Diffractive, Elastic, and Total pp Cross Sections at the LHC

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References

http://physics.rockefeller.edu/dino/my.html

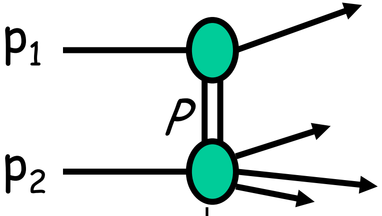
CDF	PRD 50, 5518 (1994)	σ^{el} @ 1800 & 546 GeV
CDF	PRD 50, 5535 (1994)	σ^{D} @ 1800 & 546 GeV
CDF	PRD 50, 5550 (1994)	σ^{T} @ 1800 & 546 GeV
KG-PR	Physics Reports 101, No.3 (1983) 169-219	
KG-95	PLB 358, 379 (1995)	Renormalization
	Erratum: PLB 363, 268 (1995)	
CMG-96	PLB 389, 176 (1996)	Global fit to $p^{\pm}p$, $\pi^{\pm}$, $K^{\pm}p$

Single diffraction

KG-95

Pomeron flux

s_o^{ε}



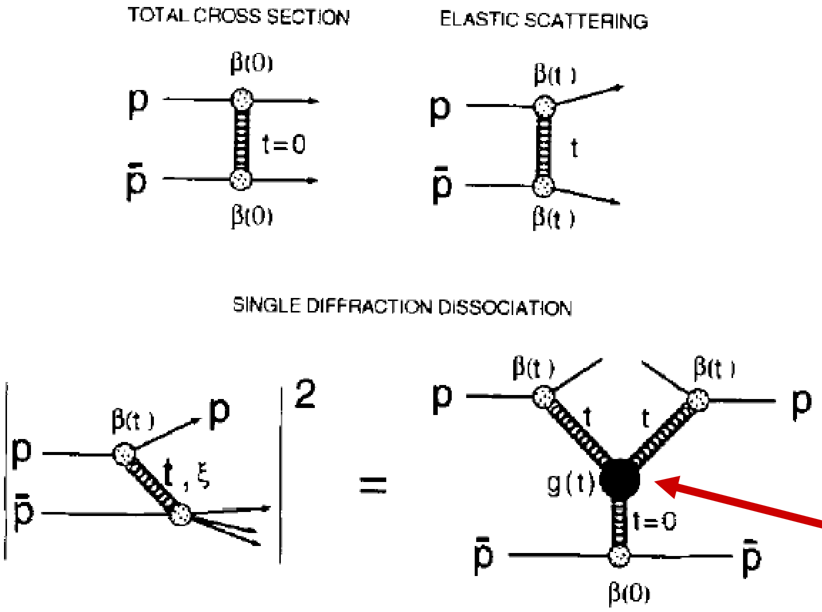
$$\frac{d^2\sigma_{SD}}{dt\,d\xi} = f_{P/p_1}(t,\xi)\cdot\sigma_{Pp}(s\xi)$$

$s_o^{-\varepsilon/2}\cdot g_{PPP}(t)$

Pomeron-proton x-section

- ☐ Two free parameters: s_o and g_{PPP}
- ☐ Obtain product $\boxed{g_{PPP}\bullet s_o^{\varepsilon/2}}$ from σ_{SD}
- ☐ Renormalized Pomeron flux determines s_o
- ☐ Get unique solution for g_{PPP}

Standard Regge theory



KG-95

$$\sigma_T = \beta_1(0)\beta_2(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1} = \sigma_0^{p\bar{p}}\left(\frac{s}{s_0}\right)^{\epsilon} \qquad (1)$$

$$\frac{d\sigma_{\rm el}}{dt} = \frac{\beta_1^2(t)\beta_2^2(t)}{16\pi}\left(\frac{s}{s_0}\right)^{2[\alpha(t)-1]}$$

$$= \frac{\sigma_T^2}{16\pi}\left(\frac{s}{s_0}\right)^{2\alpha' t} F^4(t) \approx \frac{\sigma_T^2}{16\pi} e^{b_{\rm el}(s)t} \qquad (2)$$

$$F^4(t) \approx e^{b_{0,el}t} \quad \Rightarrow \quad b_{\rm el}(s) = b_{0,el} + 2\alpha' \ln\left(\frac{s}{s_0}\right) \qquad (3)$$

$$\frac{d^2\sigma_{\rm sd}}{dt\,d\xi}$$

$$= \frac{\beta_1^2(t)}{16\pi}\,\xi^{1-2\alpha(t)}\left[\beta_2(0)\,g(t)\left(\frac{s'}{s'_0}\right)^{\alpha(0)-1}\right]$$

$$= f_{\mathcal{P}/p}(\xi,t)\;\sigma_T^{\mathcal{P}\bar{p}}(s',t) \qquad (4)$$

Parameters:
- ☐ s_o, s_o' and g(t)
- ☐ set $s_o' = s_o$ (universal) *P*
- ☐ g(t) ➔ g(0) ≡ g_{PPP} see KG-PR
- ☐ determine s_o and g_{PPP} – how?

The triple-Pomeron coupling in QCD

$$\frac{d^2\sigma}{dt\,d\Delta y} = N_{gap}\cdot\underbrace{C\cdot F_p^2(t)\cdot\left\{e^{(\varepsilon+\alpha' t)\Delta y}\right\}^2}_{P_{gap}(\Delta y,t)}\cdot\kappa\cdot\left\{\sigma_o\, e^{\varepsilon\,\Delta y'}\right\}$$

$$\Uparrow$$

$$\sigma_T = \sigma_o\cdot s^{\varepsilon}$$

Experimentally:

KG&JM, PRD 59 (114017) 1999

$$\boxed{\kappa = \frac{g_{PPP}}{\beta_{Pp}} = 0.17\pm 0.02}$$

Color factor:

$$\kappa = \mathrm{f}_g\times\frac{1}{N_c^2-1}+\mathrm{f}_q\times\frac{1}{N_c}\xrightarrow{Q^2=1}\approx 0.75\times\frac{1}{8}+0.25\times\frac{1}{3}=0.18$$

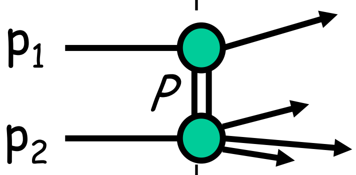
$\Uparrow$ gluon fraction

$\Uparrow$ quark fraction

σ^T_{sd} renormalized

KG-95

Pomeron flux

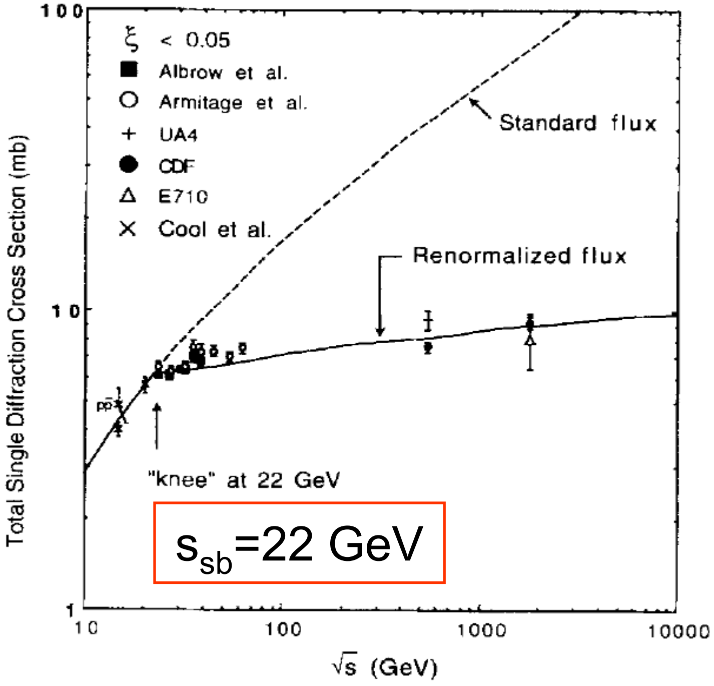


$$\frac{d^2\sigma_{SD}}{dt\ d\xi} = f_{P/p_1}(t,\ \xi)\cdot\sigma_{Pp}(s\ \xi)$$

s_0^{ε}

$s_0^{-\varepsilon/2}\cdot g_{PPP}(t)$

$$\int_{\xi_{min}}^{0.1}\int_{t=-\infty}^{0} f_{P/p}(t,\xi)\,d\xi\,dt \Rightarrow 1$$



s_{sb}=22 GeV

Renormalization

- ☐ $$\int_{\xi_{min}\approx 1/s}^{\zeta}\int_{t=-\infty}^{0} f_{P/p}(t,\xi)\,d\xi\,dt\ \approx C\cdot s^{2\varepsilon}\cdot s_0^{\varepsilon}\Rightarrow 1$$
- ☐ Flux integral depends on s and s_0
- ☐ "knee" √s-position determines the s_{sb} value where flux becomes unity ➔ get s_o
- ☐ $\delta s_o/s_o = -2\ \delta s/s = -4\ (\delta\sqrt{s})/\sqrt{s}$
- ☐ get error in s_o from error in √s-knee

Renormalization and Pumplin bound

$$\frac{d^2\sigma}{dt\,d\Delta y} = N_{gap} \cdot \underbrace{C \cdot F_p^2(t) \cdot \left\{e^{(\varepsilon+\alpha' t)\Delta y}\right\}^2}_{P_{gap}(\Delta y, t)} \cdot \kappa \cdot \left\{\sigma_o\, e^{\varepsilon\,\Delta y'}\right\}$$

$$N_{gap}^{-1}(s) = \int_{\Delta y, t} P_{gap}(\Delta y, t)\, d\Delta y\, dt \xrightarrow{s\to\infty} C' \cdot \frac{s^{2\varepsilon}}{\ln s}$$

$$\frac{d^2\sigma}{dt\,d\Delta y} = C'' \left[e^{\varepsilon(\Delta y - \ln s)} \cdot \ln s\right] e^{(b_0 + 2\alpha' \Delta y)t}$$

grows slower than s^{ε}

→ The Pumplin bound is obeyed at all impact parameters

σ^T_{sd} ($s\to\infty$) and ratio of α'/ε

arXiv:0812.4464v1[hep-ph] submitted to PLB

$$\frac{d^2\sigma(s,M^2,t)}{dM^2dt} = \left[\frac{\sigma_0^{pp}}{16\pi}\sigma_0^{\mathcal{P}p}\right]\frac{s^{2\epsilon}}{N(s)}\frac{1}{(M^2)^{1+\epsilon}}e^{bt} \xrightarrow{s\to\infty} \left[2\alpha'\, e^{\frac{\epsilon b_0}{\alpha'}}\sigma_0^{\mathcal{P}p}\right]\underbrace{\frac{\ln s^{2\epsilon}}{(M^2)^{1+\epsilon}}e^{bt}}_{b=b_0+2\alpha'\ln\frac{s}{M^2}}$$

$$\sigma_{sd} \xrightarrow{s\to\infty} \sigma_0^{\mathcal{P}p}\, e^{\frac{\epsilon}{2\alpha'}b_0}\, s^\epsilon\, \frac{\sum_{n=1}^{\infty}\frac{(\ln s^\epsilon)^n}{n\,n!}}{\sum_{n=1}^{\infty}\frac{(\ln s^{2\epsilon})^n}{n\,n!}} = 2\sigma_0^{\mathcal{P}p}\, e^{\frac{\epsilon}{2\alpha'}b_0} \;\Rightarrow\; \sigma_0^{pp}$$

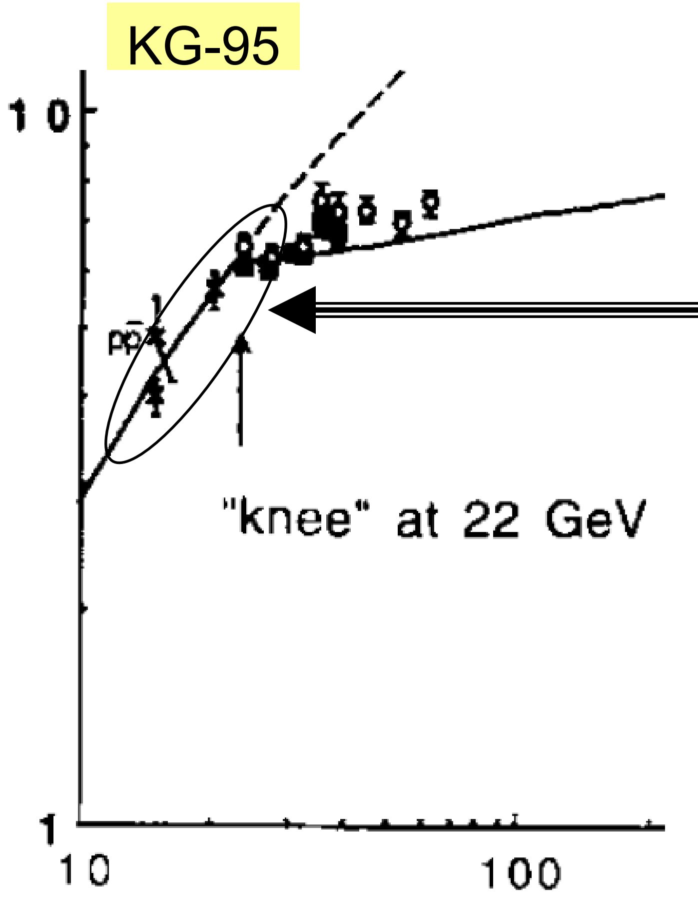
← Constant set to σ_o^{pp}

$$\sigma_0^{Pp} = \kappa\sigma_0^{pp}$$

$$\boxed{2\kappa\exp\left(\frac{\varepsilon b_o^{sd}}{2\alpha'}\right) = 1} \quad \boxed{b_o^{sd} = \frac{R_p^2}{2} = \frac{1}{2m_\pi^2}}$$

$$\alpha' = -\varepsilon\frac{1/4m_\pi^2}{4\ \ln(2\kappa)} = 0.25\ \mathrm{GeV}^{-2}\,(\text{using } \mathcal{E} = 0.08)\left[\Rightarrow \frac{\alpha'}{\varepsilon} = 3.14 \approx \pi\,!)\right]$$

The value of s_o - a bird's-eye view

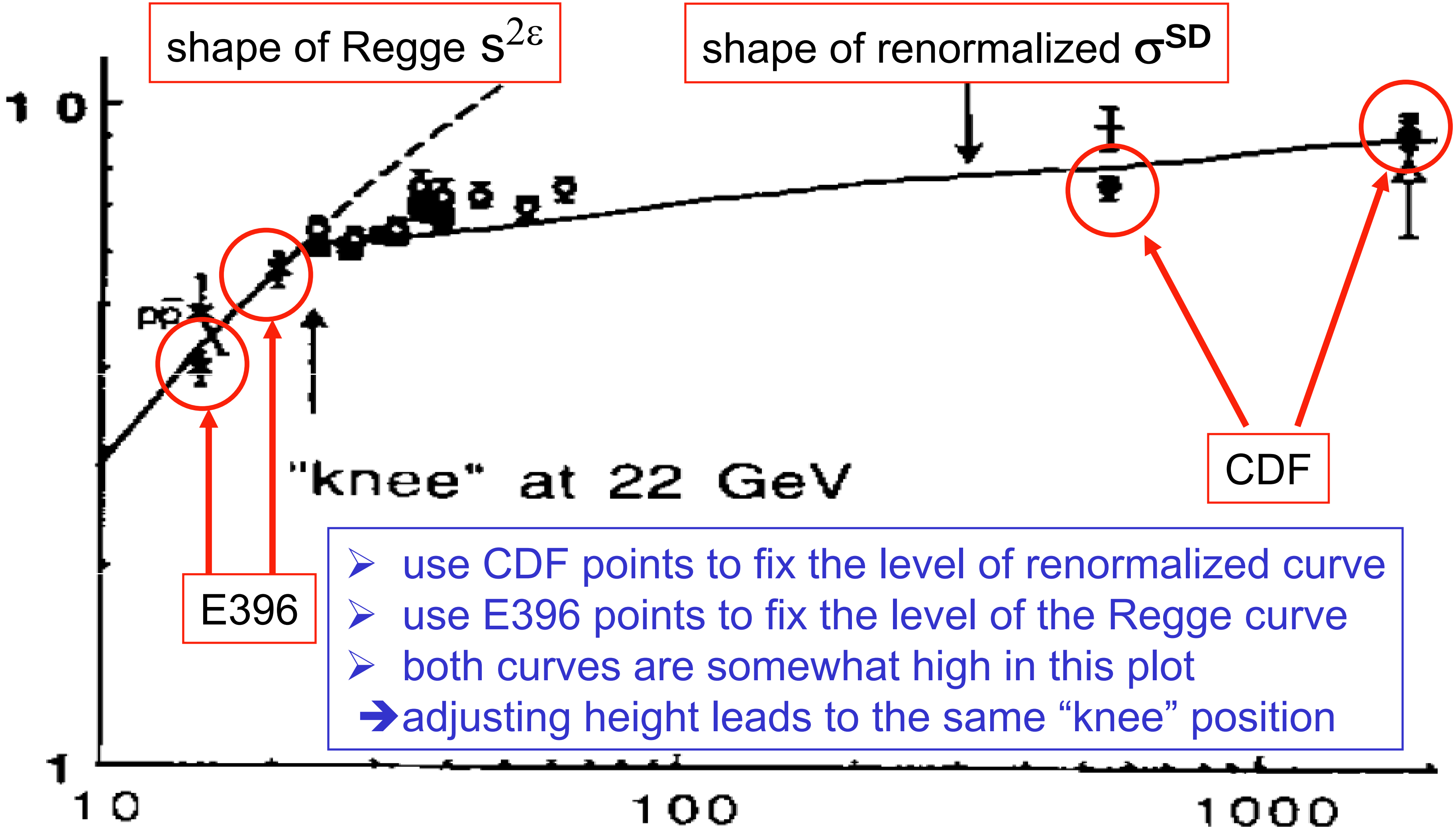


			√s (GeV)	
☒	Cool [KG]	FNAL	14	20
■	Albrow	ISR	23.3	26.9
o	Armitage	ISR	23.3	27.4

➔ error in knee position: ± 1 GeV

“knee” @ 22 ± 1 GeV ➔ $s_o = 1 \pm 0.2$ GeV

The value of s_o- limited edition



- ➢ use CDF points to fix the level of renormalized curve
- ➢ use E396 points to fix the level of the Regge curve
- ➢ both curves are somewhat high in this plot

➔adjusting height leads to the same "knee" position

The total cross section

- ☐ Froissart-Martin bound: $\sigma \leq \frac{\pi}{m^2} \cdot \ln^2 s$ (s in GeV^2)
- ☐ For $m^2 = m_\pi^2$ ➔ $\pi / m^2 \sim 10^4$ mb – very large!
- ☐ But if $m^2 = s_o = (\text{mass})^2$ of a large **SUPERglueBALL**, a $\ln^2 s$ behavior *ala* Froissart-Martin an be reached at a much lower s-value, s_{sb},

$$\Rightarrow \sigma(s > s_{sb}) = \sigma(s_{sb}) + \frac{\pi}{s_o} \cdot \ln^2 \frac{s}{s_{sb}}$$

- ☐ Determine s_{sb} and s_o from σ_T^{SD}
- ☐ Show that $\sqrt{s_{sb}} < 1.8$ TeV
- ☐ Show that at $\sqrt{s} = 1.8$ TeV Reggeon contributions are negligible
- ☐ Get cross section at the LHC from

$$\boxed{\sigma_{14000}^{LHC} = \sigma_{1800}^{CDF} + \frac{\pi}{s_0} \cdot \ln^2 \frac{s^{LHC}}{s^{CDF}}}$$

The SUPERBALL σ^T

- ❑ Froissart-Martin bound $\boxed{\sigma \leq \frac{\pi}{m^2} \cdot \ln^2 s}$
- ❑ Valid above "knee" at √s = 22 GeV and therefore at √s = 1.8 TeV
- ❑ Use superball mass

  ➔ $m^2 = s_o = (1 \pm 0.2)\ \text{GeV}^2$
- ❑ At √s 1.8 TeV Reggeon contributions are negligible (CMG-96))

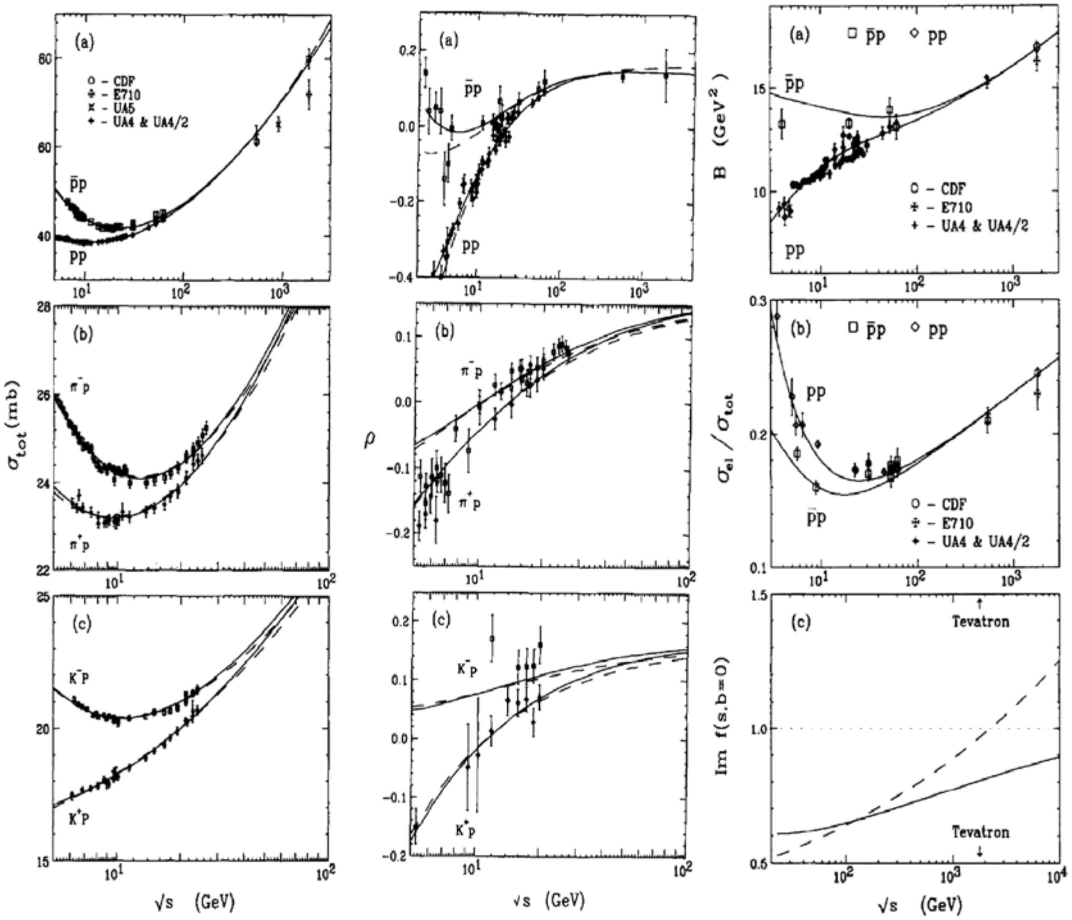
$$\sigma_{14000}^{LHC} = \sigma_{1800}^{CDF} + \frac{\pi}{s_o} \cdot \ln^2 \frac{s^{LHC}}{s^{CDF}} = (80.03 \pm 2.24) + (33 \pm 6) = 113 \pm 6\ \text{mb}$$

➔ compatible with CGM-96 global fit result of 114 ± 5 mb (see next 2 slides)

Global fit to p±p, π±, K±p x-sections

CMG-96 → A new determination of the soft pomeron intercept

R.J.M. Covolan [1], J. Montanha [2], K. Goulianos [3]



Use standard Regge theory

❑ INPUT

$$\alpha_{f/a} = 0.68 + 0.82\,t$$

$$\alpha_{\omega/\rho} = 0.46 + 0.92\,t$$

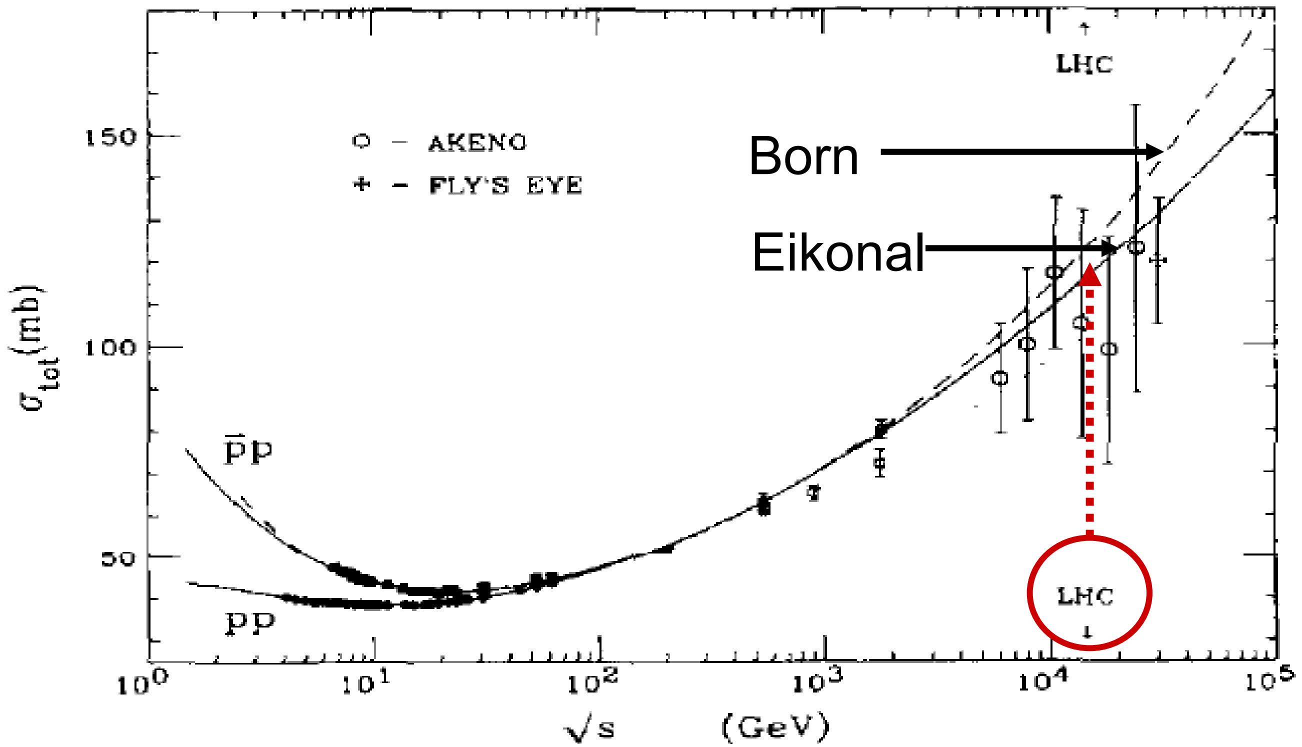
$$\alpha'_{\mathbb{P}} = 0.25\ \mathrm{GeV}^{-2}$$

❑ RESULTS

$$\alpha_{0,\mathbb{P}}^{\mathrm{Born}} = 1.104 \pm 0.002\,, \quad \alpha_{0,\mathbb{P}}^{\mathrm{Eik}} = 1.122 \pm 0.002$$

$$\sigma_{\mathrm{tot}}^{p^{\pm}p} = 16.79\, s^{0.104} + \underbrace{60.81\, s^{-0.32} \mp 31.68\, s^{-0.54}}_{\text{negligible}}$$

σ^T at LHC from global fit



- σ @ LHC √s=14 TeV: 122 ± 5 mb Born, 114 ± 5 mb eikonal

➔ error estimated from the error in ε given in CMG-96

Compare with **SUPERBALL** σ(14 TeV) = 113 ± 6 mb

caveat: s_o=1 GeV^2 was used in global fit!

The elastic cross section

- Optical theorem: obtain imaginary part of the amplitude from σ^T
- Dispersion relations: obtain real part of the amplitude from σ^T

σ_T

$\updownarrow$ **optical theorem**

Im $f_{el}(t=0)$

$\updownarrow$ **dispersion relations**

Re $f_{el}(t=0)$

- Add Coulomb amplitude
- Get differential elastic cross section and ρ-value

DISCUSSION

QUESTIONS?

thank you