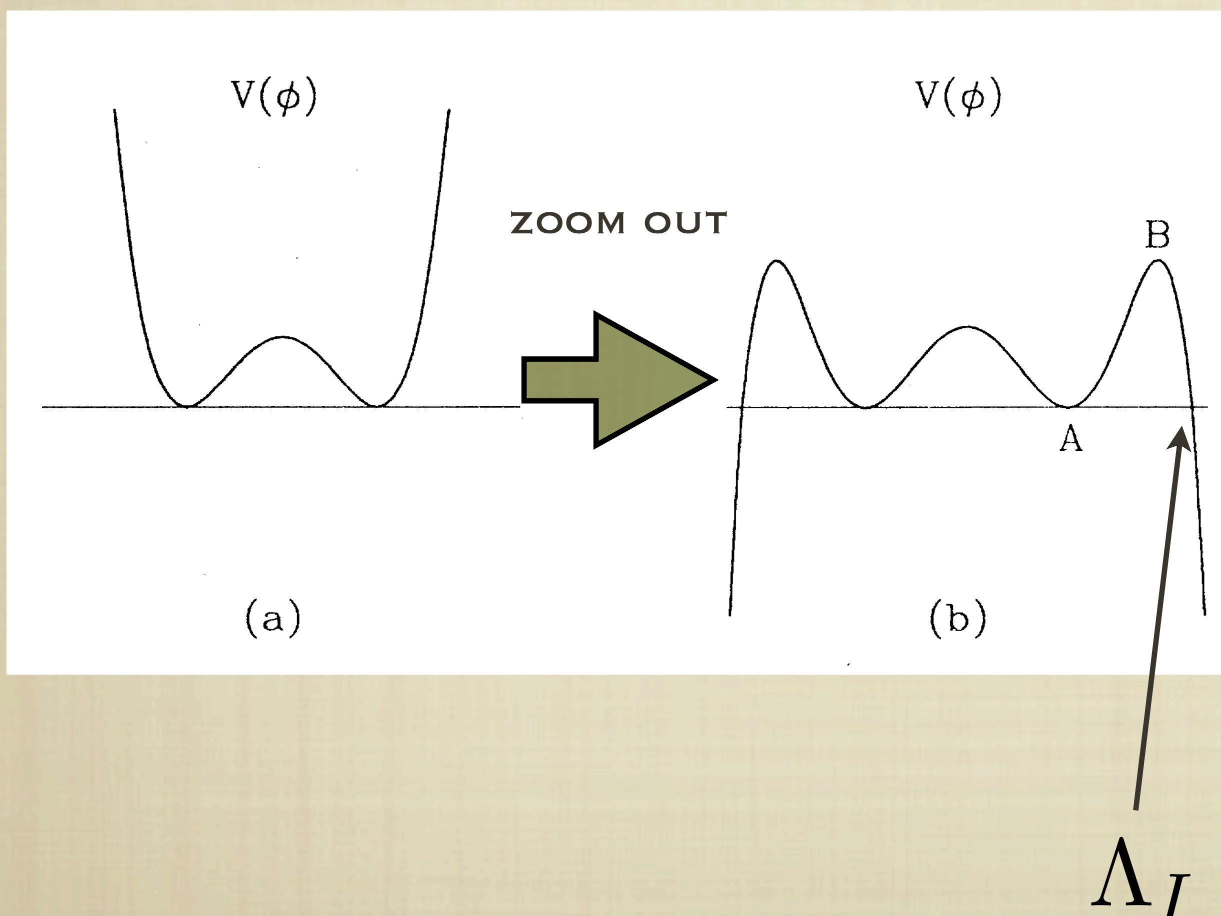
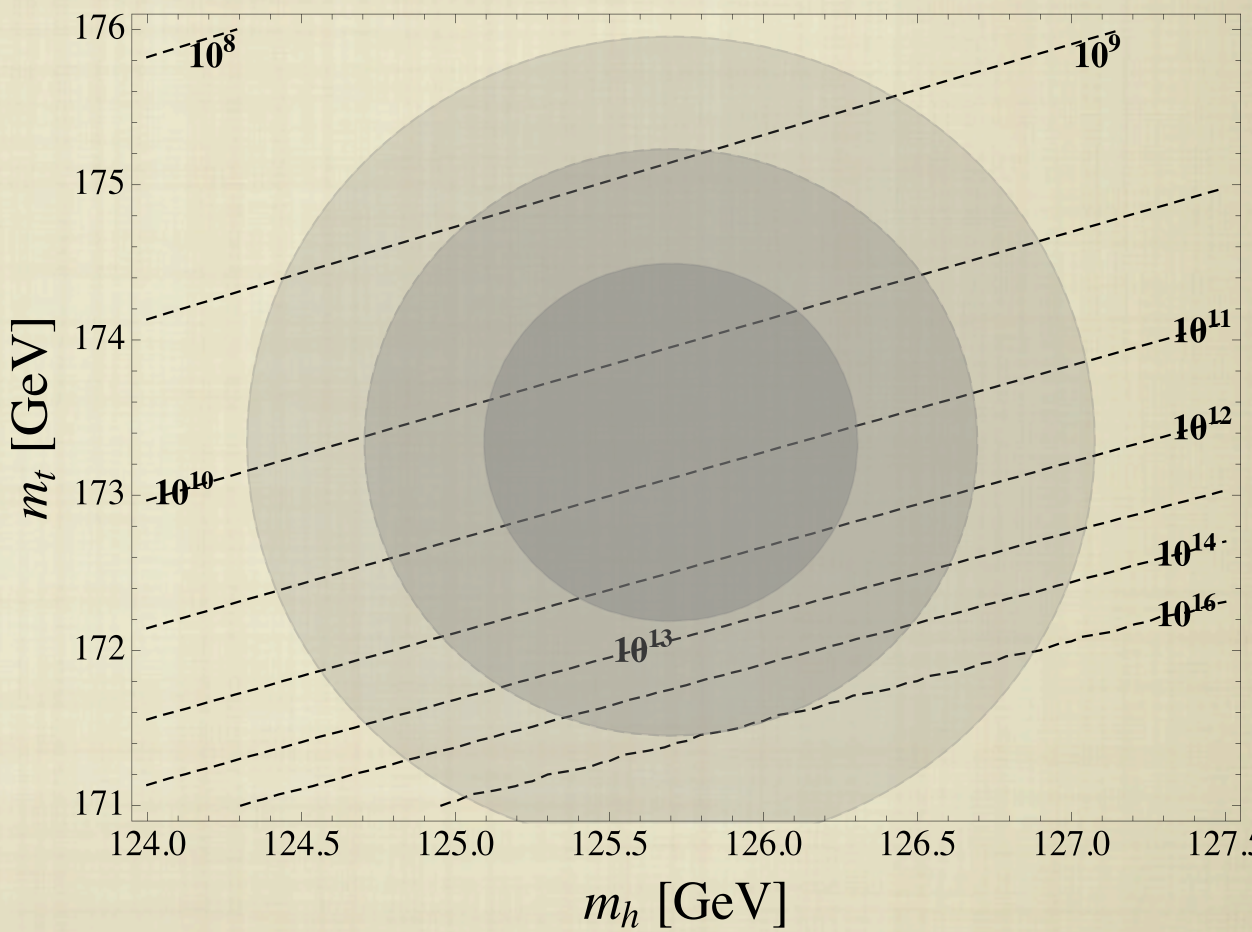
Based On Hook, Kearney, Shakya, KZ 1404.5953
Kearney, Yoo, KZ 1503.05193
Kearney, Shakya, Yoo, KZ, in progress

Higgs Instastability

- **Well-Known that SM Higgs Has Vacuum Instability Driven by Top**

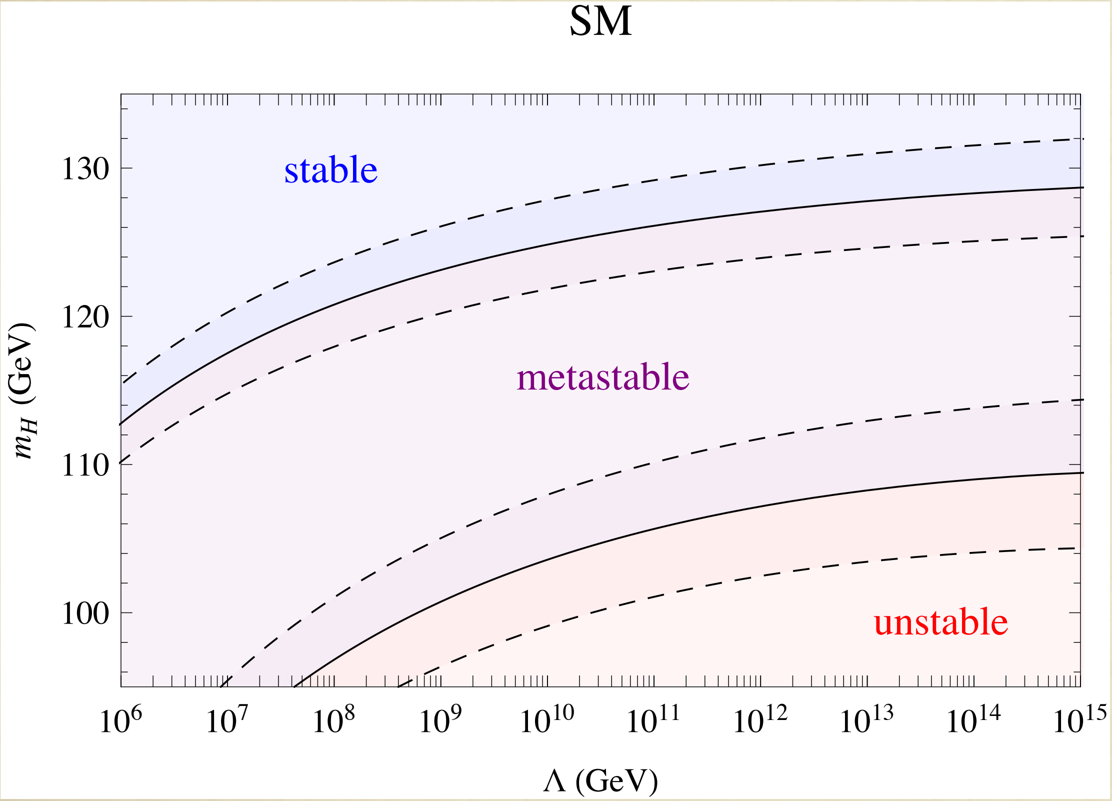
$$V_{\text{eff}}(h) = \frac{\lambda_{\text{eff}}(h)}{4} h^4$$





Higgs Metastability

- Apparently No Cosmological Implications -- Universe is "Stable Enough"

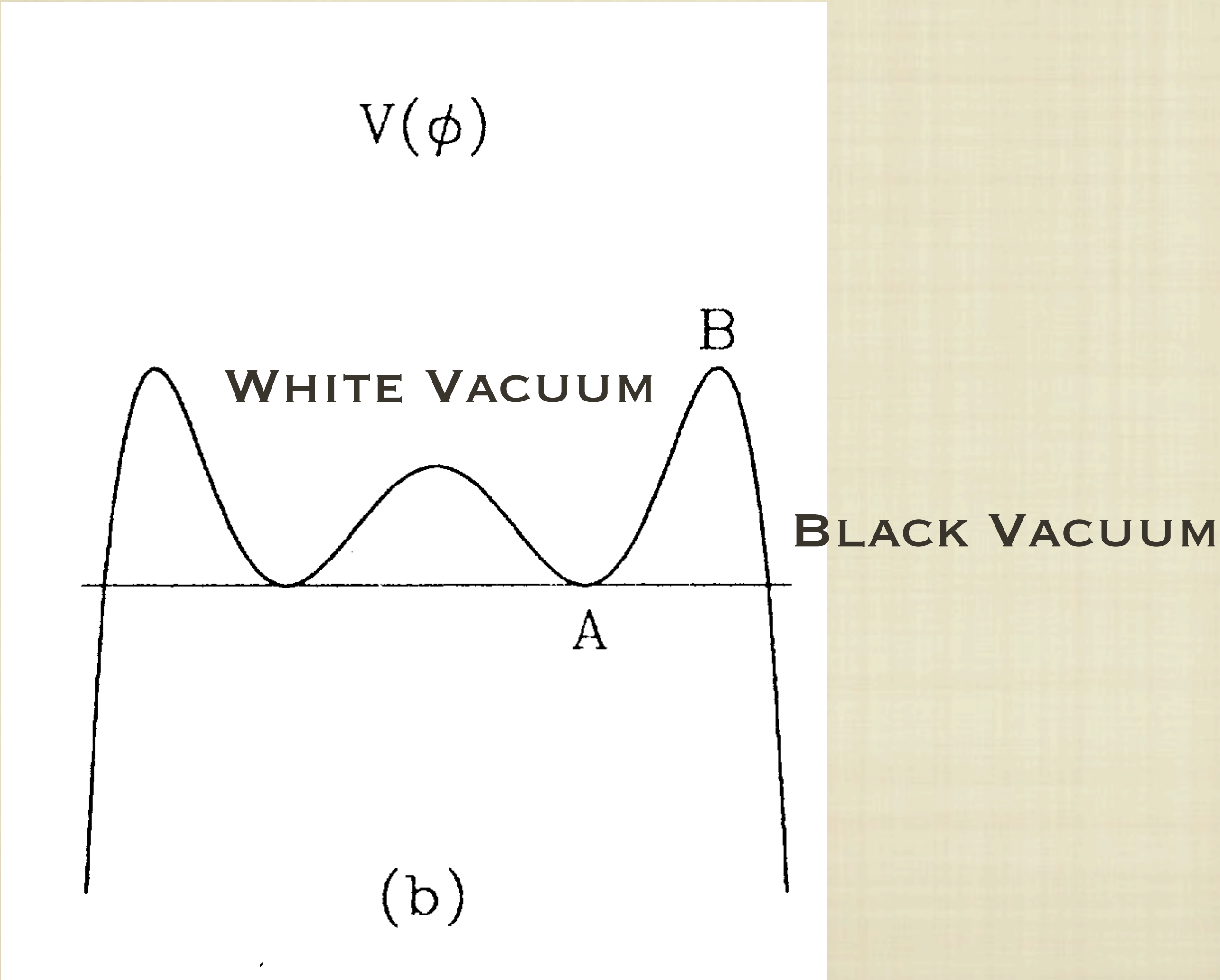


Compute Lee-Weinberg bounce

$$p = \max_{h<\Lambda}[V_U h^4 \exp\left(-8\pi^2/3|\lambda(h)|\right)]$$

Inflation

- Quantum Fluctuations During Inflation Allow to Sample Unstable Vacuum



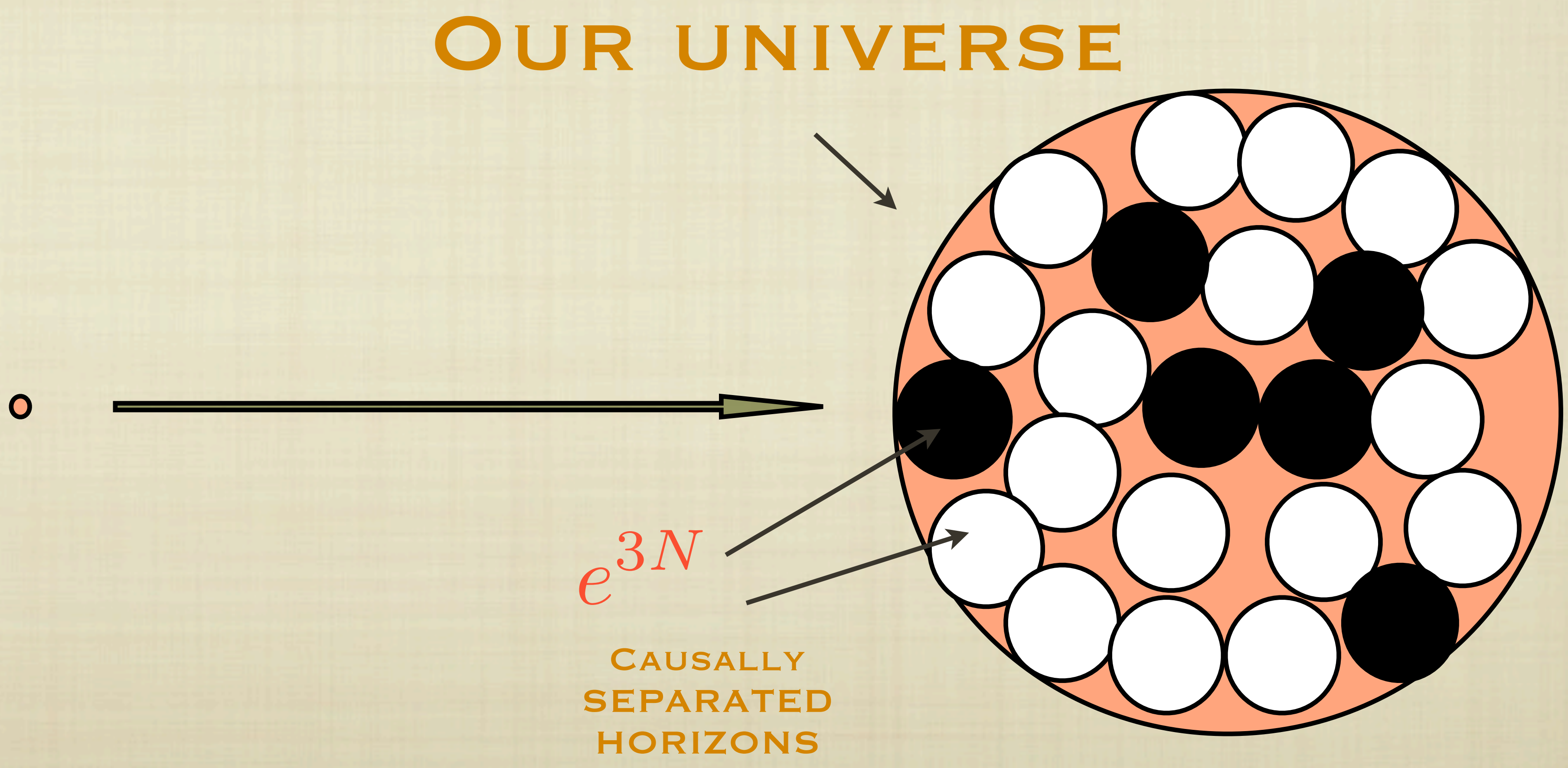
$$\delta\phi = \frac{H}{2\pi}$$

$$H > \Lambda_I$$

- Higgs in Inflation: How does this Evolve?

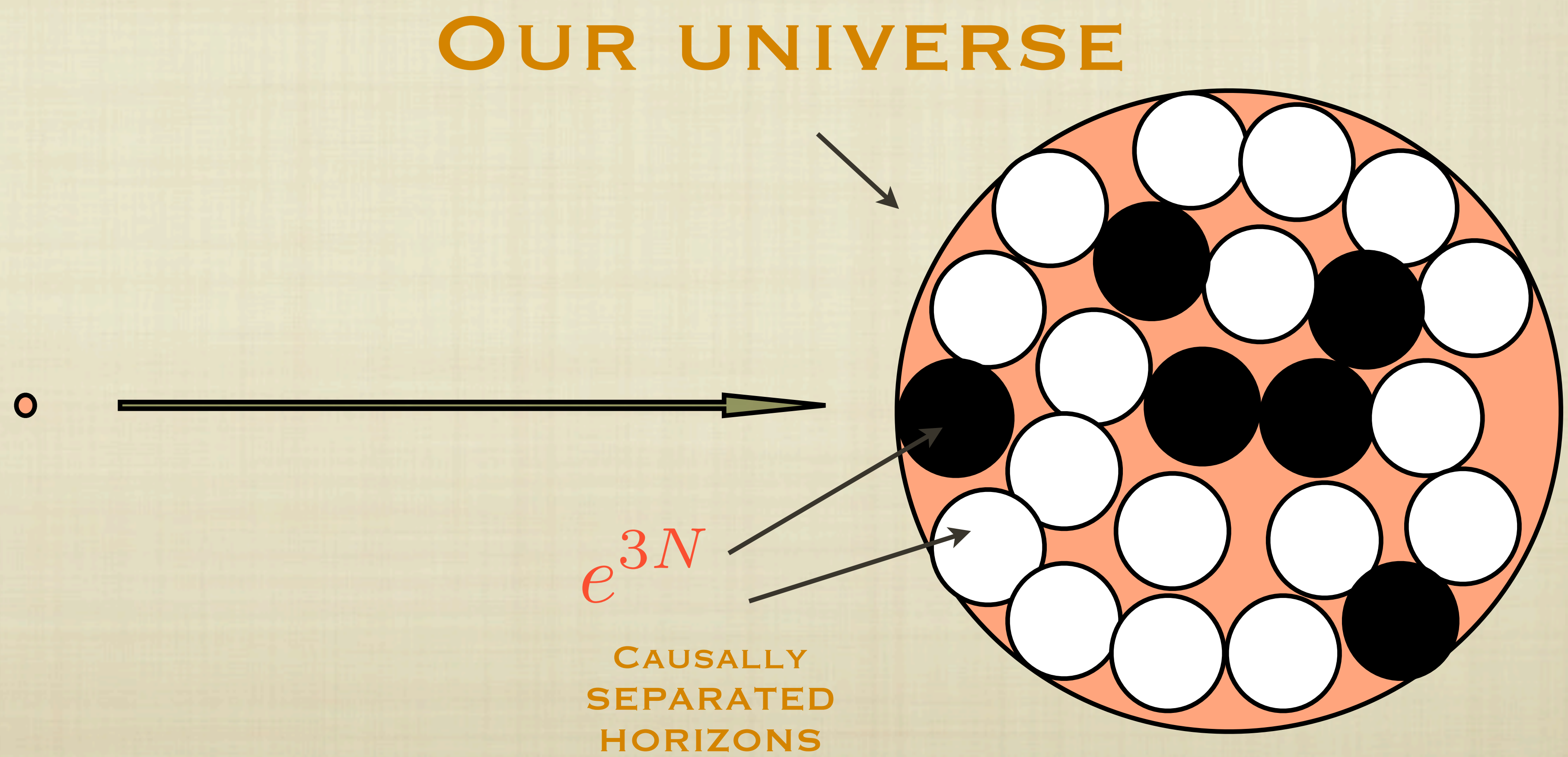
Inflation & Horizons

- Inflation Creates e^{3N} Causally Separated horizons
- Even Starting in Stable Vacuum, Fluctuations Can Spawn Black Islands



Inflation & Horizons

- Eventually Causally Separated Horizons Re-Enter Horizon
- How Does This System Evolve?

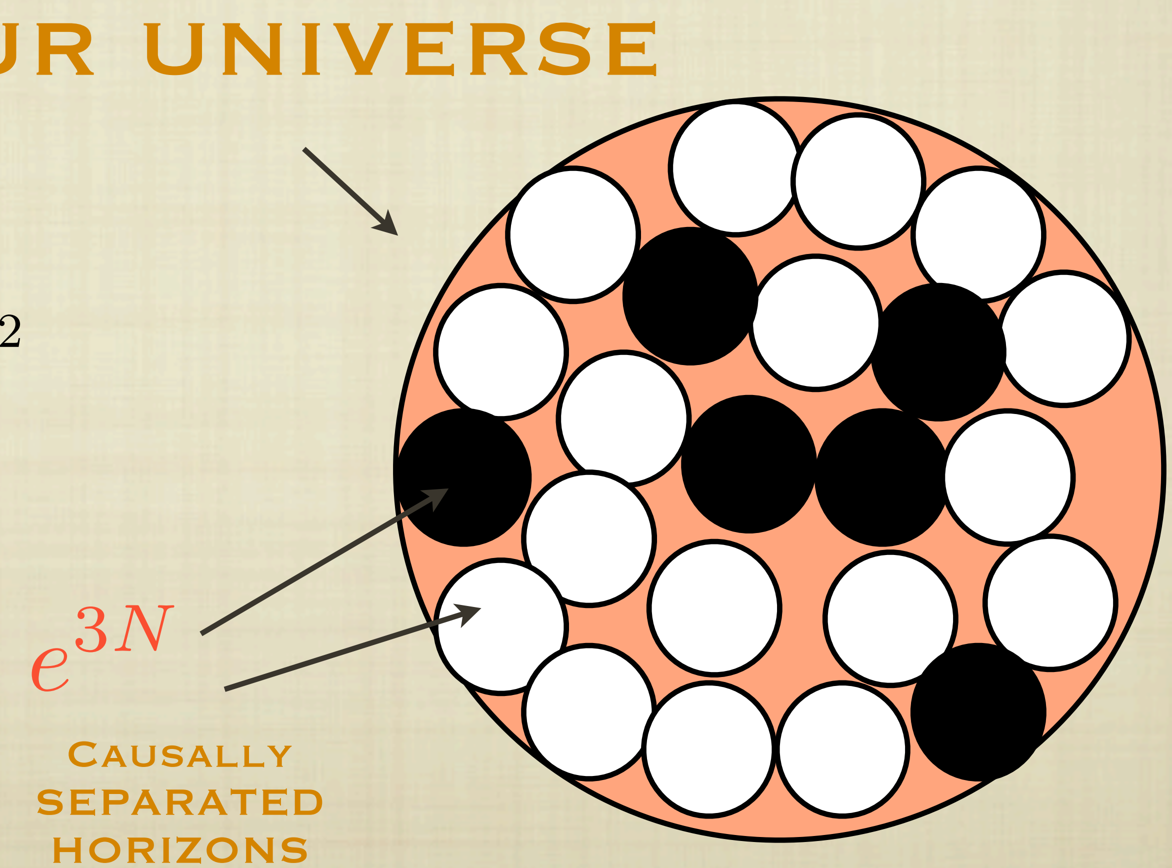


Probability

- Does the Universe Survive Inflation? How Big of a Transition Prob Can One Tolerate and Still Evolve Into a Universe Like Ours?

- Probabilities Taken with Super-Hubble Modes

$$\langle \delta h^2(t) \rangle = \int_{k=1/L}^{k=\epsilon a H} \frac{d^3k}{(2\pi)^3} \left|\delta h_k(t)\right|^2$$



Higgs EOM @ Inflation

$$\ddot{h} + 3H\dot{h} - \left(\frac{\vec{\nabla}}{a}\right)^2 h + V'(h) = 0$$

$$\bar{h}(0) = 0,\ \bar{h}(t) = 0$$

$$3H\dot{\delta h}_k(t) + 3\lambda \left\langle \delta h^2(t) \right\rangle \delta h_k(t) = 0$$

$$\left\langle \delta h^2(t) \right\rangle = \int_{k=1/L}^{k=\epsilon a H} \frac{d^3k}{(2\pi)^3} \left|\delta h_k(t)\right|^2$$

$$\frac{d}{dt}\left\langle \delta h^2(t) \right\rangle = -\frac{2\lambda}{H}\left\langle \delta h^2(t) \right\rangle^2 + \frac{H^3}{4\pi^2}$$

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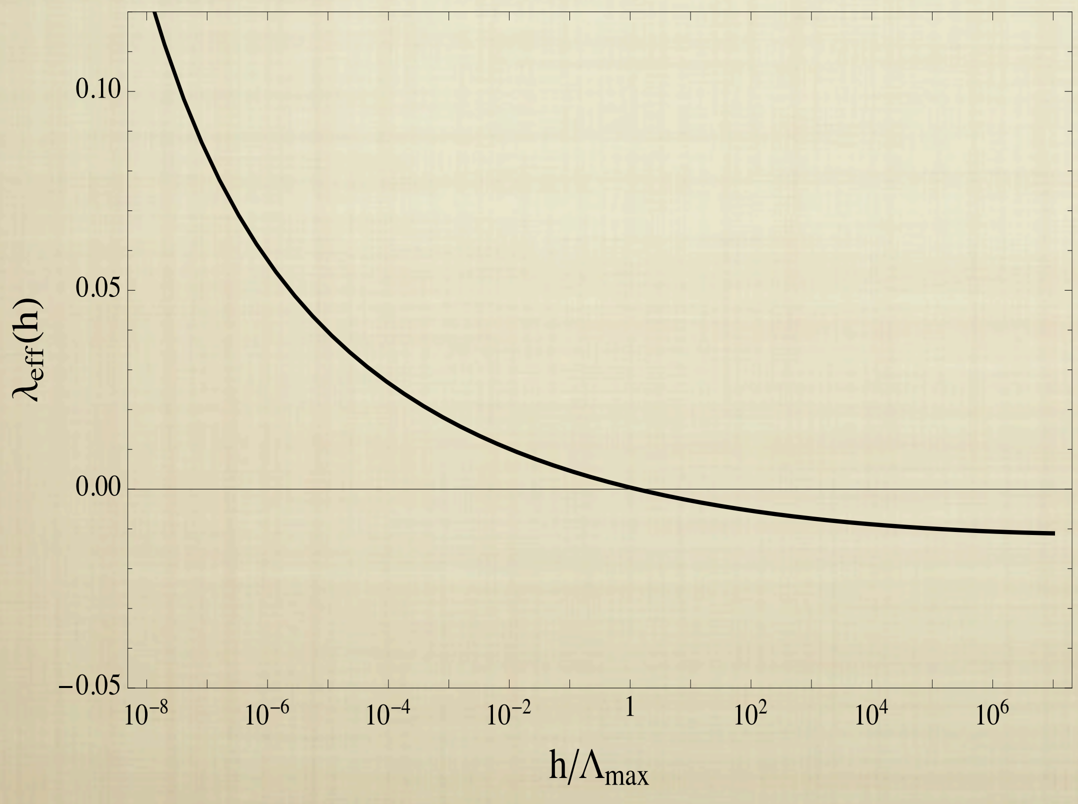
Higgs EOM @ Inflation

- Can Higgs end inflation?
- Not at least until black vacua dominate
- Takes a while

$$\langle \delta h^2(t) \rangle = \frac{1}{\sqrt{-2\lambda}} \frac{H^2}{2\pi} \tan\left(\sqrt{-2\lambda} \frac{\mathcal{N}}{2\pi} \right)$$

$$\mathcal{N}_{\rm max} = \frac{\pi^2}{\sqrt{-2\lambda}}$$



$$\left(\sqrt{-2\lambda} \frac{\mathcal{N}}{2\pi} \ll 1 \Rightarrow \langle \delta h^2(t) \rangle = \frac{H^2 \mathcal{N}}{4\pi^2}\right)$$

2sigma on mt, mh:

$$50 \lesssim \mathcal{N}_{\rm max} \lesssim 90$$

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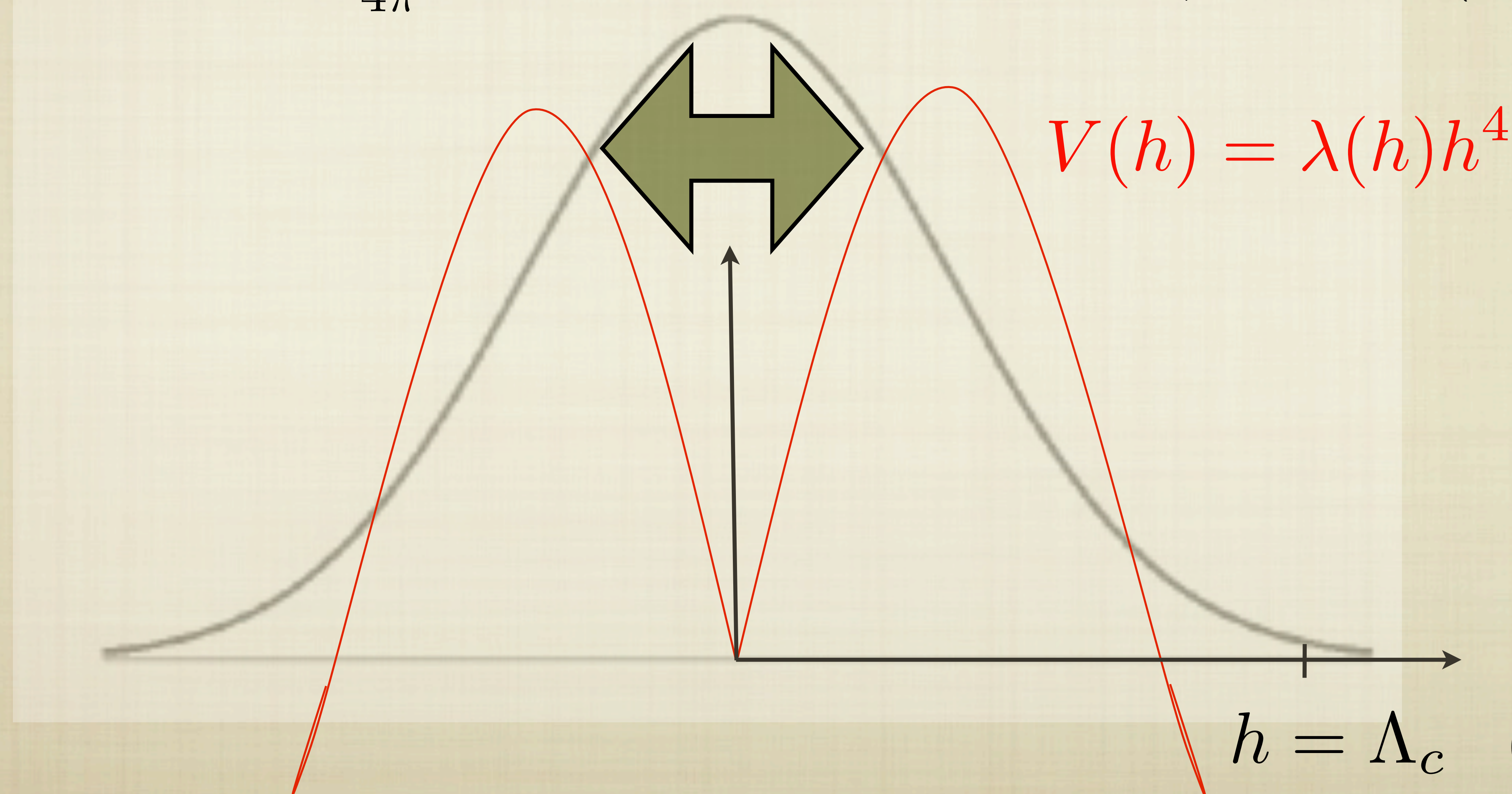
Higgs EOM @ Inflation

- Picture -- Slow Diffusion Process Until Classical Run Away

$$\langle \delta h^2(t) \rangle = \frac{H^2 \mathcal{N}}{4\pi^2} \quad \longrightarrow \quad \langle \delta h^2(t) \rangle = \frac{1}{\sqrt{-2\lambda}} \frac{H^2}{2\pi} \tan\left(\sqrt{-2\lambda} \frac{\mathcal{N}}{2\pi} \right)$$



$$\left(-V'_{\rm eff}(\Lambda_c) = \frac{3H^3}{2\pi} \right)$$

What is the Right Calculation?

- Be Careful About Regime of Validity of Calculation!

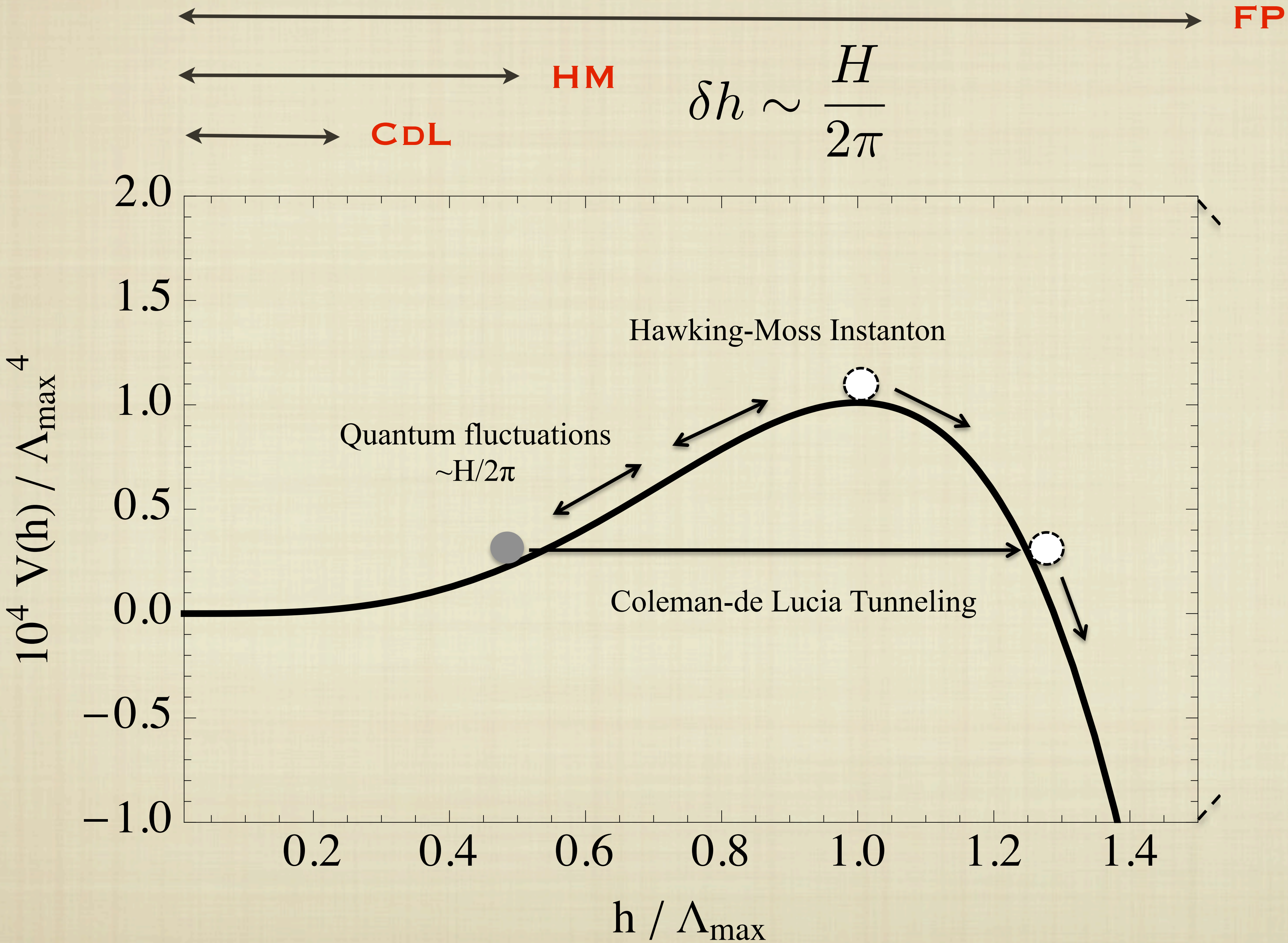
- How Does H Compare to Higgs Potential?

- Smallest Fluctuations (Coleman-DeLuccia) — Rarest Transitions $H^2 \lesssim V''_{\rm eff}(\Lambda_{\rm max})$
- Moderate Fluctuations (Hawking-Moss) — Rare Transitions $V''_{\rm eff}(\Lambda_{\rm max}) \lesssim H^2 \lesssim (V_{\rm eff}(\Lambda_{\rm max}))^{1/2}$
- Big Fluctuations (Fokker-Planck) — Not Rare Transitions $H \gtrsim \sqrt{|\lambda|}\Lambda_I$

Details: Hook, Kearney, Shakya, KZ 1404.5953

Regimes of Validity

$$\delta h \sim \frac{H}{2\pi}$$



Regimes of Validity

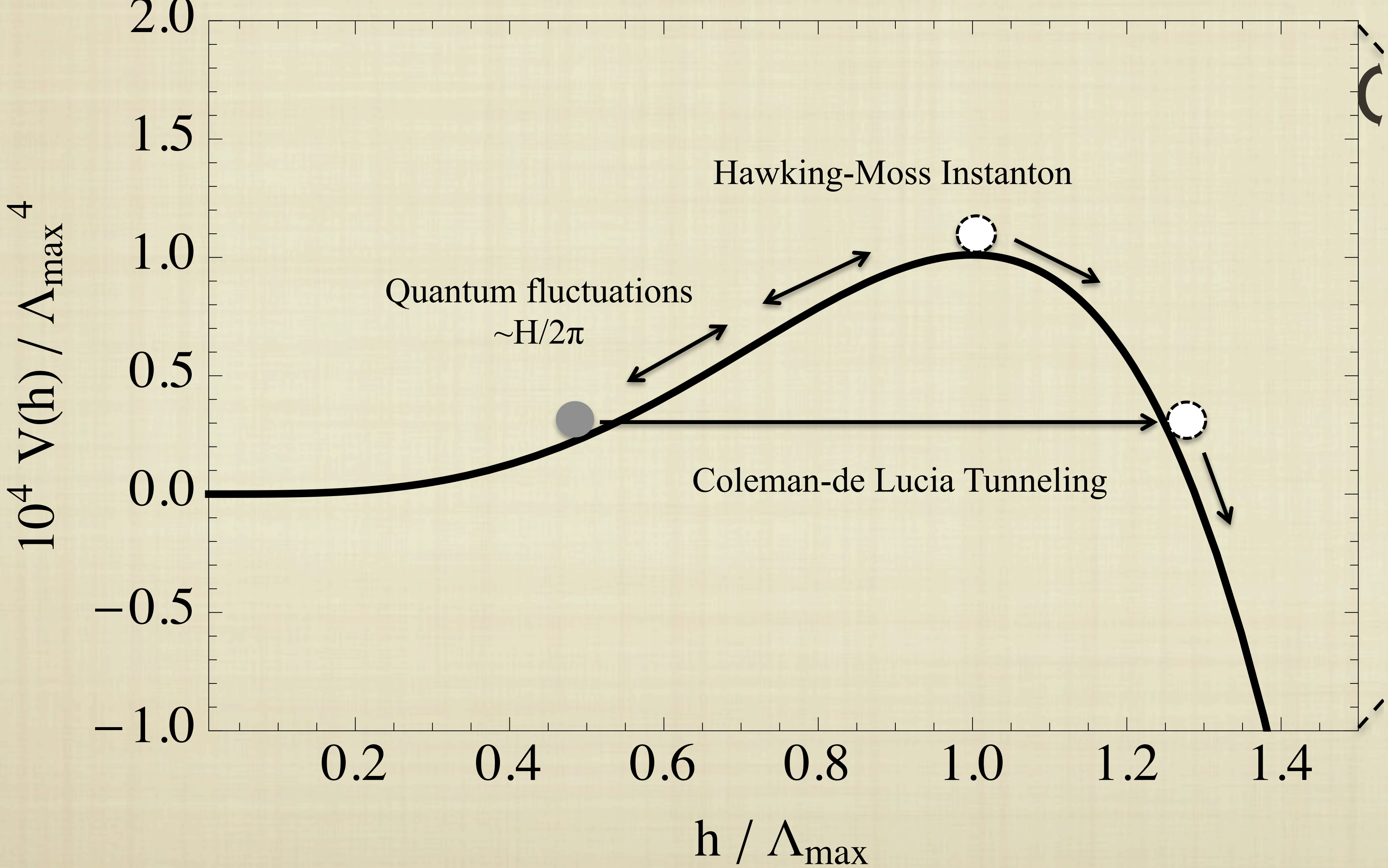
FP

HM

CdL

$$\delta h \sim \frac{H}{2\pi}$$

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \delta h}\left[\frac{V'(\delta h)}{3H}P + \frac{H^3}{8\pi^2}\frac{\partial P}{\partial \delta h}\right]$$



$$\left(\frac{d}{dt}\left\langle \delta h^2(t)\right\rangle = -\frac{2\lambda}{H}\left\langle \delta h^2(t)\right\rangle^2 + \frac{H^3}{4\pi^2}\right)$$

$$p \sim e^{-B_{\mathrm{HM}}}$$

$$B_{\mathrm{HM}} = \frac{8\pi^2}{3}\frac{\Delta V}{H^4}$$

Probability

- Correct Calculation in Limit $H > \Lambda_I$ is a Statistical Treatment Via Fokker-Planck Eq

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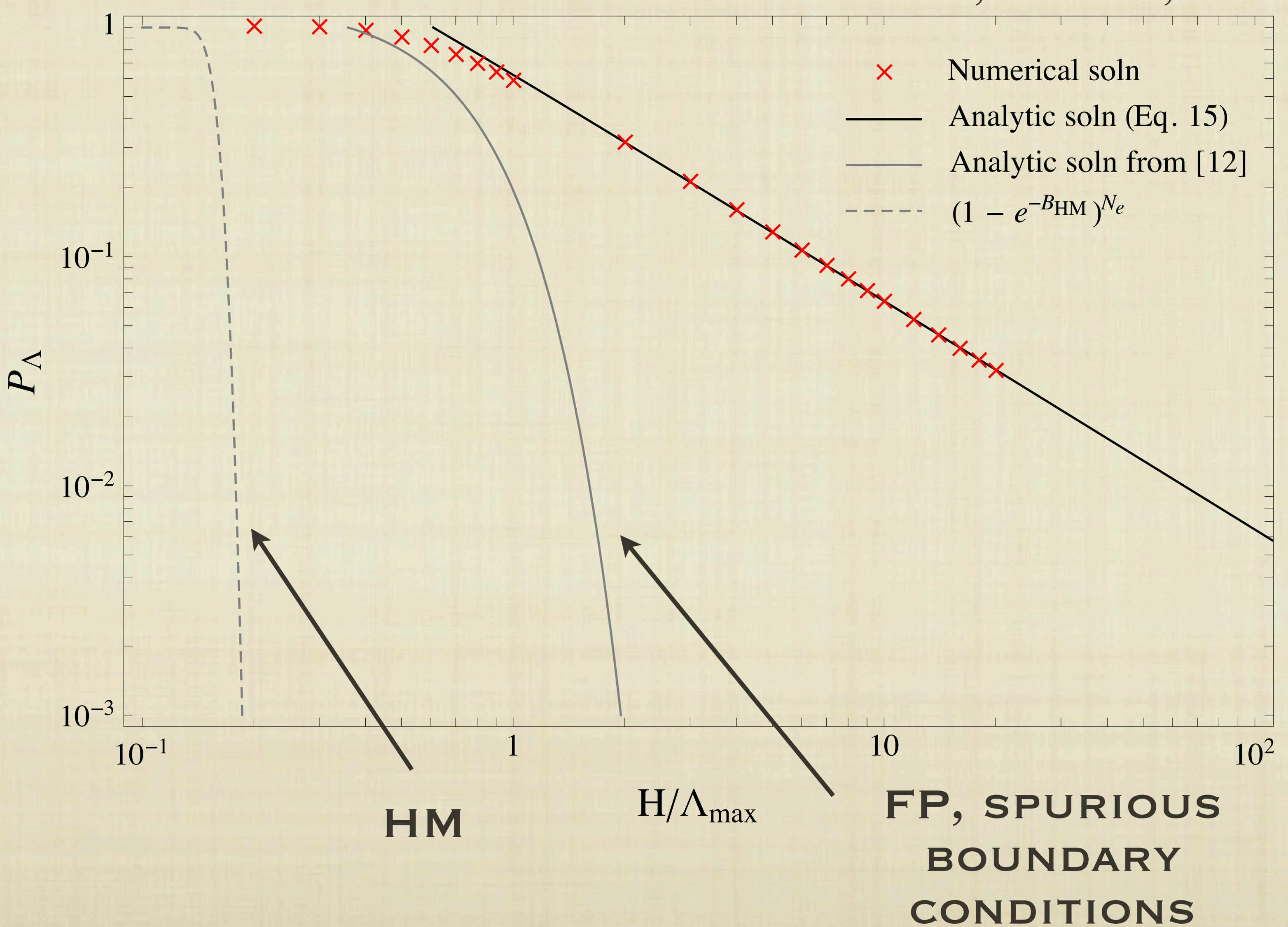
$$P_\Lambda \equiv \int_{-\Lambda_{max}}^{\Lambda_{max}} dh\, P(h, t_e)$$



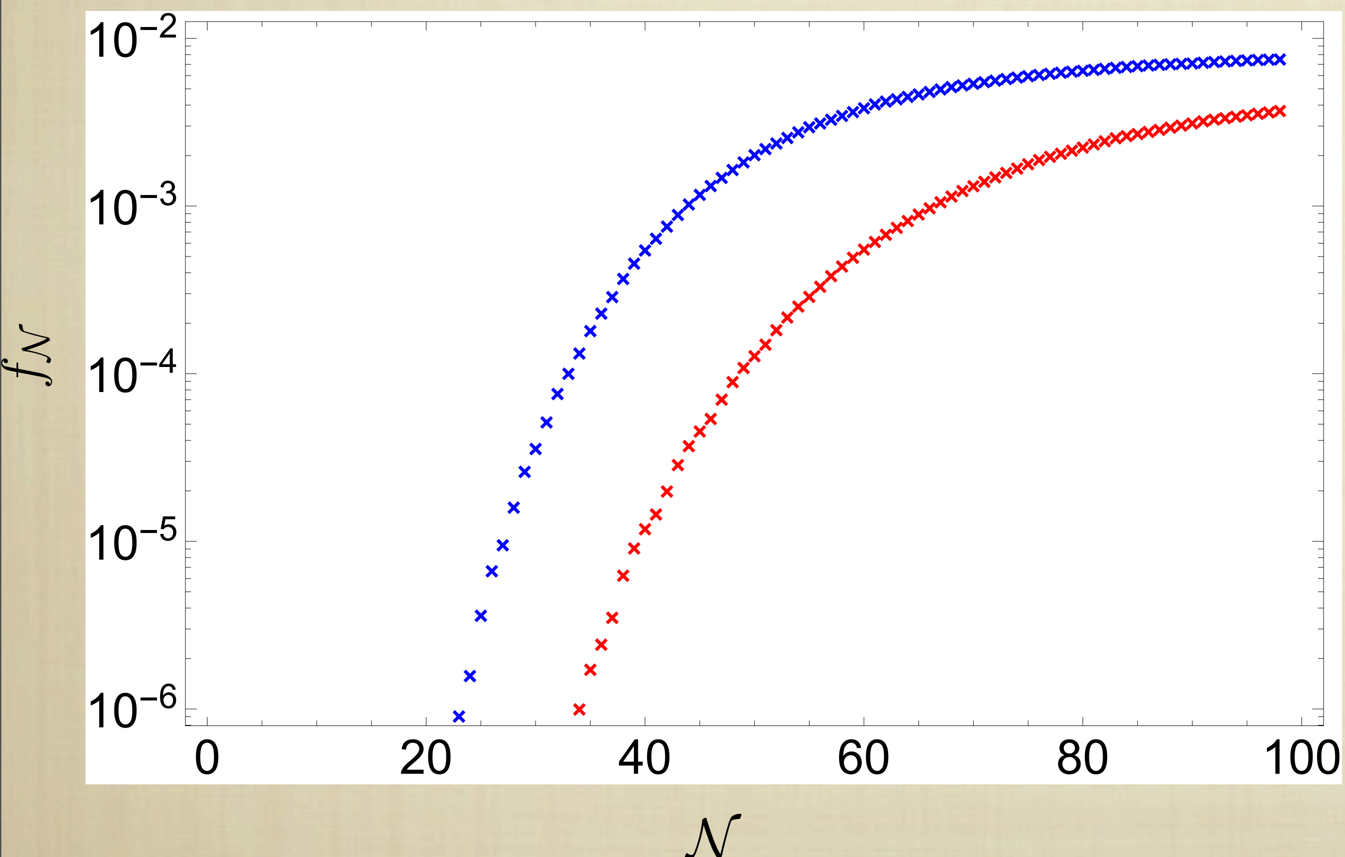
HM: Kobakhidze & Spencer-Smith 1301.2846; Fairbairn & Hogan 1406.6786
FP: Espinosa et al 0712.2484

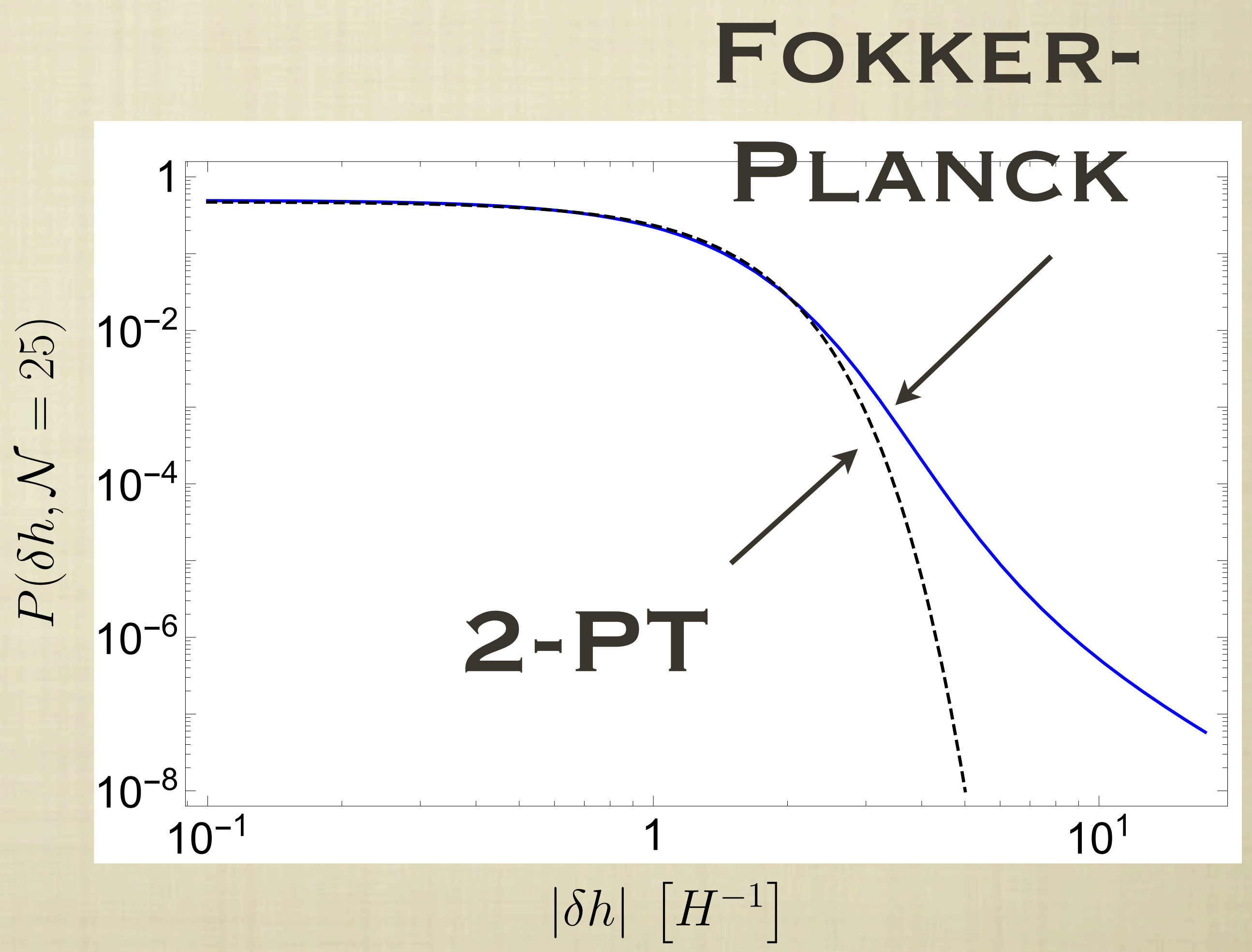
Condition to Terminate

- Fraction of Black Vacua

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \delta h}\left[\frac{V'(\delta h)}{3H}P + \frac{H^3}{8\pi^2}\frac{\partial P}{\partial \delta h}\right]$$

$$\frac{d}{dt}\left\langle \delta h^2(t)\right\rangle = -\frac{2\lambda}{H}\left\langle \delta h^2(t)\right\rangle^2 + \frac{H^3}{4\pi^2}$$





Condition to Terminate

- Terminate Inflation

$$\langle \delta h^2(t) \rangle = \frac{1}{\sqrt{-2\lambda}} \frac{H^2}{2\pi} \tan\left(\sqrt{-2\lambda} \frac{\mathcal{N}}{2\pi} \right)$$

(Runaway)

$$\mathcal{N}_{\max} = \frac{\pi^2}{\sqrt{-2\lambda}}$$

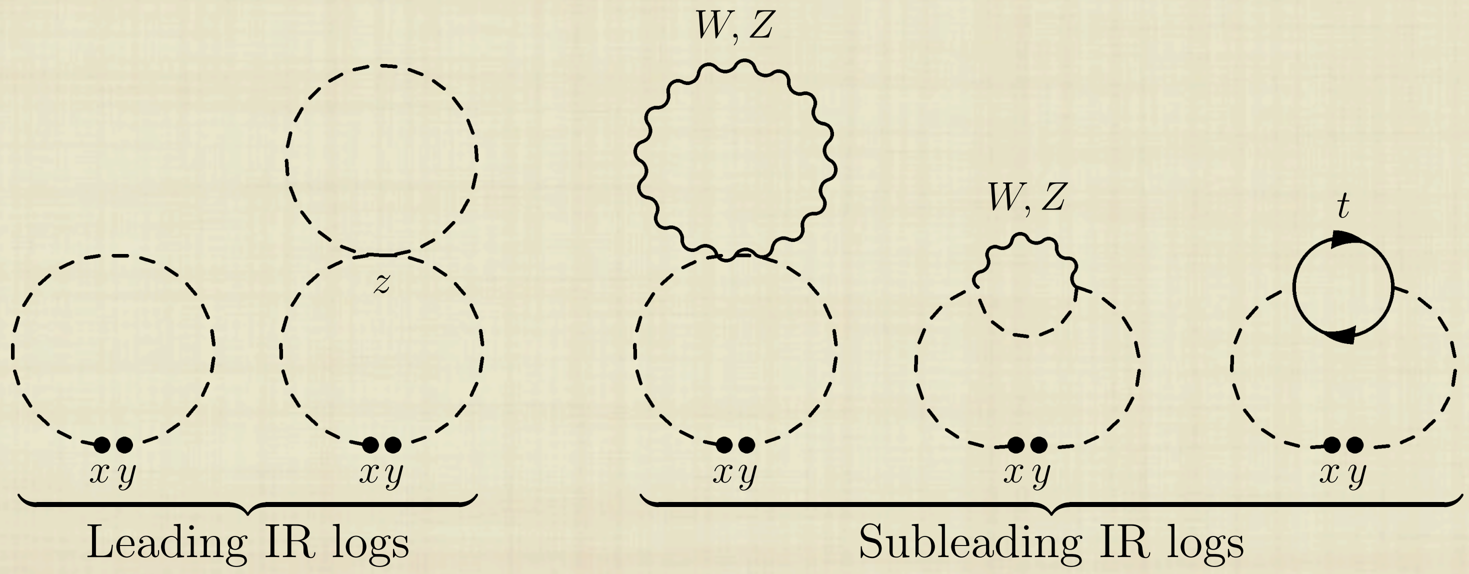
2sigma on mt, mh:

$$50 \lesssim \mathcal{N}_{\max} \lesssim 90$$

Scale of Quartic?

- $\lambda_{\rm eff}(h)$ is not a gauge invariant quantity

- Compute Two-Point Correlation in In-In Formalism $F(x,y) = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} h_k(t_x) h_k^*(t_y) e^{i\vec{k}\cdot(\vec{x}-\vec{y})} + \text{c.c.}$



$$3\lambda F(z,z) + \delta m^2 + \delta\xi R = \frac{3\lambda(\mu) H^2}{8\pi^2}\left(2\mathcal{N} + \ln\frac{\mu^2}{H^2}\right) \quad \Longrightarrow \quad \langle \delta h^2(t)\rangle_{\rm HF} \approx \frac{H^2}{4\pi^2}\mathcal{N} - \frac{\lambda H^2}{24\pi^4}\mathcal{N}^3$$

(Reproduces Leading Terms In $\langle \delta h^2(t)\rangle = \frac{1}{\sqrt{-2\lambda}}\frac{H^2}{2\pi}\tan\left(\sqrt{-2\lambda}\frac{\mathcal{N}}{2\pi}\right)$)

END INFLATION

- ASSUME INFLATION ENDS BEFORE BLACK VACUA DOMINATE
- Re-Heating Starts As Usual
- THEN WHAT?

RE-HEATING

- IF HIGH ENOUGH RE-HEAT TEMP, FINITE TEMP EFFECTS FORCE HIGGS BACK TO EW VACUUM
- Finite Temperature Effects: $m_{eff}^2 \sim T_R^2$
- Require: $m_{eff}^2 \gtrsim \lambda \langle \delta h \rangle^2$
- Easily Satisfied because $T_R^{\max} \sim \sqrt{H M_P}$

POST INFLATION

- But, Some Black Vacua Were Created During Inflation. What Happens to Them Post Inflation?
- HERE, MOST TRICKY. CALCULATE EVOLUTION OF BUBBLES IN RADIATION DOMINATED, MINKOWSKI BACKGROUND
- BLACK VACUA CRUNCH? -- CREATE DEFECTS
- OR, THEY EXPAND

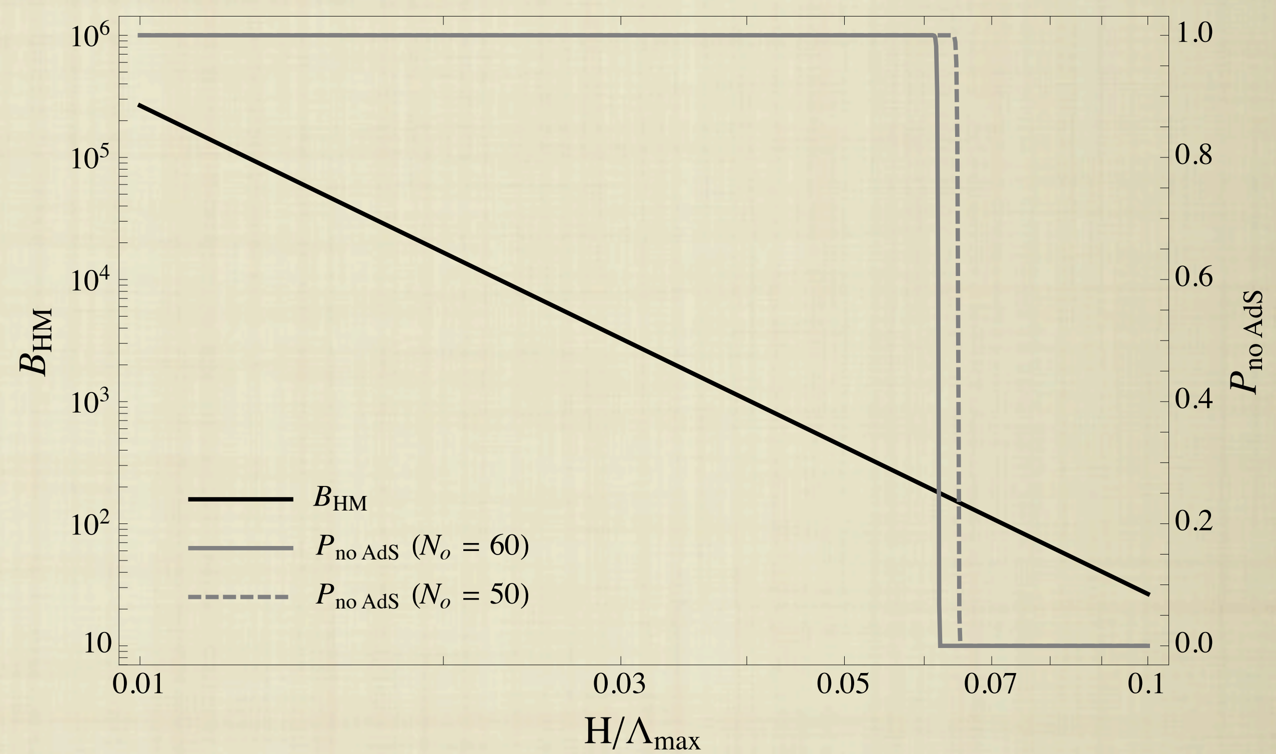
IF THEY EXPAND

- THEN A SINGLE BLACK VACUA IS ENOUGH TO DESTROY UNIVERSE

$$P_{\text{noAdS}} \sim \prod_{N_e=1}^{N_o} (e^{-p})^{e^{3N_e}}$$

- SO $p < e^{-3\mathcal{N}}$

- SO, MUST STABILIZE POTENTIAL IF $H \gtrsim \Lambda_{max}$



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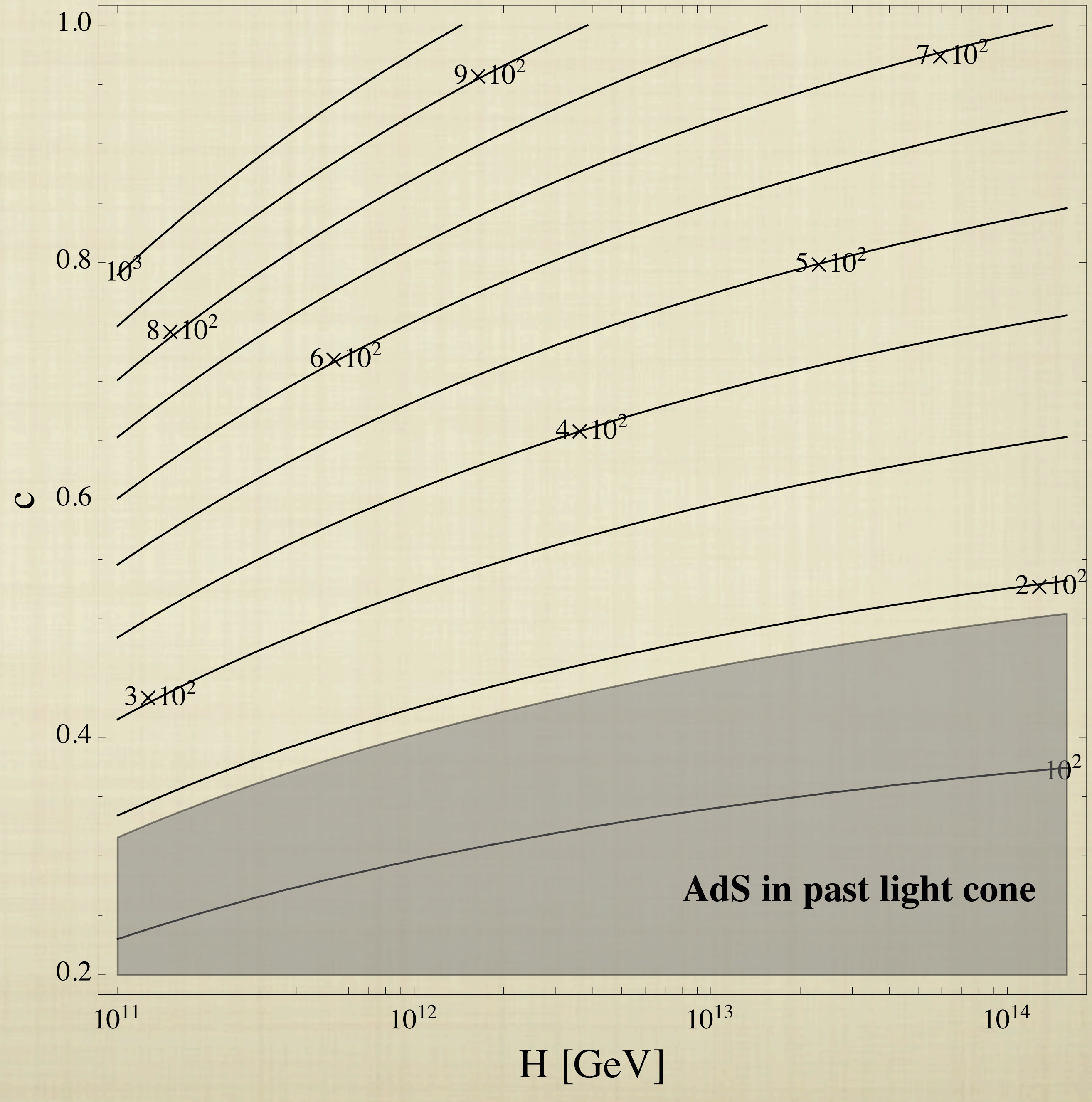
STABILIZE POTENTIAL

- PLANCK SUPPRESSED CORRECTIONS ARE SUFFICIENT

$$V(h) = \frac{c}{2} H^2 h^2 + \frac{\lambda_{\text{eff}}(h)}{4} h^4$$



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SO WHAT?

- If we can Always Stabilize Potential Sufficiently with Planck Slop, Why Study This?
- SM Higgs + Inflation Both Appear to Be Real; Worth Understanding the Dynamics
- One Tantalizing Fact:

During Inflation

- Higgs is Stable Enough to Allow Us to Inflate Long Enough to Give Rise to A Universe Like Ours .....

- But Not Much Longer

$$50 \lesssim \mathcal{N}_{\rm max} \lesssim 90 \qquad \mathcal{N}_{\rm max} = \frac{\pi^2}{\sqrt{-2\lambda}}$$

- High Scale Quartic Needed to Be Small for that to Happen

Summary

- Following Higgs Evolution During Inflation Requires Application of Correct Probability Evolution
- Post-Inflation Evolution Depends on How AdS Bubbles Evolve -- if They Expand, Require None in Past Lightcone
- Can Stabilize Potential Sufficiently with Planck Suppressed Corrections
- Higgs Instability: Tantalizing Hint or Coincidence?