

UV fixed points - from gauge theories to quantum gravitation

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standard model

local QFT for fundamental interactions

strong nuclear force

weak force

electromagnetic force

degrees of freedom

spin 0 (the **Higgs** has finally arrived)

spin 1/2 (quite a few)

spin 1

perturbatively renormalisable & **predictive**

standard model

local QFT for fundamental interactions

strong nuclear force

weak force

electromagnetic force

challenges

Higgs, QED: maximum UV extension?

complete asymptotic freedom?

how does **quantum gravity** fits in?

...

interacting UV fixed points

UV fixed points

perturbation theory

theory with coupling α :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = -B \alpha^2$$

$$\alpha_* \ll 1$$

perturbation theory


theory with coupling α :

$$t = \ln \mu / \Lambda$$

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free fixed point


$$\alpha_* = 0$$

perturbation theory


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free fixed point


$$\alpha_* = 0$$

QED, Higgs

$$B < 0$$

IR fixed point

perturbative UV Landau pole
predictive up to maximal UV extension

asymptotic freedom


theory with coupling α :

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free fixed point


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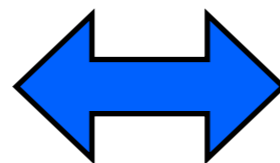
QCD

$$B > 0$$

UV fixed point

perturbative renormalisability & asymptotic freedom
predictive up to highest energies

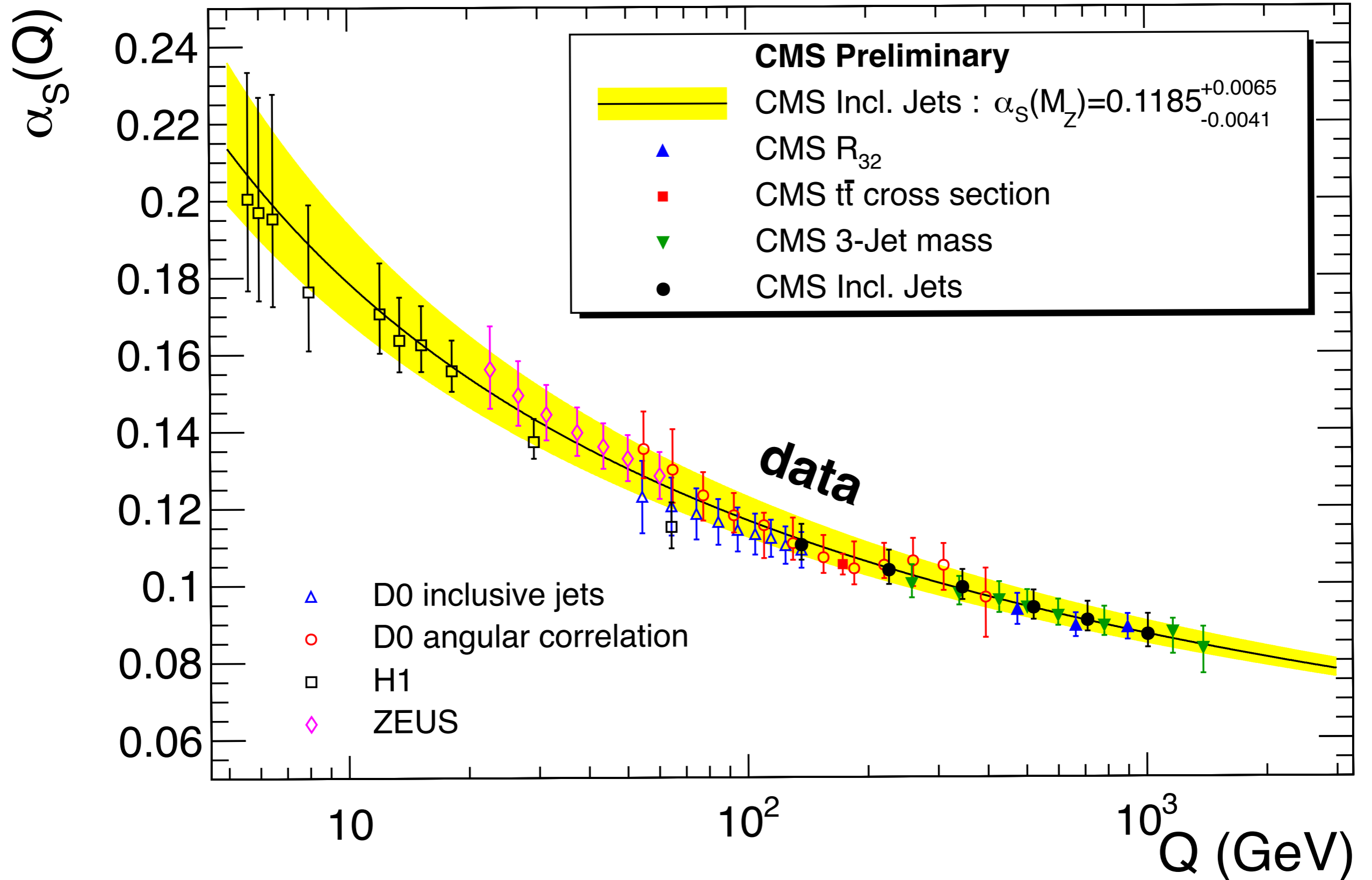
fundamental
definition of QFT



UV fixed point

Wilson '71

asymptotic freedom



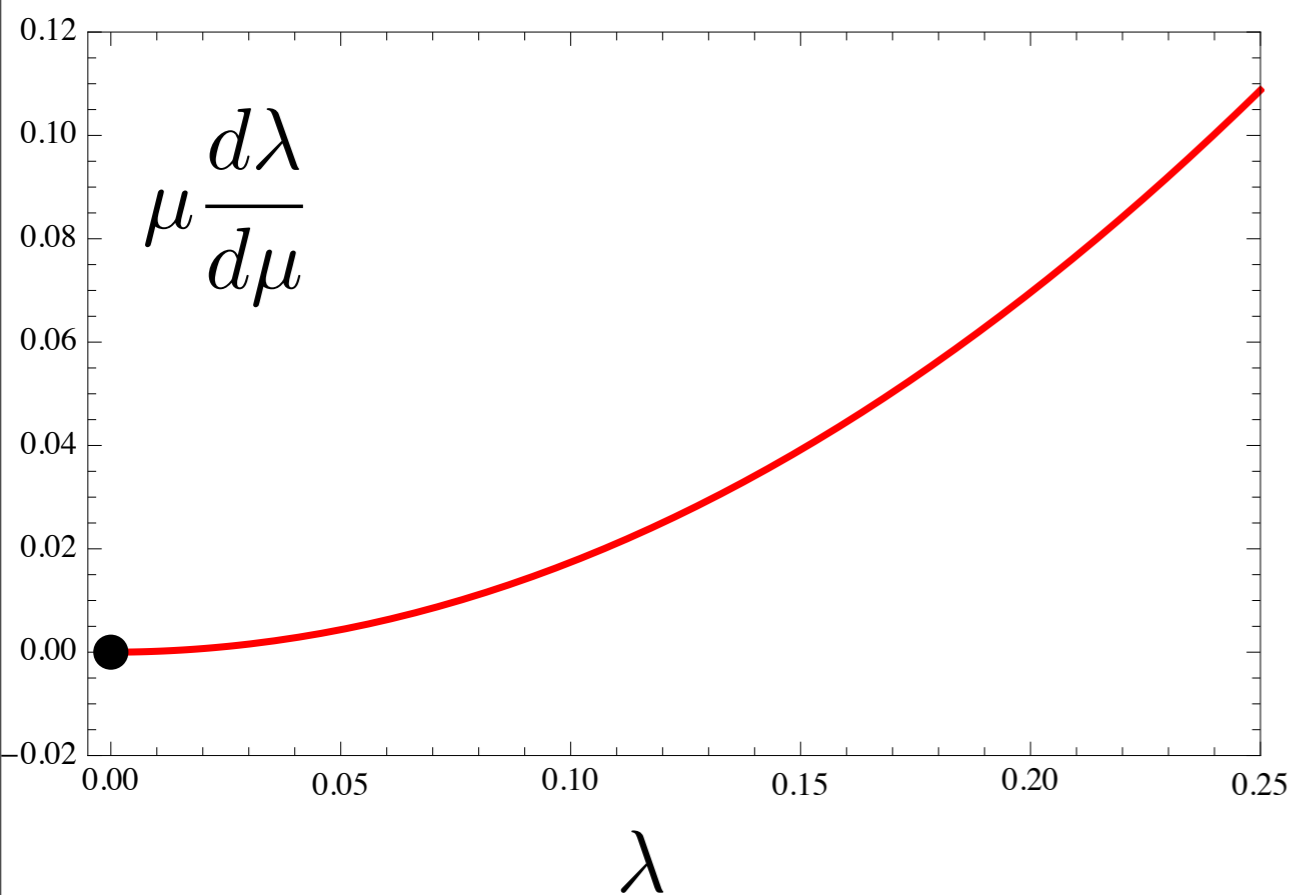
asymptotic freedom

$$\mu \frac{d\alpha_{e.m.}}{d\mu} \propto \alpha_{e.m.}^2 > 0$$

QED beta function

$$\mu \frac{d\lambda}{d\mu} \propto \lambda^2 > 0$$

Higgs self-coupling
Yukawa couplings



perturbative UV Landau pole:
maximal UV extension

cure:
complete asymptotic freedom

interacting fixed point

theory with coupling α :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

perturbative non-renormalisability: $A > 0$

interacting fixed point

theory with coupling α :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

fixed points

$$\alpha_* = 0$$

$$\alpha_* = A/B$$



interacting fixed point

theory with coupling α :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$



fixed points
if $A > 0, B > 0$:

$$\alpha_* = 0$$

IR

$$\alpha_* = A/B$$

UV

interacting fixed point

theory with coupling α :

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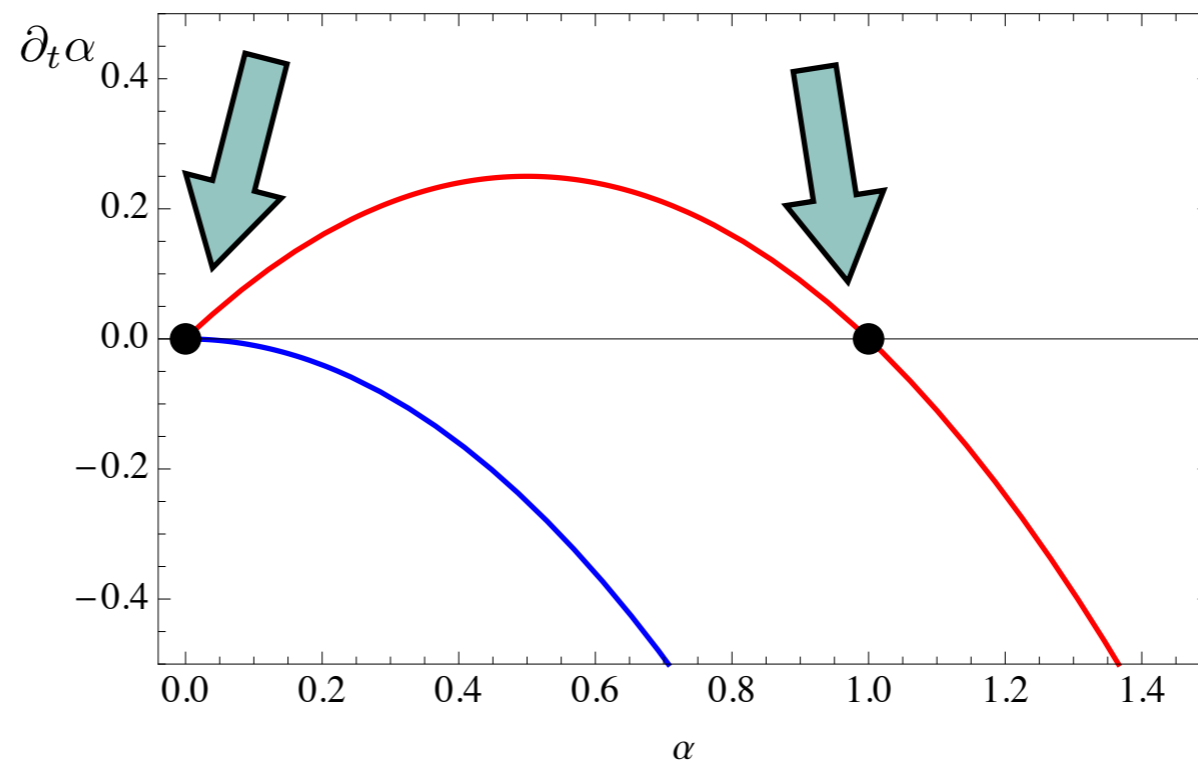
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interacting fixed point

theory with coupling α :

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$$\partial_t \alpha = A \alpha - B \alpha^2$$

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fixed points

$$\alpha_* = 0$$

$$\alpha_* = A/B$$

epsilon expansion:

$$\epsilon = D - D_c$$

large-N expansion:

many fields

perturbation theory

theory with coupling α :

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha = A \alpha - B \alpha^2$$

$$\alpha_* \ll 1$$

gravitons

$$D = 2 + \epsilon : \quad \alpha = G_N(\mu) \mu^{D-2}$$

Gastmans et al '78
Weinberg '79
Kawai et al '90

fermions

$$D = 2 + \epsilon : \quad \alpha = g_{\text{GN}}(\mu) \mu^{2-D}$$

Gawedzki, Kupiainen '85
de Calan et al '91

gluons

$$D = 4 + \epsilon : \quad \alpha = g_{\text{YM}}^2(\mu) \mu^{4-D}$$

Peskin '80
Morris '04

scalars

$$D = 2 + \epsilon : \quad \alpha = g_{NL}(\mu) \mu^{D-2}$$

Brezin, Zinn-Justin '76
Bardeen, Lee, Shrock '76

non-perturbative
renormalisability

$$A = \epsilon \ll 1, \quad B = \mathcal{O}(1) > 0$$

UV fixed points in 4D quantum gauge theories

DL, F Sannino, JHEP1214(2014)178 arXiv:1406.2337
DL, M Mojaza, F Sannino, arXiv:1501.03061

gauge theory with fermions

SU(**NC**) YM with **NF** fermions: $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$

gauge theory with fermions


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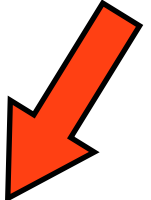
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$$\alpha_* = 0 \quad \alpha_g^* = B/C$$

large-NF,NC (Veneziano) limit:
 ϵ continuous

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

Veneziano '79

we consider

$$0 < -B \equiv -B(\epsilon) \ll 1$$

gauge theory with fermions

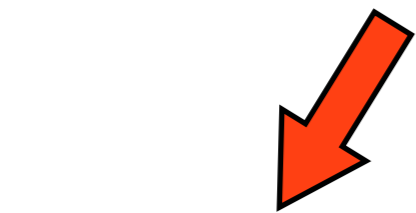
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$$\alpha_* \ll 1$$



$$\alpha_* = 0$$



$$\alpha_g^* = B/C$$

interacting
fixed points:

$B > 0$ & $C > 0$: Caswell - Banks-Zaks

IR fixed point

Caswell '74
Banks, Zaks '82

$B < 0$ & $C < 0$: **UV fixed point**

no asymptotic freedom

gauge theory with fermions


SU(**NC**) YM with **NF** fermions:

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$t = \ln \mu / \Lambda$$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$


$$\alpha_* = 0 \quad \alpha_g^* = B/C$$

we are in the regime

$$0 < \epsilon \ll 1$$

here: $B = -\frac{4\epsilon}{3} < 0$ & $C > 0$

hence:

**no physical
fixed point**


Caswell '74

gauge theory with fermions

SU(**NC**) YM with **NF** fermions: $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3$$

$$\alpha_* \ll 1$$


$$\alpha_* = 0 \quad \alpha_g^* = B/C$$

 **scalar** fields & **Yukawa** couplings required

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \quad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$t = \ln \mu / \Lambda$$

$$\alpha_* \ll 1$$

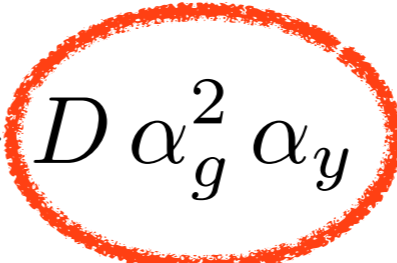

$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3 - D \alpha_g^2 \alpha_y$$



gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \quad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

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$$\partial_t \alpha_g = -B \alpha_g^2 + C \alpha_g^3 - D \alpha_g^2 \alpha_y$$


$$\partial_t \alpha_y = E \alpha_y^2 - F \alpha_g \alpha_y$$


gauge-Yukawa theory

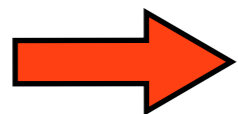
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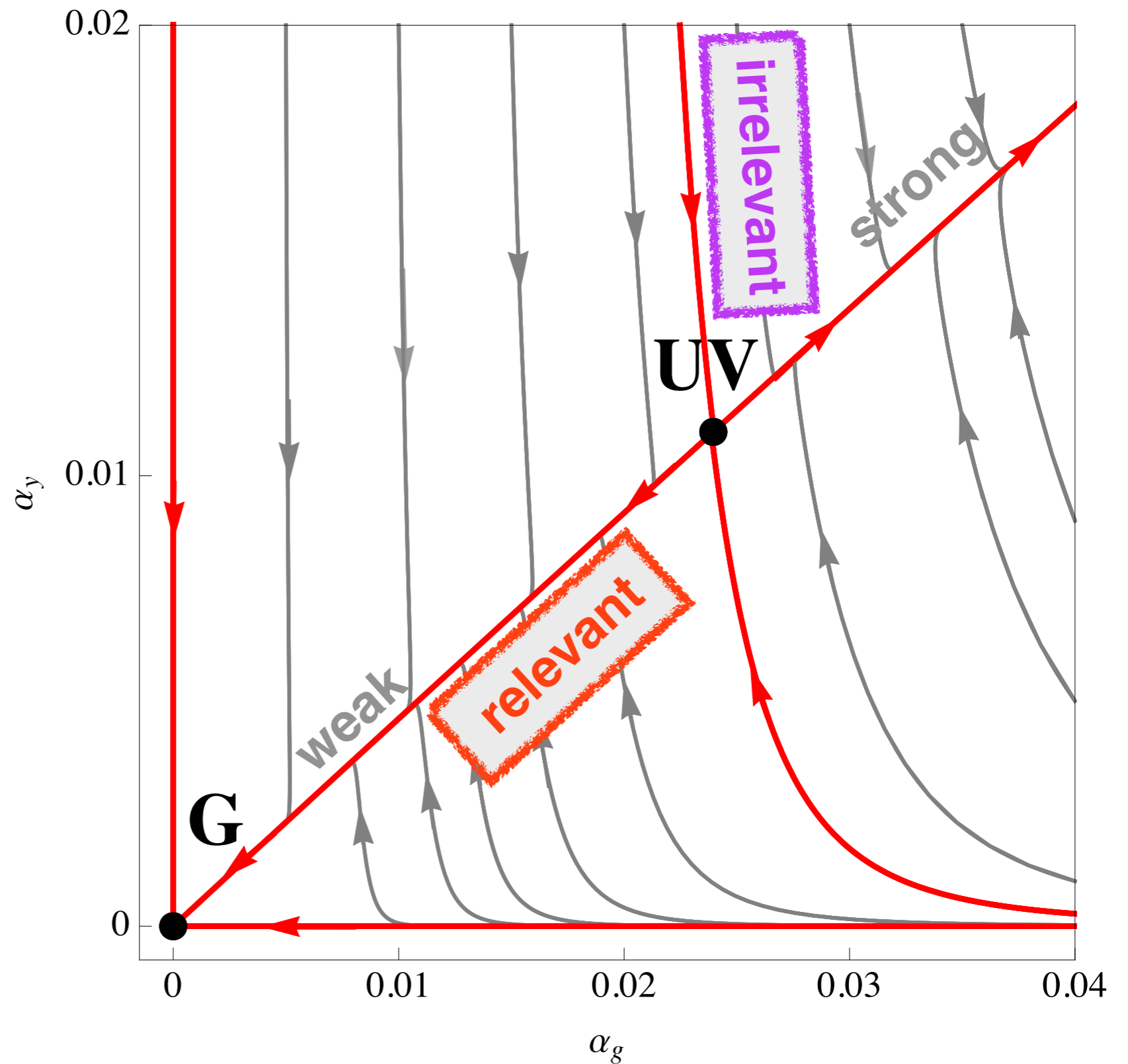


sensible interacting UV fixed point

$$D F - C E \geq 0$$

phase diagram

UV finite theories
(weak & strong)



exact UV FP
strict perturbative control

template gauge-Yukawa theory

Lagrangian

$$L_{\text{YM}} = -\frac{1}{2} \text{Tr} F^{\mu\nu} F_{\mu\nu}$$

$$L_F = \text{Tr} (\bar{Q} i \not{D} Q)$$

$$L_Y = y \text{Tr} (\bar{Q} H Q)$$

$$L_H = \text{Tr} (\partial_\mu H^\dagger \partial^\mu H)$$

$$L_U = -u \text{Tr} (H^\dagger H)^2$$

$$L_V = -v (\text{Tr} H^\dagger H)^2.$$

small parameter

global symmetry

$$SU(N_F) \times SU(N_F) \times U(1)$$

couplings

$$\alpha_g = \frac{g^2 N_C}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u N_F}{(4\pi)^2}, \quad \alpha_v = \frac{v N_F^2}{(4\pi)^2}.$$

no asymptotic freedom

$$0 < \epsilon \ll 1 \quad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

template gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon \right) \alpha_g - 2 \left(\frac{11}{2} + \epsilon \right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon) \alpha_y - 6 \alpha_g \right\} .$$

$$\beta_h = -(11 + 2\epsilon) \alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v(\alpha_v + 4\alpha_h + \alpha_y) .$$

up to 3-, 2-, 1-loop order
in the gauge, Yukawa
and scalar couplings

coupling	order in perturbation theory		
α_g	1	2	3
α_y	0	1	2
α_h	0	0	1
α_v	0	0	1
approximation level	LO	NLO	NNLO

template gauge-Yukawa theory

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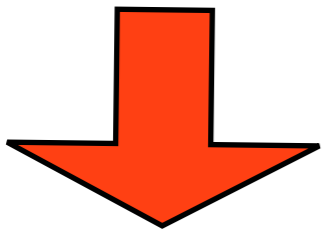
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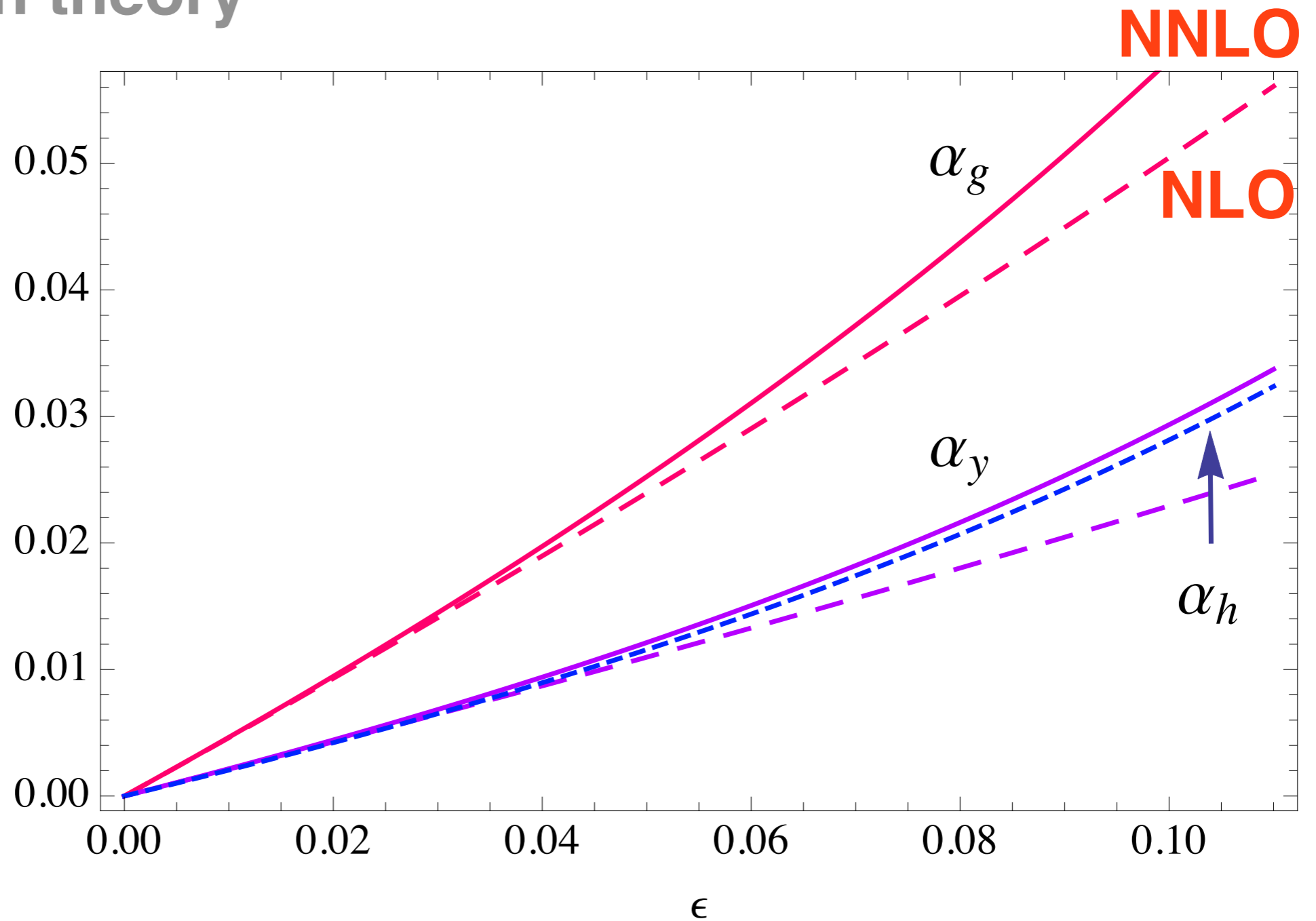


universal
UV fixed point

$$\begin{aligned} \alpha_g^* &= 0.4561 \epsilon + 0.7808 \epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105 \epsilon + 0.5082 \epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998 \epsilon + 0.5042 \epsilon^2 + \mathcal{O}(\epsilon^3). \\ \alpha_v^* &= -0.1373 \epsilon + \mathcal{O}(\epsilon^2) \end{aligned}$$

results

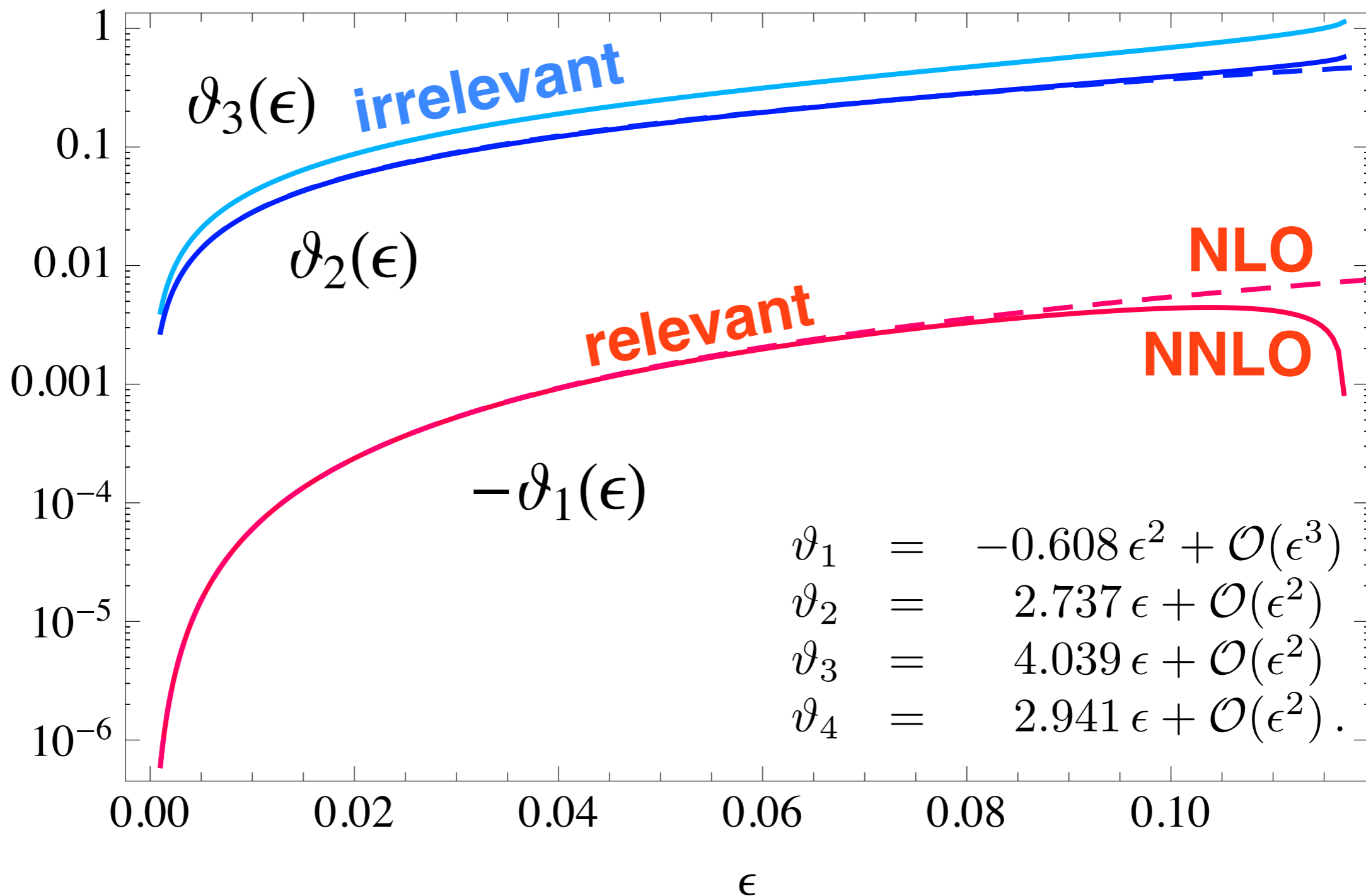
UV fixed point from perturbation theory



results

UV scaling exponents

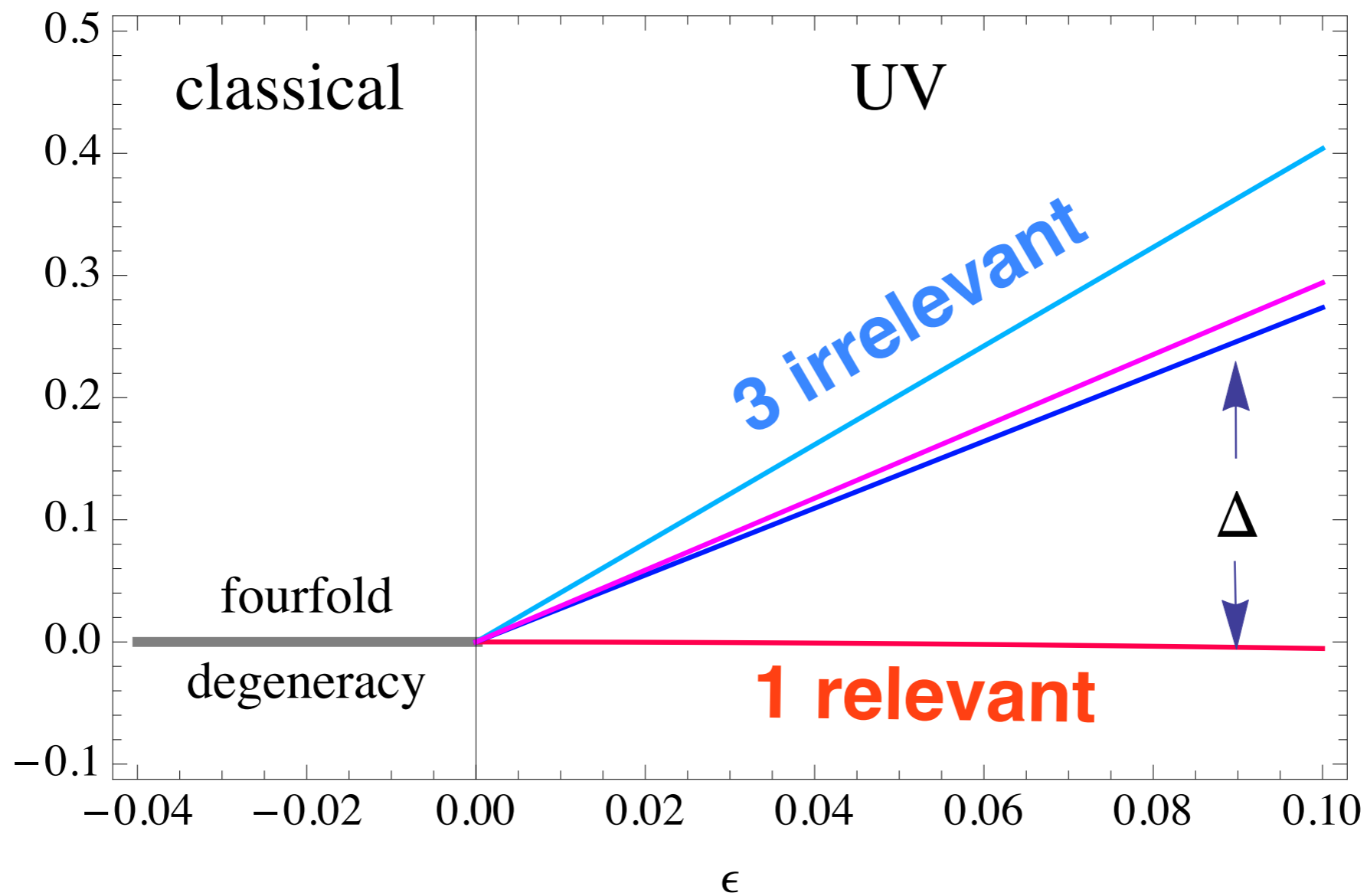
$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$



results

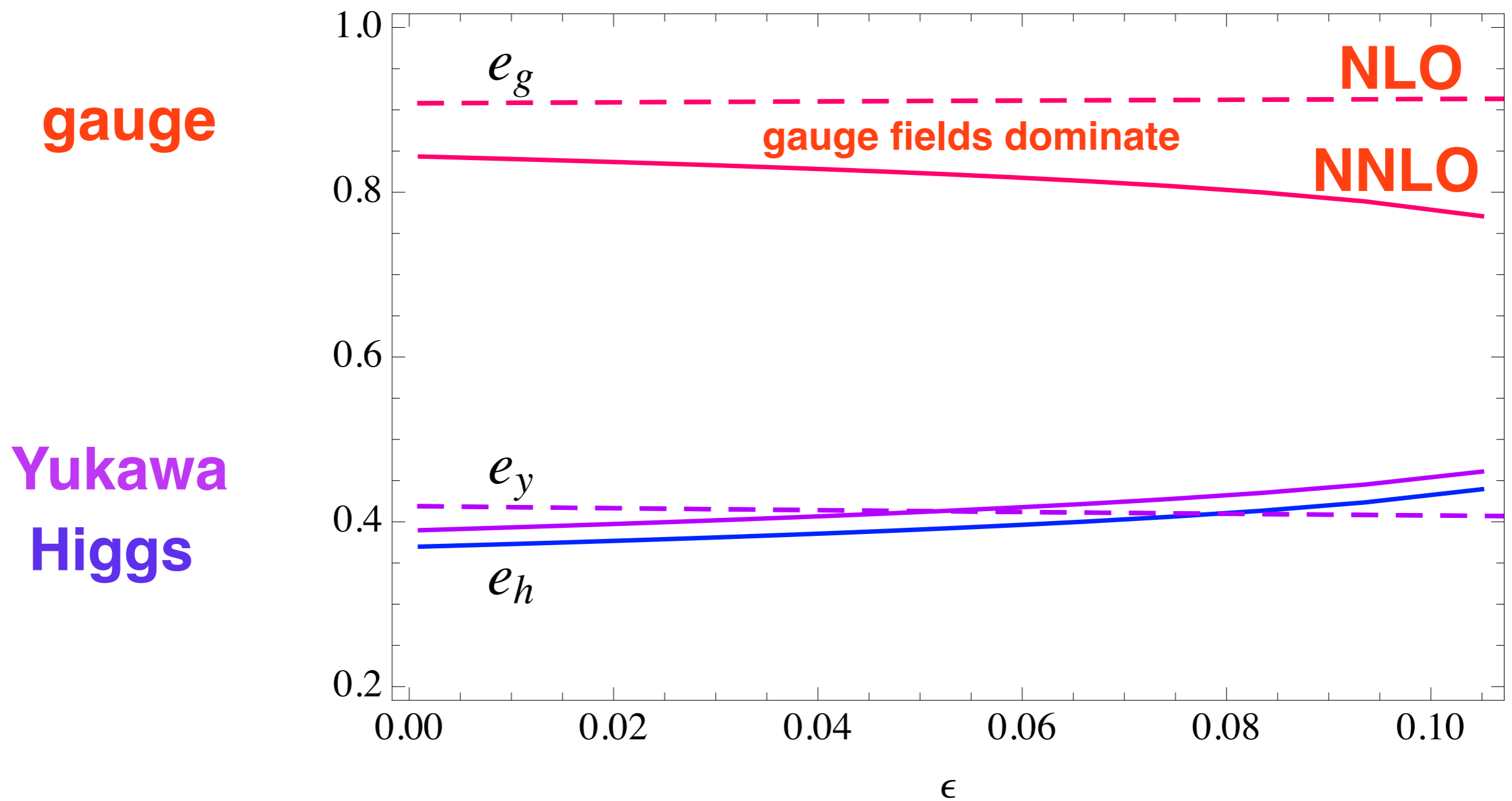
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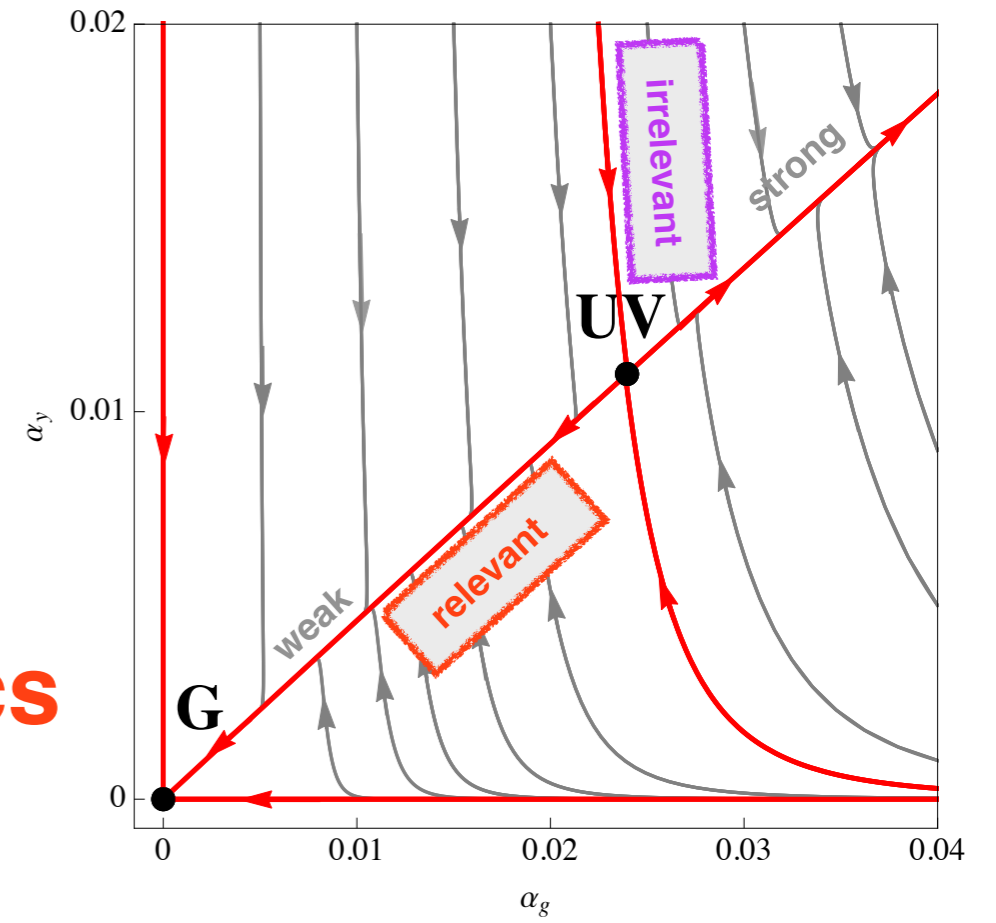
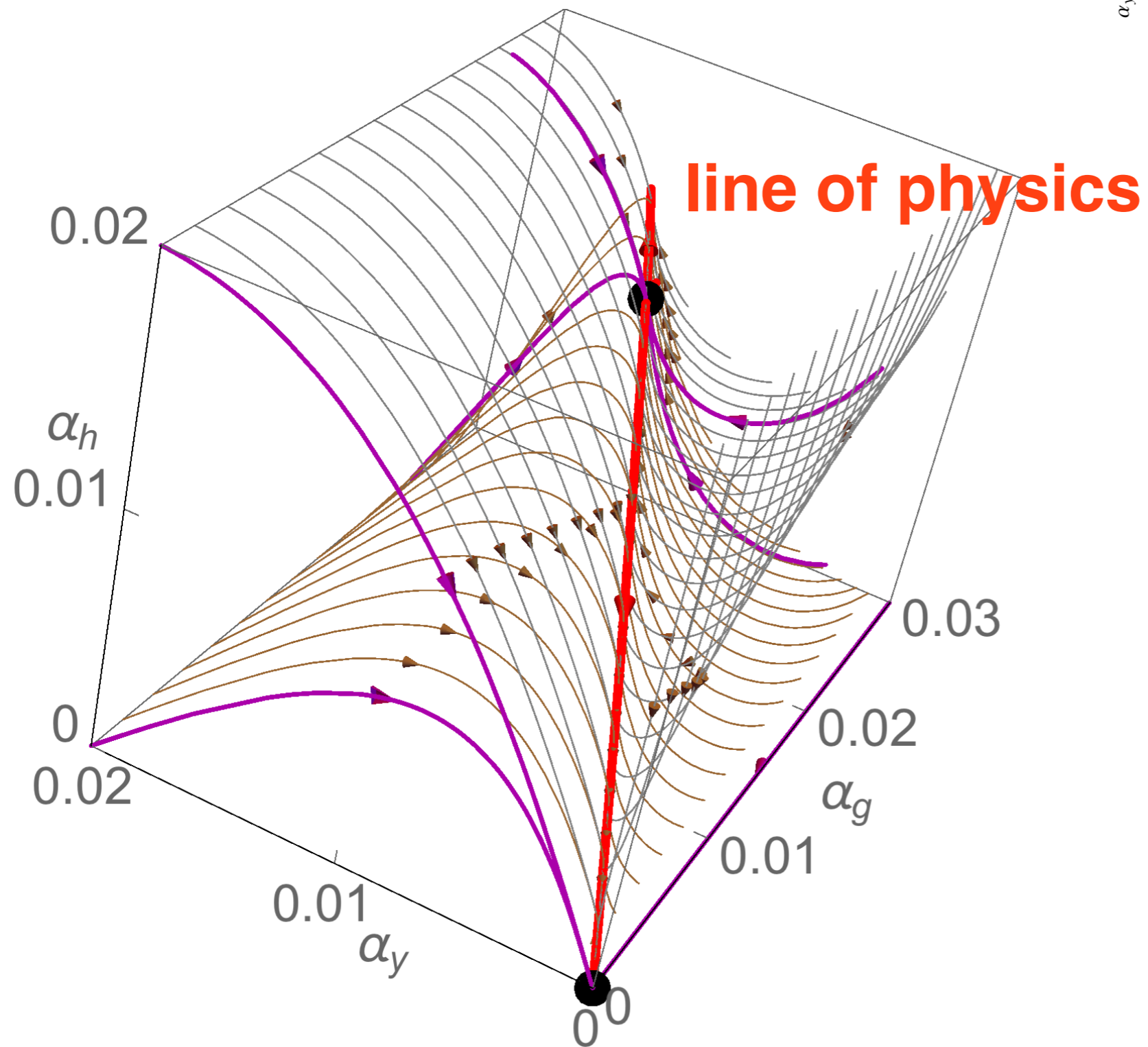
 ϑ 

results

UV-relevant
eigendirection



phase diagram

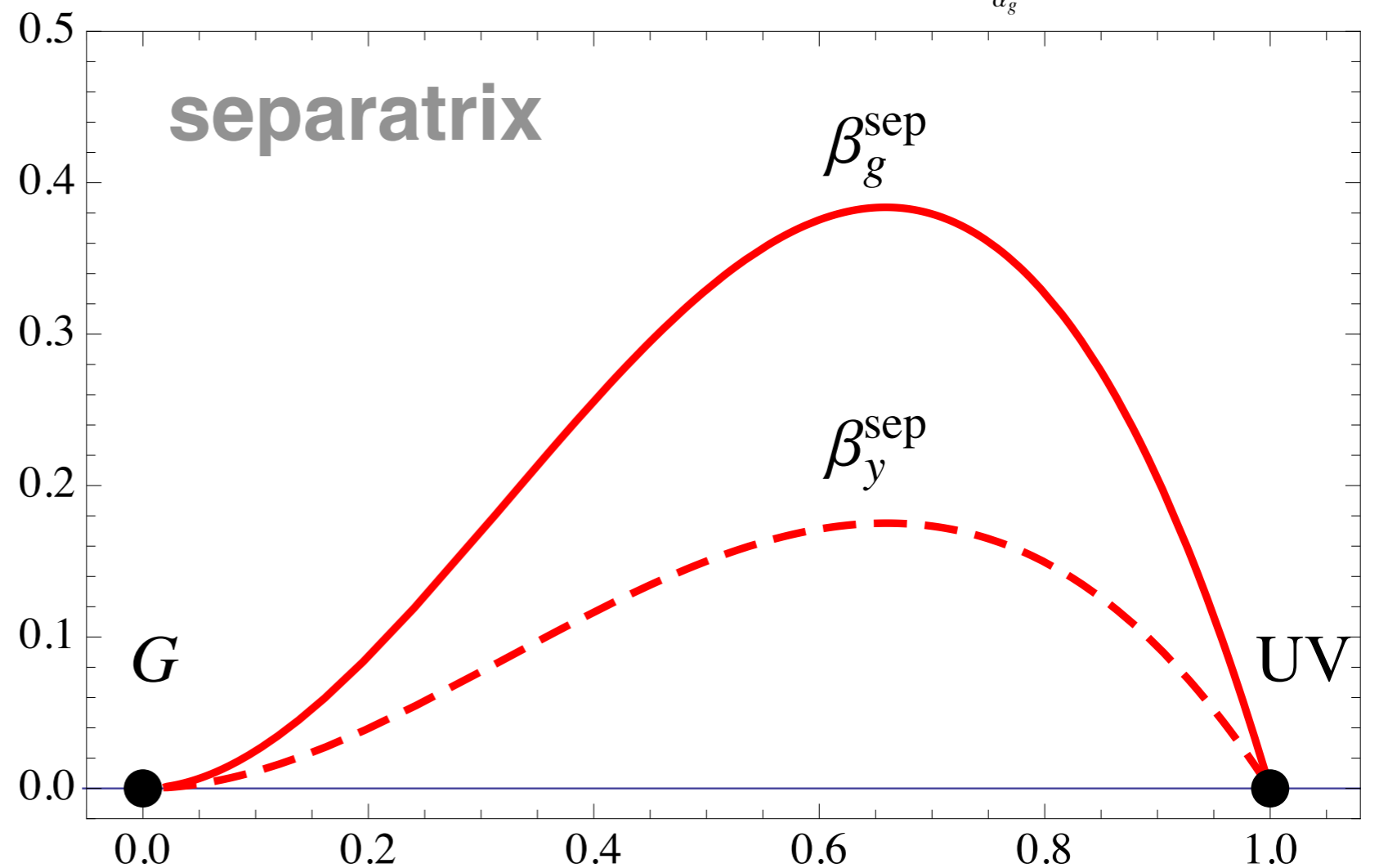
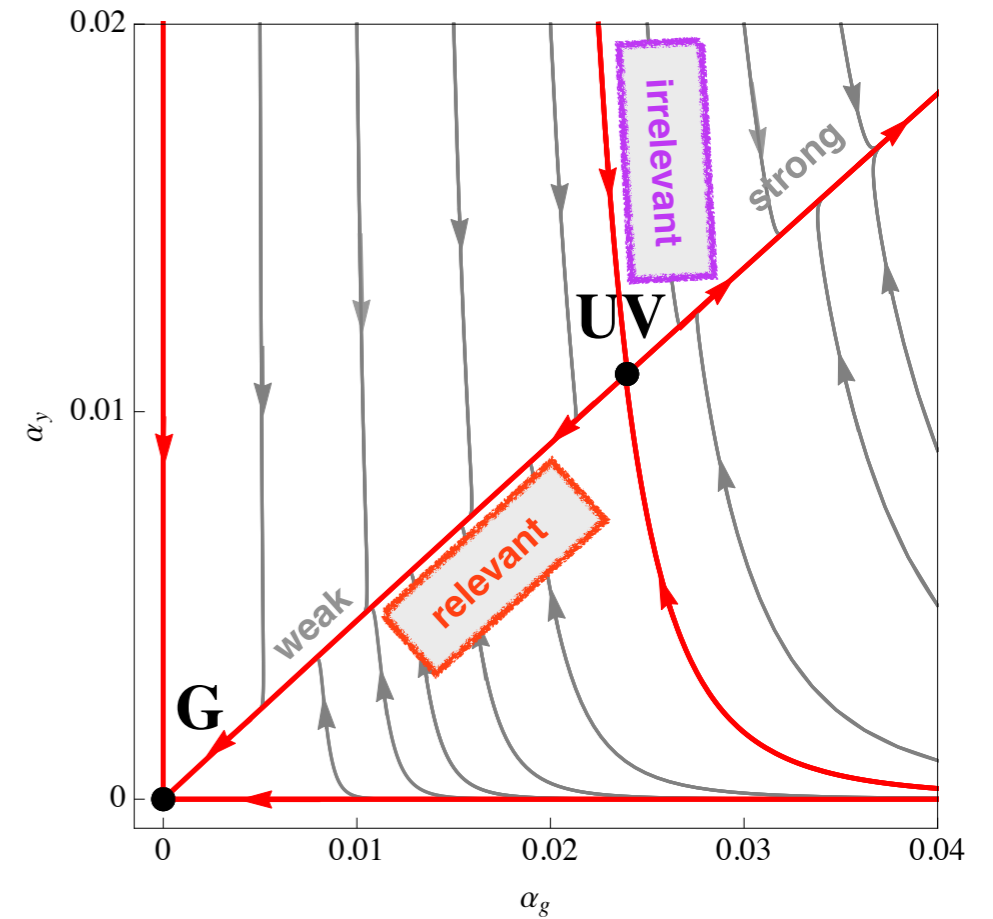


lines of physics

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

$$W(\mu) = W_{\text{Lambert}}[z(\mu)]$$

$$z = \left(\frac{\mu_0}{\mu}\right)^{-B \cdot \alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1\right) \exp\left(\frac{\alpha_*}{\alpha_0} - 1\right).$$



vacuum stability

vacuum must be stable classically
and quantum-mechanically

$$V \propto \alpha_v \text{Tr}(H^\dagger H)^2 + \alpha_h (\text{Tr} H^\dagger H)^2$$

stability

$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0$$

$$H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v/N_F \geq 0$$

$$H_c \propto \delta_{i1}$$

UV FP:

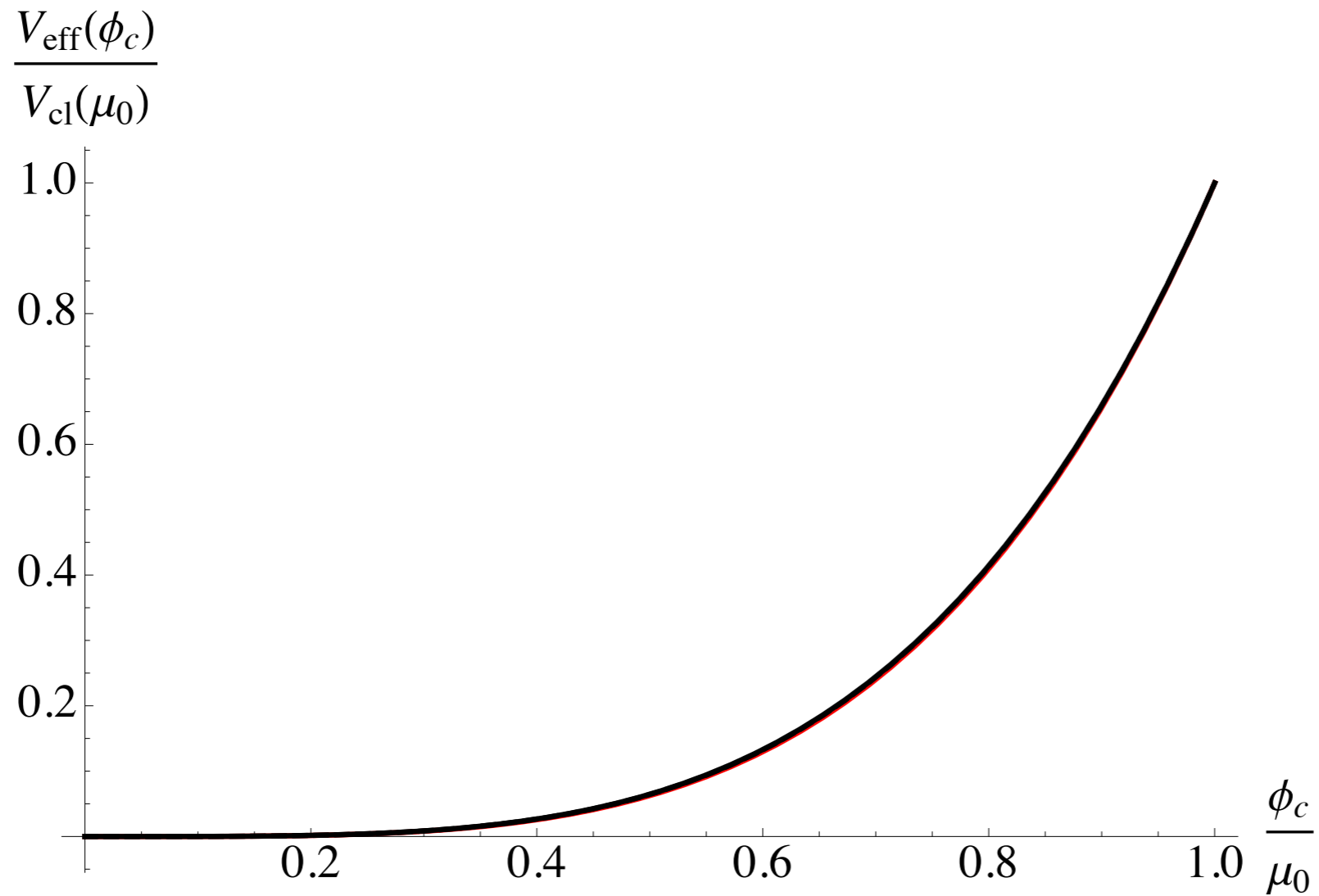
$$0 < \alpha_h^* + \alpha_v^* \quad \text{ok}$$

quantum effects: integrate exact RG

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j) \phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i} \right) V_{\text{eff}}(\phi_c, \mu_0, \alpha_j) = 0$$

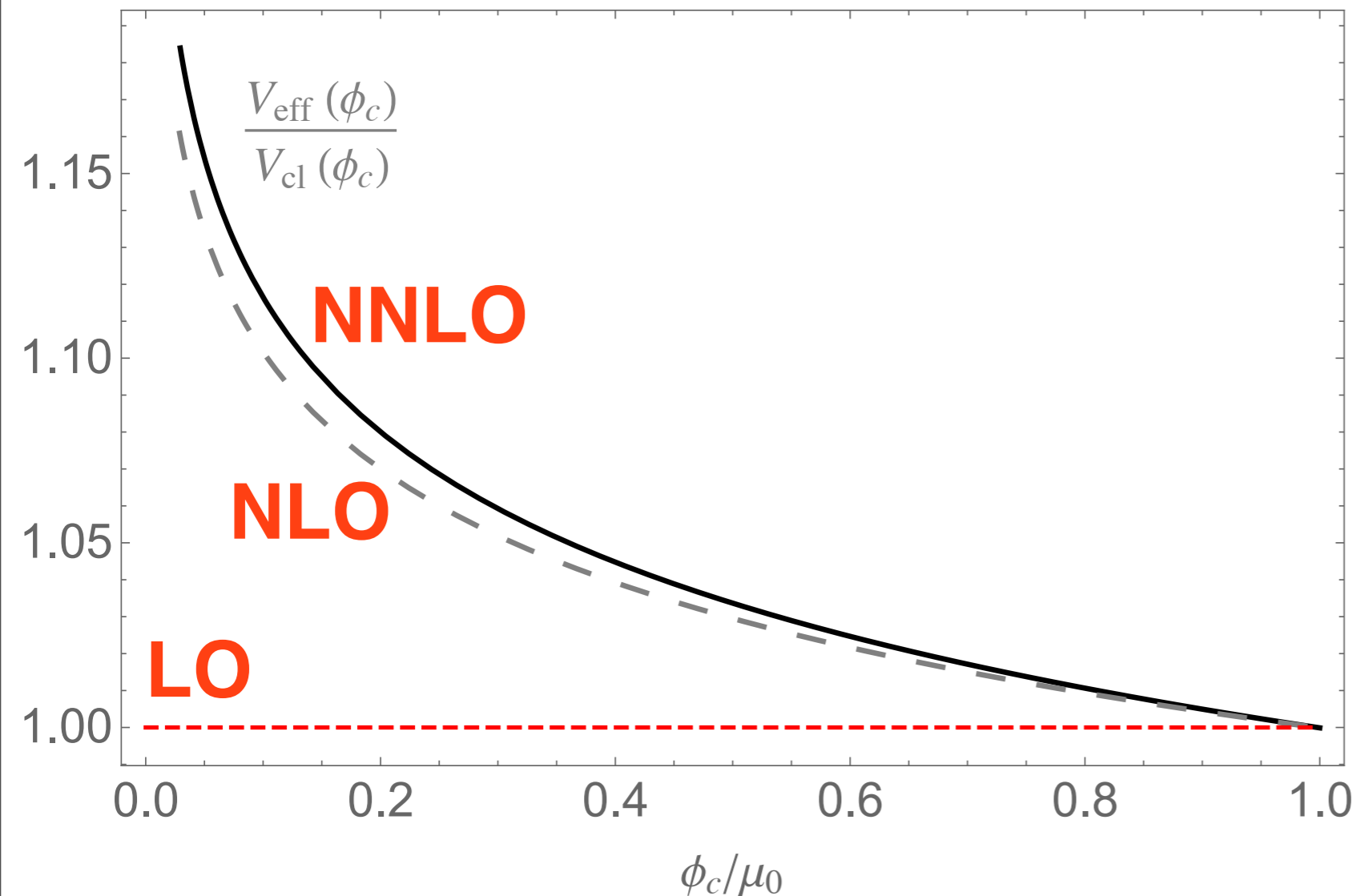
vacuum stability

Coleman-Weinberg potential



vacuum stability

quantum stability: resummation of logarithms



scalar effective potential defined for all scales
quantum vacuum is stable

summary

QFTs beyond asymptotic freedom

4D matter-gauge theories

exact **perturbative** proof of existence

all types of fields required

sensible interacting & UV finite theory

no supersymmetry

UV fixed points in 4D quantum gravity

quantum gravity

running coupling $g(k) = G_N(k)k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N) g$$

$$t = \ln k / \Lambda_c$$

quantum gravity

running coupling

$$g(k) = G_N(k)k^{D-2}$$

$$\partial_t g = (D - 2 + \eta_N) g$$

$$t = \ln k / \Lambda_c$$



$$g_* \neq 0$$

UV



$$g_* = 0$$

IR

fixed points


non-trivial anomalous
dimension

quantum gravity


running coupling $g(k) = G_N(k)k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N) g$$

$$t = \ln k / \Lambda_c$$


$$g_* \neq 0$$

UV


$$g_* = 0$$

IR

fixed points

large anomalous dimension

$$\eta_N = \eta_N(g, \text{all other couplings})$$

large UV scaling exponents

$$\vartheta \approx \mathcal{O}(1)$$

strong coupling effects

$$g_* \approx \mathcal{O}(1)$$

relevant vs **irrelevant**

invariants not known a priori

asymptotic freedom

vs

asymptotic safety

$$g_* = 0$$

anomalous dimensions

$$\eta_A = 0$$

canonical power counting

$\{\mathcal{V}_{G,n}\}$ are known

F^{256} irrelevant !

$$g_* \neq 0$$

anomalous dimensions

$$\eta_N \neq 0$$

non-canonical power counting

$\{\mathcal{V}_n\}$ are **not** known

R^{256}

relevant
marginal
irrelevant



bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\mathcal{V}_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D , and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

$f(R)$

$$\Gamma_k \propto f(R)$$

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n k^{d_n} \int d^4x \sqrt{g} R^n$$

effective action with invariants up to mass dimension $D = 2(N - 1)$

technicalities: functional renormalisation

$$k \frac{d\Gamma_k}{dk} = \frac{1}{2} \text{Tr} \left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi \delta\phi} + R_k \right)^{-1} k \frac{dR_k}{dk} \right] = \frac{1}{2} \text{Diagram}$$

here:

M Reuter hep-th/9605030

DL [hep-th/0103195](#)
[hep-th/0312114](#)

Falls, DL, Nikolakopoulos, Rahmede

Falls, DL, Nikolakopoulos, Rahmede

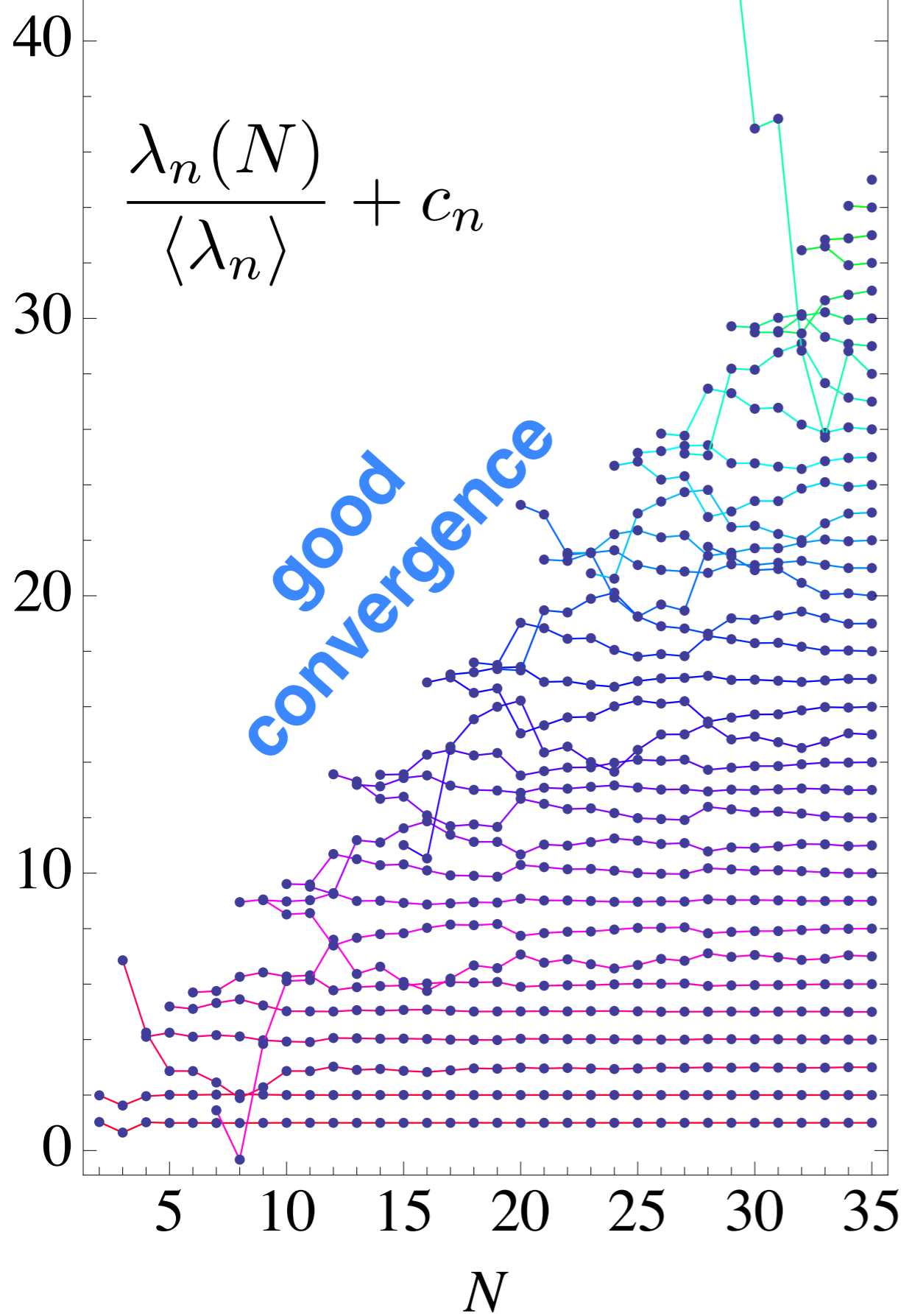
A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909
P Machado, F Saueressig 0712.0445

[1301.4191.pdf](#)

1410.4815

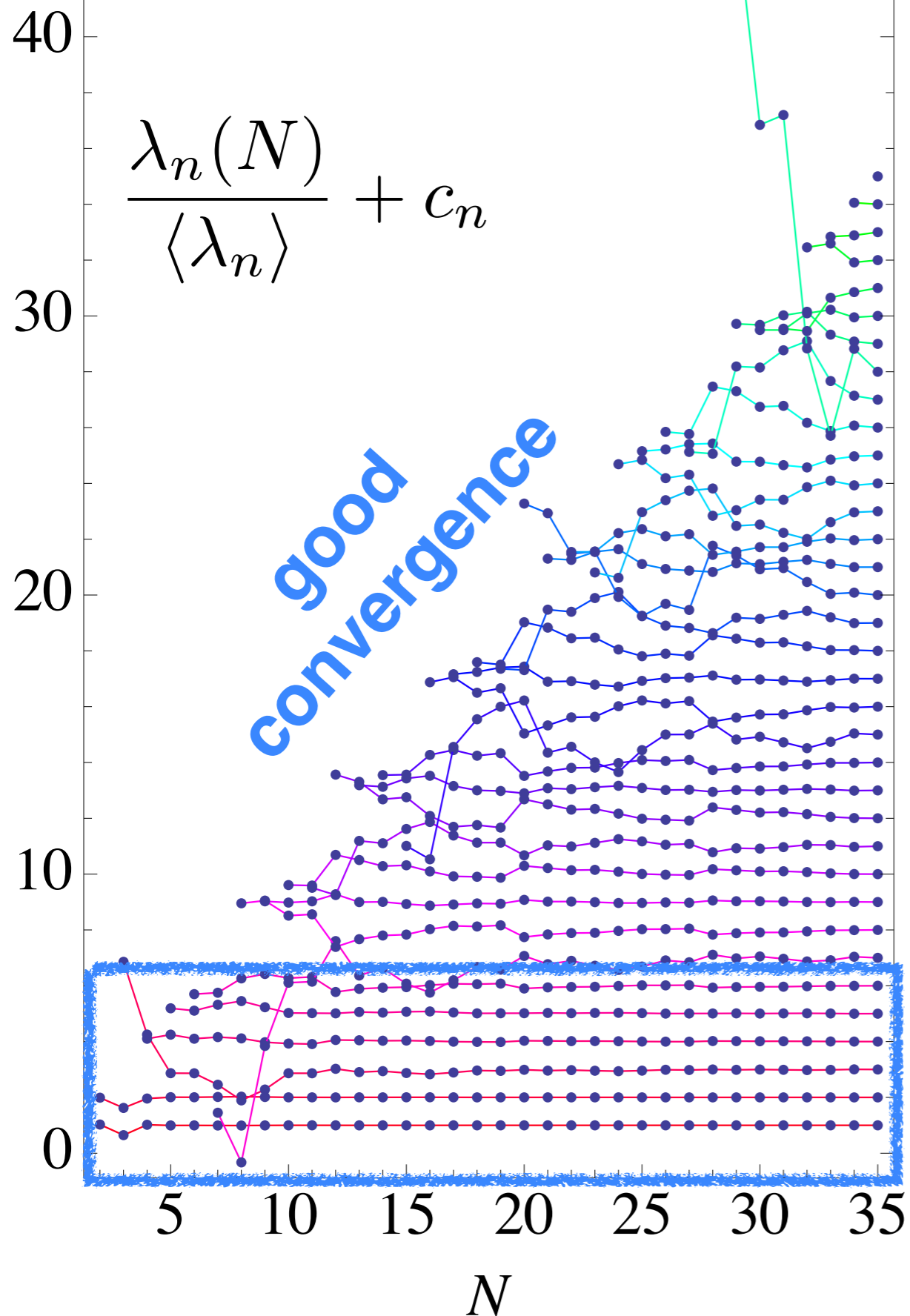
UV fixed point

$$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$$

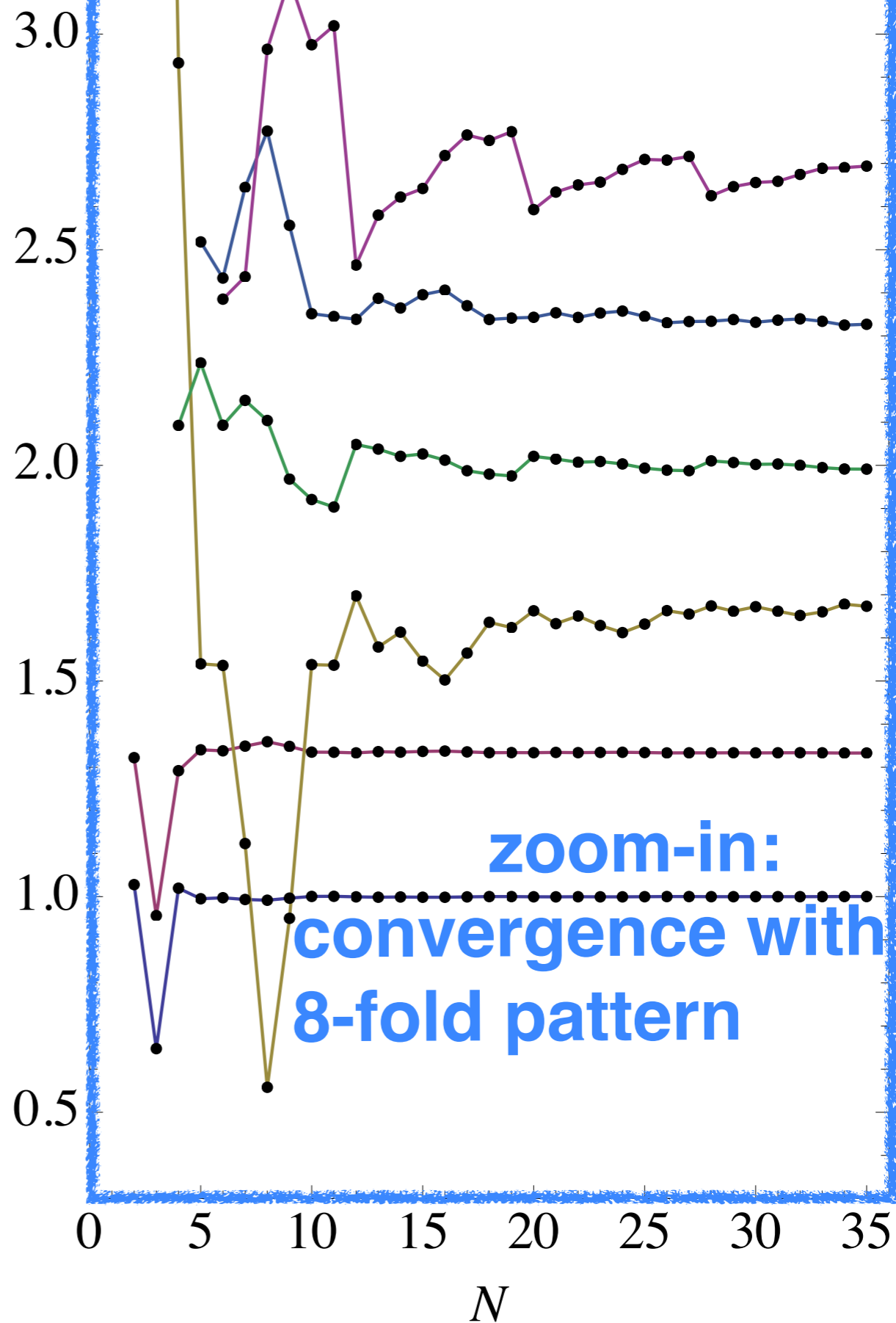


UV fixed point

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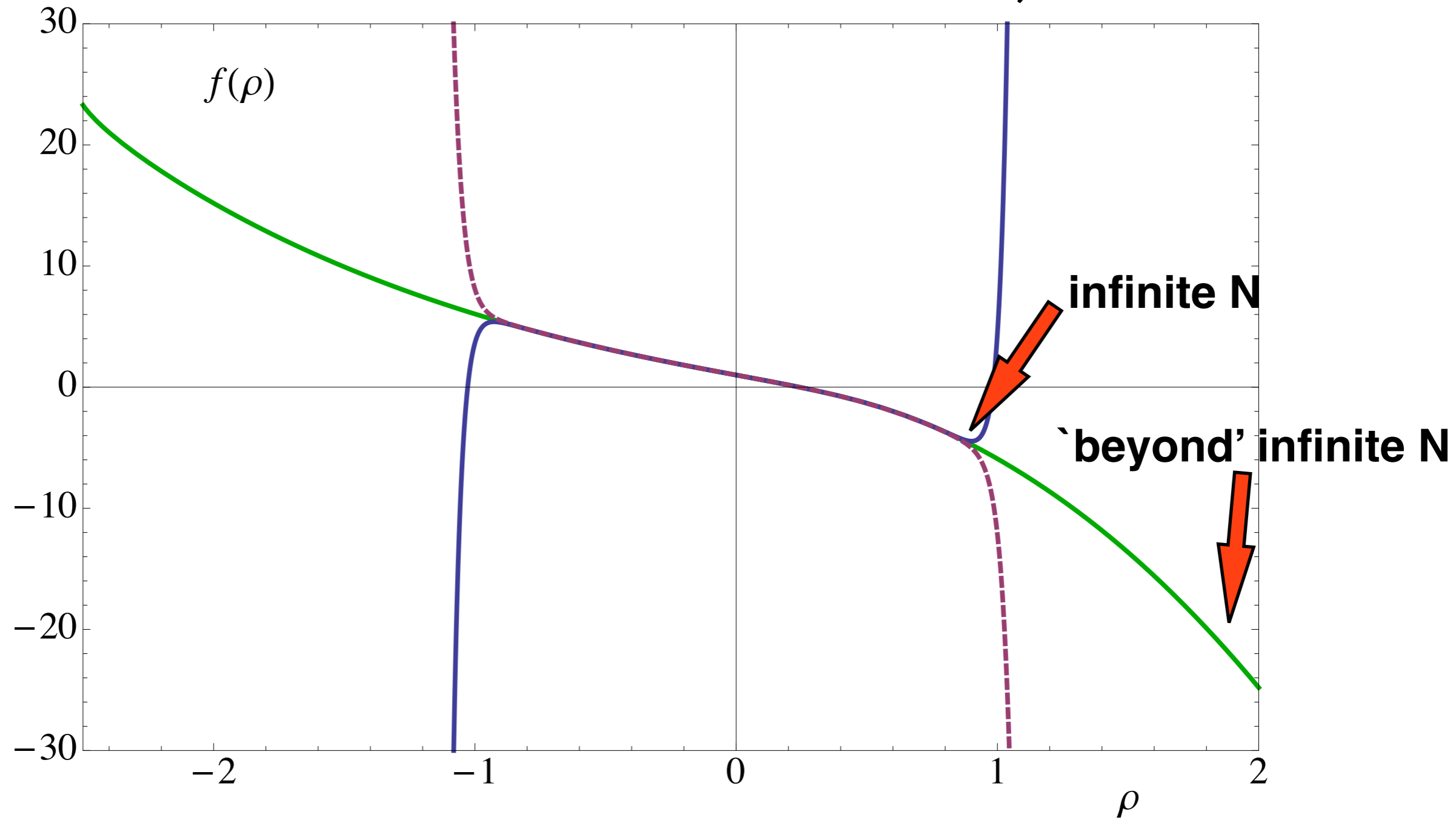


UV fixed point



f(R) quantum gravity

UV scaling solution

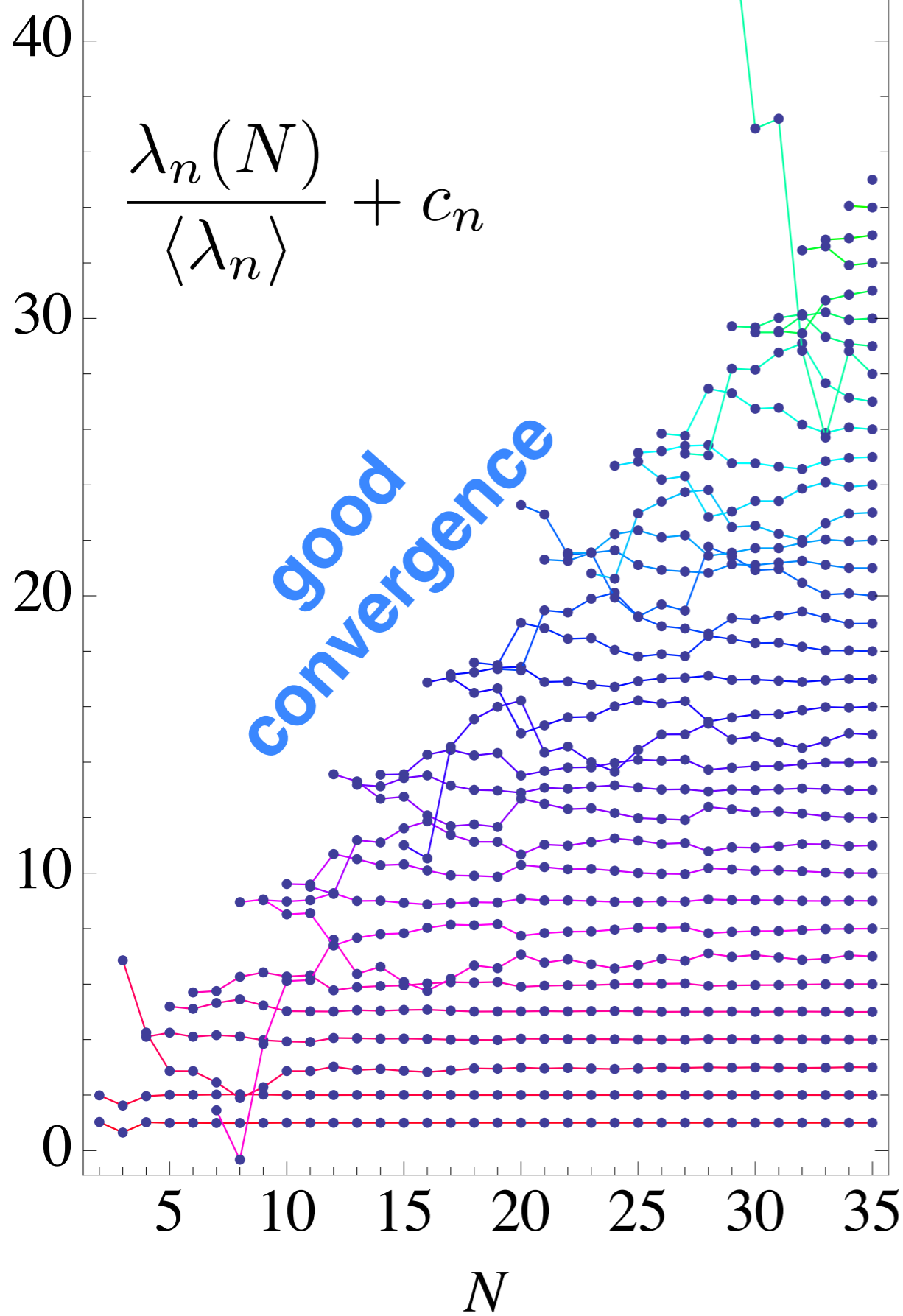


radius of convergence

$$\rho_c \approx 0.82 \pm 5\%$$

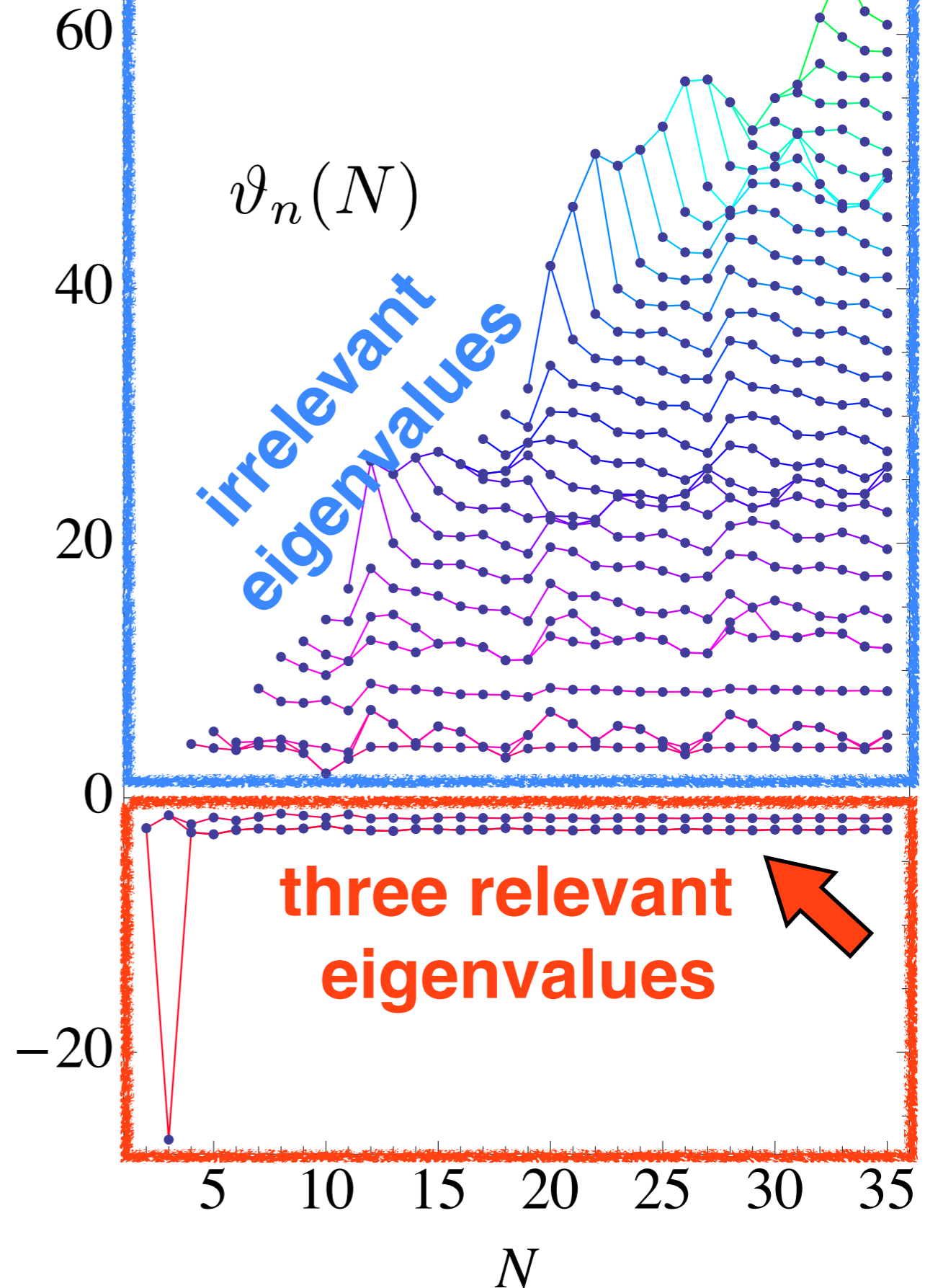
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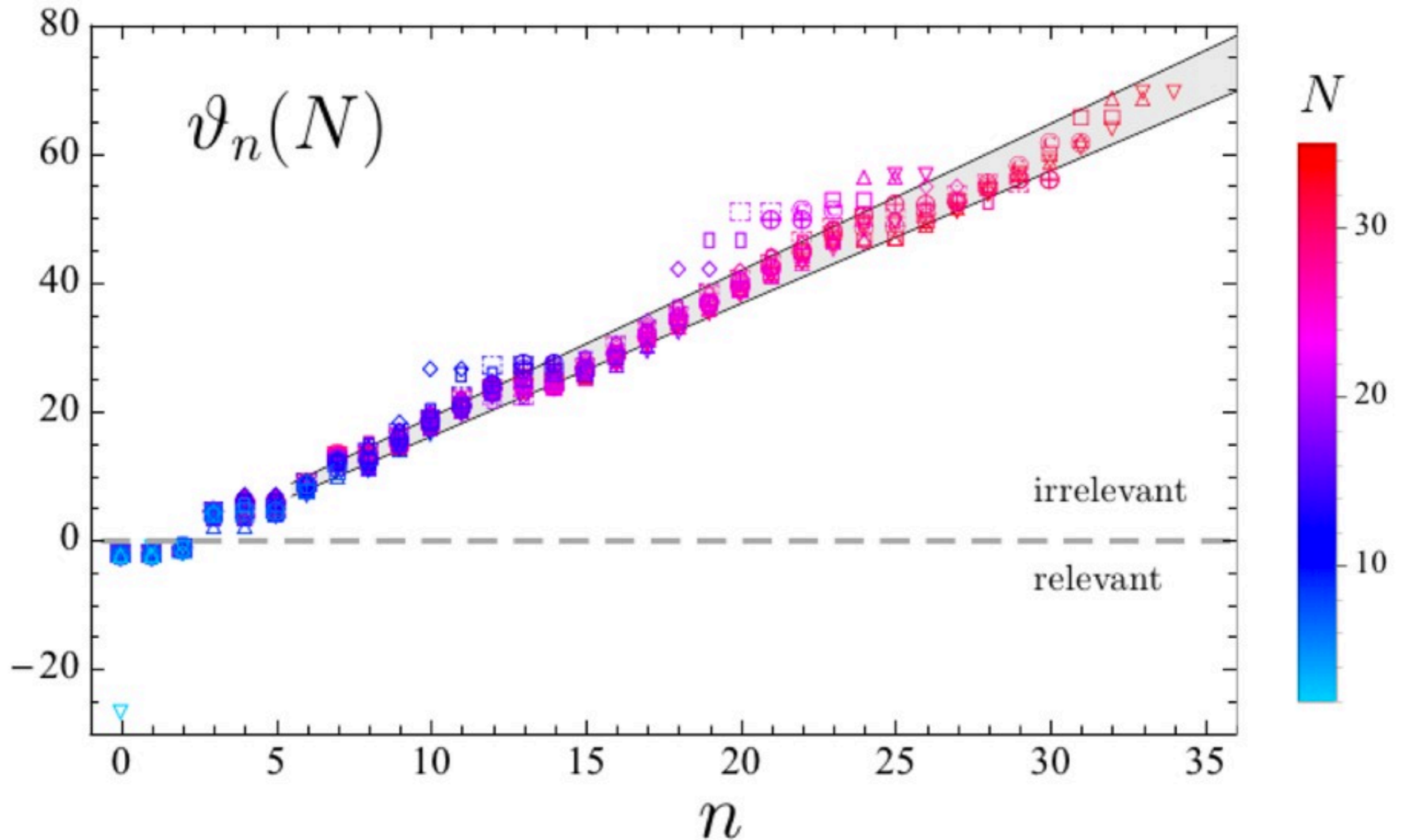


UV eigenvalues

$$\vartheta_n(N)$$



near-Gaussian



f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g} [f_k(R_{\mu\nu} R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu} R^{\mu\nu})]$$

$$\begin{aligned} \partial_t \Gamma[\bar{g}, \bar{g}] = & \frac{1}{2} \text{Tr}_{(2T)} \left[\frac{\partial_t \mathcal{R}_k^{h^T h^T}}{\Gamma_{h^T h^T}^{(2)}} \right] + \frac{1}{2} \text{Tr}'_{(1T)} \left[\frac{\partial_t \mathcal{R}_k^{\xi\xi}}{\Gamma_{\xi\xi}^{(2)}} \right] + \frac{1}{2} \text{Tr}''_{(0)} \left[\frac{\partial_t \mathcal{R}_k^{\sigma\sigma}}{\Gamma_{\sigma\sigma}^{(2)}} \right] + \frac{1}{2} \text{Tr}_{(0)} \left[\frac{\partial_t \mathcal{R}_k^{hh}}{\Gamma_{hh}^{(2)}} \right] \\ & + \text{Tr}''_{(0)} \left[\frac{\partial_t \mathcal{R}_k^{\sigma h}}{\Gamma_{\sigma h}^{(2)}} \right] - \text{Tr}_{(1T)} \left[\frac{\partial_t \mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma_{\bar{c}^T c^T}^{(2)}} \right] - \text{Tr}_{(0)'} \left[\frac{\partial_t \mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma_{\bar{\eta}\eta}^{(2)}} \right] - \text{Tr}''_{(0)} \left[\frac{\partial_t \mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma_{\bar{\lambda}\lambda}^{(2)}} \right] \\ & + \frac{1}{2} \text{Tr}''_{(0)} \left[\frac{\partial_t \mathcal{R}_k^{\omega\omega}}{\Gamma_{\omega\omega}^{(2)}} \right] - \text{Tr}'_{(1T)} \left[\frac{\partial_t \mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma_{\bar{c}^T c^T}^{(2)}} \right] + \frac{1}{2} \text{Tr}'_{(1T)} \left[\frac{\partial_t \mathcal{R}_k^{\zeta^T \zeta^T}}{\Gamma_{\zeta^T \zeta^T}^{(2)}} \right] + \text{Tr}'_{(0)} \left[\frac{\partial_t \mathcal{R}_k^{\bar{s}s}}{\Gamma_{\bar{s}s}^{(2)}} \right] \end{aligned}$$

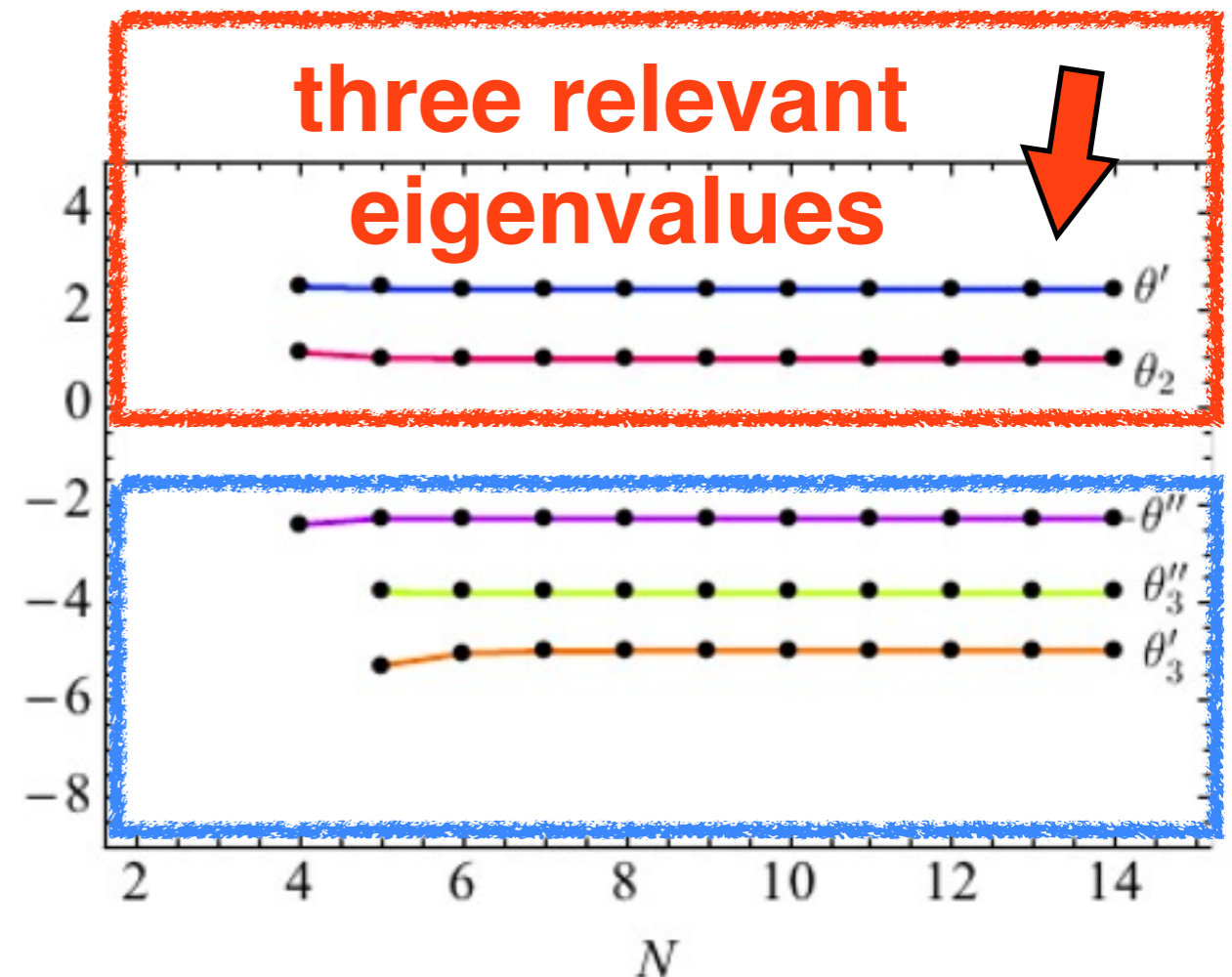
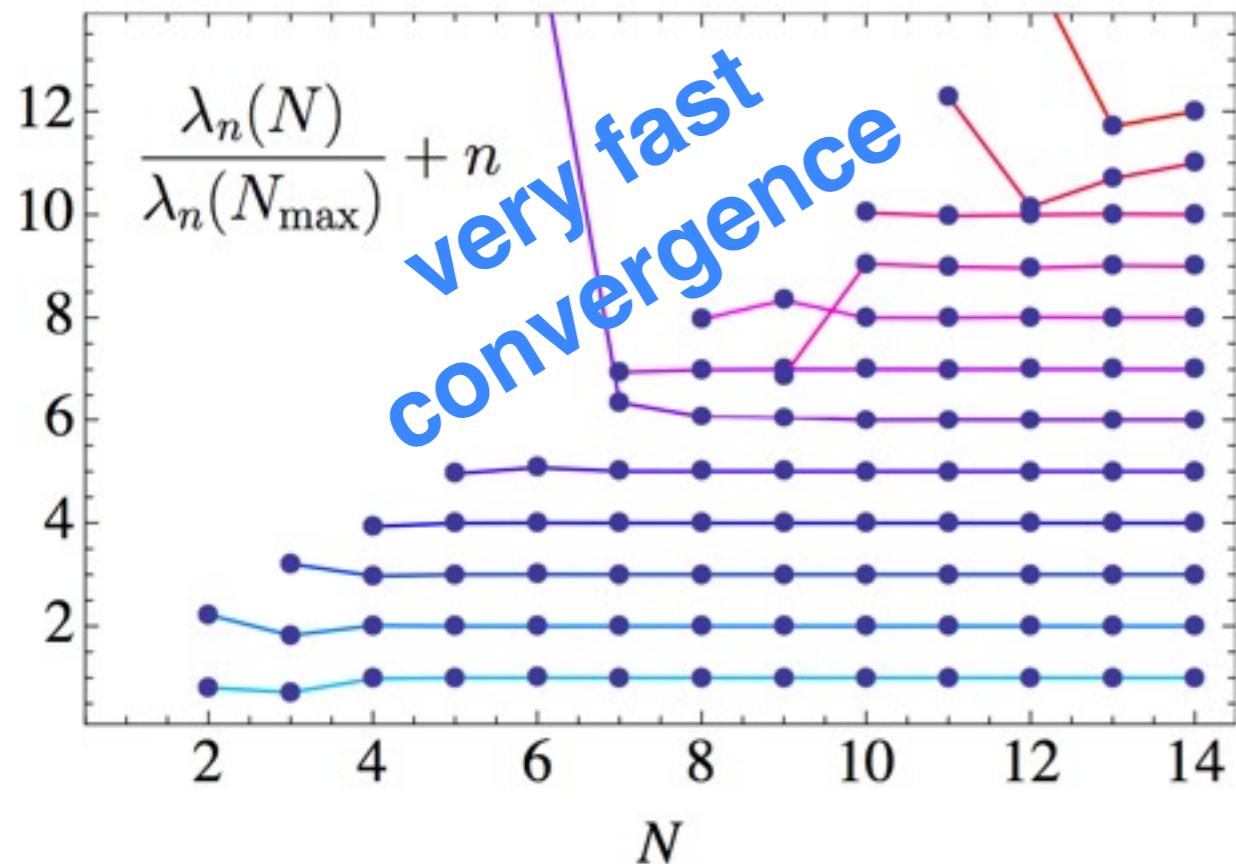
K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

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fixed point



conclusions

QFTs beyond asymptotic freedom

4D matter-gauge theories

exact **proof of existence**

requires **elementary** scalars, fermions, vectors

no additional (super)symmetry

4D quantum gravity

systematic **non-perturbative** search strategies

strong hints towards interacting UV fixed point

field-dependent **anomalous dimension**

conclusions

what's next?

4D matter-gauge theories

composite operators

UV FP beyond perturbation theory?

realistic models beyond SM?

4D quantum gravity

test further curvature invariants

include **matter fields**

combined FP for **gravity-matter** theories?