High intensity laser matter interaction

15th January 2015

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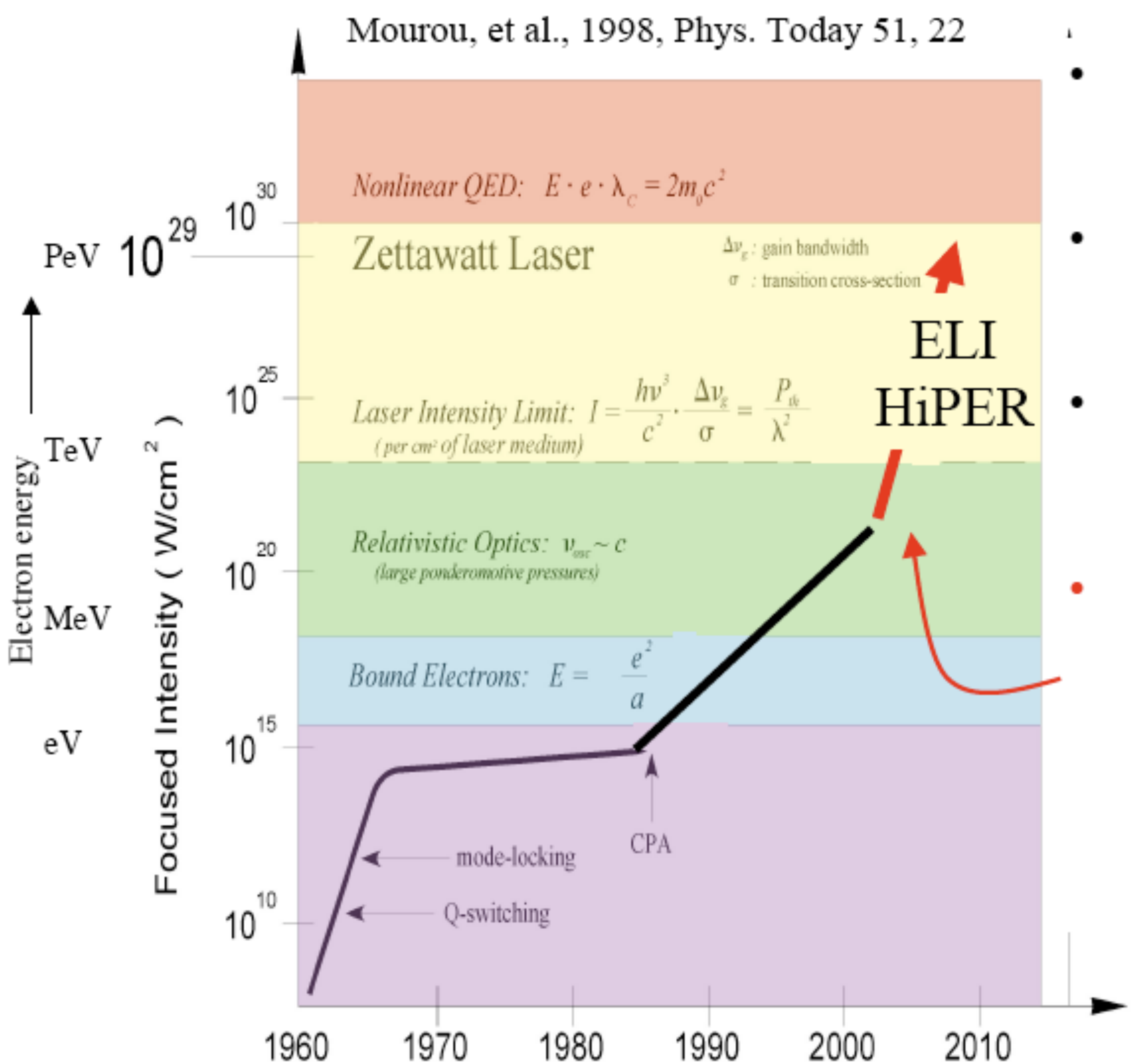
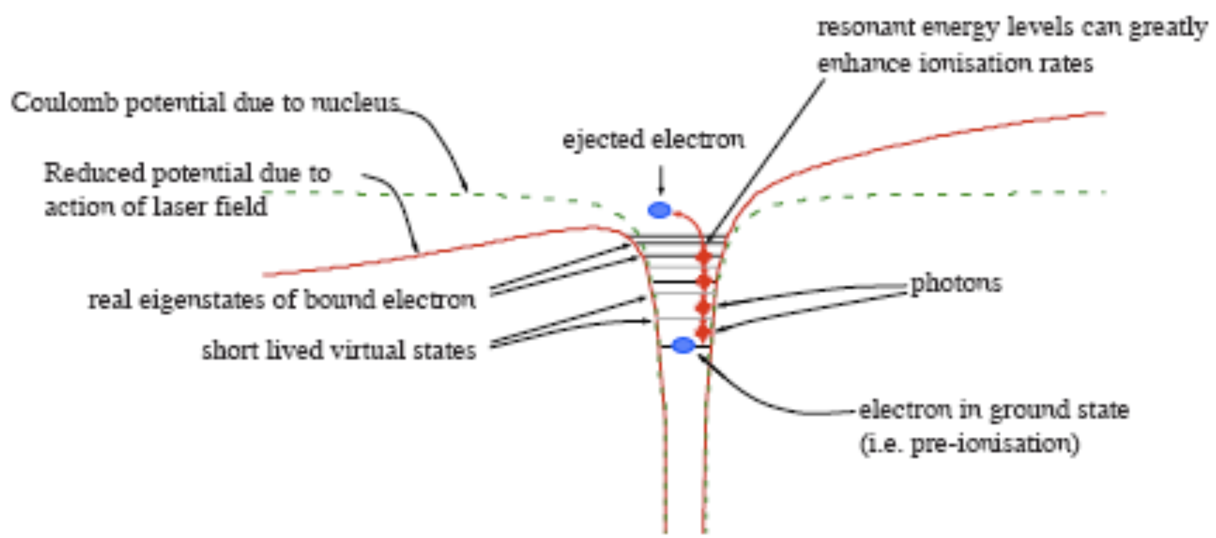
High Intensity Lasers

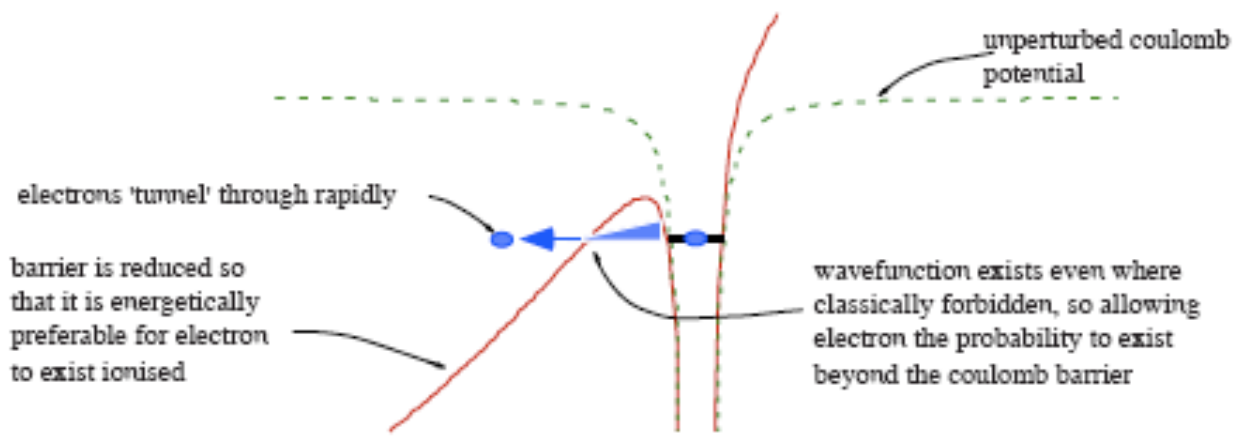


Field-ionisation

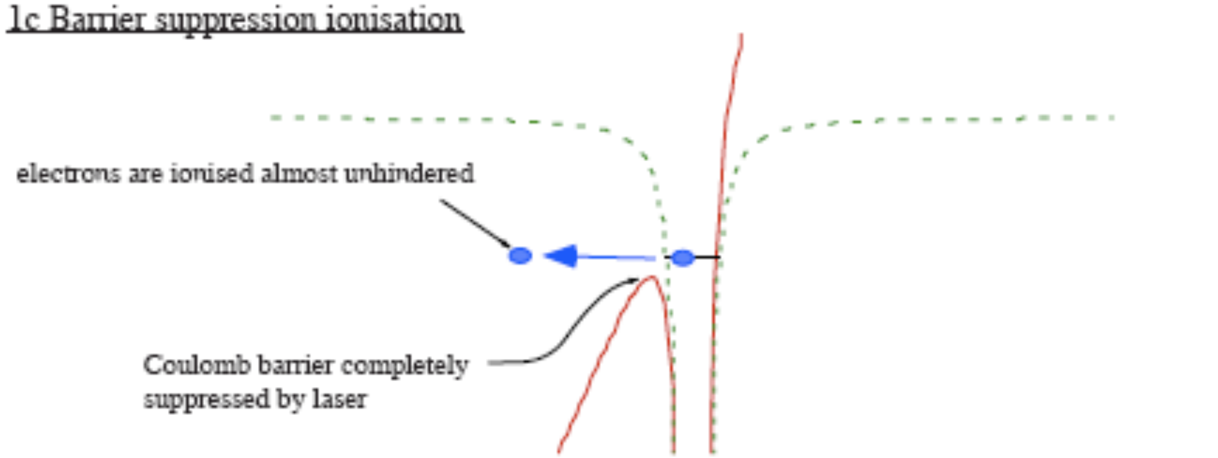
1a Multi-photon ionisation (γ»1)



1b Tunnel ionisation (γ«1)

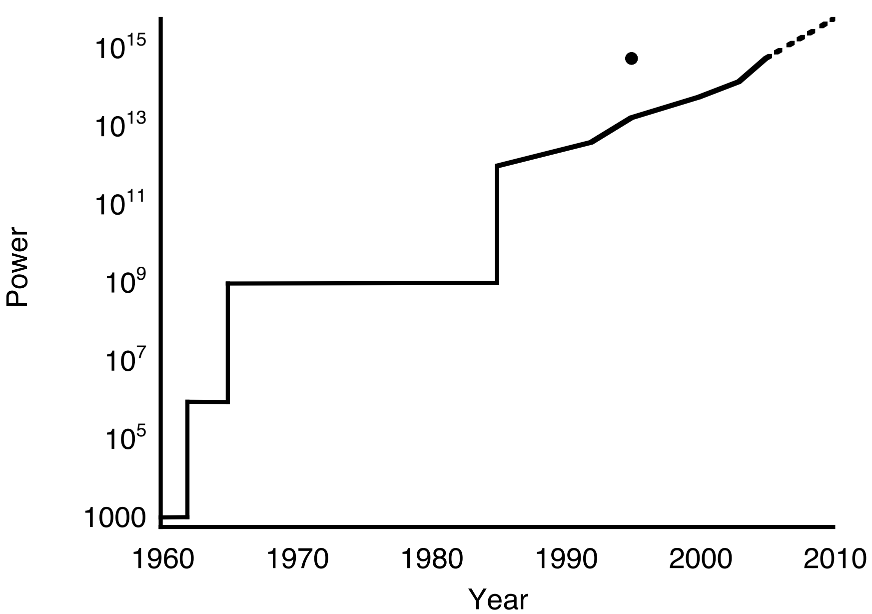


1c Barrier suppression ionisation



High intensity lasers

The evolution of Power of small diameter laser systems



State of art e.g. Vulcan Petawatt, 400 J in 400 fs = 1 PW
(cf. UK power output $\sim$ 100 GW)
focused to diameter spot $\phi = 5\mu$m,
intensity $I \sim 1 \cdot 10^{21}$ Wcm^{-2} ($= 1 \cdot 10^{25}$ Wm^{-2})

Poynting vector: $\mathbf{I} = \mathbf{E} \times \mathbf{H} = \mathbf{E} \times \mathbf{B}/\mu_\mathbf{0} = \mathbf{E^2\hat{k}/c\mu_0}$
For $E = E_0 \cos\omega t$, $< \mathbf{I} >= \frac{1}{2} E_0^2/c\mu_0$, so $E_0 = \sqrt{2c\mu_0 I}$

NB for VulcanPW, $\mathbf{E} \sim 9 \times 10^{11}$ Vcm^{-1}, *cf* 5×10^7 Vcm^{-1} $= \mathbf{E_{Bohr}}$

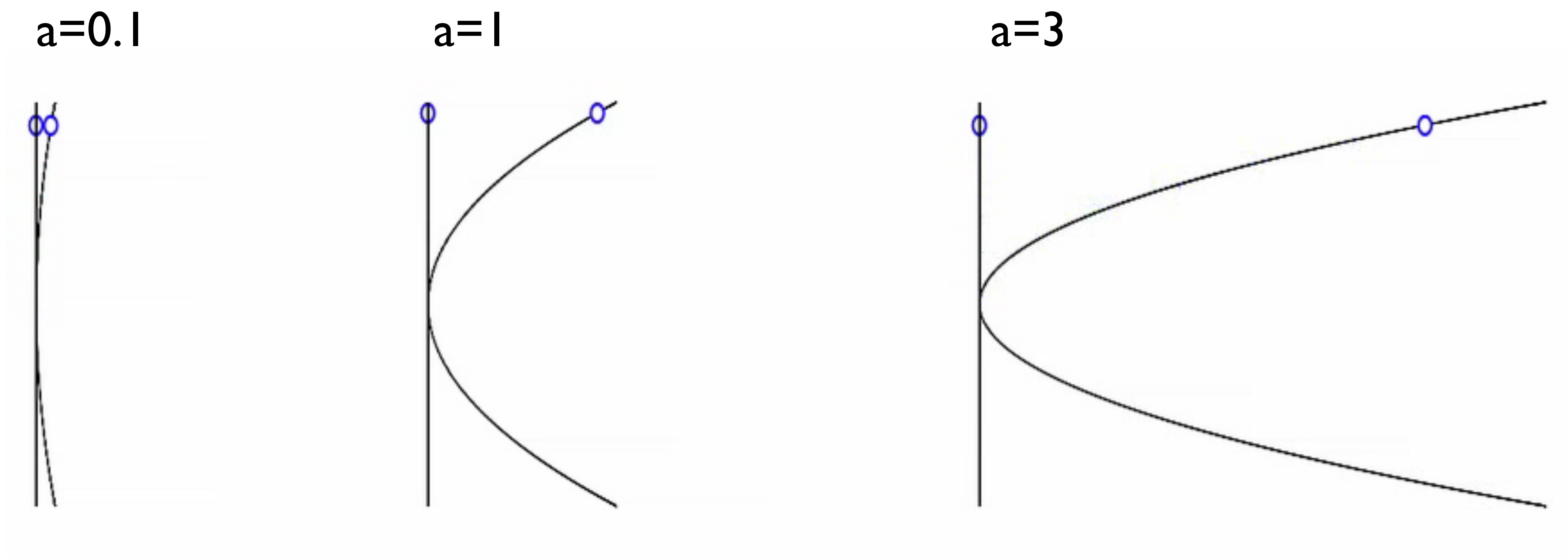
Since $\dot{p_x} = -eE_0 \cos\omega t$, $p_x = -eE_0 \cos\omega t/\omega$,
Define $a_0 = eE_0/m_e c\omega$,
i.e. $\gamma v_x = -a_0 c \cos\omega t$,

so a_0 is "normalised momentum", or "normalised vector potential",

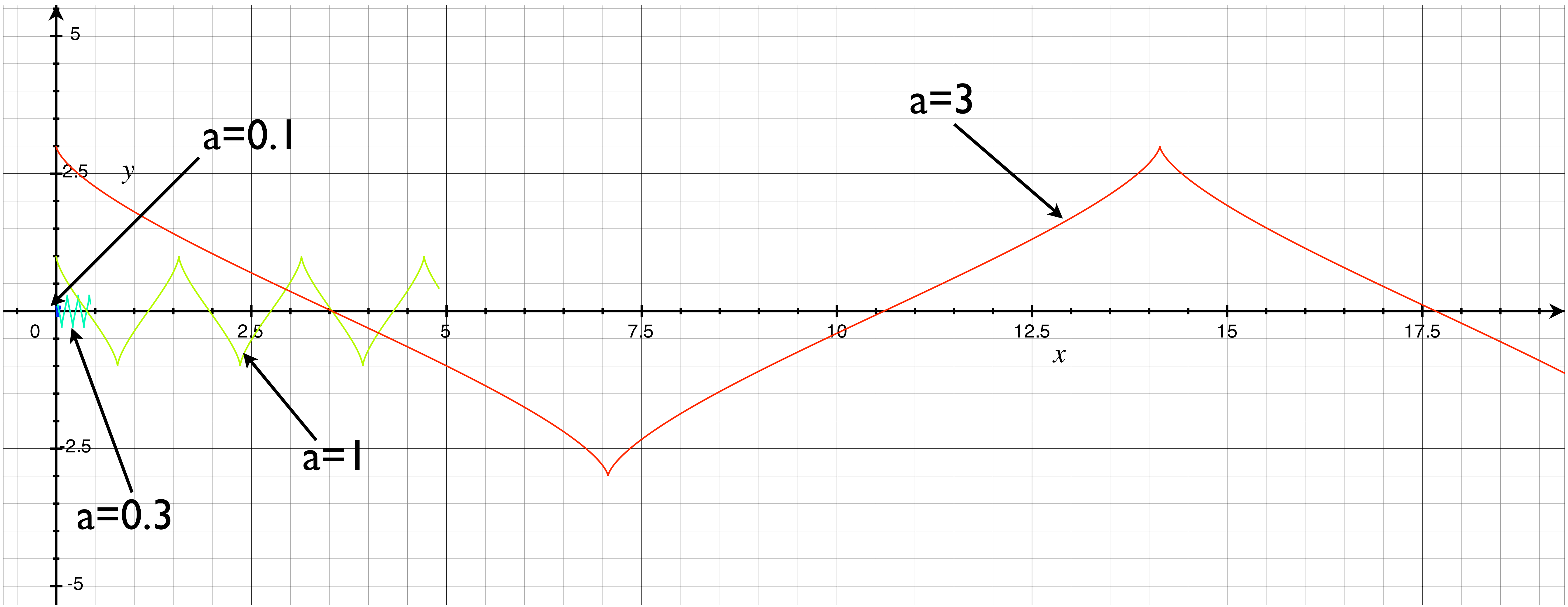
numerically $a_0 \simeq 0.856(I\lambda^2)^{\frac{1}{2}}$

Some phase space trajectories

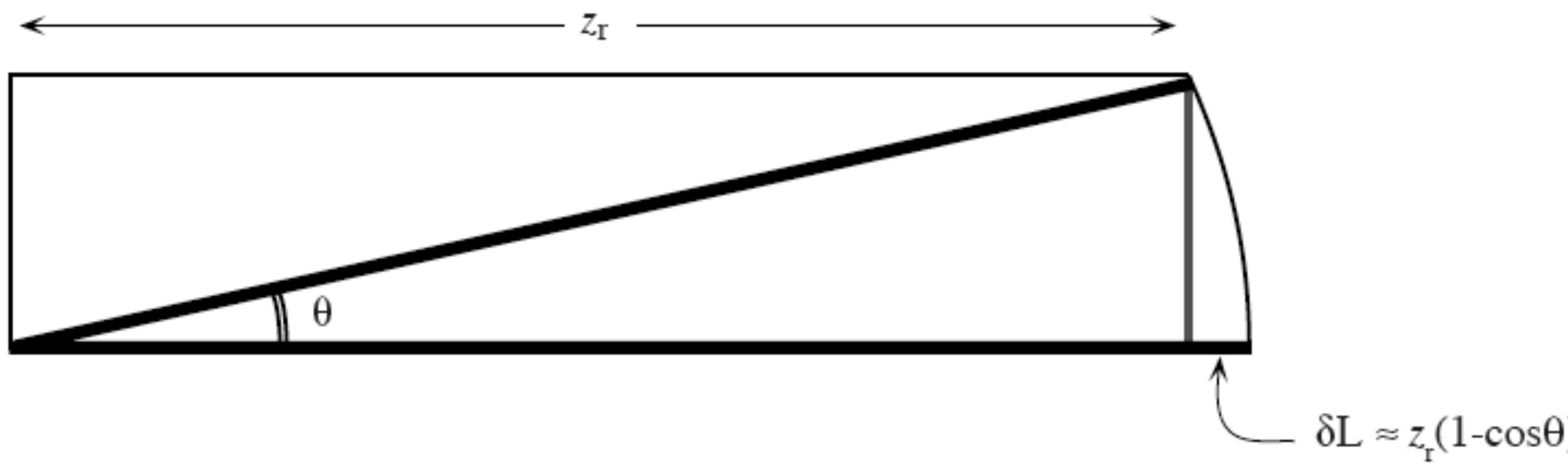
$$p_x = a_x,\ p_y = a_y \text{ and } p_z = \frac{1}{2}a_z^2$$



Trajectory of relativistic electrons



Relativistic self-focusing



Plasma Propagation

$$\nabla^2 \mathbf{E} - \frac{\eta^2}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

density

relativity

$$\eta_R \simeq 1 - \frac{{\omega_p}^2}{2\omega^2}\frac{n(r)}{n_0\gamma_r} \simeq 1 - \frac{{\omega_p}^2}{2\omega^2}\left(1 + \frac{\delta n}{n_0} - \frac{a^2}{2}\right)$$

where we used $\gamma = \sqrt{1 + {a_0}^2}$

For a gaussian pulse of beam width *R*

defocusing

focusing

$$\frac{d^2 R}{dz^2} = \frac{1}{{z_R}^2 R^3}\left(1 - \frac{P}{P_c}\right).$$

$$P_{cr} \simeq 17\,(n_e/n_{cr})\,\mathrm{GW}$$

P. Sprangle et al, PRL, **59**, 202 (1987)

Plasma Propagation

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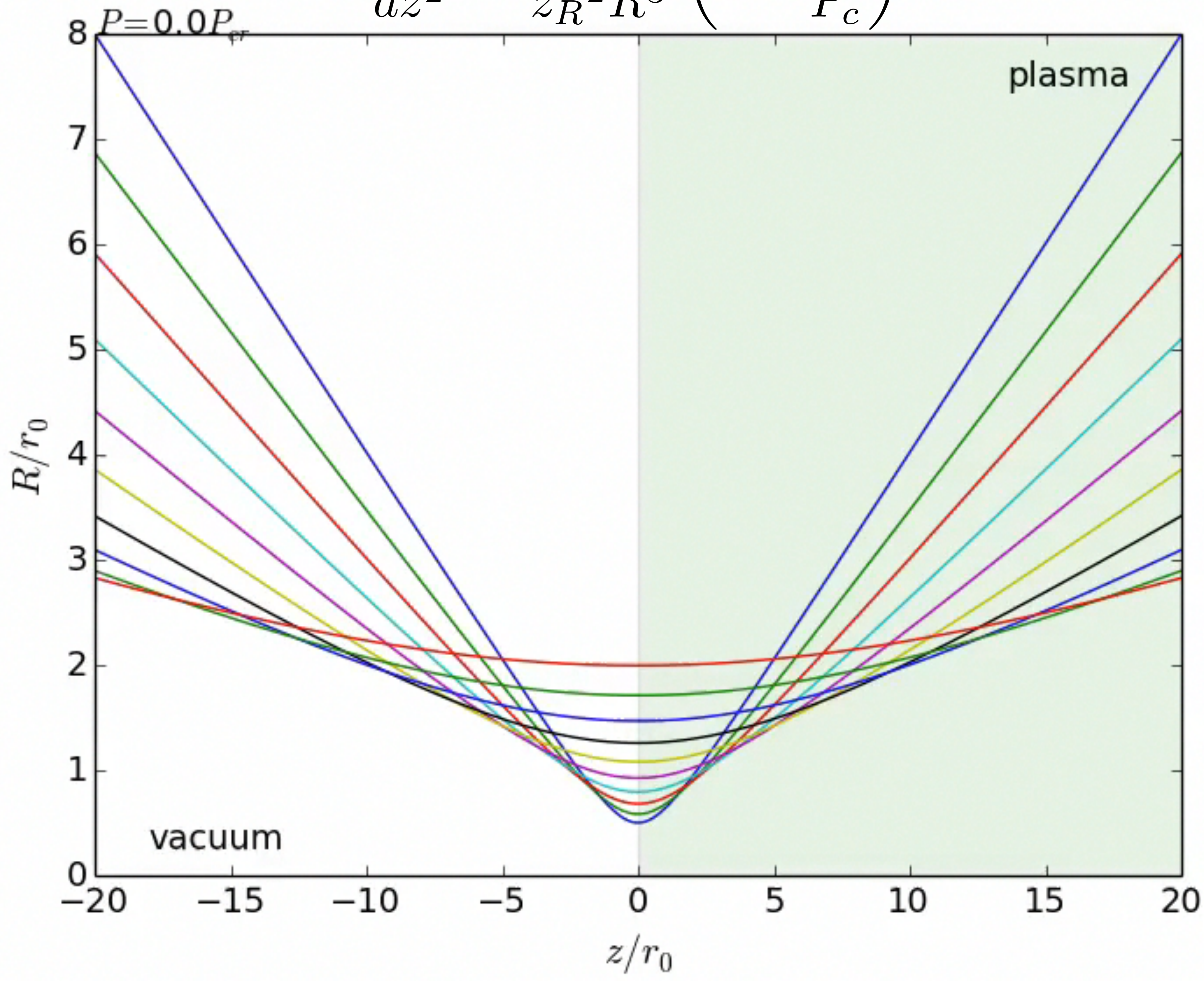

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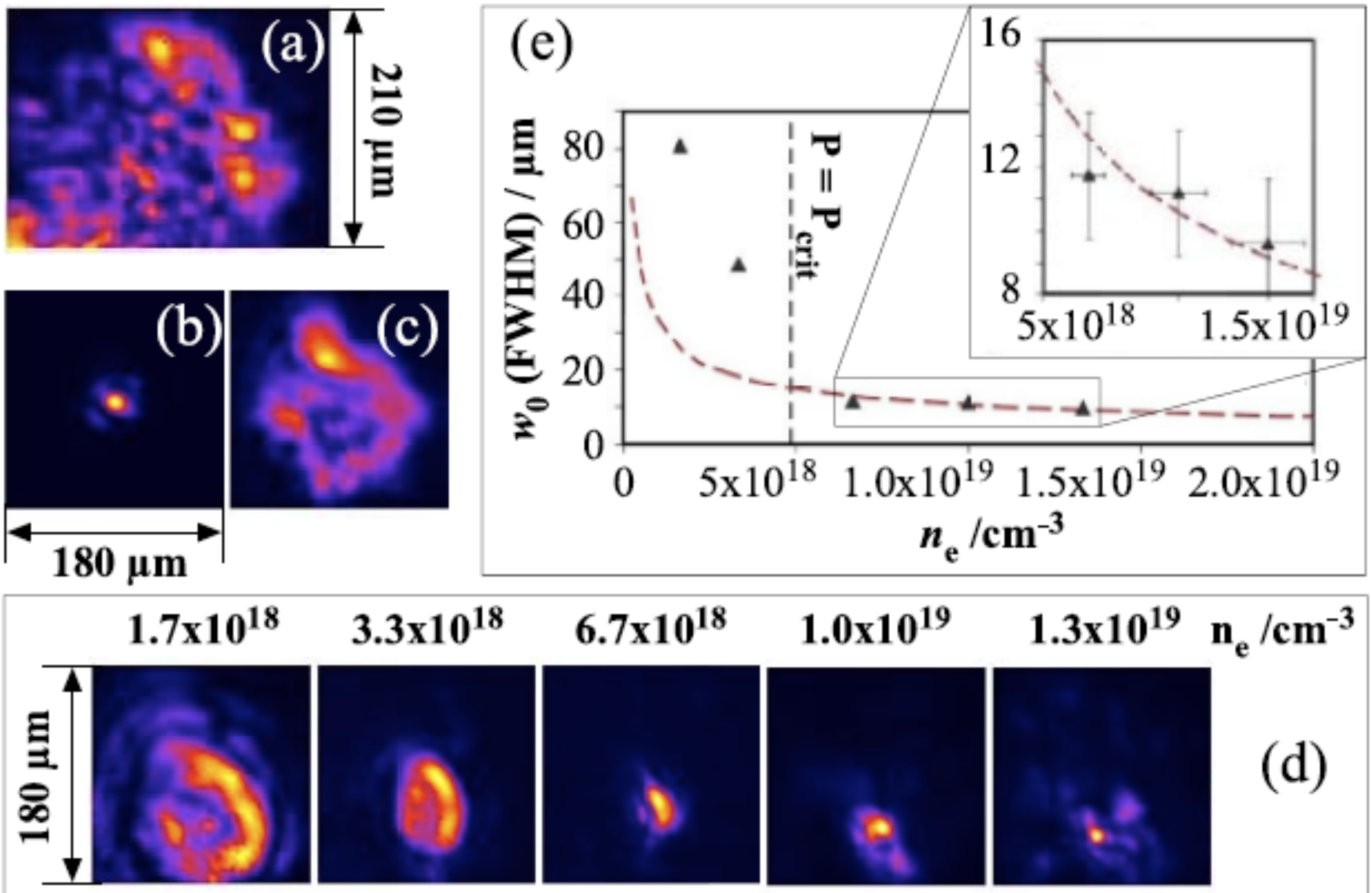
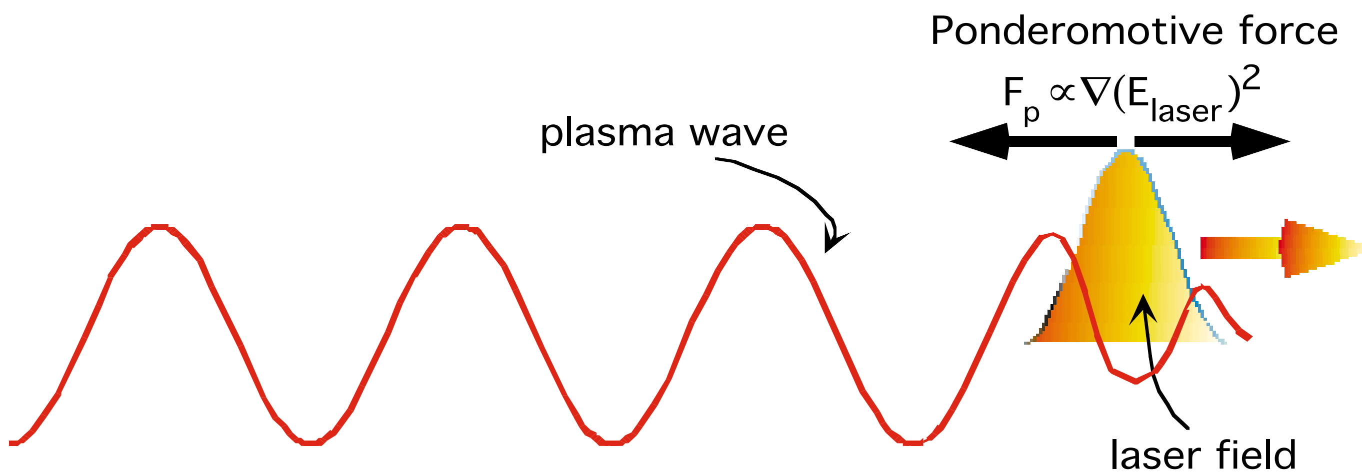


Wakefield generation



we start with the equation of motion, the continuity equation and Gauss's law

$$m\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = -e(\mathbf{E} + \mathbf{u}\times\mathbf{B})$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0$$

$$\nabla\cdot\mathbf{E} = \rho/\epsilon_0 = -e(n_e - n_i)/\epsilon_0$$

The non-linear force terms can be grouped by noting that $\mathbf{B} = \nabla\times\mathbf{A}$, and that to first order $\mathbf{u} = (e/m)\mathbf{A}$ (which is the conservation of canonical momentum). So,

$$\begin{aligned} -m(\mathbf{u}\cdot\nabla)\mathbf{u} - e(\mathbf{u}\times\mathbf{B}) &\simeq -(e^2/m)\left[(\mathbf{A}\cdot\nabla)\mathbf{A} + (\mathbf{A}\times(\nabla\times\mathbf{A})\right] \\ &= (e^2/2m)\nabla A^2 \\ &= \frac{1}{2}mc^2\nabla a^2 \end{aligned}$$

Thus the non-linear terms together combine to make the ponderomotive force, by use of a vector relation.

Wakefield generation

Solving (in 1D):

$$\frac{\partial E}{\partial \zeta} = -n_1 \qquad \text{(Gauss' Law)}$$

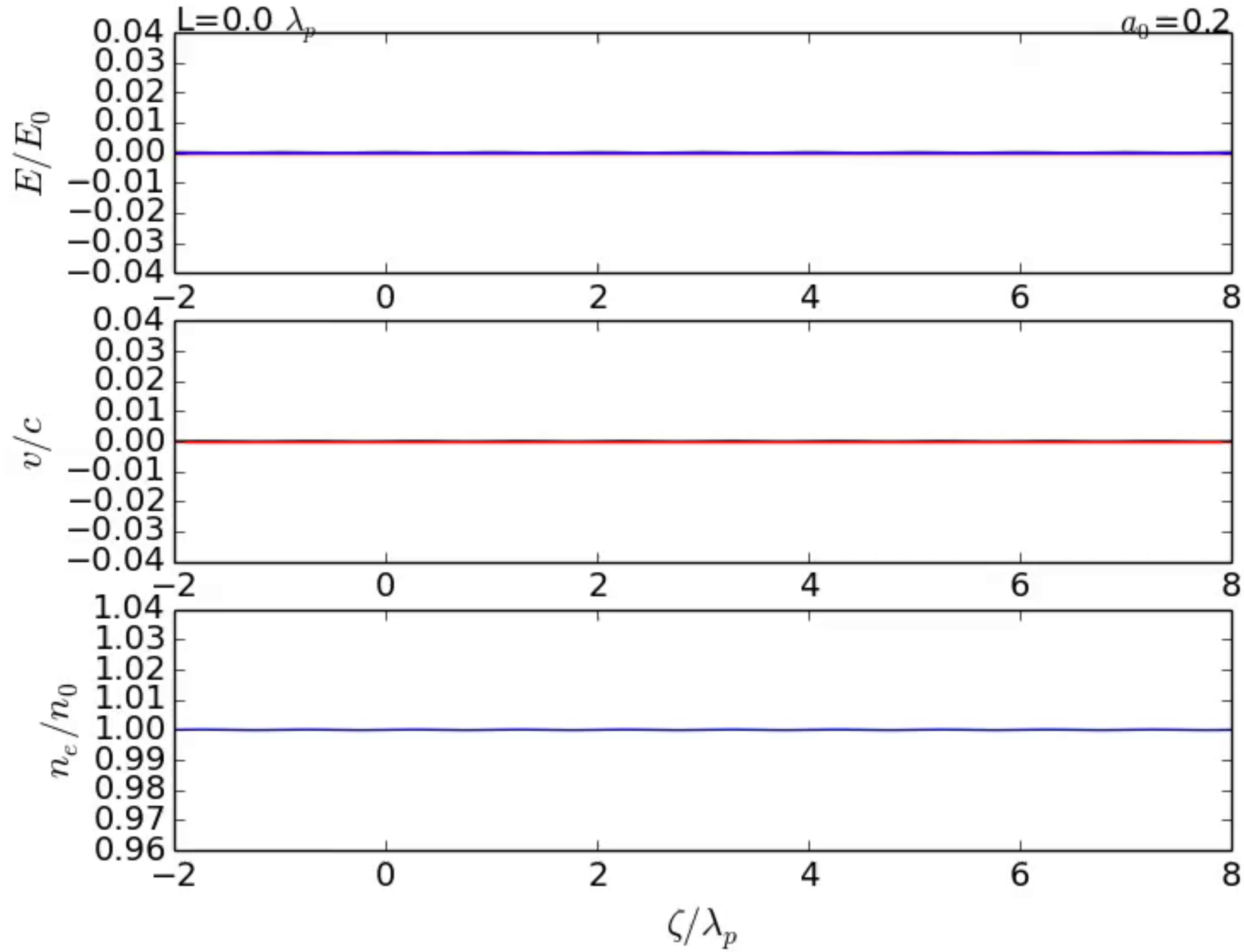
$$n_1 = n_0 \beta \qquad \text{(Continuity)}$$

$$\frac{\partial \beta}{\partial \zeta} = eE - \frac{\partial (a^2)}{\partial \zeta} \qquad \text{(Motion)}$$

Assuming beta ≪ *1*, $n_1 \ll n_0$ $\qquad n_e = n_0(1+\beta)$

Have coupled equations in *E* and β to solve

Wakefield generation



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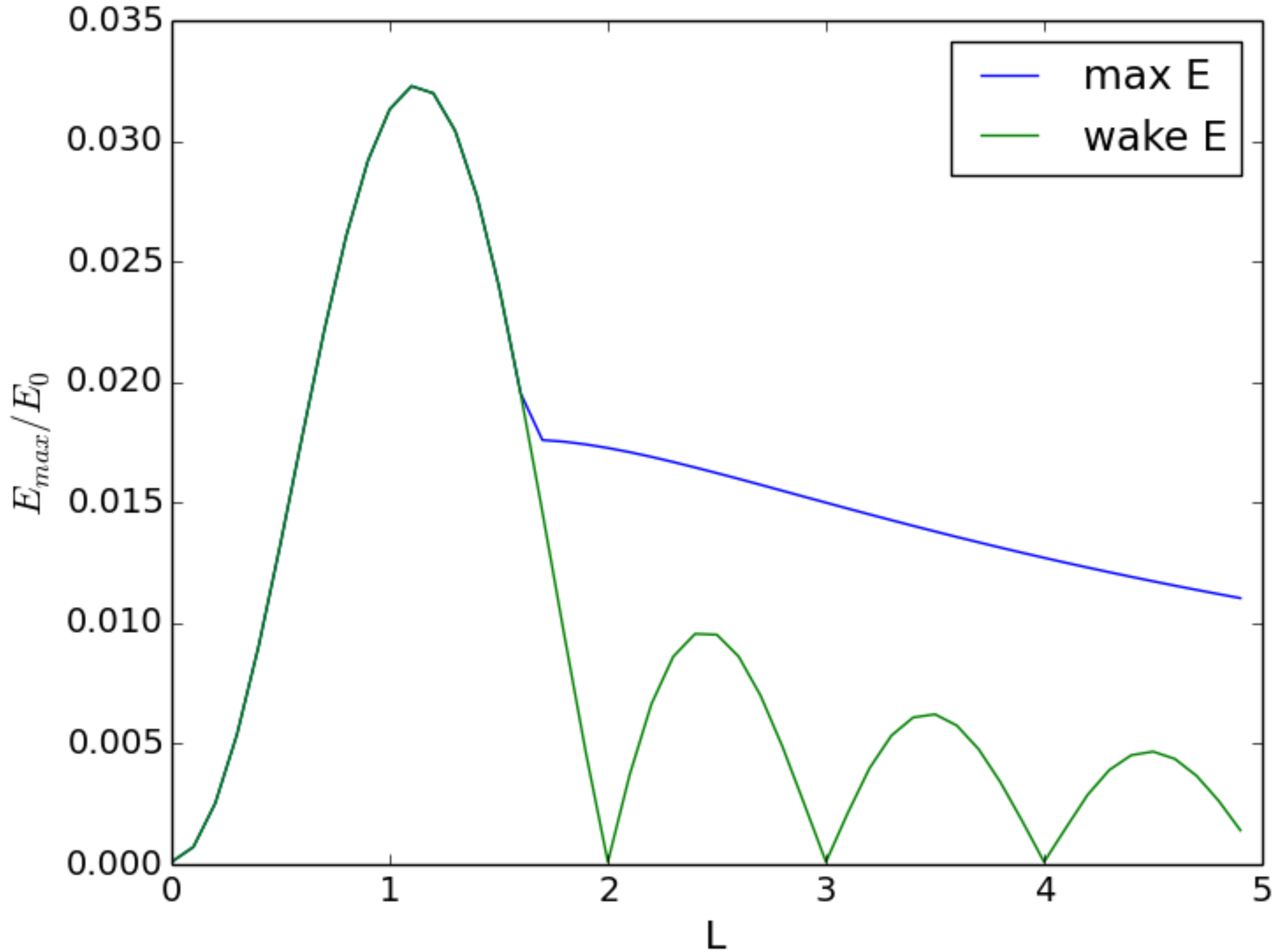
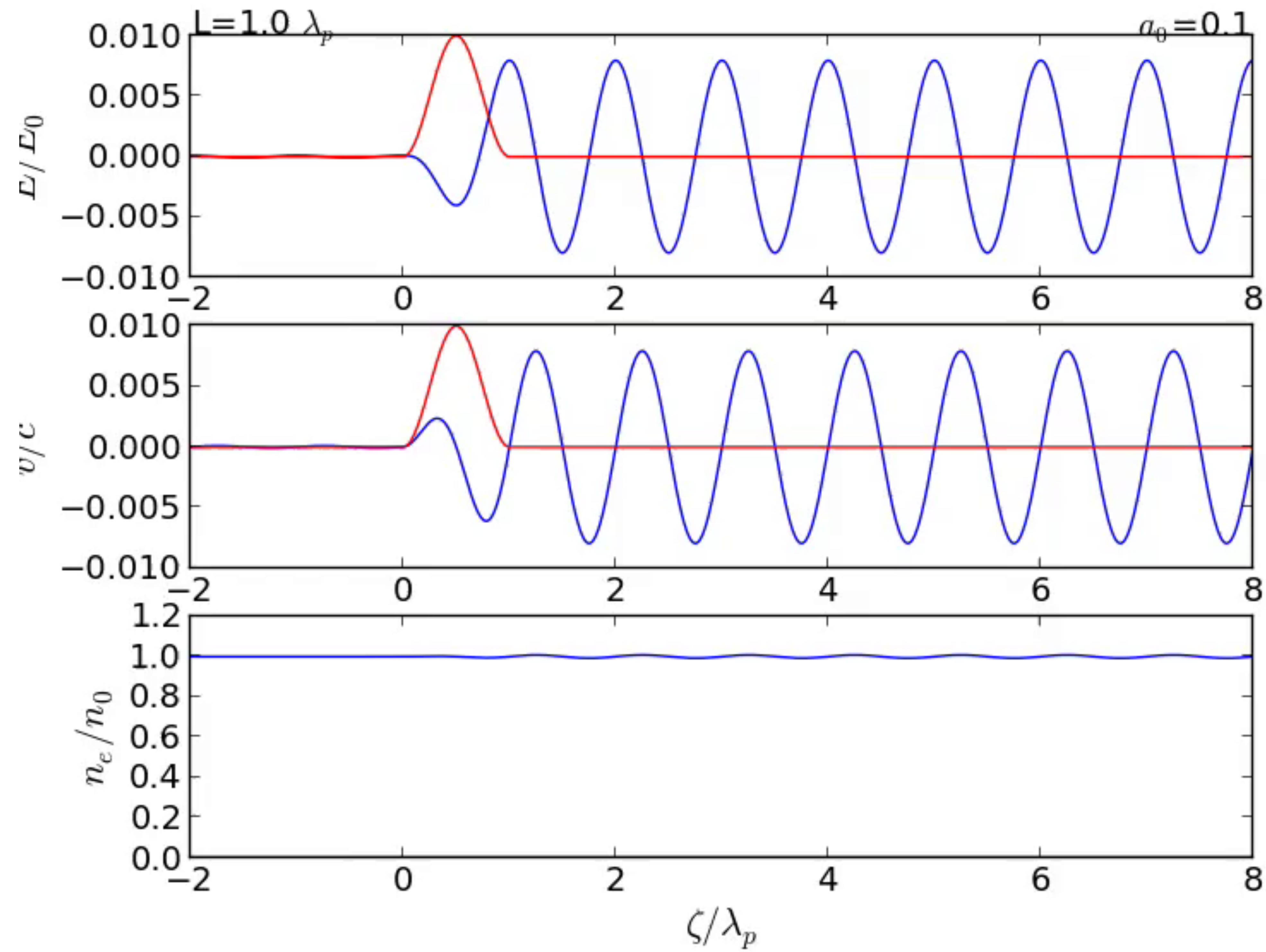
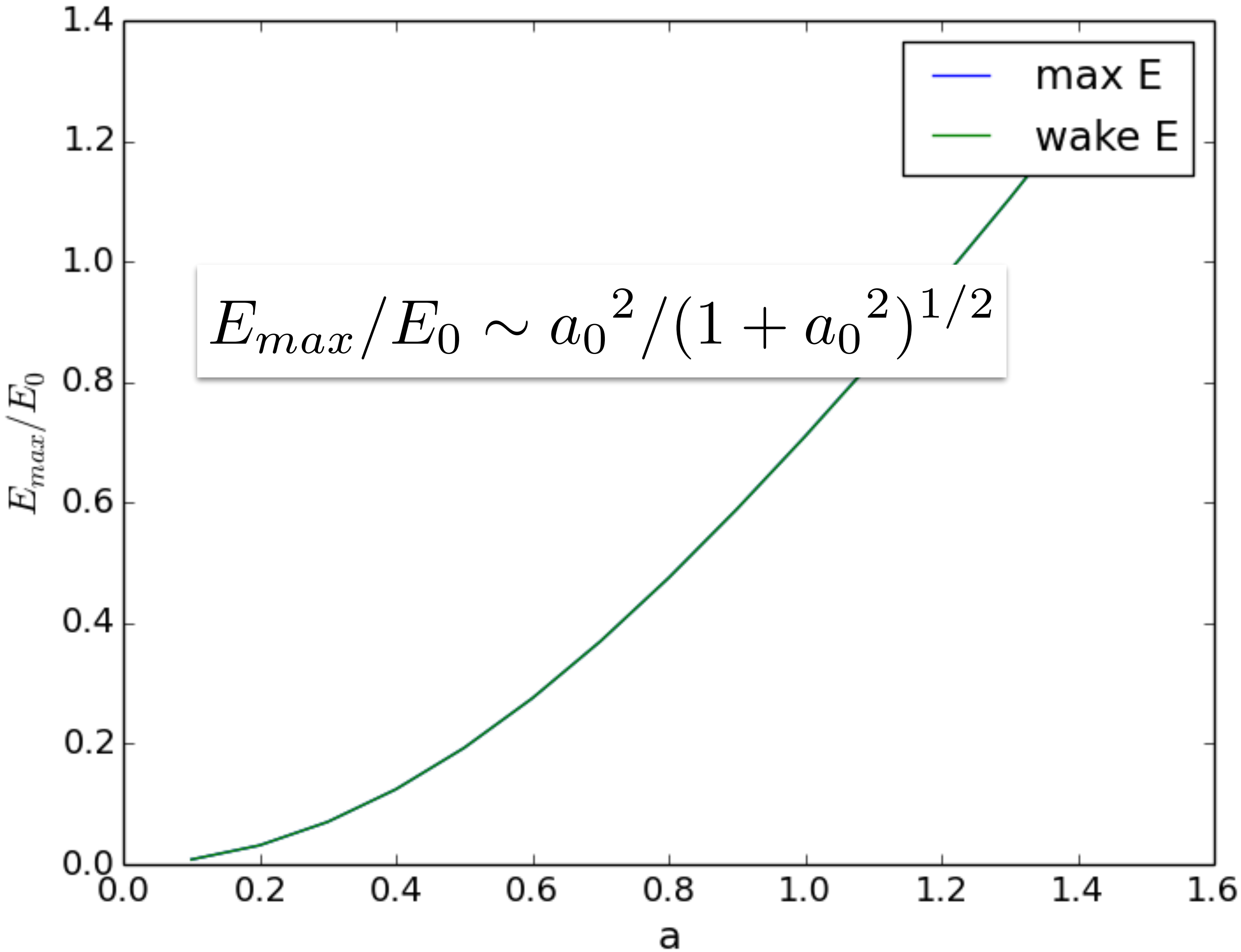


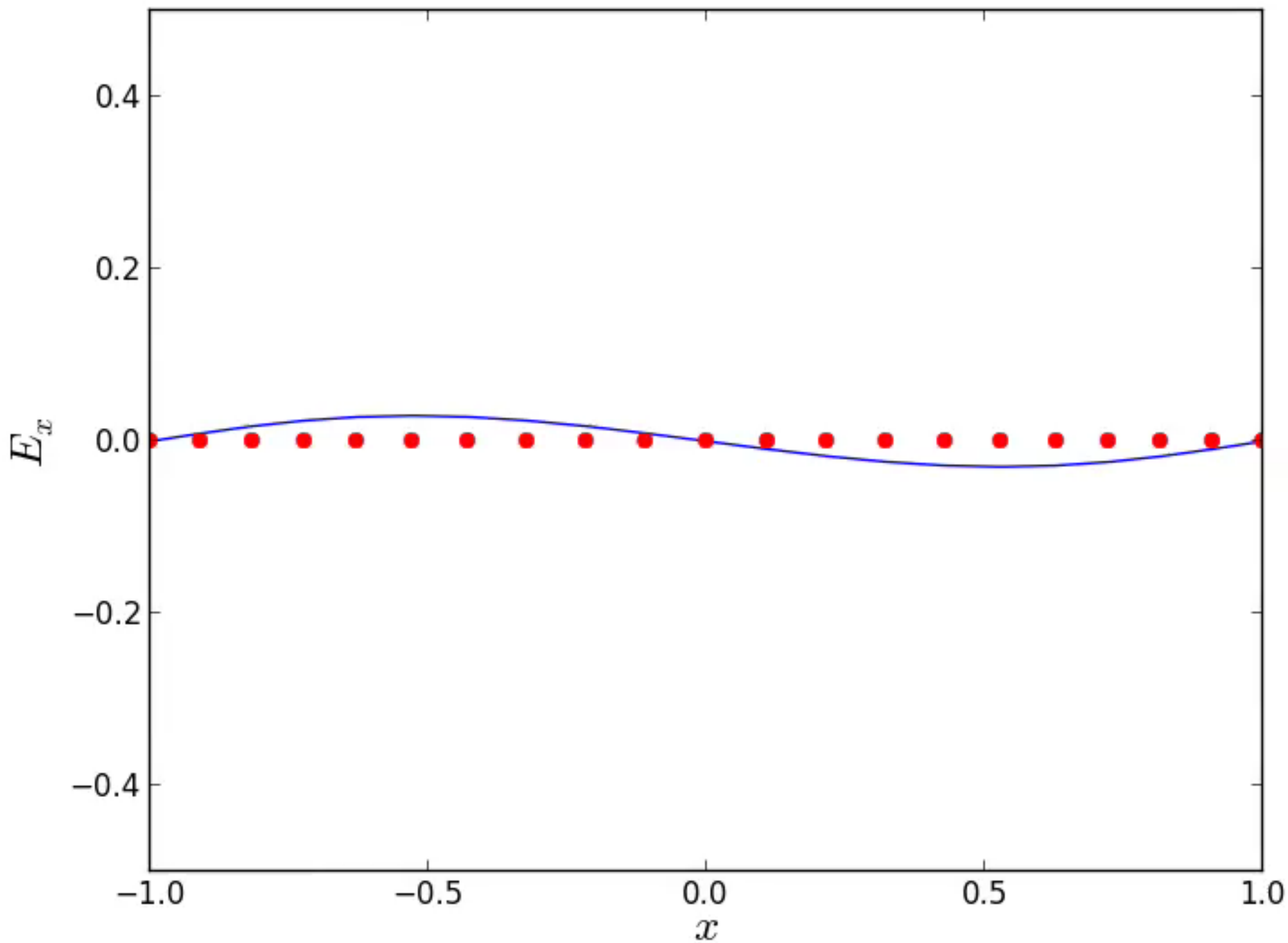
Wakefield generation



Wakefield generation



small amplitude oscillations



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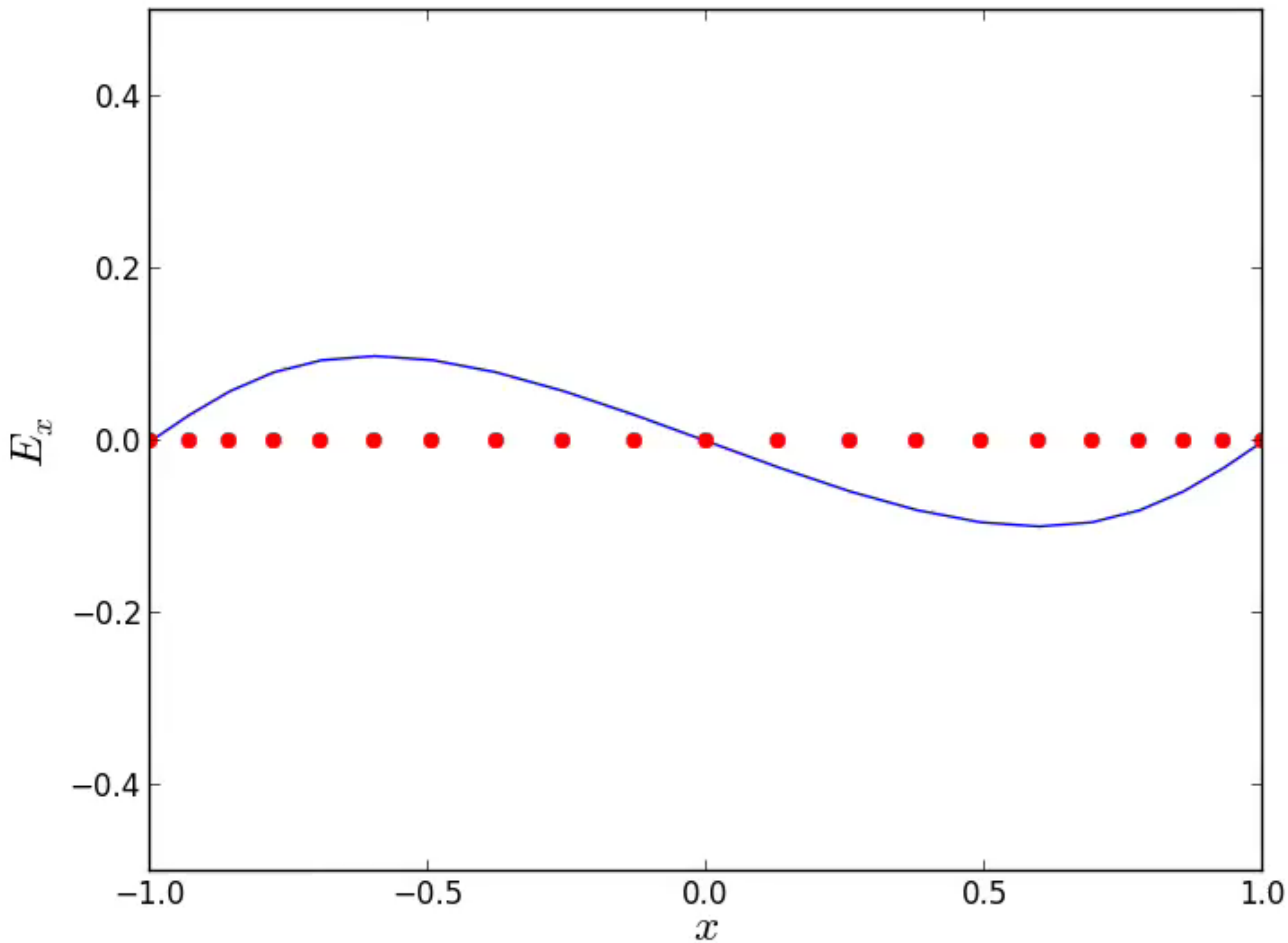

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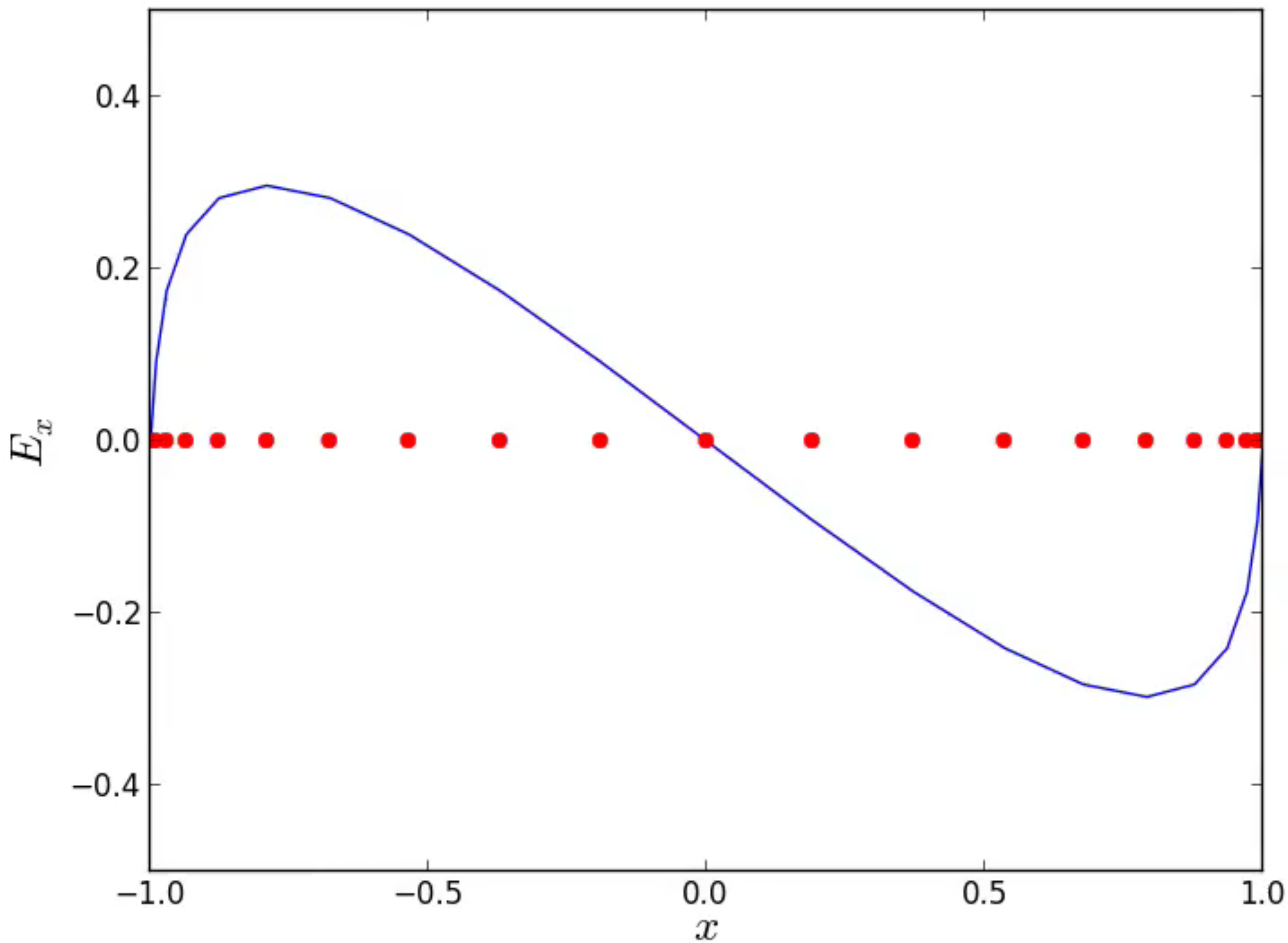
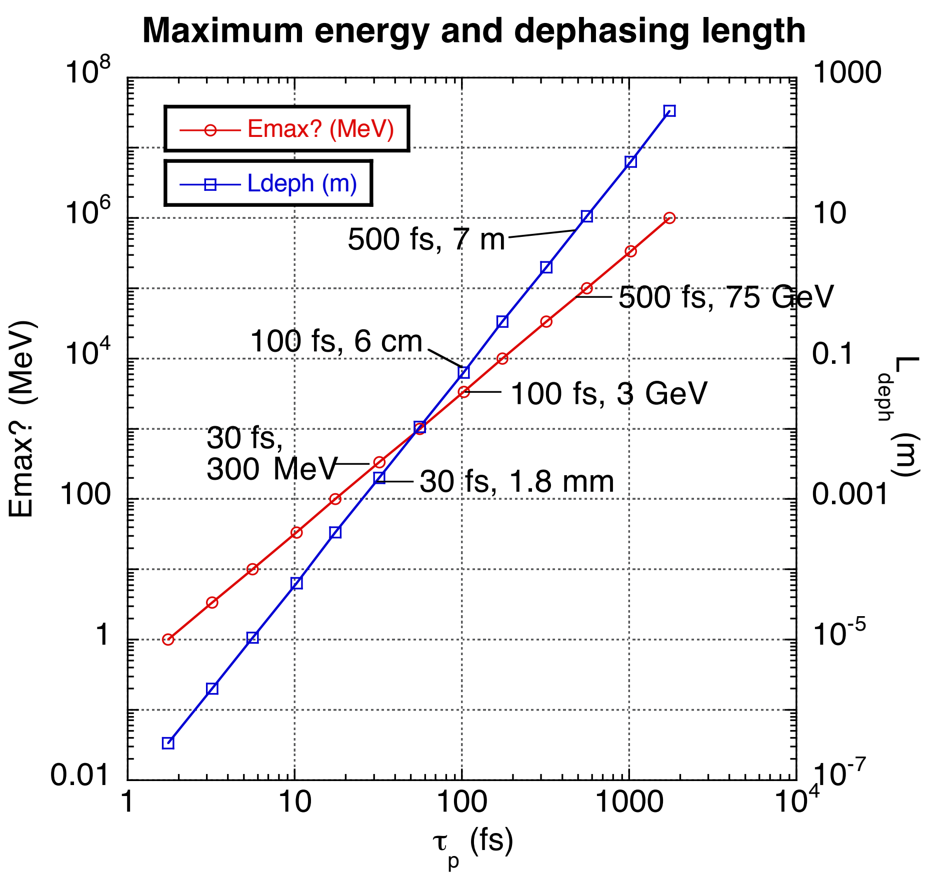
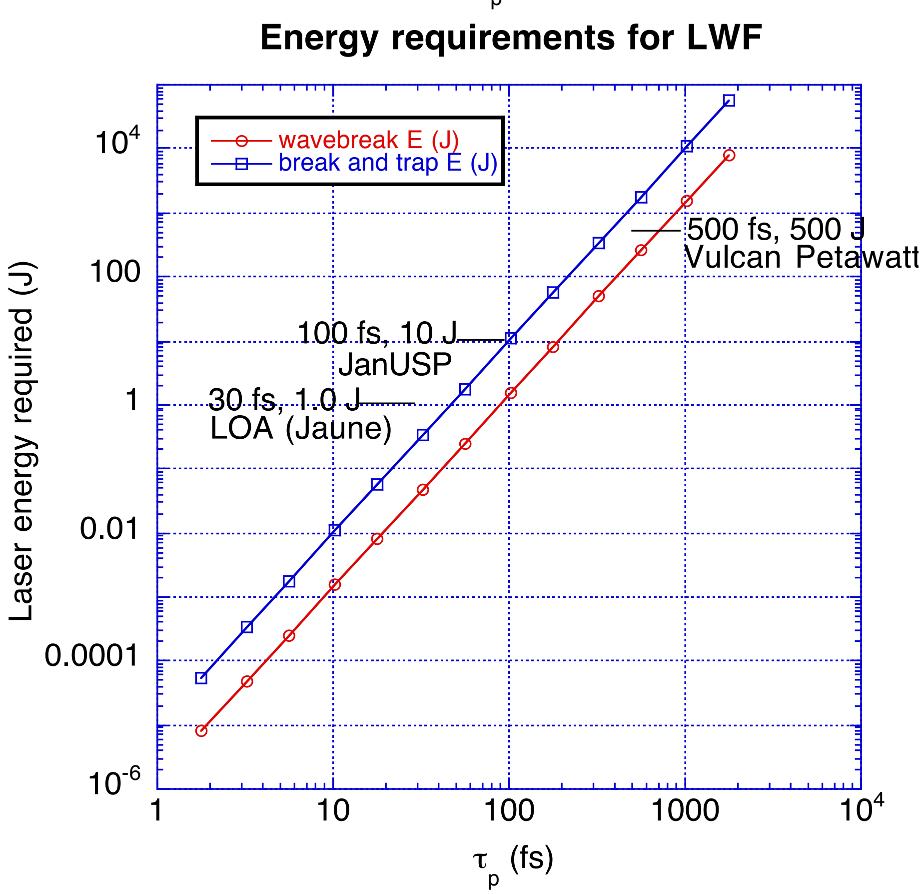


Energy gain





The maximum electric field that a wakefield can support is when $n = n_0$, and from Gauss's law, then the maximum electric field is given by;

$$E_m = -\left(\frac{e}{\epsilon_0}\right)\int n_0 sin(k_p\xi) \quad \rightarrow \quad E_{max} = n_0 e/k_p\epsilon_0 = \left(\frac{mc\omega_p}{e}\right)$$

For $a_0 \rightarrow 1$, electrons can overrun the wave oscillation and become injected into a acceleration phase.

For a field of $E = (mc\omega_p/e)\sin(k_p x - w_p t)$ in wave frame (Lorentz lengthened),

$$E' = (mc\omega_p/e)\sin(k_p x'/\gamma)$$

the potential is

$$\phi' = (\gamma mc\omega_p/k_p e)\cos(k_p x'/\gamma) = (\gamma mc^2/e)\cos(k_p x'/\gamma)$$

Energy gain in only 1/4 of wave,

$$\rightarrow W' = [e\phi]_{x=0}^{x=\pi\gamma/2k_p} = \gamma mc^2$$

and the momentum is $\sim p' = \gamma m\beta$ But the Lorentz transform

$$W = \gamma(E' + \beta cp') = \gamma^2 mc^2(1+\beta^2)$$

which for $\beta \rightarrow 1$ gives

$$W_{max} = 2\gamma^2 mc^2$$

NB the length over which this acceleration happens is $\gamma^2\lambda_p$

Wakefield simulation