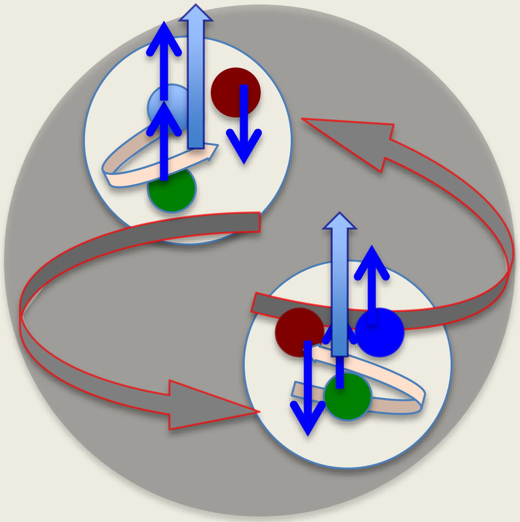
Exploring the transverse partonic structure of nucleons and nuclei at LHeC



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LHeC Workshop
Chavannes de Bogie, June 24-26, 2015

Outline

- ✓ GPDs and transverse spatial degrees of freedom
- ✓ Role of transverse spatial d.o.f. in nucleons and nuclei
- ✓ Role of partonic intrinsic transverse momentum/off-shellness in nucleons nuclei
- ✓ From transverse degrees of freedom to accessing Angular Momentum and Orbital Angular Momentum (including the Deuteron)

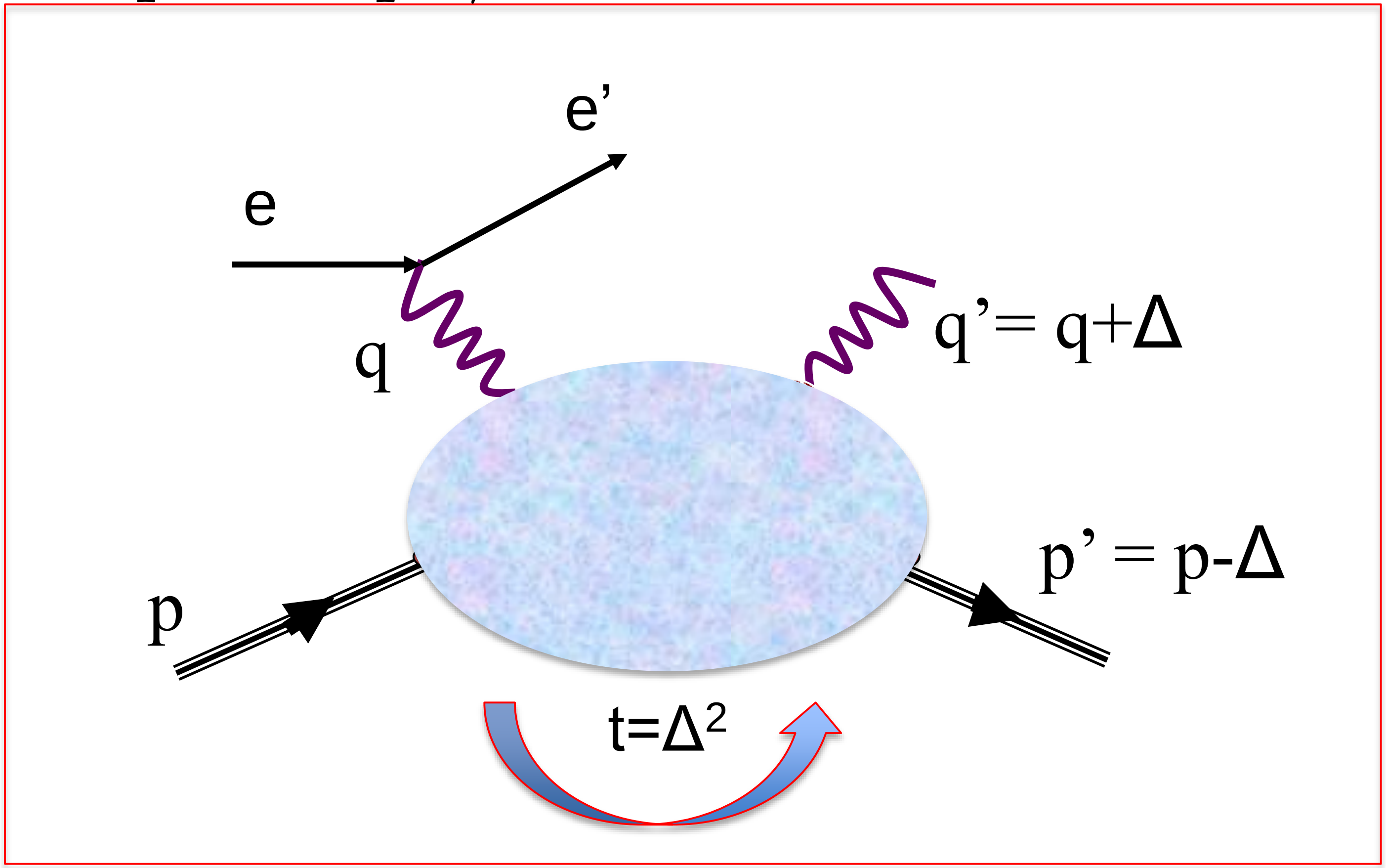
GPDs and Impact Parameter Space: where are the partons located?

M. Burkardt

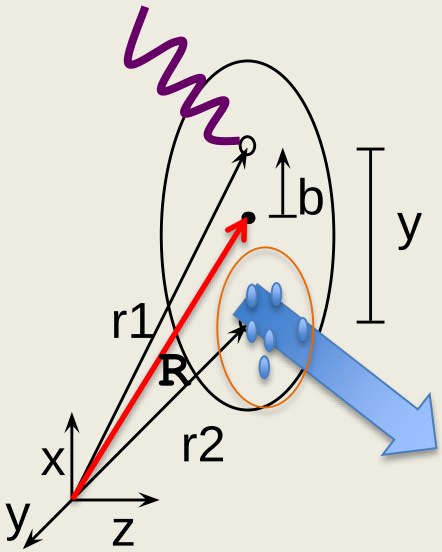
$$q_i(x, \mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{\Delta}} H_i(x, 0, -\mathbf{\Delta}^2)$$

Joint probability of finding a parton with LONG. momentum fraction x located at a TRANSV. Distance b from the proton's CoM (P^+)

DVCS➔ $ep \to e'p'\gamma$



Partonic transverse distances



In the parton model/IMF/LC the subgroup of the Poincarè group that leaves x^+=const. invariant is isomorphic to the Galileian group in the (transverse) plane

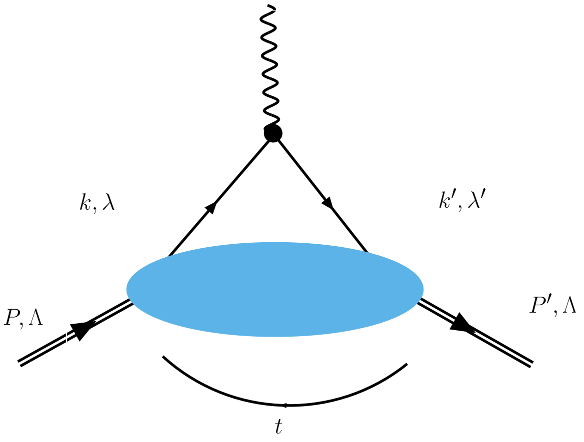
x^+ ➔ time
P^+ ,k_n^+ ➔ masses
P^+**R** ➔ boost generator in the plane

D. Soper (1977) M. Burkardt (2001)

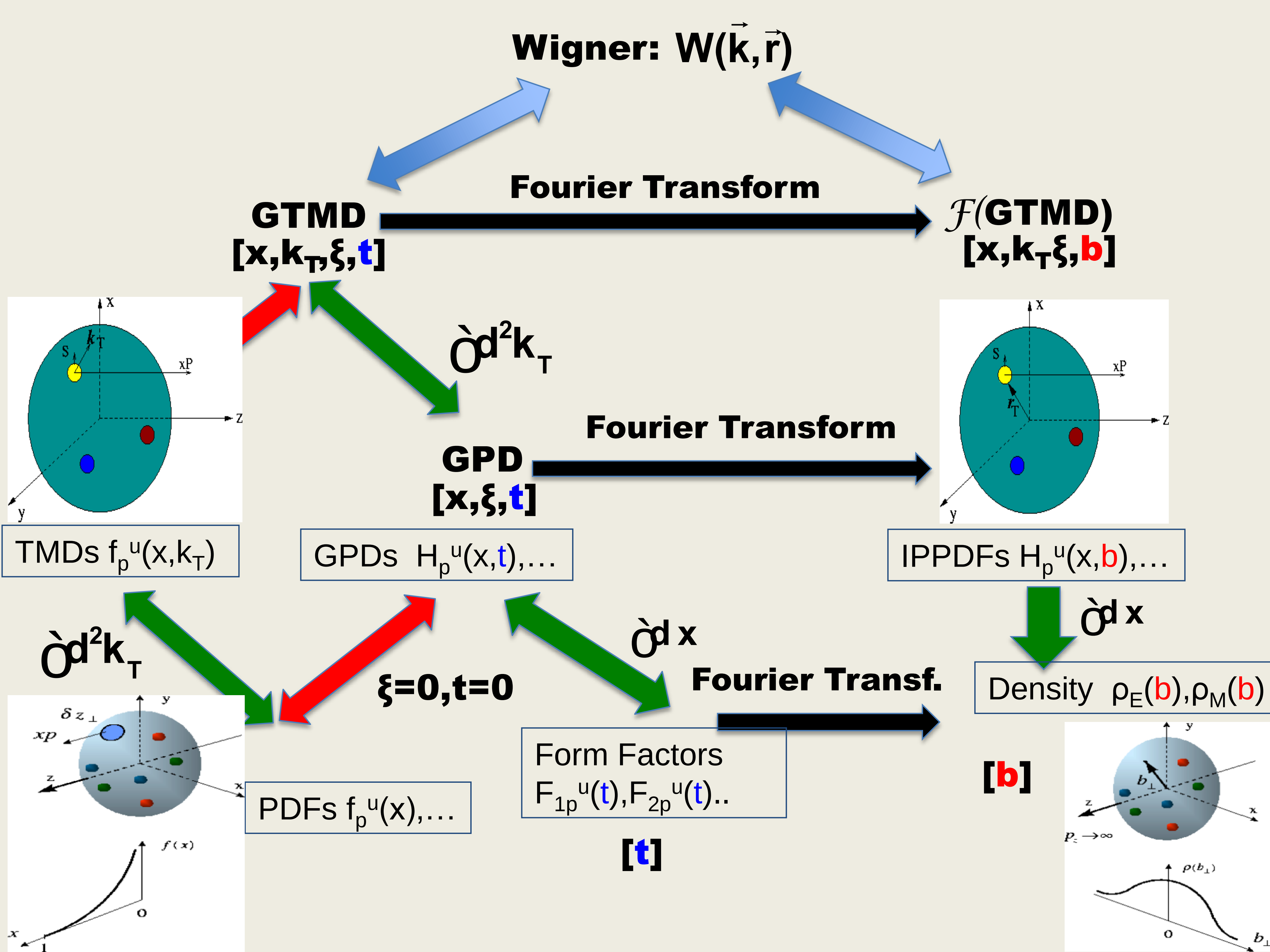
$$\mathbf{R} = \frac{1}{P^+} \sum k_n^+ \, \mathbf{r}_n$$

- struck parton $k_1^+/P^+=x$
- spectators $k_2^+/P^+=1-x$
- CoM position $\mathbf{R}=x\,\mathbf{r}_1+(1-x)\,\mathbf{r}_2$
- relative position $\mathbf{y}= \mathbf{r}_1-\mathbf{r}_2 = \mathbf{b}/(1-x)$

Because GPDs are formulated on the LC, they allow us to access physics in the transverse plane, or qualitatively different information than from the nucleon form factors (G. Miller)



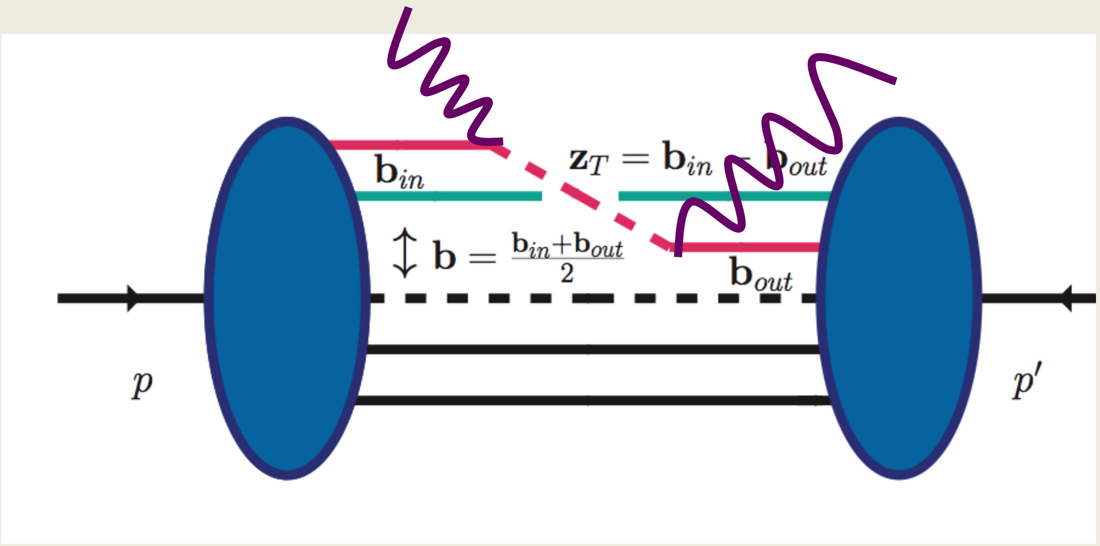
Deeply virtual exclusive experiments allow us to access spatial transverse degrees of freedom in both nucleons and in nuclei



Wigner: $W(\vec{k},\vec{r})$
Fourier Transform
GTMD
$[x,k_T,\xi,t]$
$\mathcal{F}$(GTMD)
$[x,k_T\xi,b]$
Òd^2k_T
Fourier Transform
GPD
$[x,\xi,t]$
TMDs $f_p^u(x,k_T)$
GPDs $H_p^u(x,t)$,…
IPPDFs $H_p^u(x,b)$,…
Òd x
Òd^2k_T
Òd x
$\xi=0,t=0$
Fourier Transf.
Density $\rho_E(b),\rho_M(b)$
PDFs $f_p^u(x)$,…
Form Factors
$F_{1p}^u(t),F_{2p}^u(t)$..
[b]
[t]

Wigner Distributions

$$\mathcal{W}^{\mathcal{U}}(x,\mathbf{k}_T,\mathbf{b}) = \int \frac{d^2\mathbf{\Delta}_T}{2\pi}\, e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} \left\{ \int d^2\mathbf{z_T} dz^- e^{ixP^+z^- - i\mathbf{k}_T\cdot\mathbf{z}_T} \langle p',\Lambda' \mid \bar{\psi}(0)\gamma^+\mathcal{U}(0,z)\psi(z) \mid p,\Lambda\rangle \,|_{z^+=0} \right\}$$



Deeply virtual exclusive experiments allow us to explore the partonic structure of nucleons and nuclei at the amplitude level

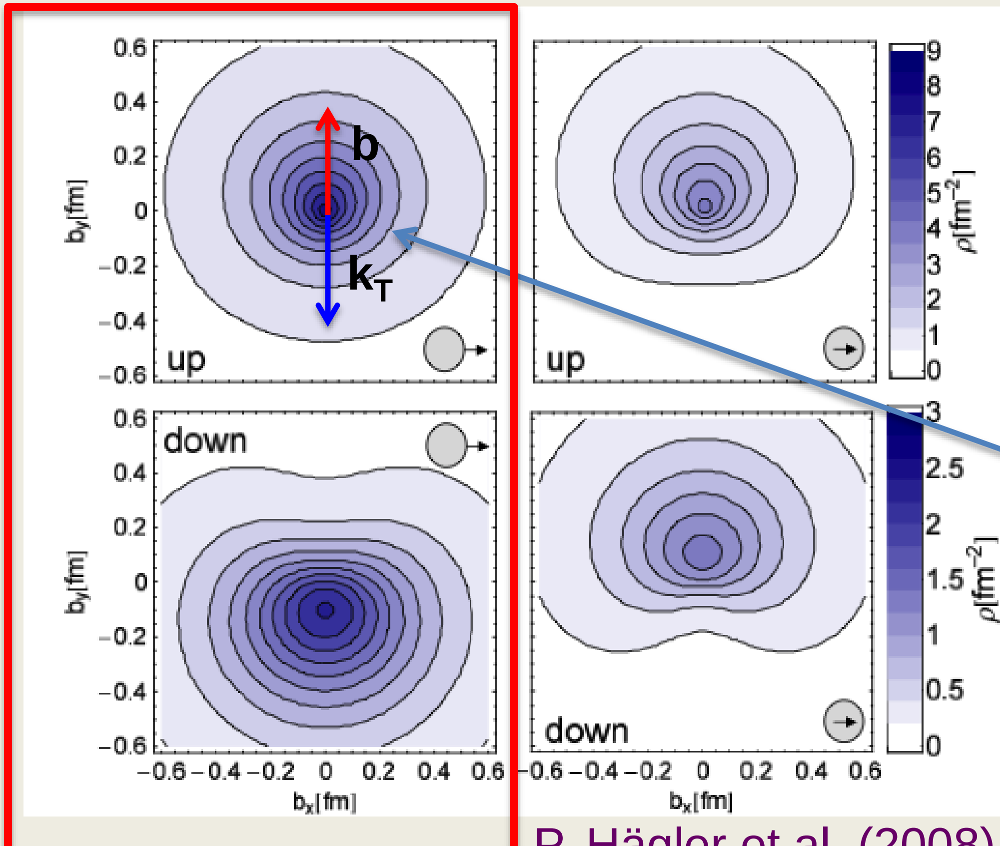
GPD E: connecting b and k_T

$$W^{\gamma^+}_{\Lambda,\Lambda'}(X,\zeta,t) = \frac{1}{2\overline{P}^+}\left[\overline{U}(P',\Lambda')\left(\gamma^+ H(X,\zeta,t) + \frac{i\sigma^{+\mu}(-\Delta_\mu)}{2M}E(X,\zeta,t)\right)U(P,\Lambda)\right],$$

spin independent

spin flip

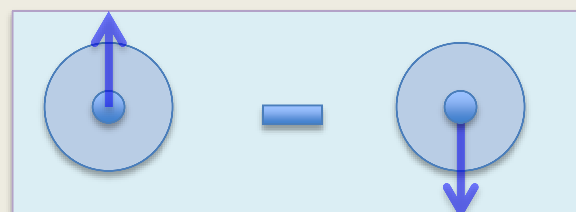
GPD E: connecting b and k_T



P. Hägler et al. (2008)

E $\rightarrow$ $s^{+j}\Delta_j \Rightarrow \vec{S}_T \times \vec{\Delta}$

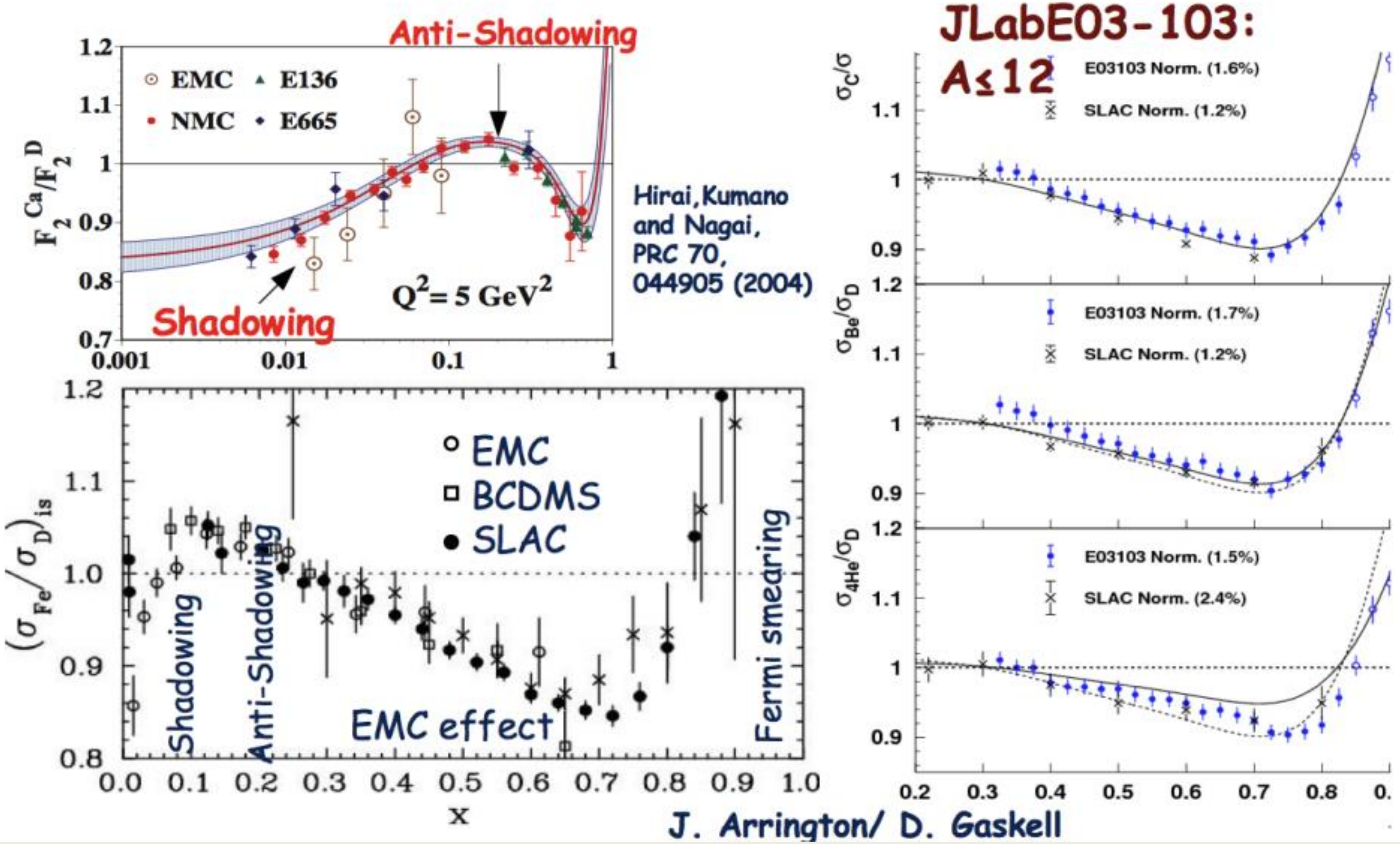
The net b corresponds to net k_T in the opposite direction (attractive color force due to FSI) M. Burkardt



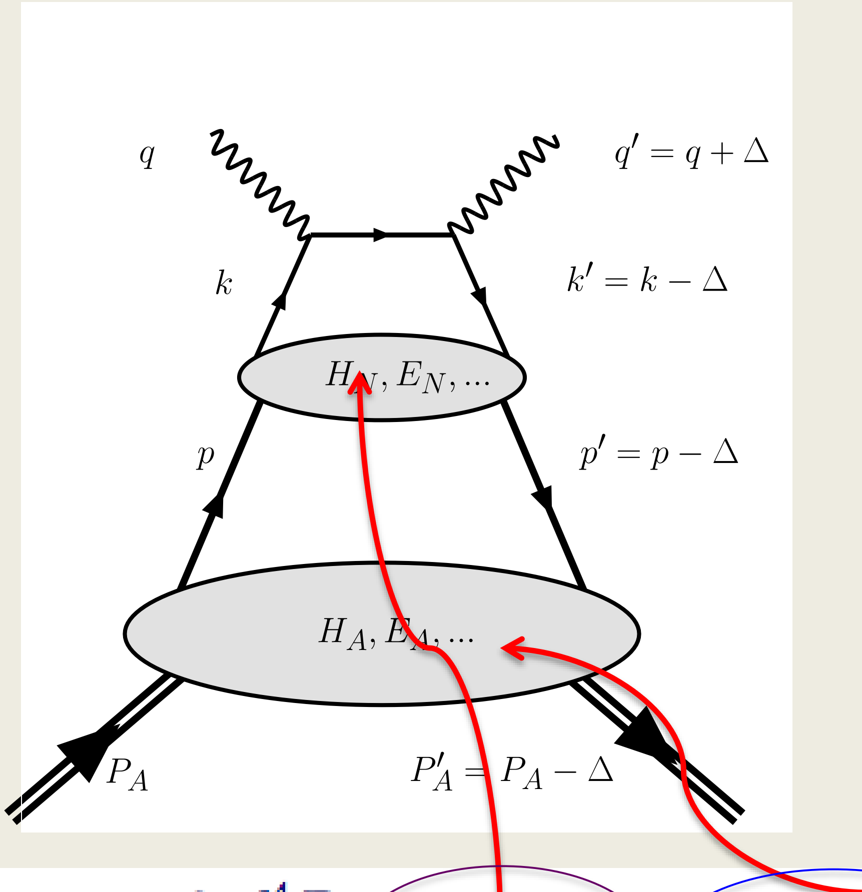
In terms of quark-proton helicity amplitudes

$$H - i\Delta_2 E = \left(A^X_{++,++} + A^X_{+-,+-} + A^X_{-+,-+} + A^X_{--,--}\right) + \left(A^X_{++,++} + A^X_{+-,+-} - A^X_{-+,-+} - A^X_{--,--}\right)$$

Over 30 yrs of experiments



DVCS in Nuclei: where is the EMC effect located?



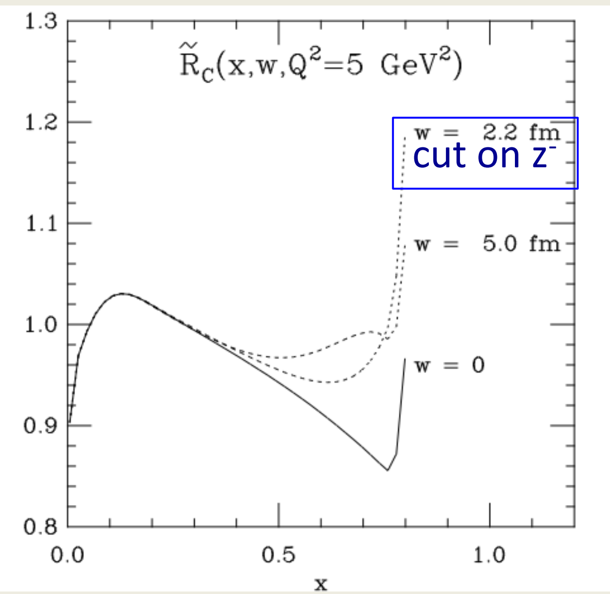
$$T^A_{\mu\nu}(P_A, \Delta) = \int \frac{d^4P}{(2\pi)^4} T^N_{\mu\nu}(k, P, \Delta)\, \mathcal{M}^A(P, P_A, \Delta),$$

$$\mathcal{M}^A_{ij}(P, P_A, \Delta) = \int d^4y\, e^{iP\cdot y} \langle P'_A | \overline{\Psi}_{A,j}(-y/2) \Psi_{A,i}(y/2) | P_A \rangle.$$

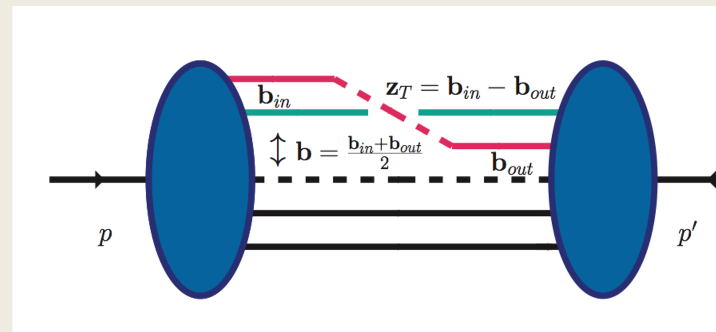
Fourier transform of $T_{\mu\nu}^{A}$

$$\mathcal{W}^{\mathcal{U}}(x,\mathbf{k}_T,\mathbf{b}) = \int \frac{d^2\mathbf{\Delta}_T}{2\pi}\, e^{i\mathbf{\Delta}_T\cdot\mathbf{b}} \int d^2\mathbf{z_T}\, dz^- e^{ixP^+z^- - i\mathbf{k}_T\cdot\mathbf{z}_T} \Big\{ \langle p',\Lambda' \mid \bar{\psi}(0)\gamma^+\mathcal{U}(0,z)\psi(z) \mid p,\Lambda\rangle \Big\}_{z^+=0},$$

Hoyer and Vanttinen (1996)

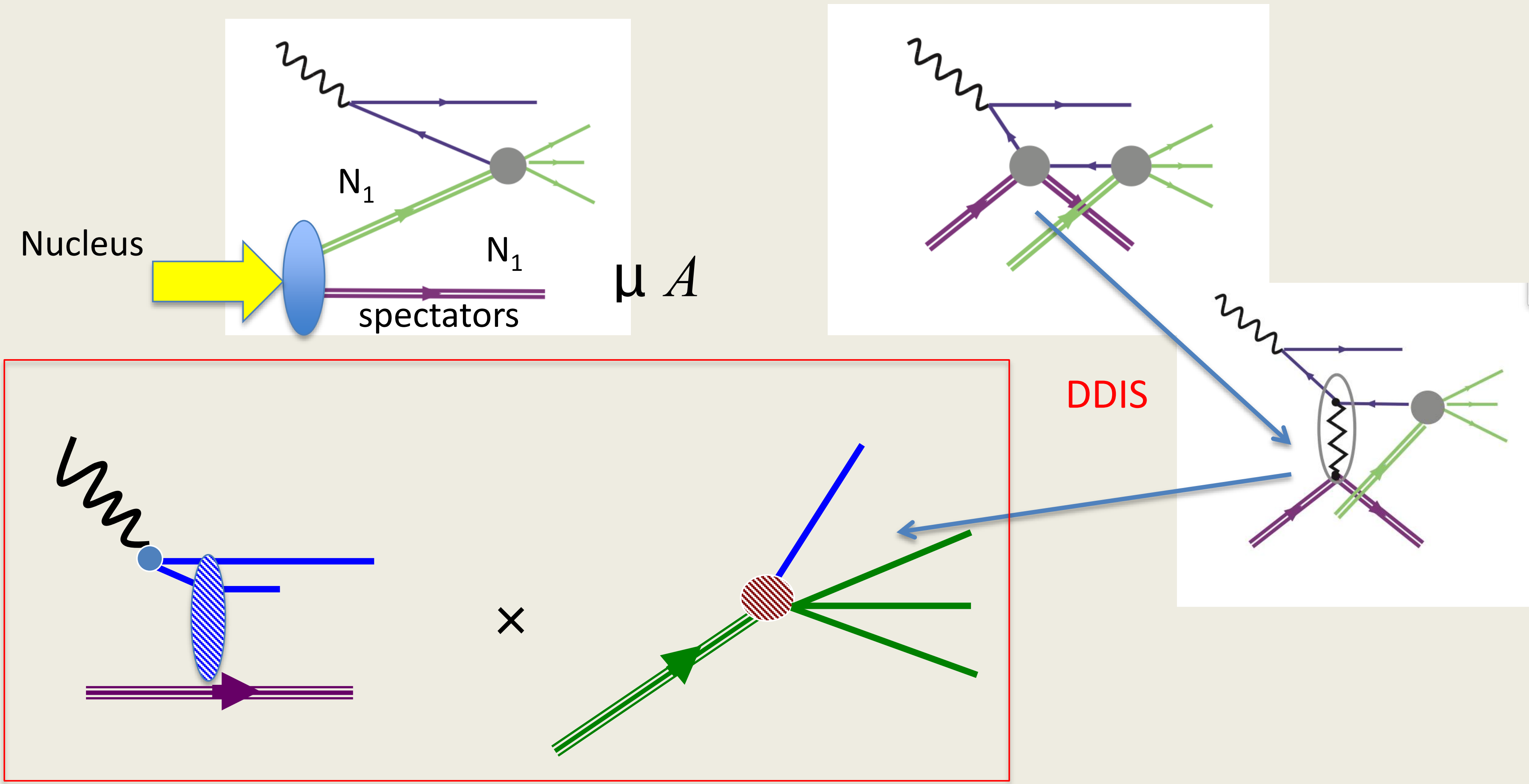


shadowing/antishadowing phenomena + some of the EMC effect are governed by large LC distances:
$z^- \approx 1/2M_N x$



Large coherence length allows for reinteraction between different nucleons(FSI)

Brodsky, Hoyer, et al, PRD65 (2002), Brodsky, Schmidt, Yang, PRD (2004), Stan Brodsky's talk



Interference between ISI/FSI (DDIS) and tree level (DIS) diagrams generates the imaginary (Coulomb-like) phase

Are the nuclear parton distributions density distributions?

What are the consequences on the baryon number, momentum, angular momentum sum rules in nucleons/nuclei?

Need to define the Wilson lines in nuclei

$$W_{\Lambda,\Lambda}^{\gamma^+,OFF} = \int \frac{dz^-}{2\pi} \frac{d^2\mathbf{z}_T}{(2\pi)^2} \exp i \left[\frac{x}{y} P^+ z^- - \left(\mathbf{k}_T - \frac{x}{y} \mathbf{P}_T \right) \cdot \mathbf{z}_T \right] \langle P, \Lambda \mid \overline{\psi}(0) \mathcal{U}(0,\infty) \gamma^+ \mathcal{U}(\infty, z) \, \psi(z) \mid P, \Lambda$$

(study of sum rules, S. Brodsky and S.L. in progress)

From M^N to M^A (Spin 0)

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$$\mathcal{M}_{ij}^{A} = \overline{U}_{A-1}(P'_A, S)\overline{\Gamma}_A(P', P'_A)\,\frac{(\not{P}' + M)}{P'^2 - M^2}\,\frac{(\not{P} + M)}{P^2 - M^2}\Gamma_A(P, P_A)\,U_{A-1}(P_A, S)$$

$U_{A-1} \rightarrow$ spectator $A-1$ nucleons with mass M^*_{A-1}

$\Gamma_A \rightarrow$ nuclear vertex function

$$\mathcal{M}_{ij}^{A} = \mathcal{N}_A \left(\sum_S U_i(P, S)\overline{U}_j(P', S) \right) \rho_A(P^2, P'^2)$$

Off-forward nuclear LC distribution

$$\begin{aligned} \rho_A(P^2, P'^2) &\approx S_A(|\,\mathbf{P}\,|, |\,\mathbf{P}'\,|, E) \\ &= \sum_f \Phi_f(|\,\mathbf{P}\,|)\Phi_f^*(|\,\mathbf{P}'\,|)\delta\left(E - (E_{A-1}^f - E_A)\right) \end{aligned}$$

S.L. and Taneja (2005)

Spin 0

Off-forward joint LC and P_T momentum distribution

$$
\begin{aligned}
H^A(X,\zeta,t) &= \int \frac{d^2P_\perp dY}{2(2\pi)^3}\mathcal{A}\rho_A(Y,P_\perp^2,\zeta,t) \\
&\times \; c_1(\zeta,t)\left[H^N\left(\frac{X}{Y},\frac{\zeta}{Y},P^2,t\right) - \frac{1}{4}\frac{(\zeta/Y)^2}{1-\zeta/Y}E^N\left(\frac{X}{Y},\frac{\zeta}{Y},P^2,t\right)\right] \\
&+ \; c_2(\zeta,t)\sqrt{\frac{t-t_o}{2M}}E^N\left(\frac{X}{Y},\frac{\zeta}{Y},P^2,t\right)
\end{aligned}
$$

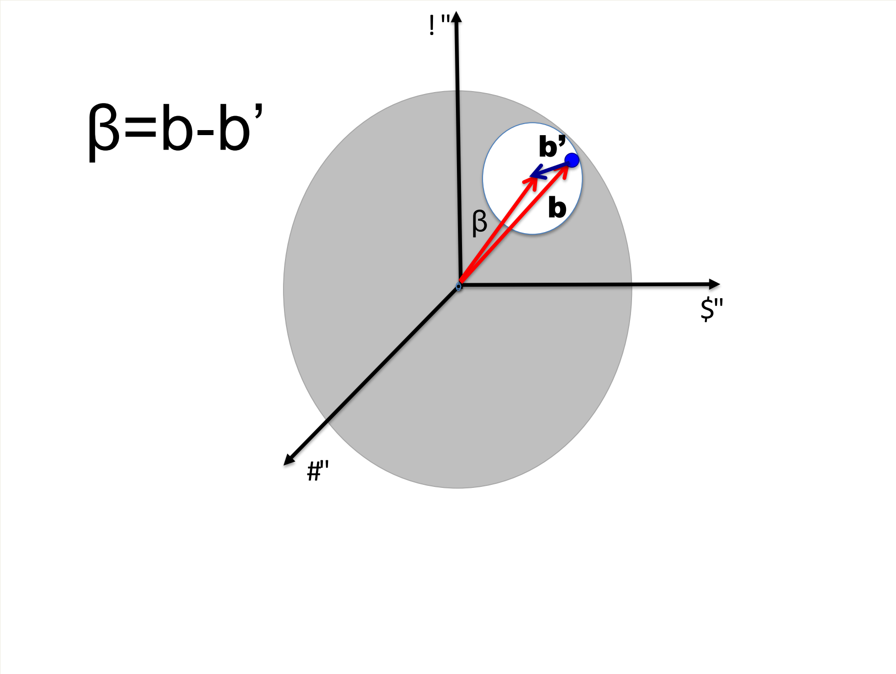
Off-shell nucleon GPDs

In impact parameter space

$$q_N(x,\mathbf{b}) = \int \frac{d^2\Delta}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\boldsymbol{\Delta}}\, H_N(x,0,\Delta^2) \qquad q_N(x) = \int d^2b\, q_N(x,b)$$

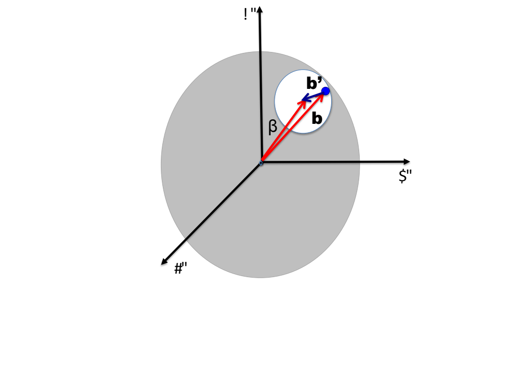
$$q_A(x,\mathbf{b}) = \int \frac{d^2\Delta}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\boldsymbol{\Delta}}\, H_A(x,0,\Delta^2) \qquad q_A(x) = \int d^2b\, q_A(x,b)$$

$$\widetilde{\rho}_A(y,\beta) = \int \frac{d^2\Delta}{(2\pi)^2}\, e^{i\beta\cdot\boldsymbol{\Delta}}\, \rho_A(y,0,\Delta^2) \qquad f_A(y) = \int d^2\beta\, \widetilde{\rho}_A(y,\beta)$$



Where are quarks located inside a nucleus?

$$\langle b_A^2 \rangle = \frac{\int d^2b\, q_A(x,b)\, b^2}{\int d^2b\, q_A(x,b)}$$



Additive relation

In medium nucleon size

"pointlike nucleon radius

$$\langle b_A^2 \rangle = \frac{1}{q_A(x)} \int_x^A dz \left[\langle b_N^2(x/z) \rangle^A + \langle \beta^2(z) \rangle \right] q_N(x/z) f_A(z)$$

$$\langle b_N^2(x) \rangle^A = \frac{1}{q_A(x)} \int_x^A dz\, f_A(z) \int d^2b\, b^2\, q_N(x/z, b)$$

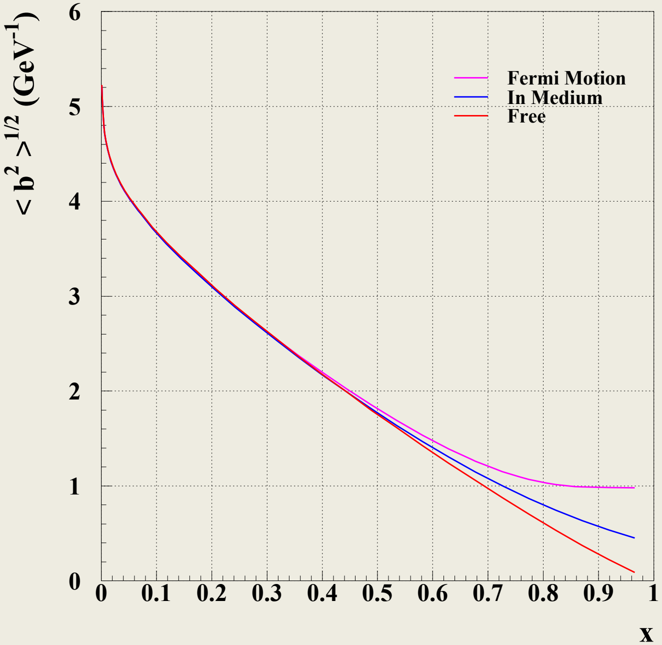
In simple convolution formula the in medium nucleon and free nucleon sizes are the same:

$$\int dx \langle b_N^2(x) \rangle^A \equiv \int \langle b_N^2(x) \rangle^N$$

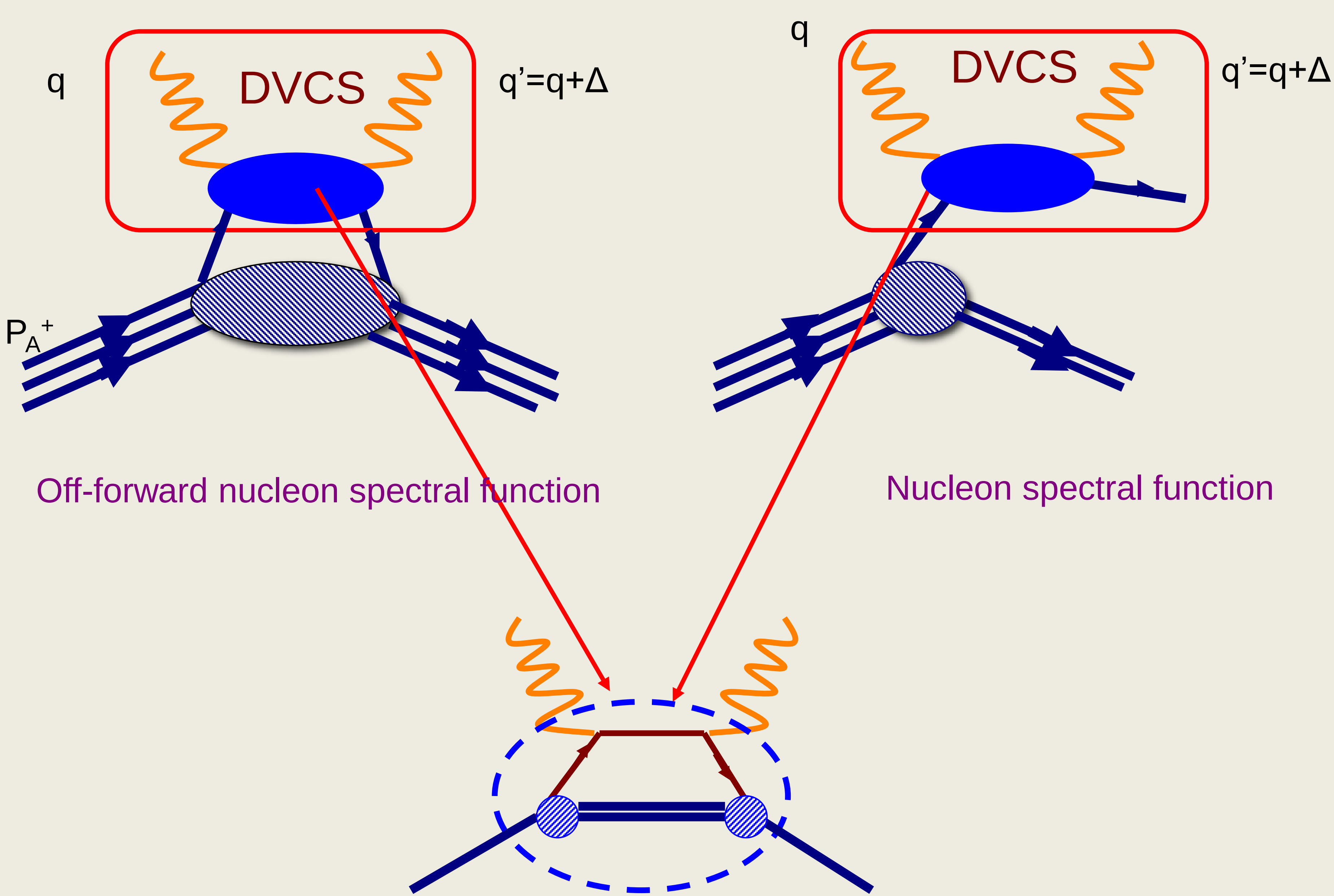
Off-shell effects/parton reinteractions will modify this relation

$$H_A(x,\Delta) = \int_x^A dz\, \rho_A(z,\Delta) H_N(x/z,\Delta) = \int_x^A dz \int d^2b \left[\int d^2b'\, \widetilde{\rho}_A(z, \mid \mathbf{b} - \mathbf{b}' \mid)\, q_N(x/z, b') \right] e^{i\mathbf{b}\cdot\mathbf{\Delta}}$$

$$\rho_A(z,\Delta) = 2\pi M \int dE \int_{P_{min}(z,E)} dP\, P \Phi(P) \Phi^*(\mid \mathbf{P} + \mathbf{\Delta} \mid)$$



Observables: Coherent vs. Incoherent processes (S.L. and S.K. Taneja, 2006)



4He: Spin 0 (Belitsky-Muller-Kirchner)

Bethe-Heitler

$$c_0^{BH} = \left[\left\{ (2-y)^2 + y^2(1+\epsilon^2)^2 \right\} \left\{ \frac{\epsilon^2 Q^2}{t} + 4(1-x_A) + (4x_A + \epsilon^2)\frac{t}{Q^2} \right\} \right.$$
$$+2\epsilon^2 \left\{ 4(1-y)(3+2\epsilon^2) + y^2(2-\epsilon^4) \right\} - 4x_A^2(2-y)^2(2+\epsilon^2)\frac{t}{Q^2}$$
$$\left. +8K^2 \frac{\epsilon^2 Q^2}{t} \right] F_A^2, \qquad (24)$$

$$c_1^{BH} = -8(2-y)K \left\{ 2x_A + \epsilon^2 - \frac{\epsilon^2 Q^2}{t} \right\} F_A^2, \qquad (25)$$

$$c_2^{BH} = 8K^2 \frac{\epsilon^2 Q^2}{t} F_A^2, \qquad (26)$$

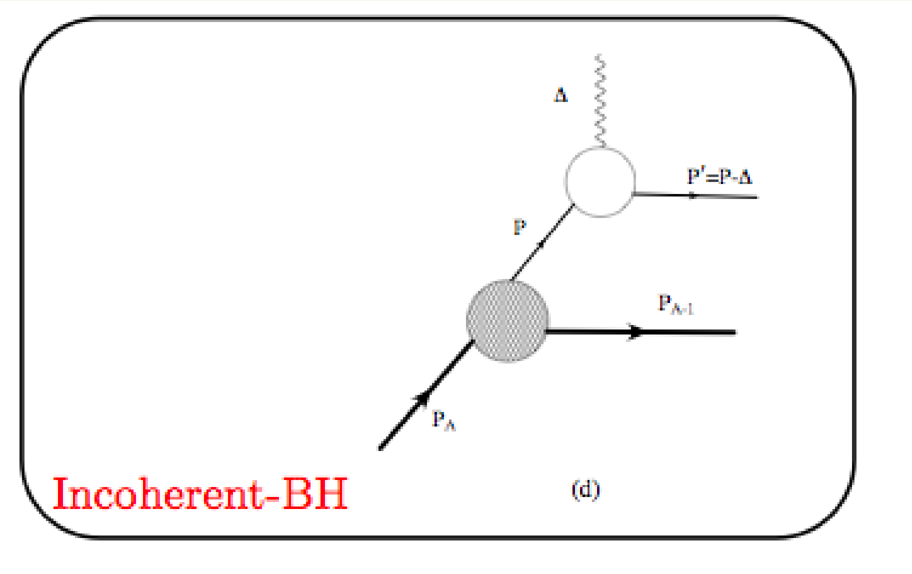
DVCS

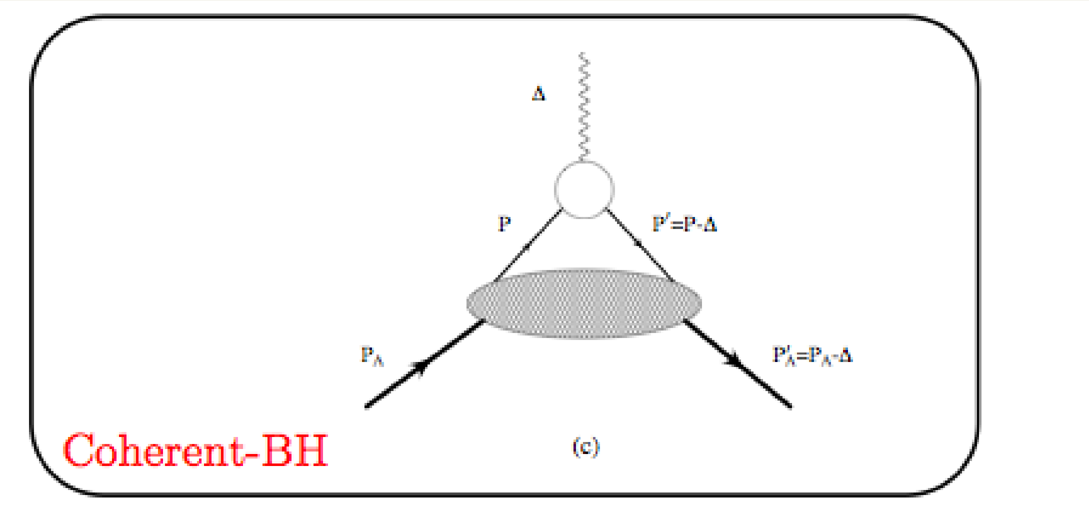
$$c_0^{DVCS} = 2(2-2y+y^2)\mathcal{H}_A \mathcal{H}_A^*,$$

Interference

$$c_0^{\mathcal{I}} = -8(2-y)\frac{t}{Q^2} F_A \Re\mathrm{e}\{\mathcal{H}_A\}$$
$$\times \left\{ (2-x_A)(1-y) - (1-x_A)(2-y)^2 \left(1 - \frac{t_{min}}{Q^2} \right) \right\},$$
$$c_1^{\mathcal{I}} = 8K(2y - y^2 - 2) F_A \Re\mathrm{e}\{\mathcal{H}_A\},$$
$$s_1^{\mathcal{I}} = 8Ky(2-y) F_A \Im\mathrm{m}\{\mathcal{H}_A\}.$$

Coherent/Incoherent contributions also in Bethe Heitler processes from nuclei





⇒ **Interference Term for Coeherent DVCS & BH**

$$\mathcal{I}_{coh}(\zeta,t) = \mathcal{K}\, H^A(\zeta,t) \times Z^2 F^A(t)$$

Non-forward spectral function

$$H^A(\zeta,t) = \int \frac{d^2P_\perp dY}{2(2\pi)^3} \mathcal{N}\, \rho^A(Y,P^2;\zeta,t) H^N\left(\frac{\zeta}{Y},\frac{\zeta}{Y},t;P^2;\right)$$

↑ **off-forward EMC-effect** ↑

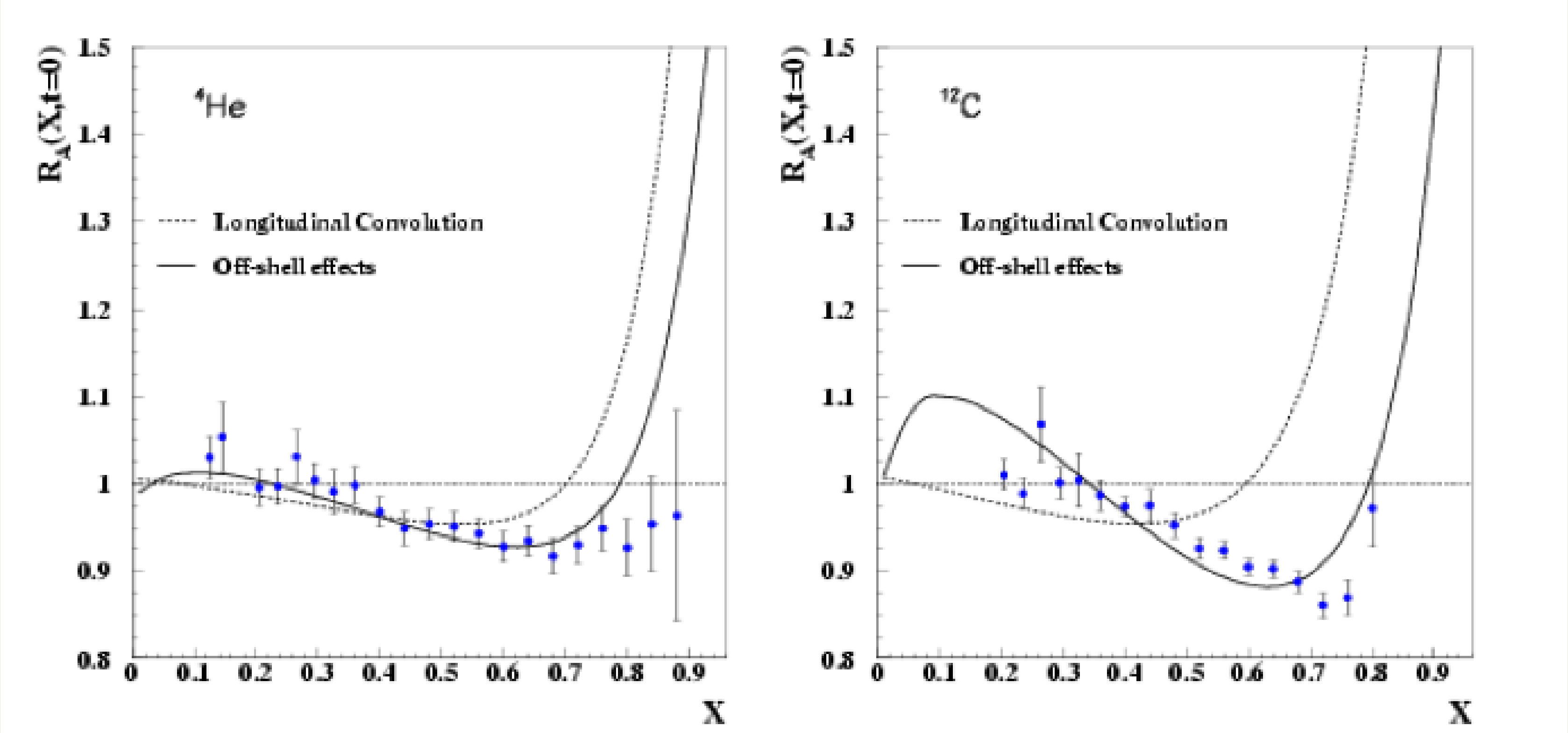
⇒ **Interference Term for Incoeherent DVCS & BH**

$$\mathcal{I}_{inc}(\zeta,t) = \mathcal{K} H_0^A(\zeta,t) \times Z F_1^N(t)$$

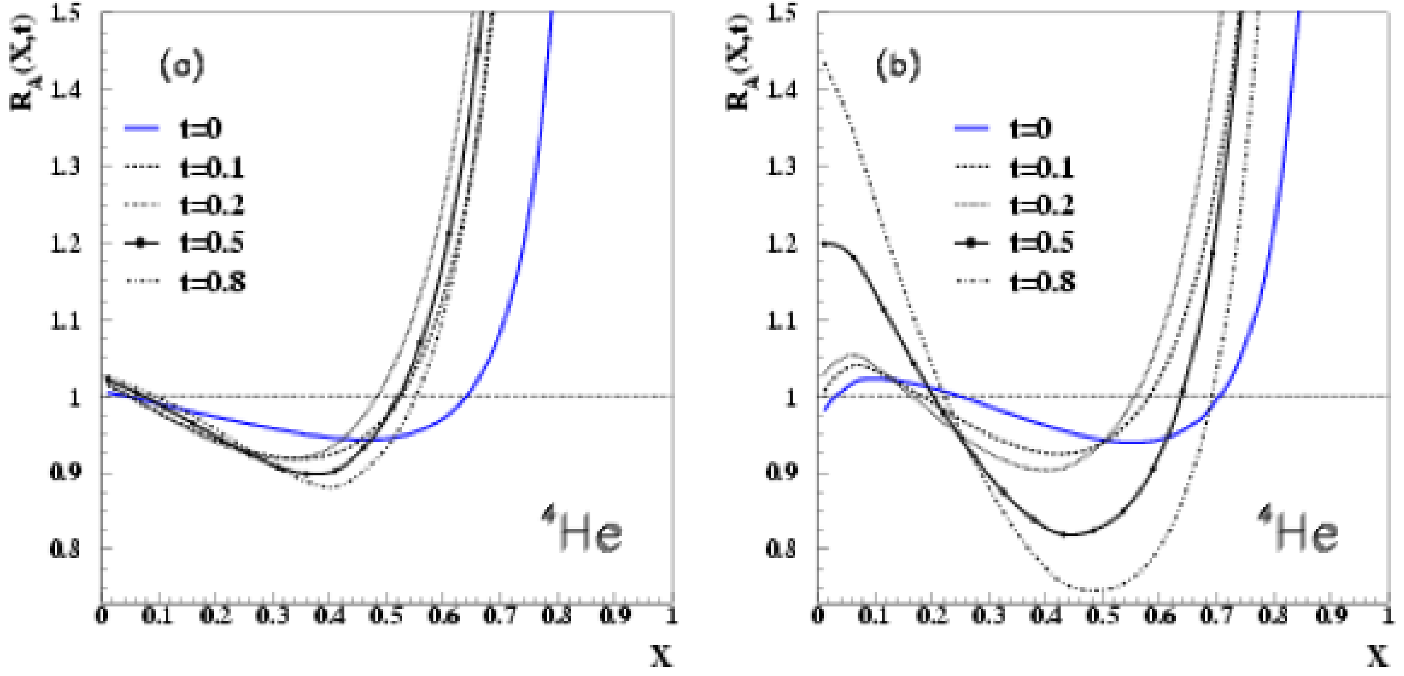
Forward spectral function

$$H_0^A(\zeta,t) = \int \frac{d^2P_\perp dY}{2(2\pi)^3} \mathcal{N}\, \rho_0^A(Y,P^2) H^N\left(\frac{\zeta}{Y},\frac{\zeta}{Y},t;P^2\right)$$

Liuti and Taneja (2005)



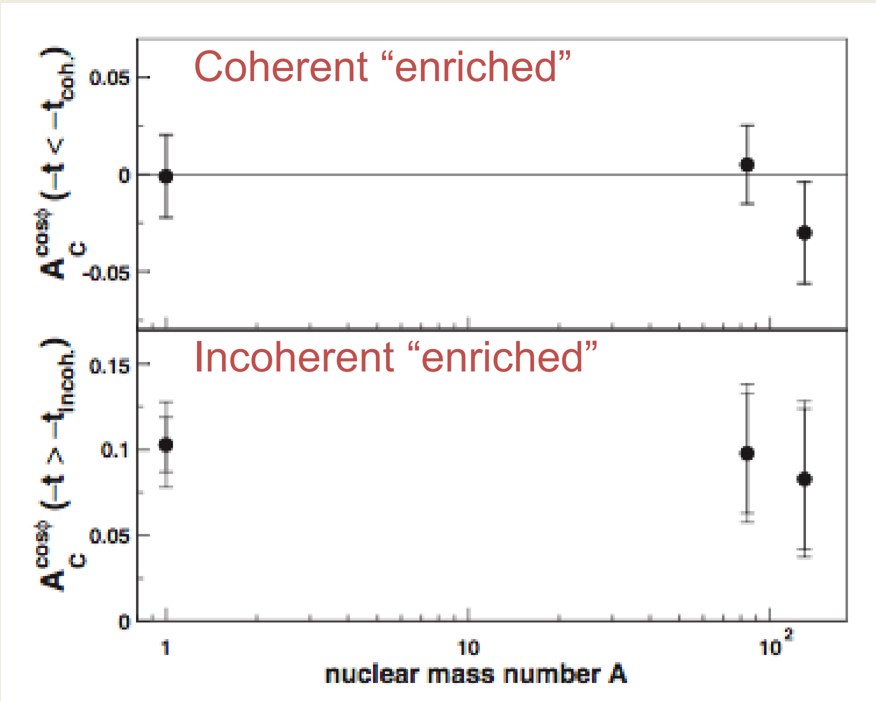
Effect is related to <u>transverse motion of quarks</u>



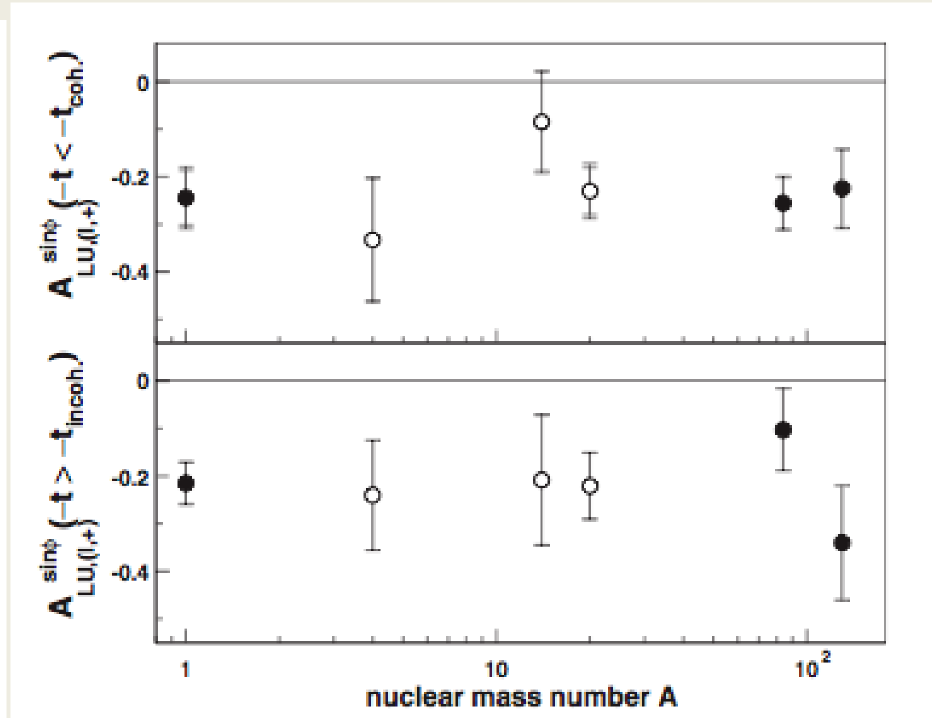
(a)
(b)
$R_A(X,t)$
t=0
t=0.1
t=0.2
t=0.5
t=0.8
^{4}He
X

Hermes first data

Phys.Rev.C81 (2010)



Beam Charge Asymmetry

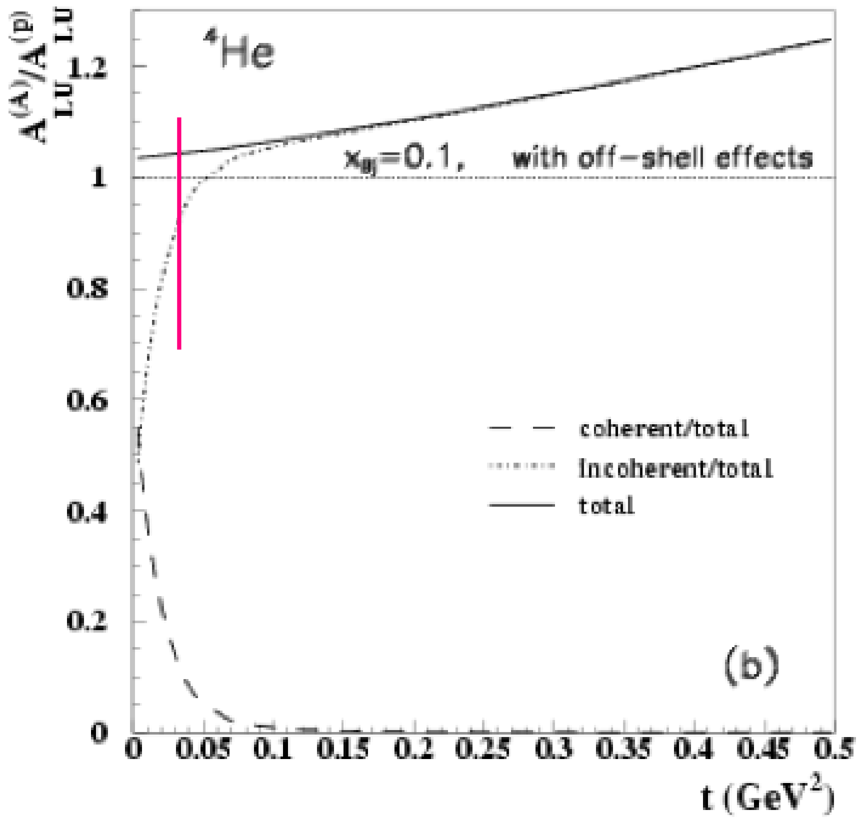


Beam Spin Asymmetry

$R_{LU}^{\sin\varphi}(A/p) = 0.91 \pm 0.19$ coherent

$R_{LU}^{\sin\varphi}(A/p) = 0.93 \pm 0.23$ incoherent

Hermes data



$$R_{LU}^{(A)}(\zeta,t) = A_{LU}^{A} / A_{LU}^{p} = \frac{Z^2 \mathcal{I}_{coh}^{A} + Z \mathcal{I}_{incoh}^{A}}{\mathcal{F}_{DVCS}^{p}(\zeta,t) F_1(t)} \times \frac{F_1^2(t)}{Z^2 F_A^2(t) + Z F_1^2(t)}$$

$\mathcal{I}_{coh}^{A} = \mathcal{F}_{DVCS}^{A}(\zeta,t) F_A(t)$ $\mathcal{I}_{incoh}^{A} = \mathcal{F}_{DVCS,0}^{A}(\zeta,t) F_1(t)$

Spin One Angular MomentumSum Rule

Taneja, Kathuria, Liuti, Goldstein (2009)

Energy Momentum Tensor Matrix Element

gravitomagentic form factors

$$
\begin{aligned}
\langle p'|T^{\mu\nu}|p\rangle = & -\frac{1}{2}P^\mu P^\nu(\epsilon'^*\epsilon)\mathcal{G}_1(t) \\
- & \frac{1}{4}P^\mu P^\nu\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_2(t) - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon) \\
\times & \mathcal{G}_3(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}\mathcal{G}_4(t) \\
+ & \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu\leftrightarrow\nu\right]\mathcal{G}_5(t) \\
+ & \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu\leftrightarrow\nu\right. \\
+ & \left. 2g_{\mu\nu}(\epsilon P)(\epsilon'^* P) - (\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu)\Delta^2\right]\mathcal{G}_6(t) \\
+ & \frac{1}{2}\left[\epsilon^{*\prime\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu\right]\mathcal{G}_7(t) + g^{\mu\nu}(\epsilon'^*\epsilon)M^2\mathcal{G}_8(t) \qquad (5)
\end{aligned}
$$

7 (conserved f.f.'s→☝📂 – ☝⌨) + 1 (non-conserved →☝🖱)

Using Ji's framework we derived…

Momentum Sum Rule

$$\left\langle p' \right| \int d^3x \, T^{0i}_{q,g} \left| p \right\rangle = p^i \left\langle p' \middle| p \right\rangle = \mathcal{G}_1^{q,g} p^i \int d^3x \, 2p^0$$

$$\Rightarrow \mathcal{G}_1^q + \mathcal{G}_1^q = 1$$

$$\left\langle p' \middle| p \right\rangle = 2p^0 \delta^3 (p' - p)$$

Angular Momentum Sum Rule

$$\left\langle p' \right| \int d^3x \left(x_1 T^{02}_{q,g} - x_2 T^{01}_{q,g} \right) \left| p \right\rangle = \mathcal{G}_5^{q,g} \int d^3x \, p^0$$

$$\Rightarrow \frac{1}{2} \mathcal{G}_5^{q,g} = J_z^{q,g}$$

Compare to spin 1/2

Momentum

$$\mathrm{A}^{q} + \mathrm{A}^{g} = 1$$

Angular Momentum

$$\frac{1}{2}\left(\mathrm{A}^{q,g} + B^{q,g}\right) = J_{z}^{q,g}$$

$$\frac{1}{2}\mathcal{G}_5^{q,g} = \frac{1}{2}\int dx\; xH_2(x,0,0) = J_z^{q,g}$$

Interpretation

If H_2=H+E

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right],$$

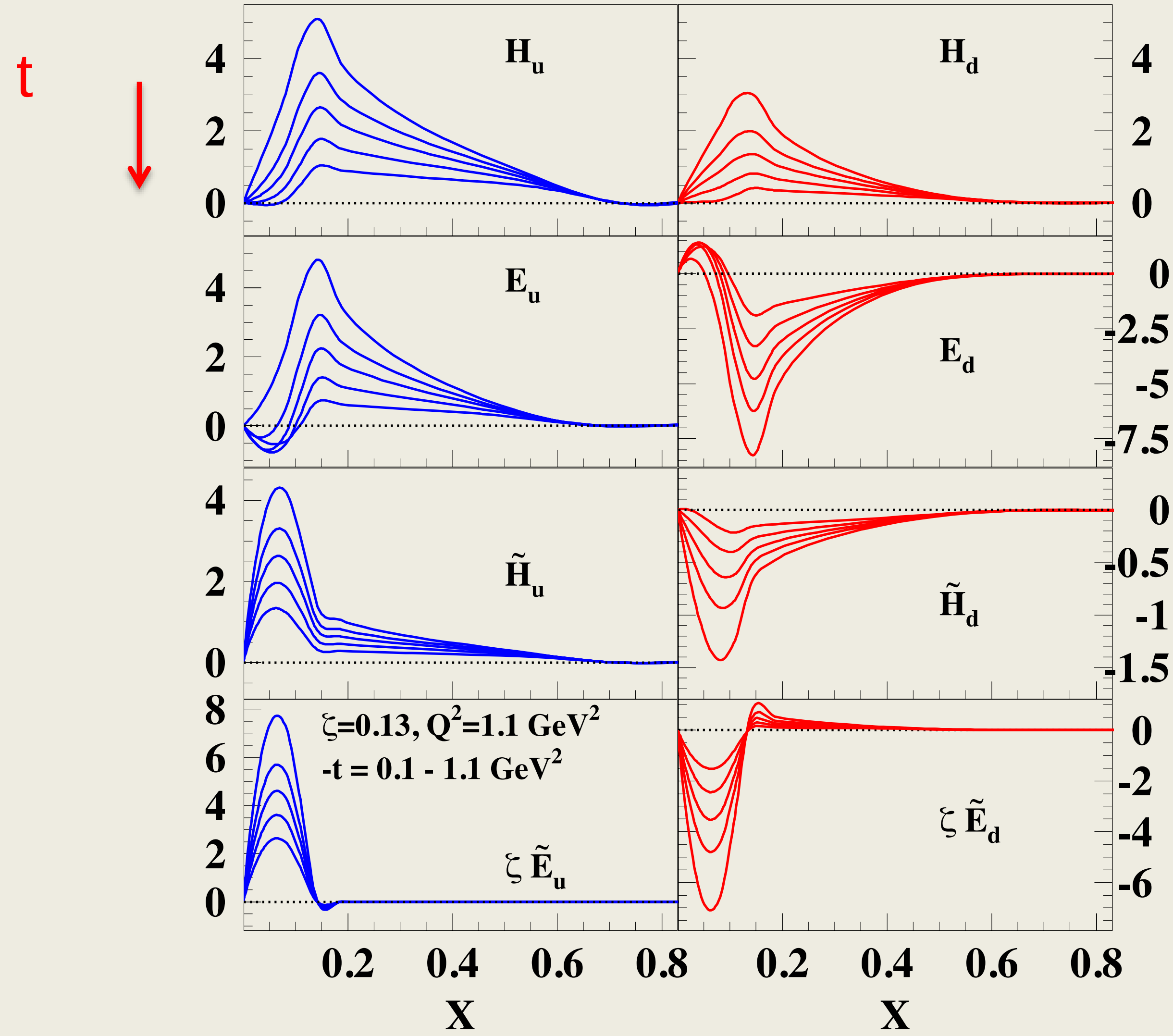
→

$$J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

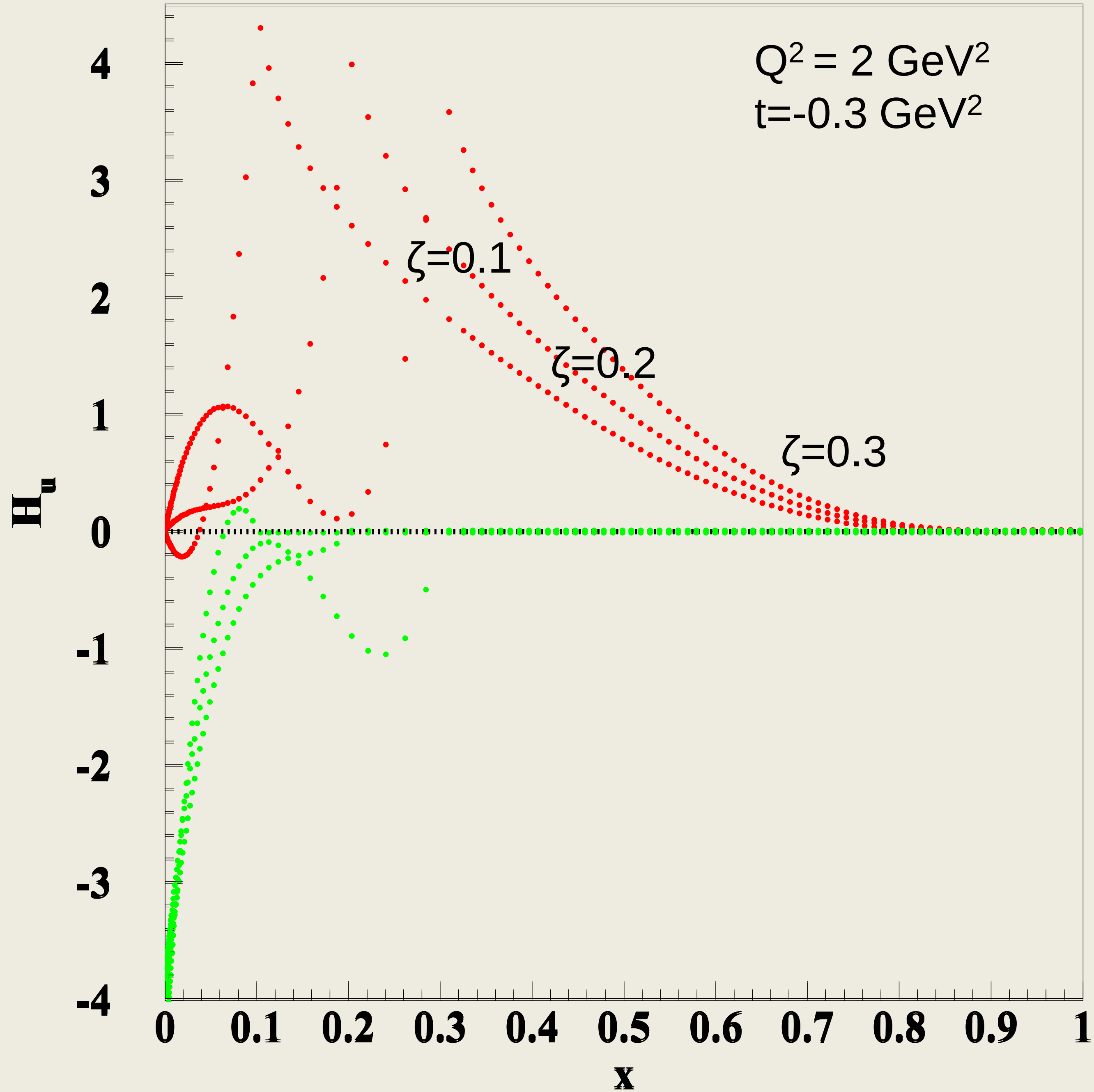
$F_1+F_2= G_M$

G_M

What is needed now: a roadmap for extracting GPDs at LHeC



Goldstein, Gonzalez-Hernandez, S.L., PRD (2015)


Q² = 2 GeV²
t=-0.3 GeV²
ζ=0.1
ζ=0.2
ζ=0.3
H_u
x
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

How do we perform a global fit -- given the enhanced complexity –

- ✓ quantitative studies/simulations using both existing DVCS, DVMP data and models

- ✓ how do we choose the “initial parametrization”?

- ✓ What is the minimal number of parameters necessary to fit x, ξ , t and Q^2 dependences?

- ✓ These issues can be addressed e.g. with a Recursive Fit

Functional form:

From DIS

$$q(x,Q_o^2) = A_q \, x^{-\alpha_q} (1-x)^{b_q} F(x, c_q, d_q, \ldots)$$

to DVCS, DVMP

$$H_q(x,\xi,t;Q_o^2) = N_q \, x^{-\left[\alpha_q + \alpha'_q (1-x)^p t\right]} G^{\alpha_1 \alpha_2 \alpha_3 ..}(x,\xi,t)$$

$$\alpha_1 = m_q, \; \alpha_2 = M_X^q, \; \alpha_3 = M_\Lambda^q, \ldots$$

✓ $H_q(x,0,0;Q^2) = f_1^q(x,Q^2)$, …

DIS data

$F_2^{p,d}, G_1, \ldots$

Total number of parameters = N

fix $n_1 < N$ parameters

Q^2 evol

t=ξ=0

$f_{L/}^{a_1,a_2,a_3,\ldots}(k,P)$ → $A_{L\pm L\pm}^{S=0,S=1}$ → $f_1^{S=0,S=1}, g_1^{S=0,S=1}$ → $f_1^{u,d}, g_1^{u,d}$

✓ Switch on t: $H_q(x,0,t;Q^2)$

fix n_2 parameters $n_1 + n_2 < N$

form factor data

$F_1(t), F_2(t), G_A, G_P$

Q^2 evol

✓ Switch on ξ: $H_q(x,\xi,t;Q^2)$

fix remaining $N-(n_1+n_2)$ parameters

DVCS data

$A_{UL}(x,t), A_{LU}(x,t), A_{LL}(x,t), \ldots$

Conclusions and Outlook

- Comparison between GPD models and data is indeed possible…GPD extraction is possible!!!
- We have seen more constraints on GPDs from nuclei…
- …and at the same time new insights on nuclear modifications from GPDs
- Re-interactions are important and emphasize transverse d.o.f.: need to explore connections between k_T and b
- Exclusive experiments at LHeC range using nuclei will provide an even better laboratory to study QCD in coordinate space: vast phenomenology…