

## *On the breaking of Lepton Flavor Universality in B decays*

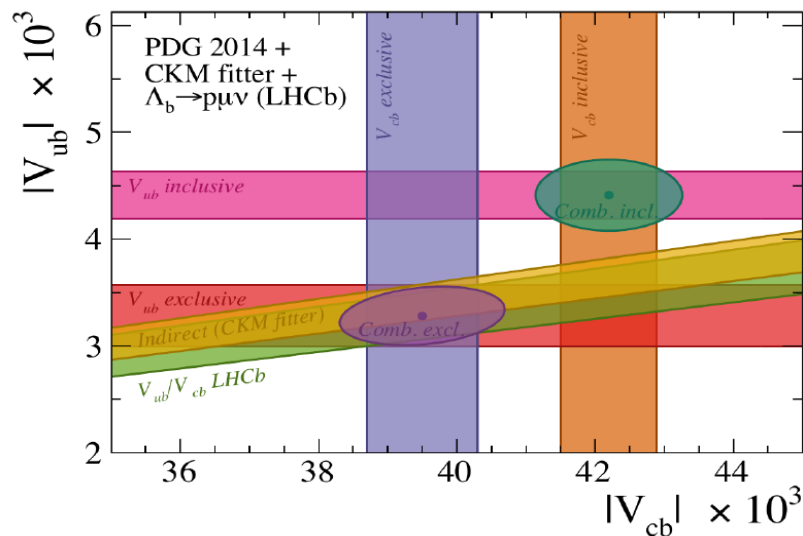
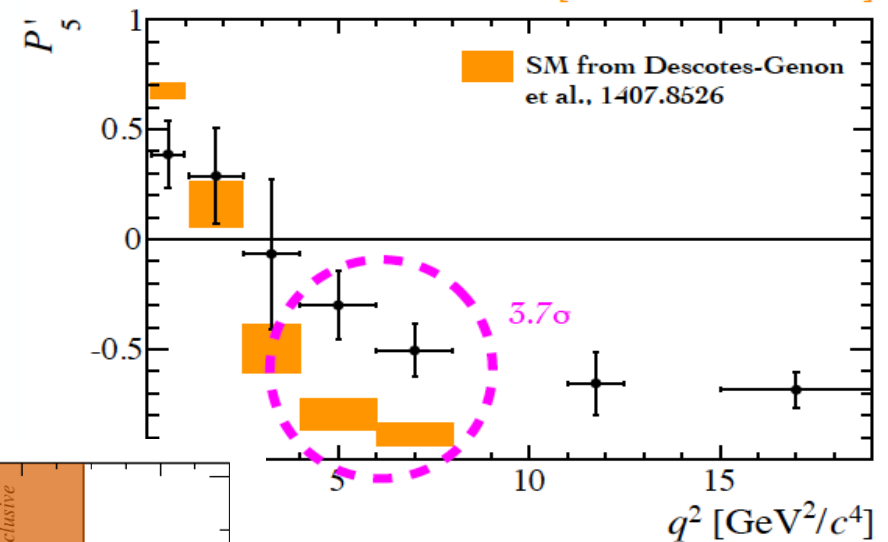
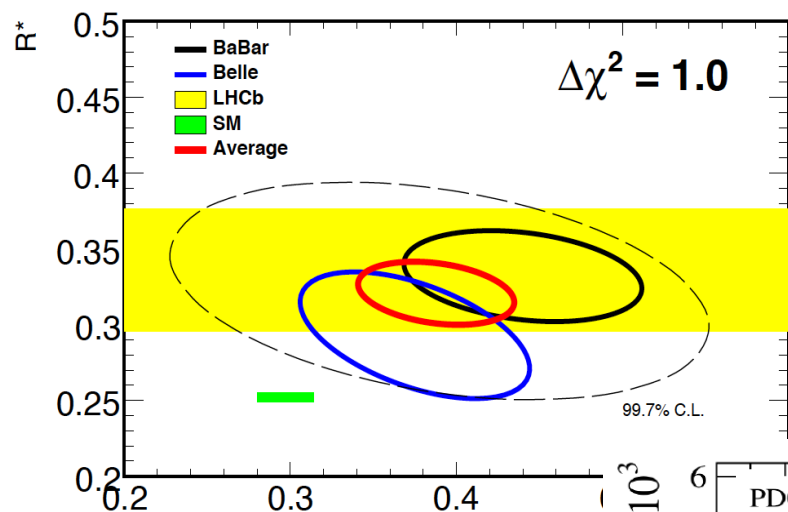
Gino Isidori

[ *University of Zürich* ]

- ▶ Some interesting new (and old) anomalies in B physics
- ▶ Speculations on the breaking of **L**epton **F**lavor **U**niversality
- ▶ Conclusions

# New and old anomalies in B physics [or better in semi-leptonic B decays...]

[LHCb-CONF-2015-002]



I.  $B \rightarrow D^{(*)} \tau \nu$  [LHCb, Belle]

Test of **LFU** in charged currents  
 [ $\tau$  vs. light leptons ( $\mu, e$ ) ]:

$$R(X) = \frac{\Gamma(B \rightarrow X \tau \bar{\nu})}{\Gamma(B \rightarrow X \ell \bar{\nu})}$$

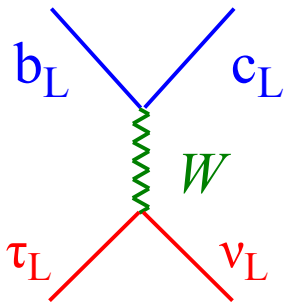
|                         | $R(D)$                              | $R(D^*)$                            |
|-------------------------|-------------------------------------|-------------------------------------|
| BaBar                   | $0.440 \pm 0.058 \pm 0.042$         | $0.332 \pm 0.024 \pm 0.018$         |
| NEW $\rightarrow$ Belle | $0.375^{+0.064}_{-0.063} \pm 0.026$ | $0.293^{+0.039}_{-0.037} \pm 0.015$ |
| NEW $\rightarrow$ LHCb  |                                     | $0.336 \pm 0.027 \pm 0.030$         |
| Average                 | $0.388 \pm 0.047$                   | $0.321 \pm 0.021$                   |
| SM expectation          | $0.300 \pm 0.010$ $\sim 1.8\sigma$  | $0.252 \pm 0.005$ $\sim 3.2\sigma$  |

- **SM** prediction quite **solid**: f.f. uncertainty cancel (*to a good extent...*) in the ratio
- Consistent exp. results by 3 (very) different experiments

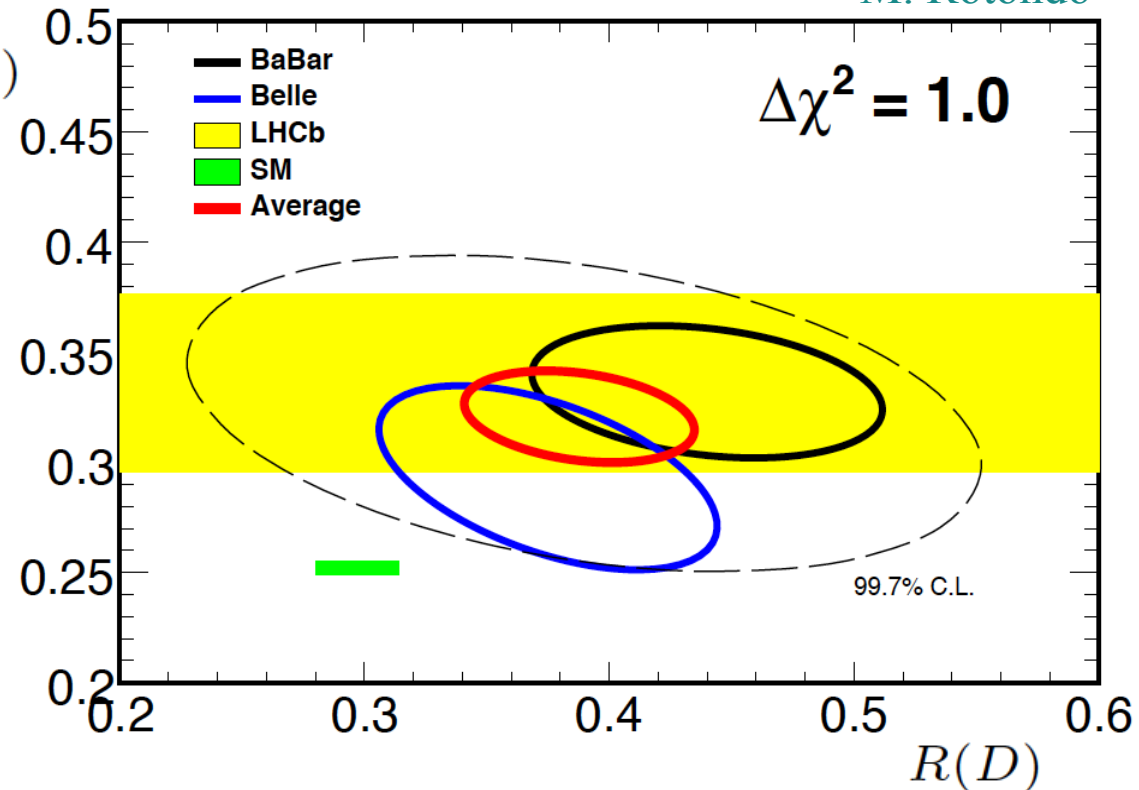
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$R(D^*)$



M. Rotondo

- **SM** prediction quite **solid**: f.f. uncertainty cancel (*to a good extent...*) in the ratio
- Consistent exp. results by 3 (very) different experiments
  - $4\sigma$  excess over SM (if D and  $D^*$  combined)
  - The two channels are well consistent with a **universal enhancement** ( $\sim 30\%$ ) of the SM  $b_L \rightarrow c_L \tau_L \nu_L$  amplitude (*RH or scalar amplitudes disfavored*)

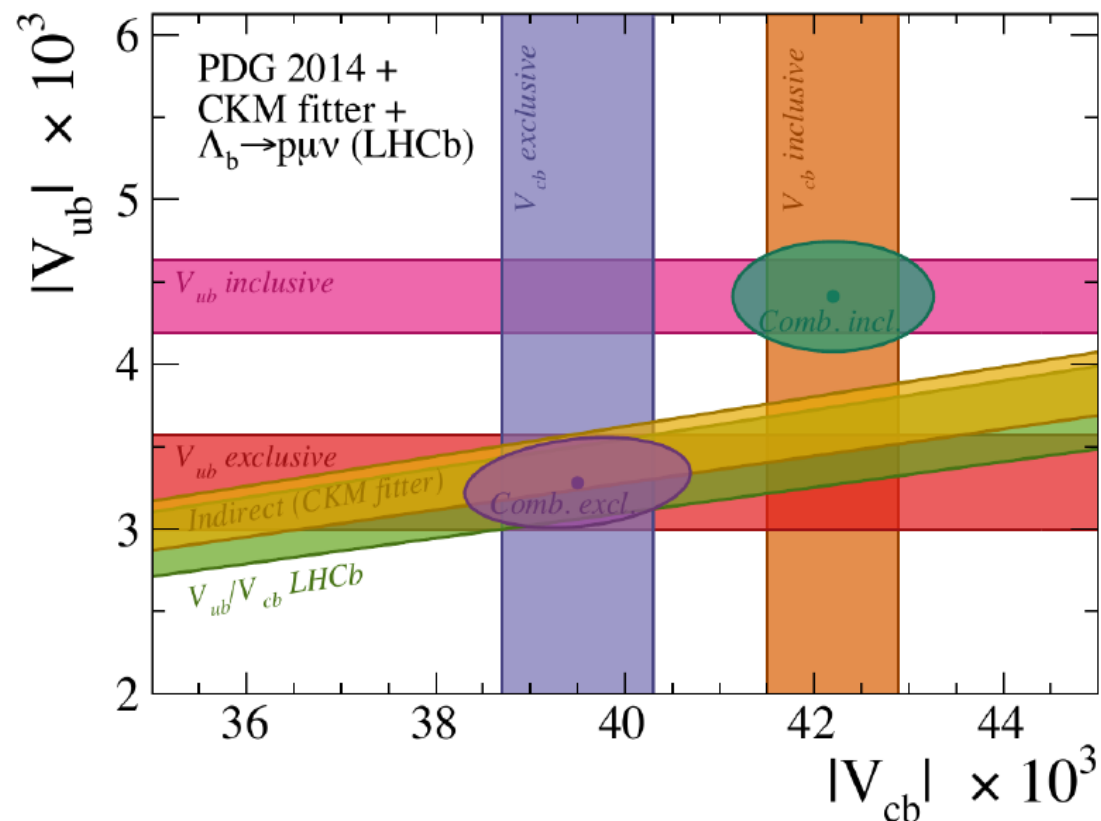
## II. $|V_{ub}/V_{cb}|$ from $B(\Lambda_b \rightarrow p\mu\nu)/B(\Lambda_b \rightarrow \Lambda_c\mu\nu)$ [LHCb]

T. Gershon

Long-standing discrepancy  
between exclusive and inclusive  
determinations of both  $|V_{ub}|$  &  $|V_{cb}|$   
(again charged currents...)

**New ingredient:**  $|V_{ub}/V_{cb}|$  from  
 $B(\Lambda_b \rightarrow p\mu\nu)/B(\Lambda_b \rightarrow \Lambda_c\mu\nu)$

→ small th. error given recent  
Lattice estimate of the  $f.f.$   
[arXiv:1503.01421]



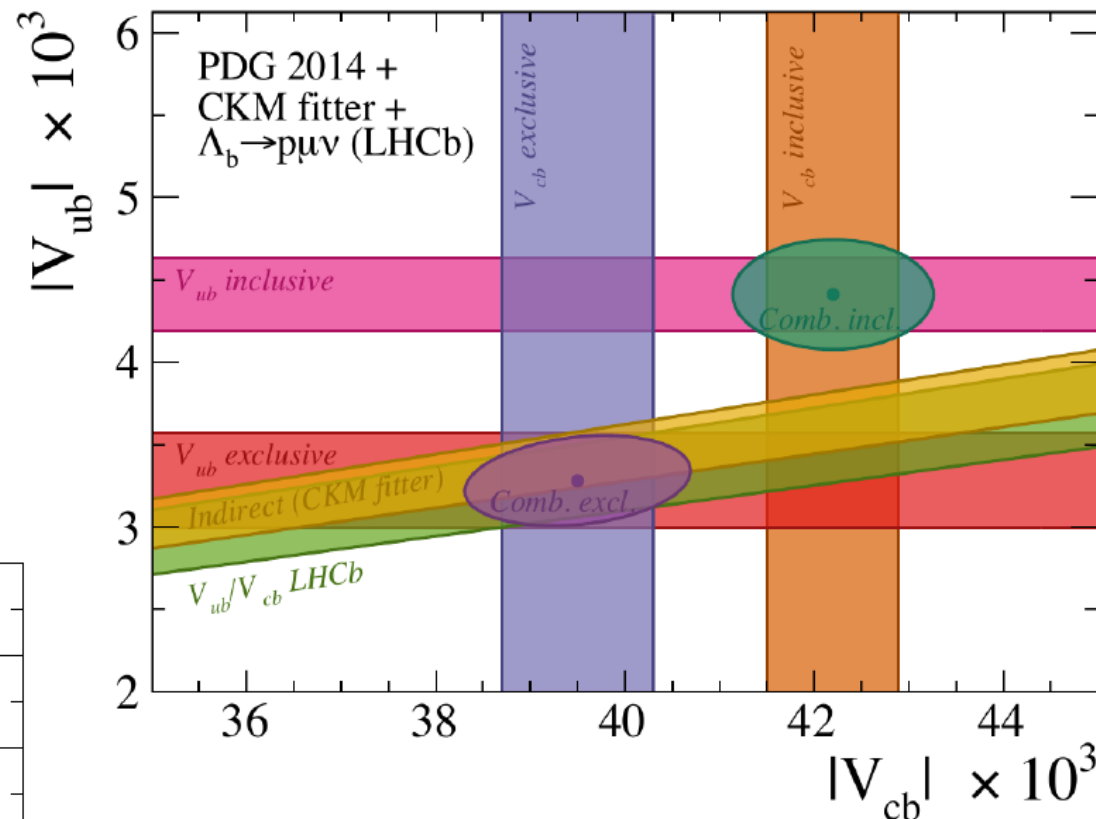
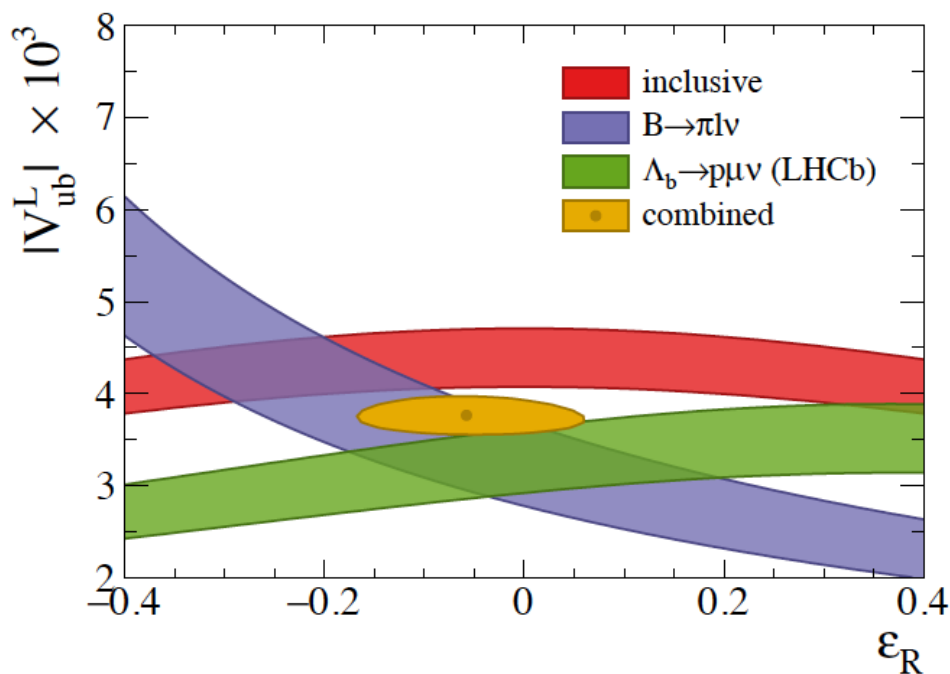
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- Consistent with other exclusive data
- Increased tension between excl. & incl.
- Rules out RH currents as possible explanation of the tension

### III. Anomalies in $B \rightarrow K^{(*)} \mu\mu / ee$ [LHCb]

The largest anomaly is the one [*obs. in 2013 and confirmed with higher stat. in 2015*] in the  $P_5'$  [ $B \rightarrow K^* \mu\mu$ ] angular distribution.

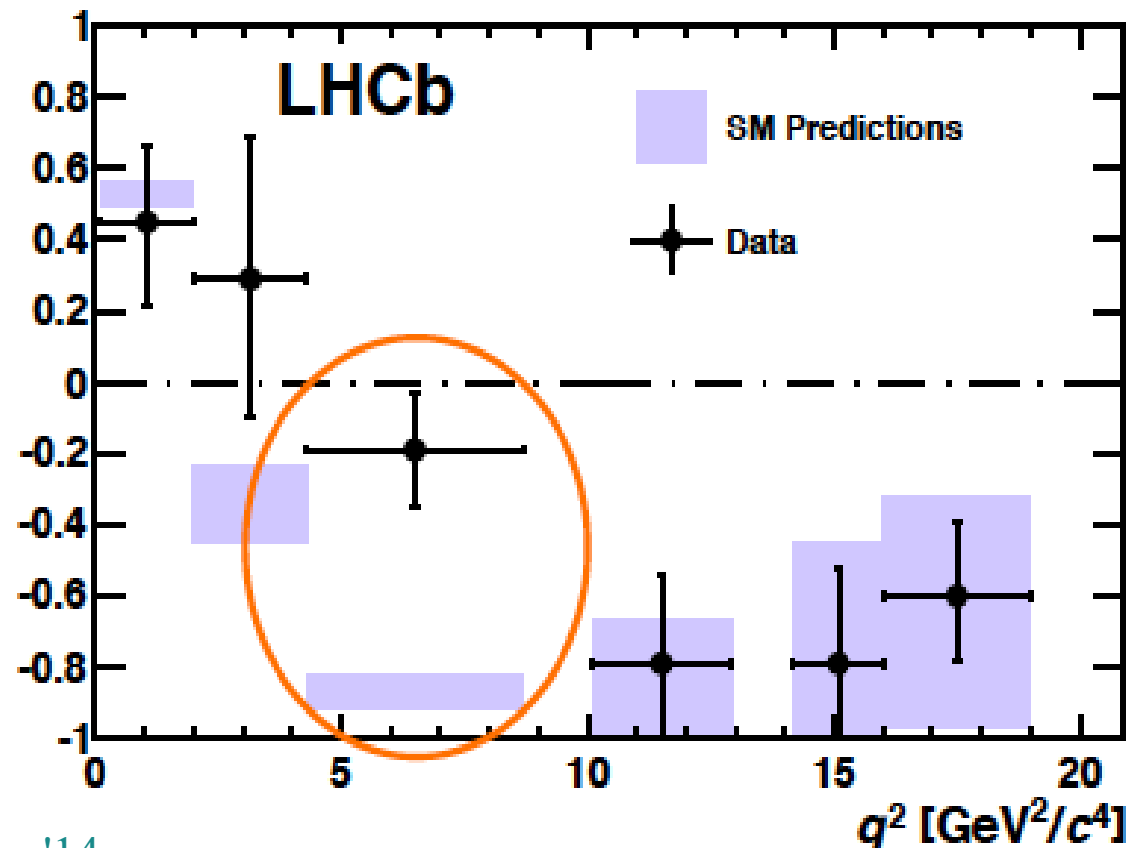
But less significant anomalies present also in other  $B \rightarrow K^* \mu\mu$  observables and also in other  $b \rightarrow s \mu\mu$  channels [*overall smallness of all  $BR(B \rightarrow \text{Hadron} + \mu\mu)$* ]

#### Pro NP:

- Reduced tension in all the observables with a unique fit of non-standard short-distance Wilson coefficients

#### Against NP:

- Main effect in  $P_5'$  not far from cc threshold
- Significance reduced with conservative estimates of non-factorizable corrections



Jaeger *et al.* '12 Hambrock *et al.* '13  
Hiller & Zwicky '13, Lyon & Zwicky '14

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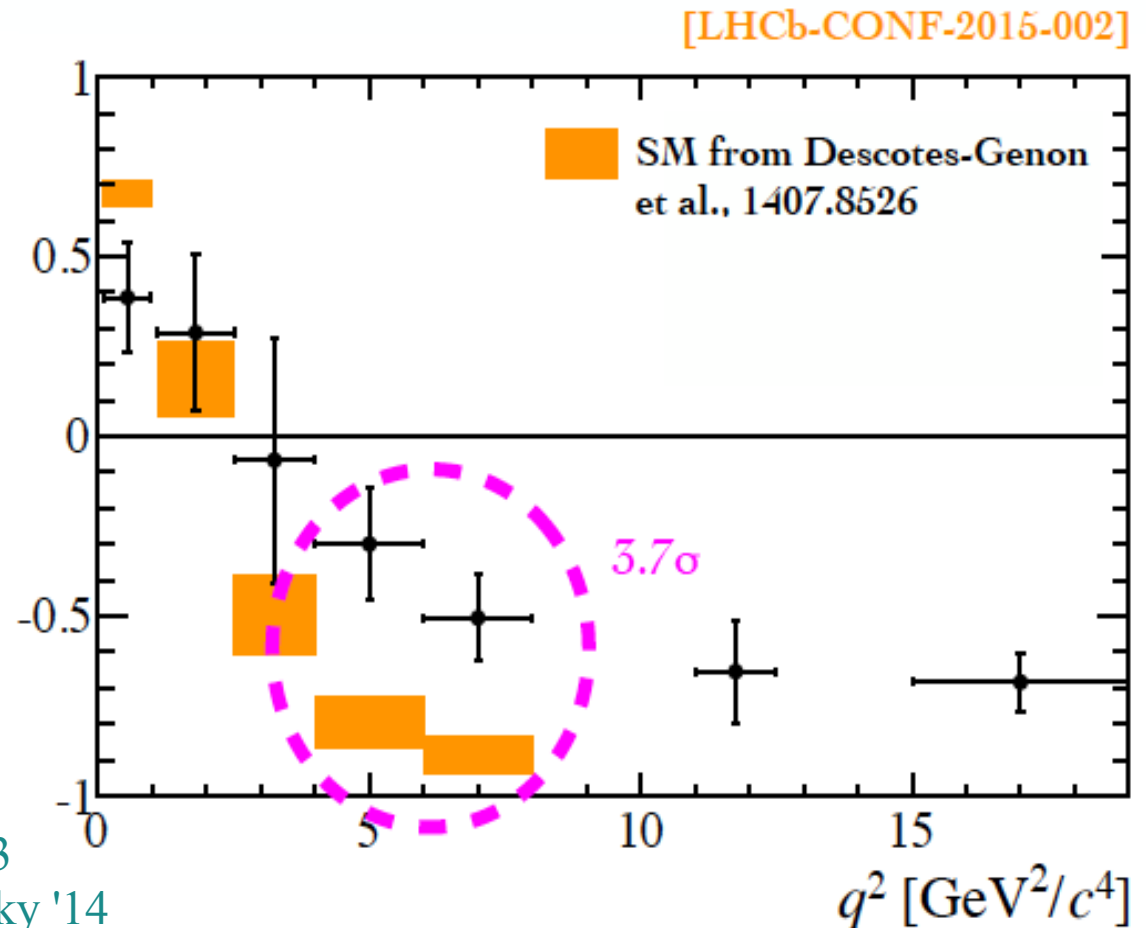
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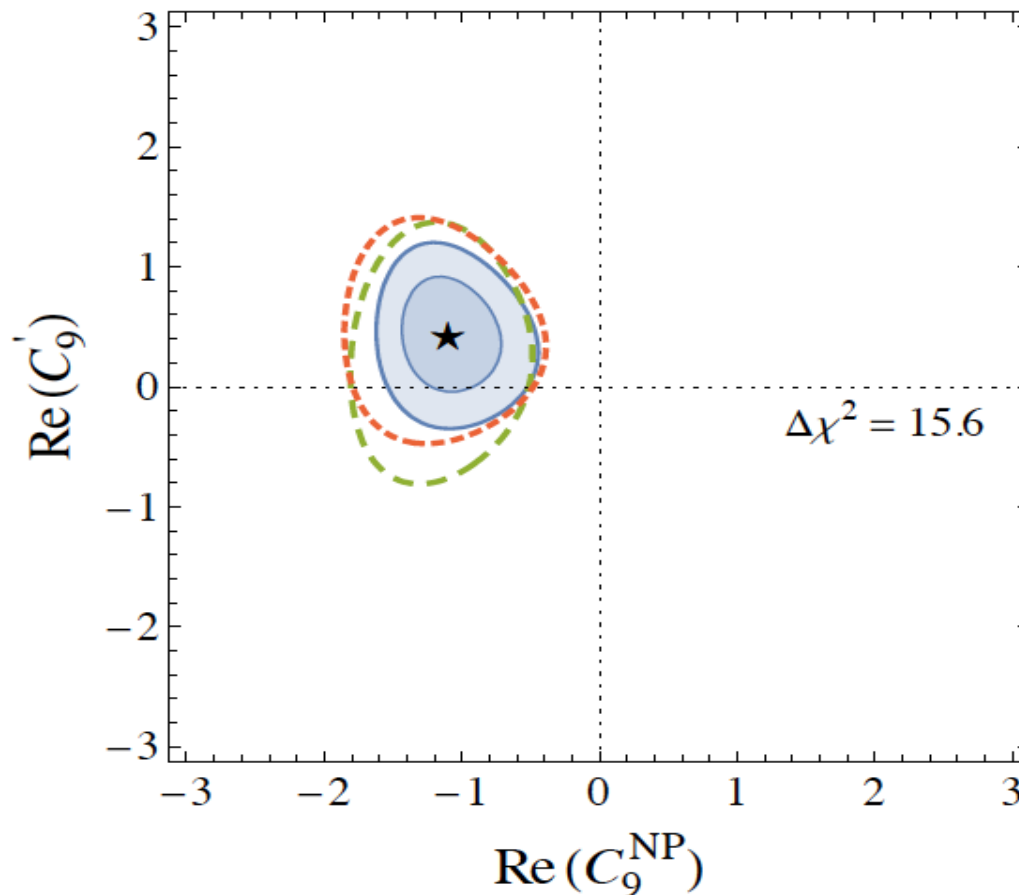




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Descotes-Genon, Matias, Virto '13  
 Altmannshofer & Straub '13, '15  
 Beaujean, Bobeth, van Dyk '13  
 Horgan *et al.* '13

$$O_9^{(\prime)} \propto (\bar{s}\gamma_\mu P_{L(R)}b)(\bar{\mu}\gamma^\mu \mu)$$

muonic vector current

- ▶ NP contributions to  $C_9$  give best description of the data
- ▶ (NP with  $C_9 = -C_{10}$  works almost equally well)

### III. Anomalies in $B \rightarrow K^{(*)} \mu\mu / ee$ [LHCb]

Last but not least, a  $2.6\sigma$  deviation from the SM is observed also in the LFU ratio

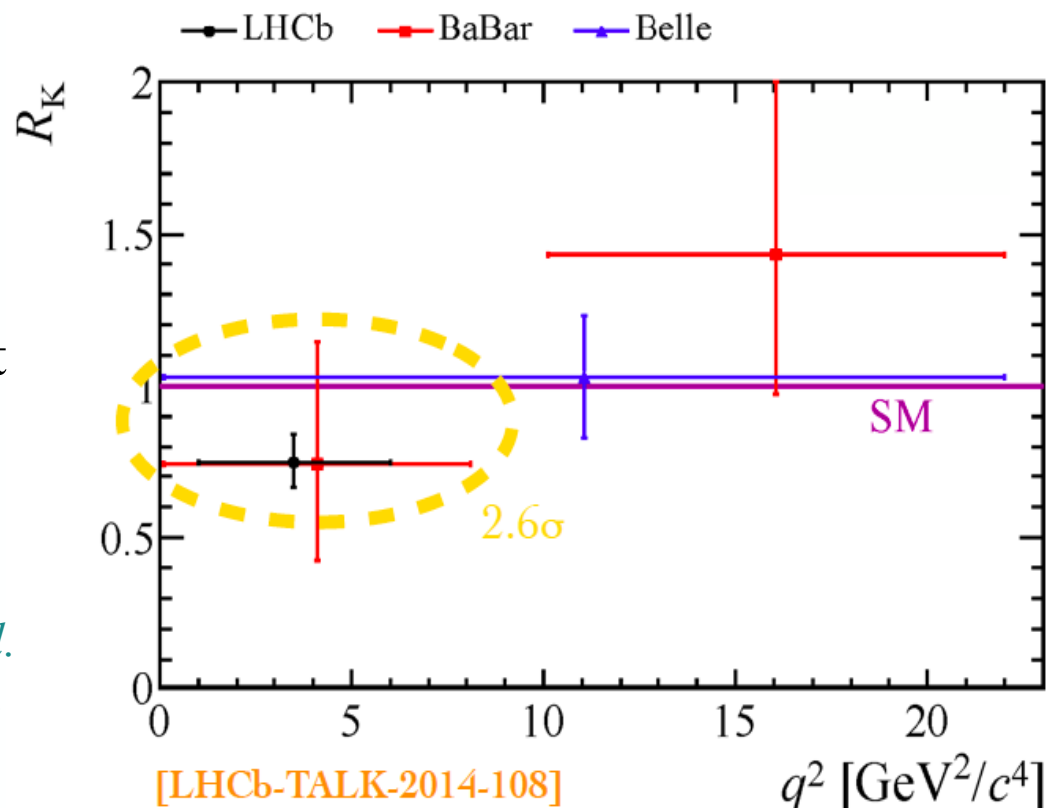
$$R_K = \frac{\int d\Gamma(B^+ \rightarrow K^+ \mu\mu)}{\int d\Gamma(B^+ \rightarrow K^+ ee)}$$

[1-6] GeV<sup>2</sup>

- Negligible th. error  $\rightarrow$  clean test of LFU (in neutral currents)

$$R_K = 1 \pm O(1\%)$$

Bordone *et al.*  
work in prog.



- The anomaly is perfectly described assuming NP only in  $b \rightarrow s \mu\mu$  [and not in  $b \rightarrow s ee$ ] consistently with the various  $b \rightarrow s \mu\mu$  anomalies

# Speculations on the breaking of **L**epton **F**lavor **U**niversality

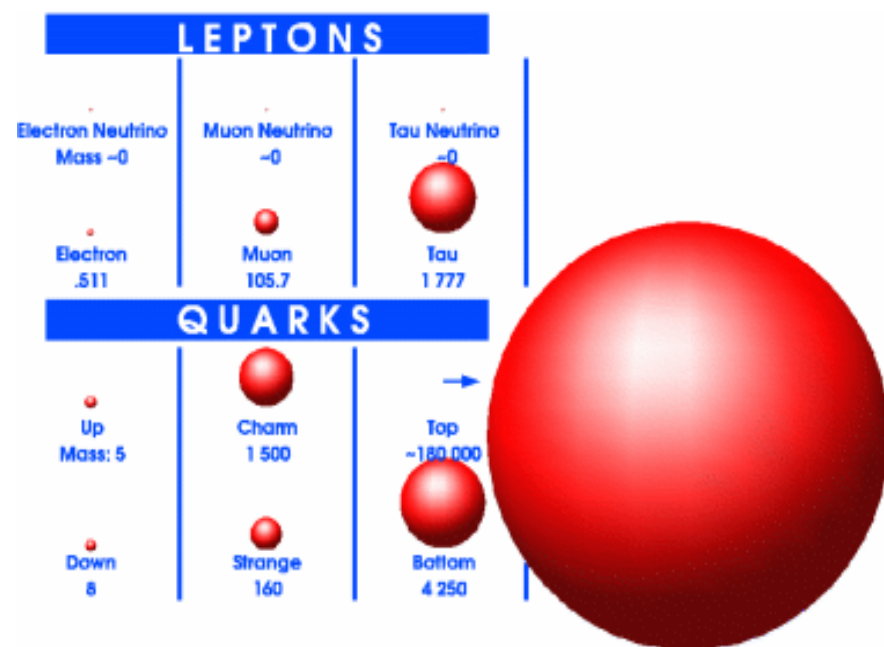
## ► Speculations on the breaking of LFU

(Some of) these recent results have stimulated a lot of theoretical activity.

Most interesting aspect: possible breaking of LFU, both in charged currents ( $b \rightarrow c\tau\nu$  vs.  $b \rightarrow c\mu\nu$ ) and in neutral currents ( $b \rightarrow s\mu\mu$  vs.  $b \rightarrow see$ )

A few general messages:

- ★ LFU is not a fundamental symmetry of the SM Lagrangian (*accidental symmetry in the gauge sector, broken by Yukawas*)
- ★ LFU tests at the Z peak are not very interesting ( $\rightarrow$  gauge sector)
- ★ Most stringent tests of LFU involve only 1<sup>st</sup>-2<sup>nd</sup> gen. quarks & leptons
  - $\rightarrow$  Natural to conceive NP models where LFU is violated more in processes with 3<sup>rd</sup> gen. quarks ( $\leftrightarrow$  hierarchy in Yukawa coupl.)



## ► Speculations on the breaking of LFU

(Some of) these recent results have stimulated a lot of theoretical activity:

S. Fajfer, J. F. Kamenik, I. Nisandzic and J. Zupan, Phys. Rev. Lett. **109** (2012) 161801 [[arXiv:1206.1872](#)].

S. Descotes-Genon, J. Matias and J. Virto, Phys. Rev. D **88** (2013) 074002 [[arXiv:1307.5683](#)].

W. Altmannshofer and D. M. Straub, Eur. Phys. J. C **73** (2013) 2646 [[arXiv:1308.1501](#)].

A. Datta, M. Duraissamy and D. Ghosh, Phys. Rev. D **89** (2014) 7, 071501 [[arXiv:1310.1937](#)].

G. Hiller and M. Schmaltz, Phys. Rev. D **90** (2014) 054014 [[arXiv:1408.1627](#)]; JHEP **1502** (2015) 055

A. Crivellin and S. Pokorski, Phys. Rev. Lett. **114** (2015) 1, 011802 [[arXiv:1407.1320](#)].

S. L. Glashow, D. Guadagnoli and K. Lane, Phys. Rev. Lett. **114** (2015) 091801 [[arXiv:1411.0565](#)].

+ many others...

...but most attempts based on EFT approaches and focused only on semi-leptonic operators [[quark](#)×[lepton](#)]

What I will discuss today is a proposal to describe all these effects within a simplified dynamical model:

- low-energy correlations among [quark](#)×[quark](#), [quark](#)×[lepton](#), [lepton](#)×[lepton](#)
- correlation between [low-energy](#) and [high-energy](#) physics

► A “prototype data-inspired” model:

Greljo, GI, Marzocca '15

Main assumptions:

- NP in both charged & neutral currents + RH currents disfavored +  $SU(2)_L \times U(1)_Y$  symmetry  $\rightarrow$   **$SU(2)_L$ -triplet effective operator**

$$\frac{g_q g_\ell}{\Lambda^2} \lambda_{ij}^q \lambda_{kl}^\ell \left( \bar{Q}_L^i T^a \gamma_\mu Q_L^j \right) \left( \bar{L}_L^k T^a \gamma^\mu L_L^l \right)$$

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- We assume this effective operator is the result of integrating-out a **heavy triplet of vector bosons (W', Z')** coupled to a single current:

$$J_\mu^a = g_q \lambda_{ij}^q \left( \bar{Q}_L^i \gamma_\mu T^a Q_L^j \right) + g_\ell \lambda_{ij}^\ell \left( \bar{L}_L^i \gamma_\mu T^a L_L^j \right) \longrightarrow \frac{1}{2m_V^2} J_\mu^a J_\mu^a$$

- low-energy correlations among **quark**×**quark**, **quark**×**lepton**, **lepton**×**lepton**
- correlation between **low-energy** and **high-energy** physics

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- **Non-Universal flavor structure** of the currents  $\rightarrow$  **mainly 3<sup>rd</sup> generations**

$$\lambda_{ij}^{q,\ell} = \delta_{i3} \delta_{3j} + \text{small corrections for 2<sup>nd</sup> (& 1<sup>st</sup>) generations}$$

*(hierarchy determined by CKM in the quark sector)*



► A “prototype data-inspired” model:

A closer look to the flavor structure of the model:

$$J_\mu^a = g_q \lambda_{ij}^q \left( \bar{Q}_L^i \gamma_\mu T^a Q_L^j \right) + g_\ell \lambda_{ij}^\ell \left( \bar{L}_L^i \gamma_\mu T^a L_L^j \right)$$

→ Coupling to 3<sup>rd</sup> generations not suppressed [*dynamical assumption*]

→ Coupling to light generations controlled by small  $U(2)_q \times U(2)_l$  breaking spurions related to subleading terms in the Yukawa couplings

Barbieri *et al.* '11

*Connection to CKM matrix in the quark sector:*

$$\lambda^q \simeq \begin{pmatrix} |\epsilon|^2 V_{3\alpha}^* V_{3\beta} & \epsilon^* V_{3\alpha}^* \\ \epsilon V_{3\beta} & 1 \end{pmatrix}$$

$$\lambda_{bd} \ll \lambda_{bs} \ll \lambda_{bb} = 1$$

$$\lambda_{ss} \sim \lambda_{bs}^2$$

down-type  
mass basis

$$\epsilon \lesssim 1$$

► Effects in charged currents:

$$\frac{\mathcal{A}(b \rightarrow c \ell^i \bar{\nu}^i)_{\text{SM+NP}}}{\mathcal{A}(b \rightarrow c \ell^i \bar{\nu}^i)_{\text{SM}}} = 1 + R_0 \lambda_{ii}^\ell$$

$$R_0 \equiv \frac{g_\ell g_q}{g^2} \frac{m_W^2}{m_V^2}$$

I. From  $R(D^*)$  &  $R(D)$  data  $[\Gamma(b \rightarrow c \tau \nu)/\Gamma(b \rightarrow c \mu \nu)] \rightarrow R_0 = 0.14 \pm 0.04$

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II. In principle, it should be possible to get a strong bound on the sub-leading leptonic coupling ( $\lambda_{\mu\mu}$ ) from  $\Gamma(b \rightarrow c \mu \nu)/\Gamma(b \rightarrow c e \nu)$ , but surprisingly it is not so stringent ( $|\lambda_{\mu\mu}| \lesssim 0.1$ )  $\rightarrow$  *no dedicated studies @ B-factories !*

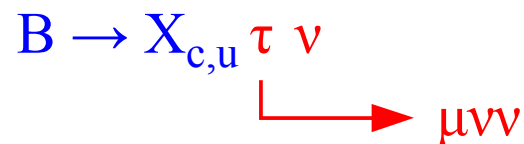
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III. Even if it is hard to quantify [*work in prog.*], this breaking of LFU in c.c could decrease the tension between exclusive & inclusive determinations of  $|V_{ub}|$  &  $|V_{cb}|$ :



Irreducible bkg. for the inclusive meas. subtracted  
(at present) assuming SM-like  $\Gamma(B \rightarrow X_{c,u} \tau \nu)$



if  $\Gamma(B \rightarrow X_{c,u} \tau \nu)$  is enhanced over the SM  $\rightarrow |V_{c(u)b}|_{\text{incl.}}$  are overestimated

► Global fit to low-energy data:

5 free parameters: 
$$\epsilon_{\ell,q} \equiv \frac{g_{\ell,q} m_W}{g m_V} \approx g_{\ell,q} \frac{122 \text{ GeV}}{m_V} + \lambda_{bs}^q, \lambda_{\mu\mu}^\ell, \lambda_{\tau\mu}^\ell$$

several constraints:

- R(D\*)
- R(D)
- R<sub>K</sub>
- P<sub>5'</sub>(B → K\* μμ)
- B(B → Kνν)
- ΔM<sub>B<sub>s</sub></sub>, ΔM<sub>B<sub>d</sub></sub>
- CPV(D-D)
- Γ(B → Xμν)/Γ(B → Xev)
- τ → 3μ
- Γ(τ → μνν)/Γ(τ → eνν)

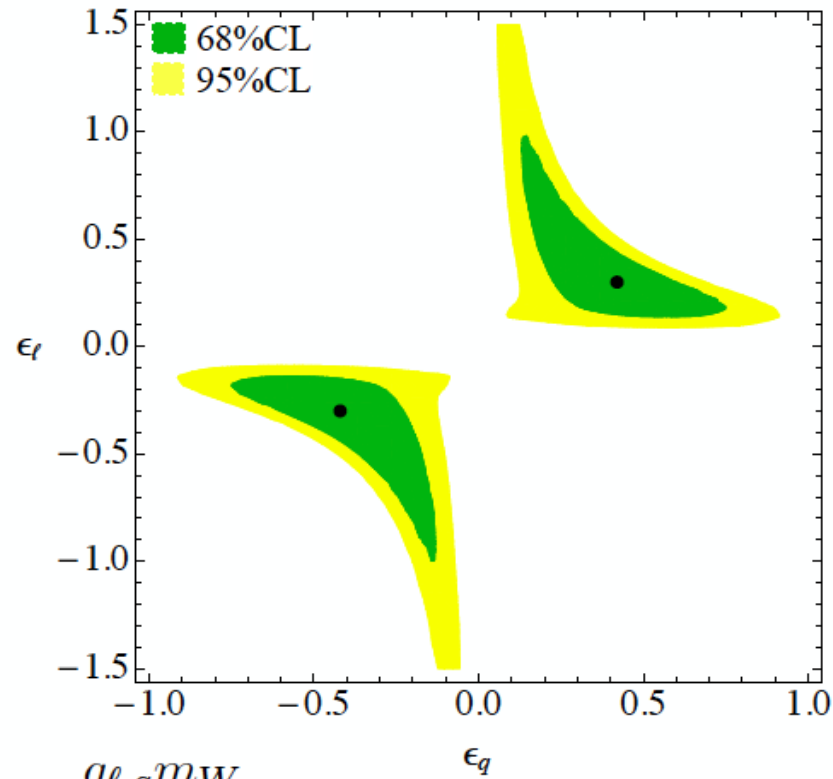


Overall good fit of low-energy data  
(non-trivial given tight constraints from ΔF=2 & LFV)

Best fit point:  $\epsilon_\ell \approx 0.37$ ,  $\epsilon_q \approx 0.38$   $p(\text{SM}) = 0.002$

(flavor structure of the sub-leading terms not really probed)

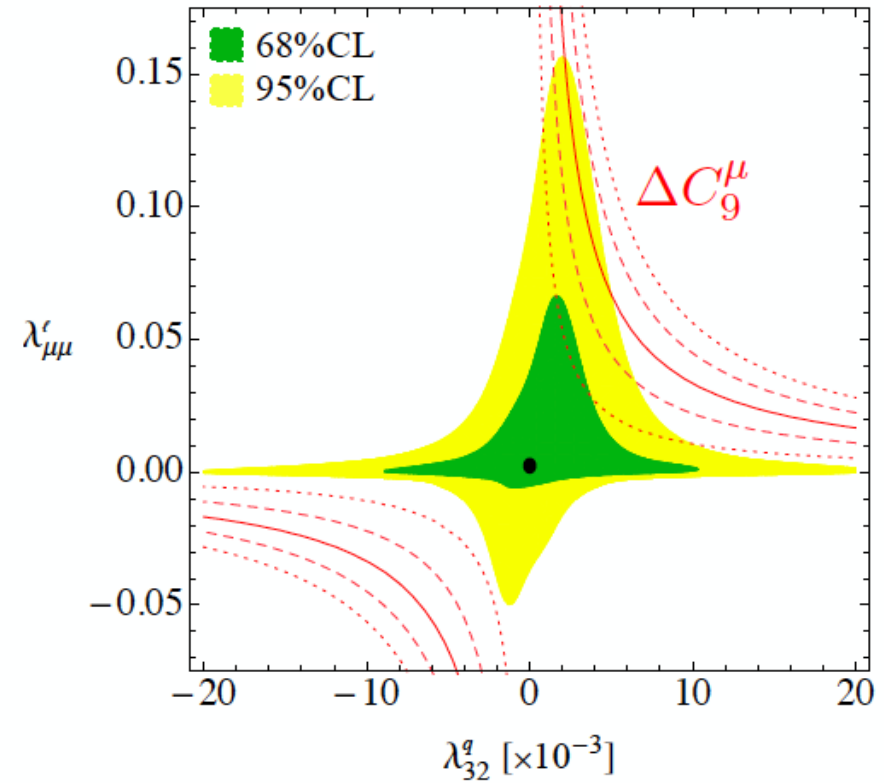
► Global fit to low-energy data:



$$\epsilon_{\ell,q} = \frac{g_{\ell,q} m_W}{g m_V}$$

$$\epsilon_{\ell}, \epsilon_q \lesssim 1$$

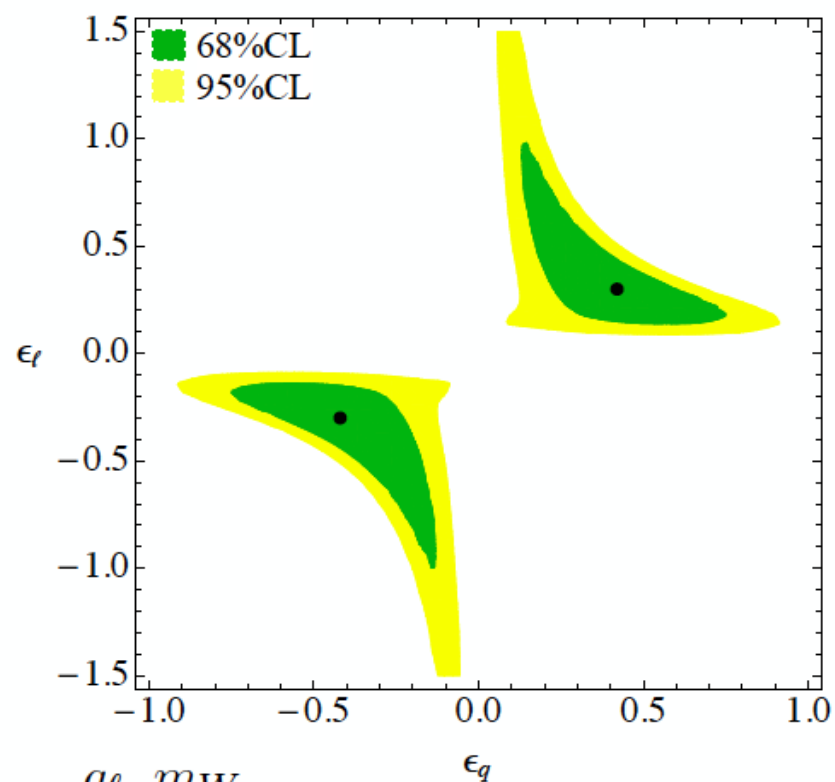
$\epsilon_q > \epsilon_{\ell}$  would improve  $b \rightarrow s \mu \mu$



$$\lambda_{\mu\mu} \lesssim 0.1$$

$$\lambda_{bs} \lesssim 0.015$$

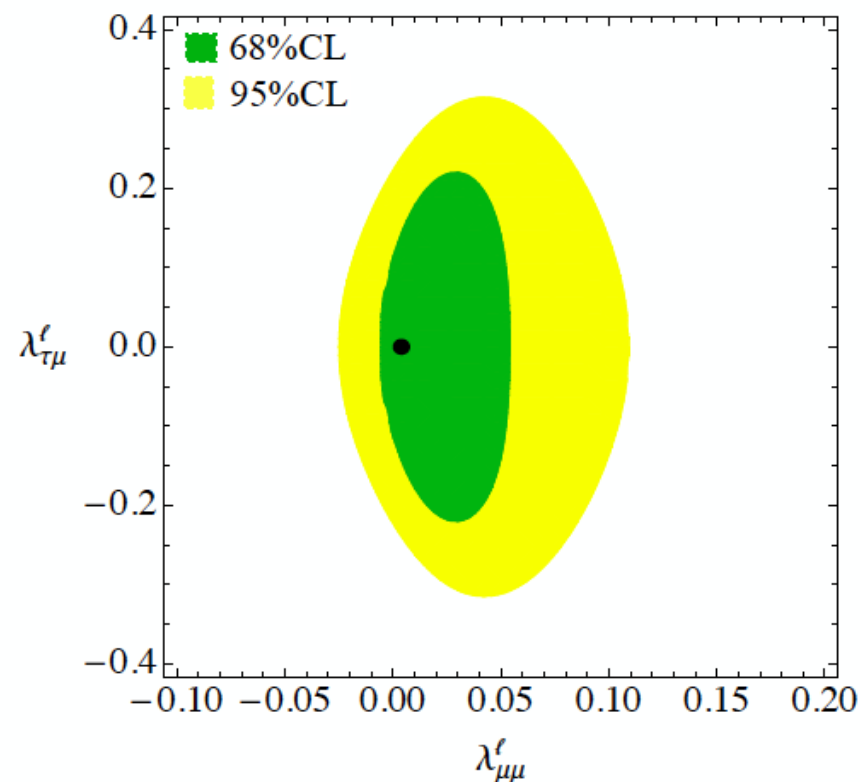
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$$\lambda_{\mu\mu} \lesssim 0.1$$

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► Future low-energy tests:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2m_V^2} J_\mu^a J_\mu^a \quad \text{works well...}$$

... and gives several clear predictions for future low-energy data:

- $b \rightarrow c \ell \nu$        $R_D^{\tau/\ell} = R_{D^*}^{\tau/\ell}$        $R_D^{\mu/e} \lesssim 10\% R_D^{\tau/\ell}$
- $b \rightarrow s \mu \mu$        $\Delta C_9^\mu = -\Delta C_{10}^\mu$       overall size of the anomaly should decrease
- $b \rightarrow s \tau \tau$        $|\text{NP}| \sim |\text{SM}| \rightarrow$  either large ( $\sim 4$ )  
enhancement or strong suppression
- **Meson mixing**       $\sim 10\%$  deviations from SM both in  $\Delta M_{B_s}$  &  $\Delta M_{B_d}$
- $\tau$  decays       $\tau \rightarrow 3\mu$  close to present exp. bound



► High-energy constraints:

The dynamical model

$$\mathcal{L}_V = -\frac{1}{4} D_{[\mu} V_{\nu]}^a D^{[\mu} V^{\nu]a} + \frac{m_V^2}{2} V_\mu^a V^{\mu a} + g_H V_\mu^a (H^\dagger T^a i \overleftrightarrow{D}_\mu H) + V_\mu^a J_\mu^a$$

The “heavy vector triplet” eff. Lagrangian [Pappadopulo, Tham, Torre, Wulzer, '14] in a rather peculiar parameter range:

- **W** and **Z** resonances in the mass range:

$$g_{l,q} \sim 1 \rightarrow m_V \sim 250 \text{ GeV}$$

$$g_{l,q} \sim \sqrt{4\pi} \rightarrow m_V \lesssim 1 \text{ TeV}$$

- Strong constraint on  $g_H$  from e.w. precision tests:

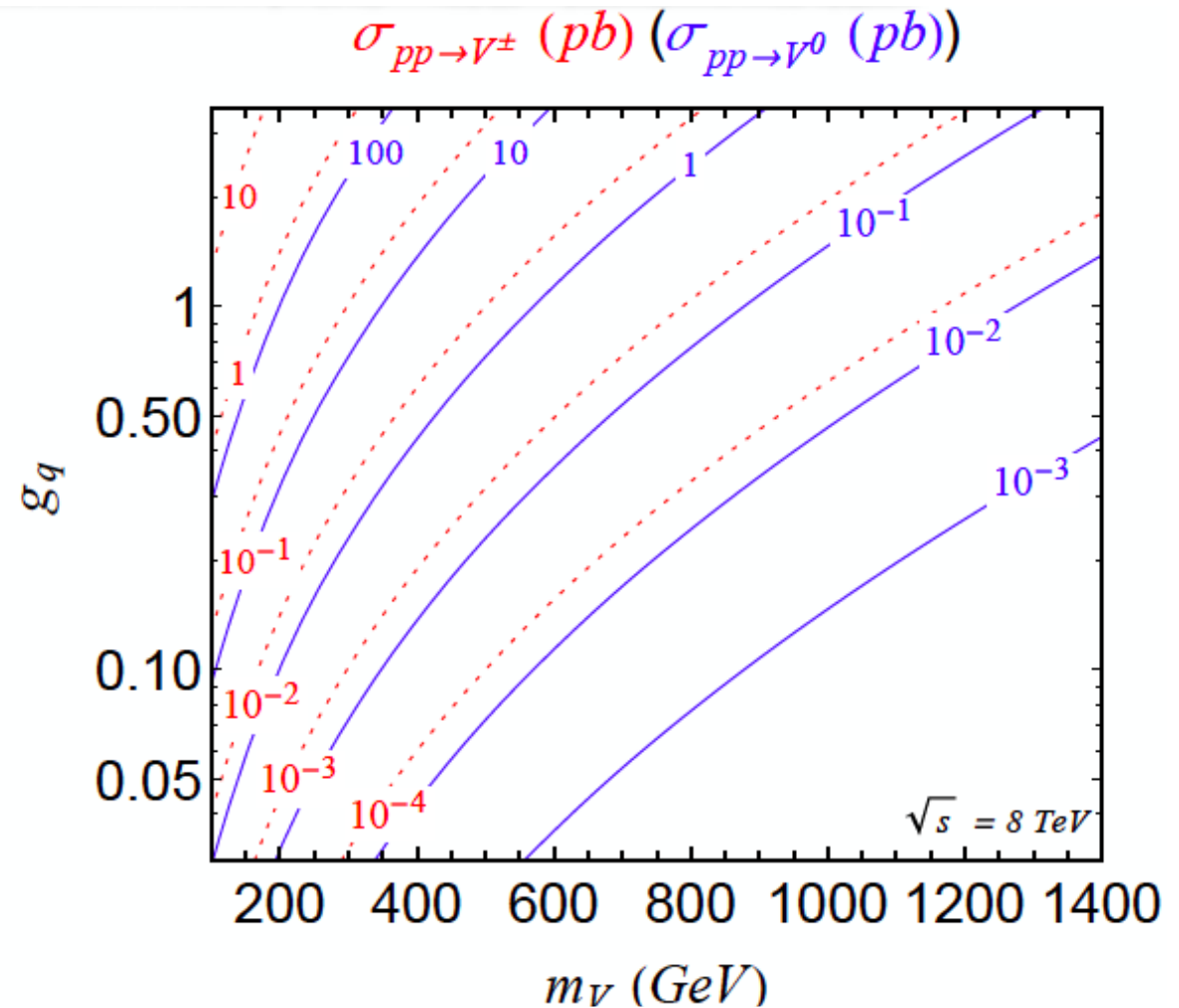
$$\epsilon_{l,q} = \frac{g_{l,q} m_W}{g m_V} \approx 0.3 \qquad \epsilon_H = \frac{g_H m_W}{g m_V} \lesssim 0.01$$

► High-energy constraints:

- The heavy vectors are produced mainly from 3<sup>rd</sup> gen. quarks ( $bb \rightarrow Z'$ ,  $bc \rightarrow W'$ ) and decay mainly in 3<sup>rd</sup> generations quarks or leptons ( $Z' \rightarrow \tau\tau, bb, tt$ ,  $W' \rightarrow tb, \tau\nu$ )



- Not a very easy signature...



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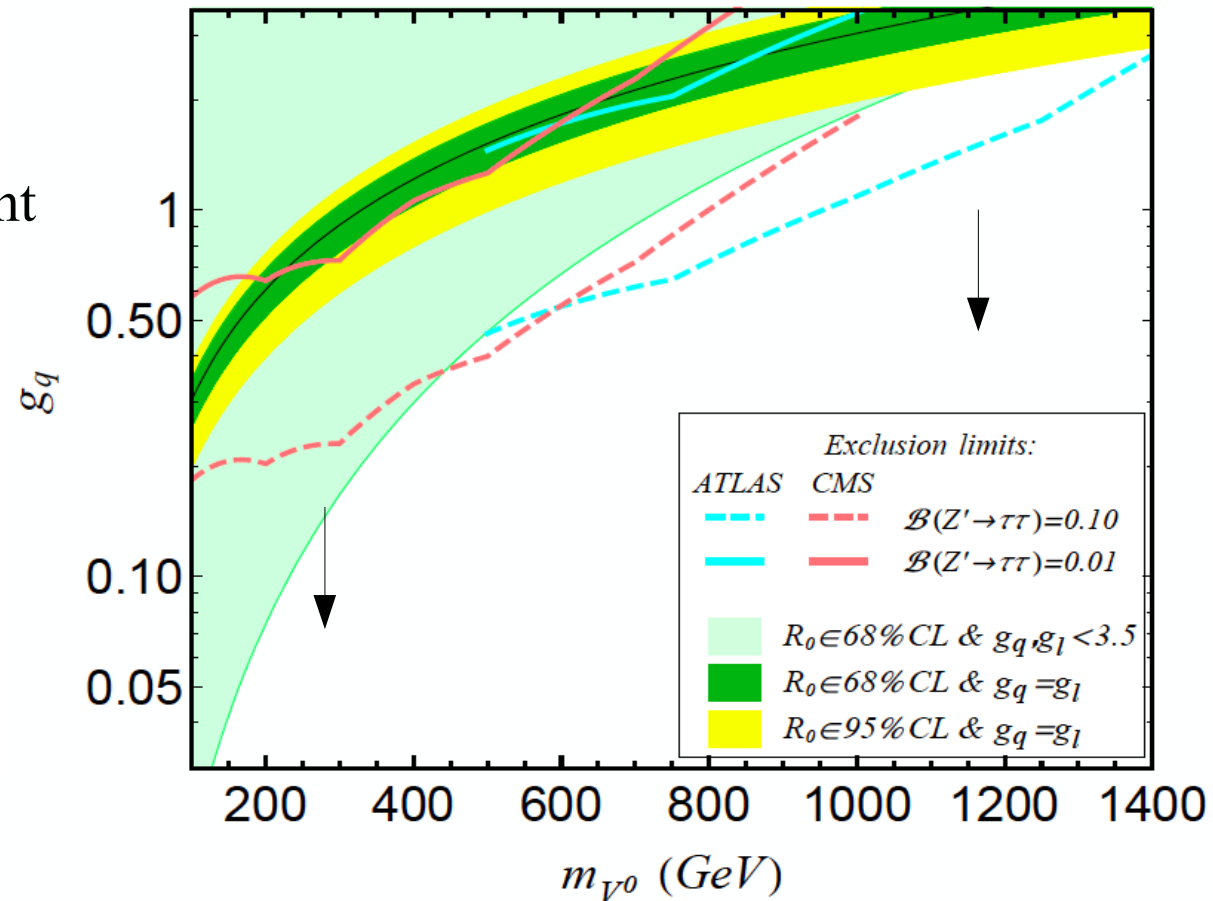
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The only really stringent constraint comes from  $Z' \rightarrow \tau\tau$



Minimal version of the model  
(no exotic decay channels)  
ruled out by direct searches

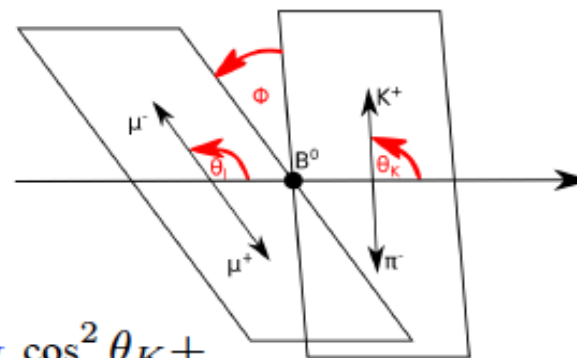
$$BR(Z' \rightarrow \bar{\tau}\tau) = \frac{g_\ell^2}{2g_\ell^2 + 6g_q^2 + \text{extra}}$$



## Conclusions

- Intriguing hints of LF non Universality in recent semi-leptonic B-physics data, but picture far from being clear → more data can help to clarify the situation
- Main messages of these recent anomalies:
  - (re)analyze B physics data without assuming LFU
  - conceive more low-energy tests of LFU (especially in B decays)
  - the search for LFV in charged leptons is extremely well motivated
  - the bounds on NP coupled mainly to 3<sup>rd</sup> generation are still relatively weak
  - the interplay of low- and high-energy searches is essential



Angular analysis of  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ 

$$\frac{d^4(\Gamma + \bar{\Gamma})}{d \cos \theta_\ell d \cos \theta_K d\phi dq^2} = \frac{9}{32\pi} \left[ \frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K + \right. \\ \left. \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_\ell - F_L \cos^2 \theta_K \cos 2\theta_\ell + \right. \\ S_3 \sin^2 \theta_K \sin^2 \theta_\ell \cos 2\phi + S_4 \sin 2\theta_K \sin 2\theta_\ell \cos \phi + \\ S_5 \sin 2\theta_K \sin \theta_\ell \cos \phi + S_6 \sin^2 \theta_K \cos \theta_\ell + \\ S_7 \sin 2\theta_K \sin \theta_\ell \sin \phi + \\ \left. S_8 \sin 2\theta_K \sin 2\theta_\ell \sin \phi + S_9 \sin^2 \theta_K \sin^2 \theta_\ell \sin 2\phi \right]$$

$$P'_{4,5} = \frac{S_{4,5}}{\sqrt{F_L(1-F_L)}}$$