Laser-plasma interactions

Reference material:

"Physics of laser-plasma interactions" by W.L.Kruer

"Scottish Universities Summer School Series on Laser Plasma Interactions"
number 1 edited by R. A. Cairns and J.J.Sanderson
number 2 edited by R. A. Cairns
number 3 - 5 edited by M.B. Hooper

Outline

1. Introduction
   Ionisation processes

1. Laser pulse propagation in plasmas
   Resonance absorption
   Inverse bremsstrahlung
   Not-so-resonant resonance absorption (Brunel effect)

3. Ponderomotive force

4. Instabilities in laser-produced plasmas
   Stimulated Raman scatter (SRS) instability
   Stimulated Brillouin scatter (SBS) instability
   Two plasmon decay
   Filamentation

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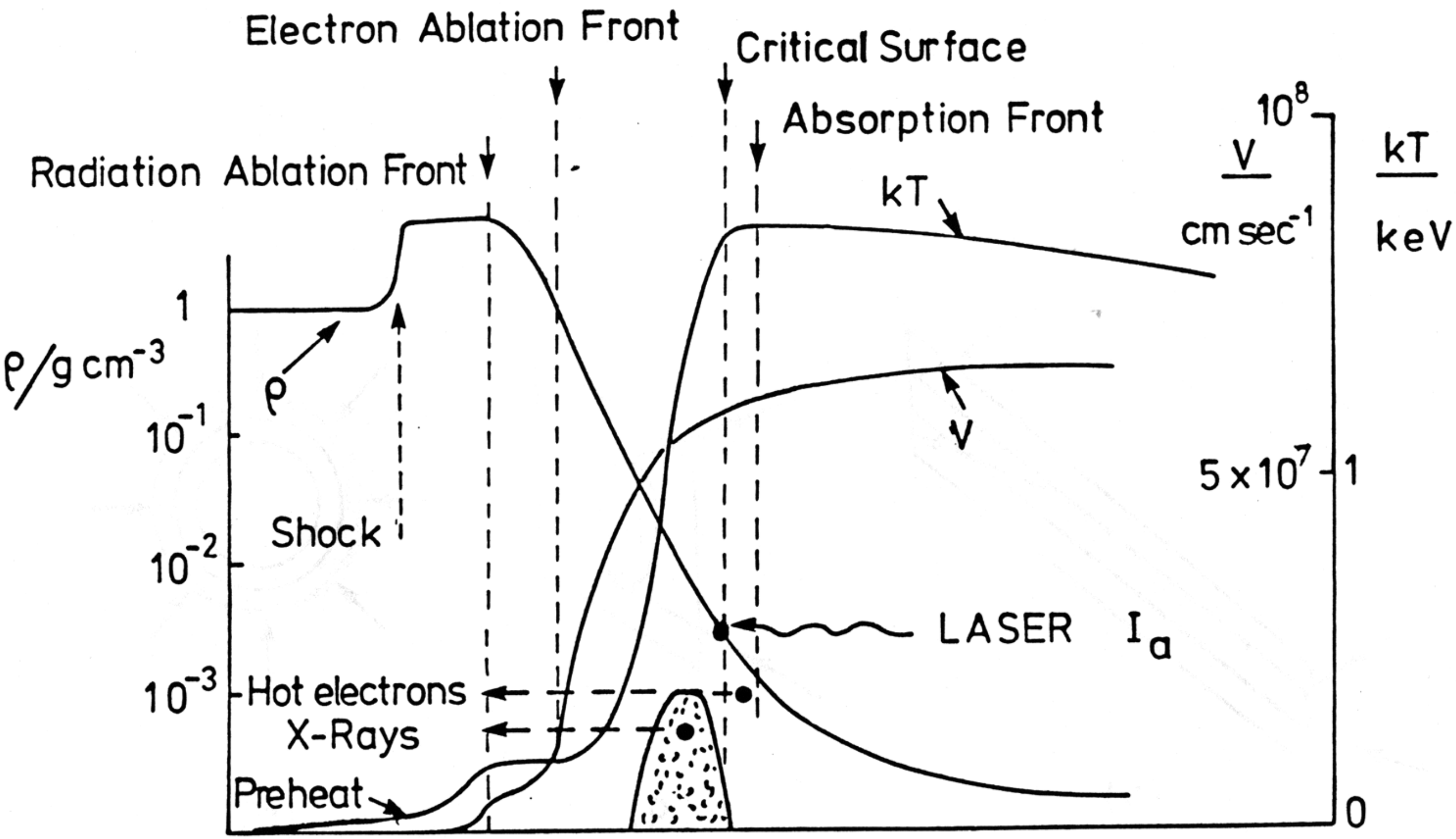
Structure of a laser produced plasma

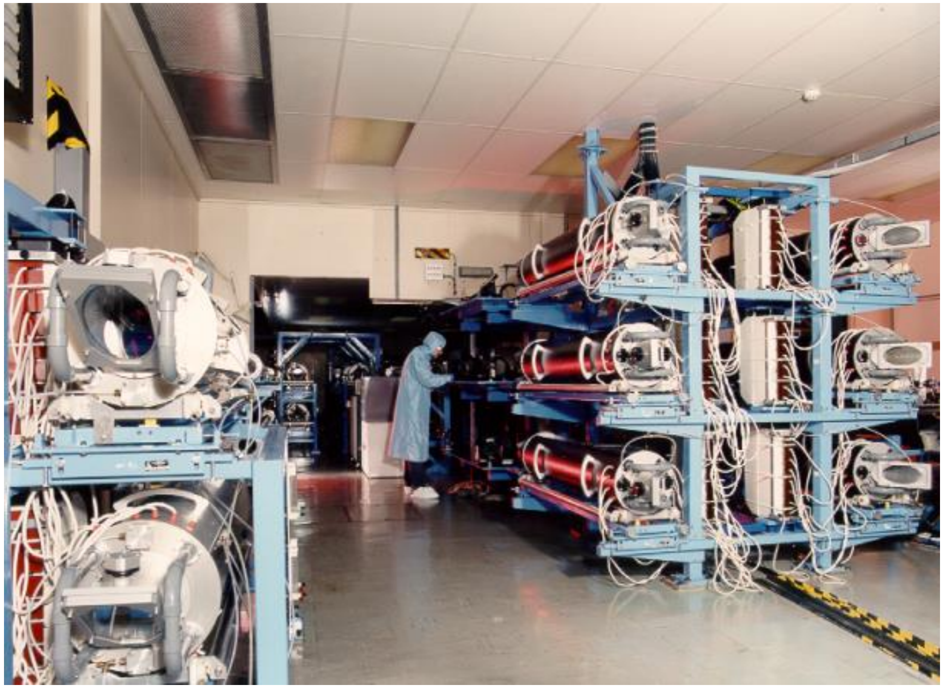


Laser systems at the CLF

- Laser systems for laser plasma research

Vulcan: Nd: glass system

- long pulse (100ps-2ns), 2.5 kJ, 8 beams
- 100TW pulse (1ps) and 10TW pulse (10ps) synchronised with long pulse
- Petawatt (500fs, 500J)
- Upgrading to 10 PW peak power



UK Flagship 2002-2008: Vulcan facility

8 Beam CPA Laser
3 Target Areas
3 kJ Energy
1 PW Power

Increased complexity, accuracy & rep-rate

UK Flagship 2008-2013: Astra-Gemini facility

Dual Beam Petawatt Upgrade of Astra (factor 40 power upgrade)
10^{22} W/cm^2 on target irradiation
1 shot every 20 seconds
Opened by the UK Science Minister (Ian Pearson MP) Dec 07
Significantly over-subscribed. Assessing need for 2nd target area.

Ian Pearson, MP
Minister of State for Science and Innovation , Dec 2007

The next step: Vulcan 10 PW (>10^{23} W/cm^2)

100 fold intensity enhancement

OPCPA design

Coupled to existing PW and long pulse beams

Introduction

SOLIDS		GASES
Non-linear optics (2^{nd} harmonic generation)	10^4	
Dielectric breakdown (collisional ionisation)	10^9	
	10^{11}	Avalanche breakdown in air
	10^{13}	Non-linear optics regime
Collisional absorption	10^{14}	Tunnel ionisation regime for hydrogen
	10^{15}	Scattering instabilities (SRS, SBS etc) become important
Resonance absorption	10^{16}	
Profile steepening	$3\ 10^{16}$	
	10^{17}	Self focusing
Brunel heating	$2\ 10^{18}$	Relativistic effects particle acceleration
Hole boring	10^{20}	
	10^{21}	

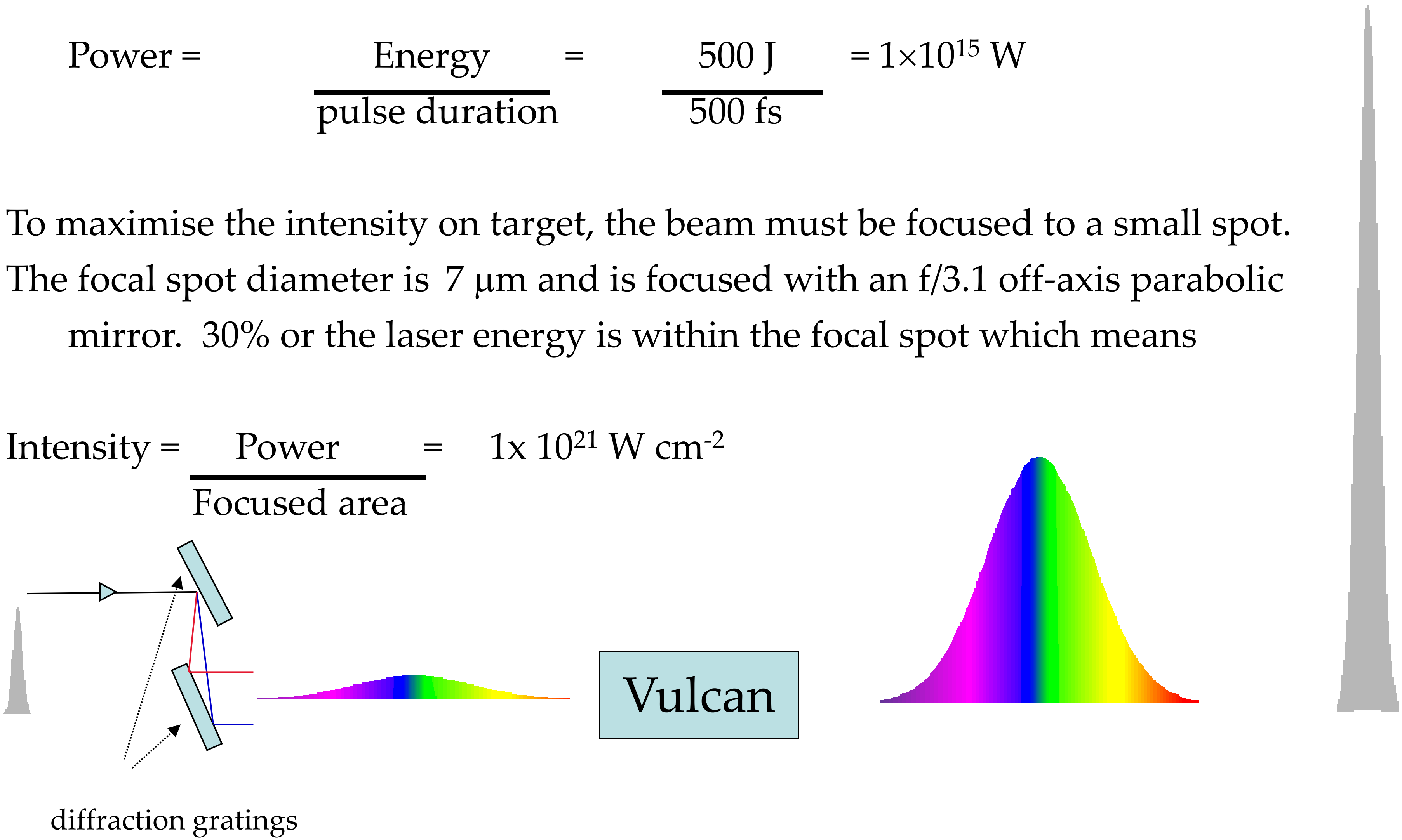
Power and Intensity

$$1\text{PW} = 10^{15}\text{W}$$

$$\text{Power} = \frac{\text{Energy}}{\text{pulse duration}} = \frac{500\text{ J}}{500\text{ fs}} = 1\times10^{15}\text{ W}$$

To maximise the intensity on target, the beam must be focused to a small spot. The focal spot diameter is 7 μm and is focused with an f/3.1 off-axis parabolic mirror. 30% or the laser energy is within the focal spot which means

$$\text{Intensity} = \frac{\text{Power}}{\text{Focused area}} = 1\text{x } 10^{21}\text{ W cm}^{-2}$$



Units

In these lecture notes, the units are defined in cgs. Conversions to SI units are given here in the hand-outs.

But note that in the literature

Energy – Joules

Pulse duration τ - ns nanoseconds 10^{-9} sec (ICF)
ps picoseconds 10^{-12} sec
fs femtoseconds 10^{-15} sec (limit of optical pulses)
as attoseconds 10^{-18} sec (X-ray regime)

Distance – microns (μm)

Power – Gigawatts 10^9 Watts
Terawatts 10^{12} Watts
Petawatts 10^{15} Watts
Exawatts 10^{18} Watts

Intensity – W cm^{-2}
Irradiance – W cm^{-2} μm^2
Pressure – Mbar – 10^{11} Pa
Magnetic field – MG – 100 T

FORMULA CONVERSION[8]

Here $\alpha = 10^2$ cm m^{-1}, $\beta = 10^7$ erg J^{-1}, $\epsilon_0 = 8.8542 \times 10^{-12}$ F m^{-1}, $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}, $c = (\epsilon_0\mu_0)^{-1/2} = 2.9979 \times 10^8$ m s^{-1}, and $\hbar = 1.0546 \times 10^{-34}$ J s. To derive a dimensionally correct SI formula from one expressed in Gaussian units, substitute for each quantity according to $\bar{Q} = \bar{k}Q$, where $\bar{k}$ is the coefficient in the second column of the table corresponding to Q (overbars denote variables expressed in Gaussian units). Thus, the formula $\bar{a}_0 = \bar{\hbar}^2/\bar{m}\bar{e}^2$ for the Bohr radius becomes $\alpha a_0 = (\hbar\beta)^2/[(m\beta/\alpha^2)(e^2\alpha\beta/4\pi\epsilon_0)]$, or $a_0 = \epsilon_0 h^2/\pi m e^2$. To go from SI to natural units in which $\hbar = c = 1$ (distinguished by a circumflex), use $Q = \hat{k}^{-1}\hat{Q}$, where $\hat{k}$ is the coefficient corresponding to Q in the third column. Thus $\hat{a}_0 = 4\pi\epsilon_0\hbar^2/[(\hat{m}\hbar/c)(\hat{e}^2\epsilon_0\hbar c)] = 4\pi/\hat{m}\hat{e}^2$. (In transforming *from* SI units, do not substitute for ϵ_0, μ_0, or c.)

Physical Quantity	Gaussian Units to SI	Natural Units to SI
Capacitance	$\alpha/4\pi\epsilon_0$	$\epsilon_0{}^{-1}$
Charge	$(\alpha\beta/4\pi\epsilon_0)^{1/2}$	$(\epsilon_0\hbar c)^{-1/2}$
Charge density	$(\beta/4\pi\alpha^5\epsilon_0)^{1/2}$	$(\epsilon_0\hbar c)^{-1/2}$
Current	$(\alpha\beta/4\pi\epsilon_0)^{1/2}$	$(\mu_0/\hbar c)^{1/2}$
Current density	$(\beta/4\pi\alpha^3\epsilon_0)^{1/2}$	$(\mu_0/\hbar c)^{1/2}$
Electric field	$(4\pi\beta\epsilon_0/\alpha^3)^{1/2}$	$(\epsilon_0/\hbar c)^{1/2}$
Electric potential	$(4\pi\beta\epsilon_0/\alpha)^{1/2}$	$(\epsilon_0/\hbar c)^{1/2}$
Electric conductivity	$(4\pi\epsilon_0)^{-1}$	$\epsilon_0{}^{-1}$
Energy	β	$(\hbar c)^{-1}$
Energy density	β/α^3	$(\hbar c)^{-1}$
Force	β/α	$(\hbar c)^{-1}$
Frequency	1	c^{-1}
Inductance	$4\pi\epsilon_0/\alpha$	$\mu_0{}^{-1}$
Length	α	1
Magnetic induction	$(4\pi\beta/\alpha^3\mu_0)^{1/2}$	$(\mu_0\hbar c)^{-1/2}$
Magnetic intensity	$(4\pi\mu_0\beta/\alpha^3)^{1/2}$	$(\mu_0/\hbar c)^{1/2}$
Mass	β/α^2	$c/\hbar$
Momentum	β/α	$\hbar^{-1}$
Power	β	$(\hbar c^2)^{-1}$
Pressure	β/α^3	$(\hbar c)^{-1}$
Resistance	$4\pi\epsilon_0/\alpha$	$(\epsilon_0/\mu_0)^{1/2}$
Time	1	c
Velocity	α	c^{-1}

MAXWELL'S EQUATIONS

Name or Description	SI	Gaussian
Faraday's law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
Ampere's law	$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$	$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{J}$
Poisson equation	$\nabla \cdot \mathbf{D} = \rho$	$\nabla \cdot \mathbf{D} = 4\pi\rho$
[Absence of magnetic monopoles]	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
Lorentz force on charge q	$q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$q\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)$
Constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{B} = \mu\mathbf{H}$	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{B} = \mu\mathbf{H}$

In a plasma, $\mu \approx \mu_0 = 4\pi \times 10^{-7}$ H m^{-1} (Gaussian units: $\mu \approx 1$). The permittivity satisfies $\epsilon \approx \epsilon_0 = 8.8542 \times 10^{-12}$ F m^{-1} (Gaussian: $\epsilon \approx 1$) provided that all charge is regarded as free. Using the drift approximation $\mathbf{v}_\perp = \mathbf{E} \times \mathbf{B}/B^2$ to calculate polarization charge density gives rise to a dielectric constant $K \equiv \epsilon/\epsilon_0 = 1 + 36\pi \times 10^9 \rho/B^2$ (SI) $= 1 + 4\pi\rho c^2/B^2$ (Gaussian), where ρ is the mass density.

The electromagnetic energy in volume V is given by

$$W = \frac{1}{2}\int_V dV\,(\mathbf{H}\cdot\mathbf{B} + \mathbf{E}\cdot\mathbf{D}) \qquad \text{(SI)}$$

$$= \frac{1}{8\pi}\int_V dV\,(\mathbf{H}\cdot\mathbf{B} + \mathbf{E}\cdot\mathbf{D}) \qquad \text{(Gaussian).}$$

Poynting's theorem is

$$\frac{\partial W}{\partial t} + \int_S \mathbf{N}\cdot d\mathbf{S} = -\int_V dV\,\mathbf{J}\cdot\mathbf{E},$$

where S is the closed surface bounding V and the Poynting vector (energy flux across S) is given by $\mathbf{N} = \mathbf{E} \times \mathbf{H}$ (SI) or $\mathbf{N} = c\mathbf{E} \times \mathbf{H}/4\pi$ (Gaussian).

http://wwppd.nrl.navy.mil/formulary/NRL_FORMULARY_07.pdf

Critical density and the quiver velocity

- The laser can propagate in a plasma until what is known as the critical density surface. The dispersion relationship for electromagnetic waves in a plasma is $\omega^2 = \omega_{pe}^2 + k^2 c^2$ so that when $\omega = \omega_{pe}$ the wave cannot propagate

<u>Critical density</u>

$$n_{critical} = \frac{1.1 \times 10^{21}\ cm^{-3}}{\lambda_{\mu m}{}^2}$$

$\lambda_{\mu m}$ is the laser wavelength in μm

- An electron oscillates in a plane electromagnetic wave according to the Lorentz force $F = -e(\underline{E} + c^{-1}\underline{v} \times \underline{B})$. Neglecting the $v \times B$ term, then the <u>quiver velocity</u> v_{osc} is

$$v_{osc} = \frac{eE_L}{m\omega}$$

- The dimensionless vector potential is defined as $a \equiv \dfrac{v_{osc}}{c}$

Note that a can be > 1. This is because a = γ β = γ $v_{electron}$ / a and

$$a = \left[\frac{I\lambda^2}{1.4 \times 10^{18} Wcm^{-2}} \right]^{1/2}$$ ~ 1 if relativity is important and $\gamma_{electron} = [\ 1 + a^2\ ]^{1/2}$

Laser-plasmas can access the high energy density regime

- The ratio of the quiver energy to the thermal energy of the plasma is given by

$$\frac{mv_{osc}^2}{2kT} = \frac{1.8\times10^{-18} I\lambda^2}{T(eV)}$$

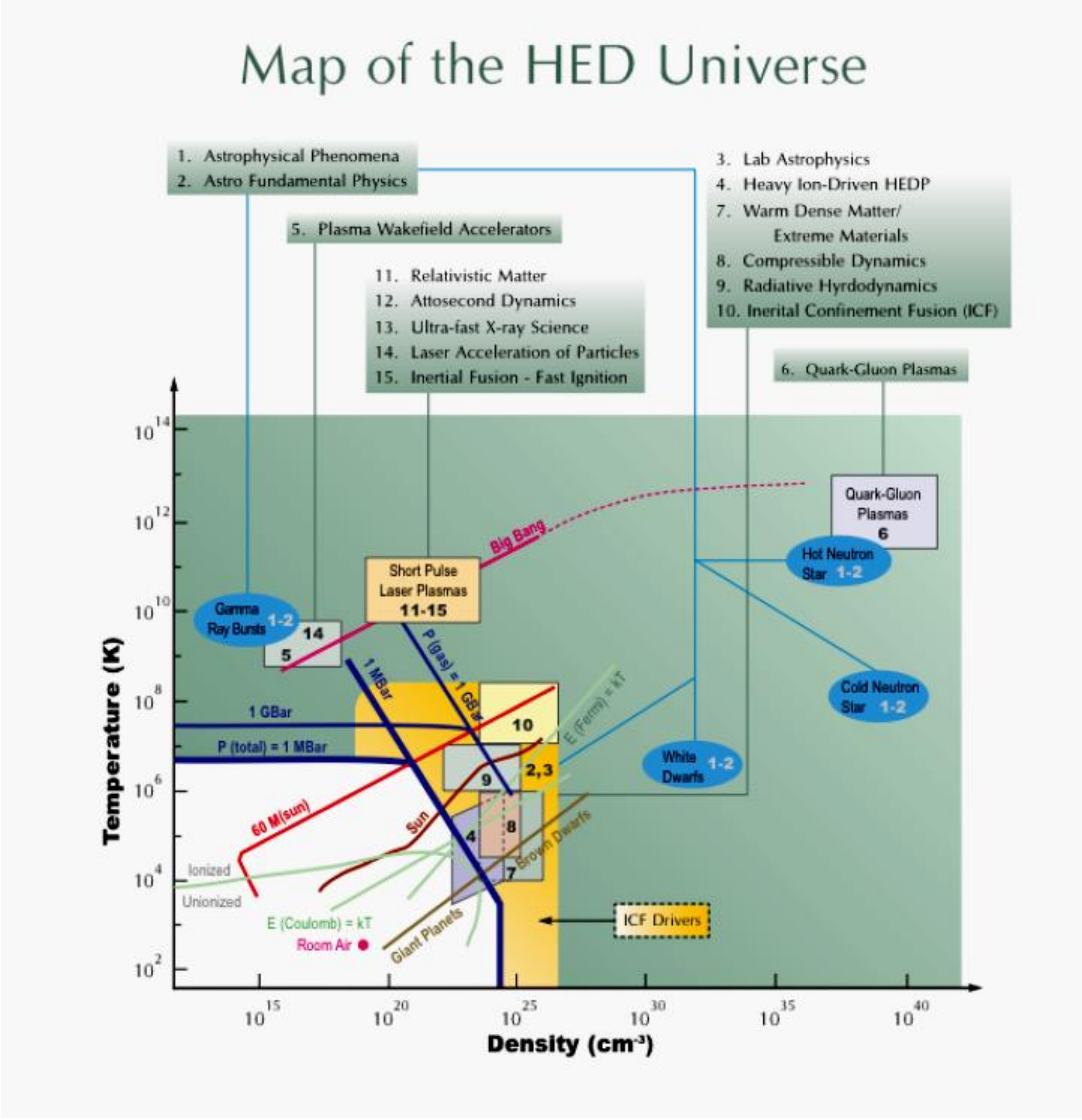
< 1 for ICF plasmas
>> 1 for relativistic intensities

- The laser electric field is given by

$$E_0 = \frac{3.2a_0}{\lambda}\left[10^{10}\right] \; Vcm^{-1}$$

- The Debye length λ_D for a laser-produced plasma is ~ µm for gas targets and ~ nm for solids. n λ_D - 50 -100

- Strongly coupled plasma regime can be accessed



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Ionisation processes

- **Multi-photon ionisation**

  Non linear process
  Very low rate
  Scales as I^n n = number of photons

- **Collisional ionisation (avalanche ionisation)**

  Seeded by multiphoton ionisation
  Collisions of electrons and neutrals $\Leftrightarrow$ plasma

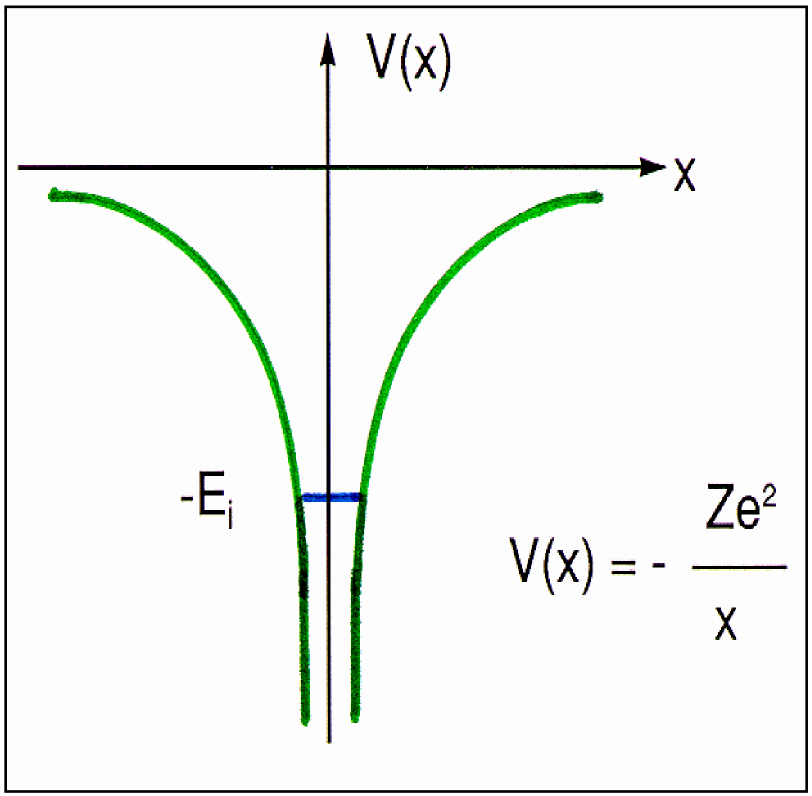
$$\nu \propto \frac{n_e^{\,2} T_e^{-3/2}}{\omega^2}$$

- **Tunnel and field ionisation**

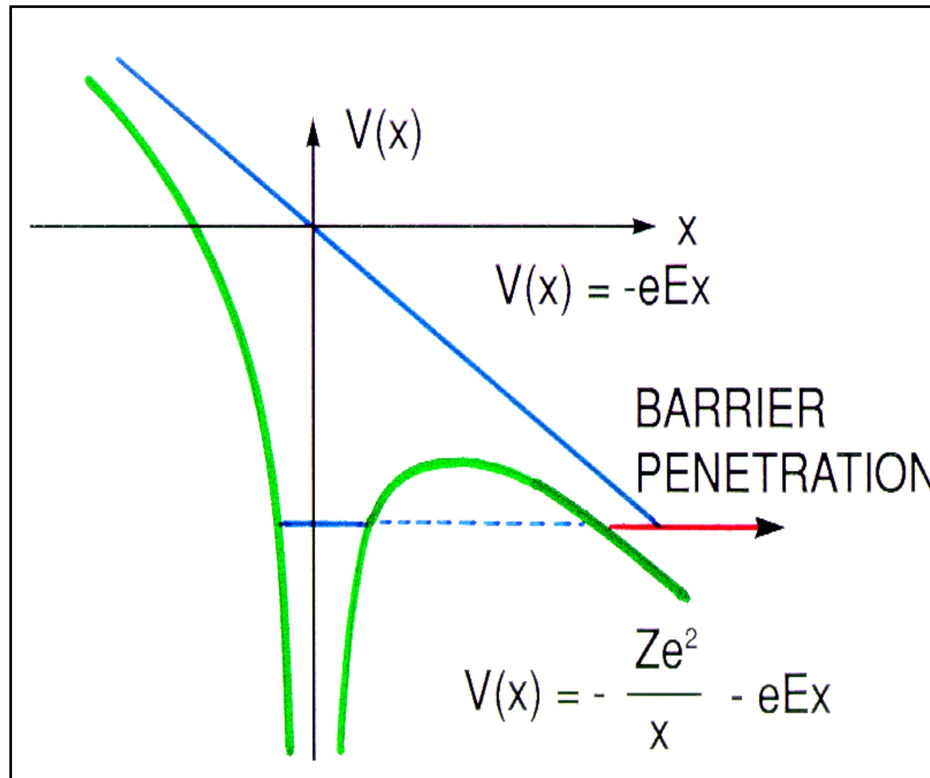
  Laser electric field “lowers” the Coloumb barrier which confines the electrons in the atom

Tunnelling

Isolated atom



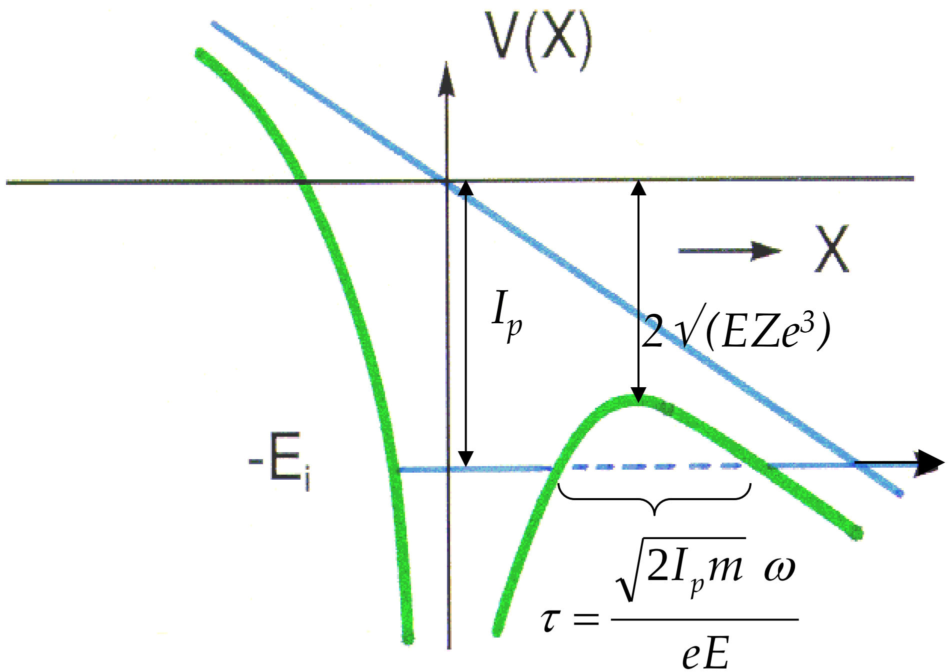
Atom in an electric field



Atom in an oscillating field

Keldysh parameter γ = time to tunnel out / period of laser field

$$= \frac{\sqrt{2I_p m}}{eE}\omega = 2.3\times10^6\left(\frac{I_p}{I_\lambda}\right)^{1/2}\frac{1}{\lambda}$$

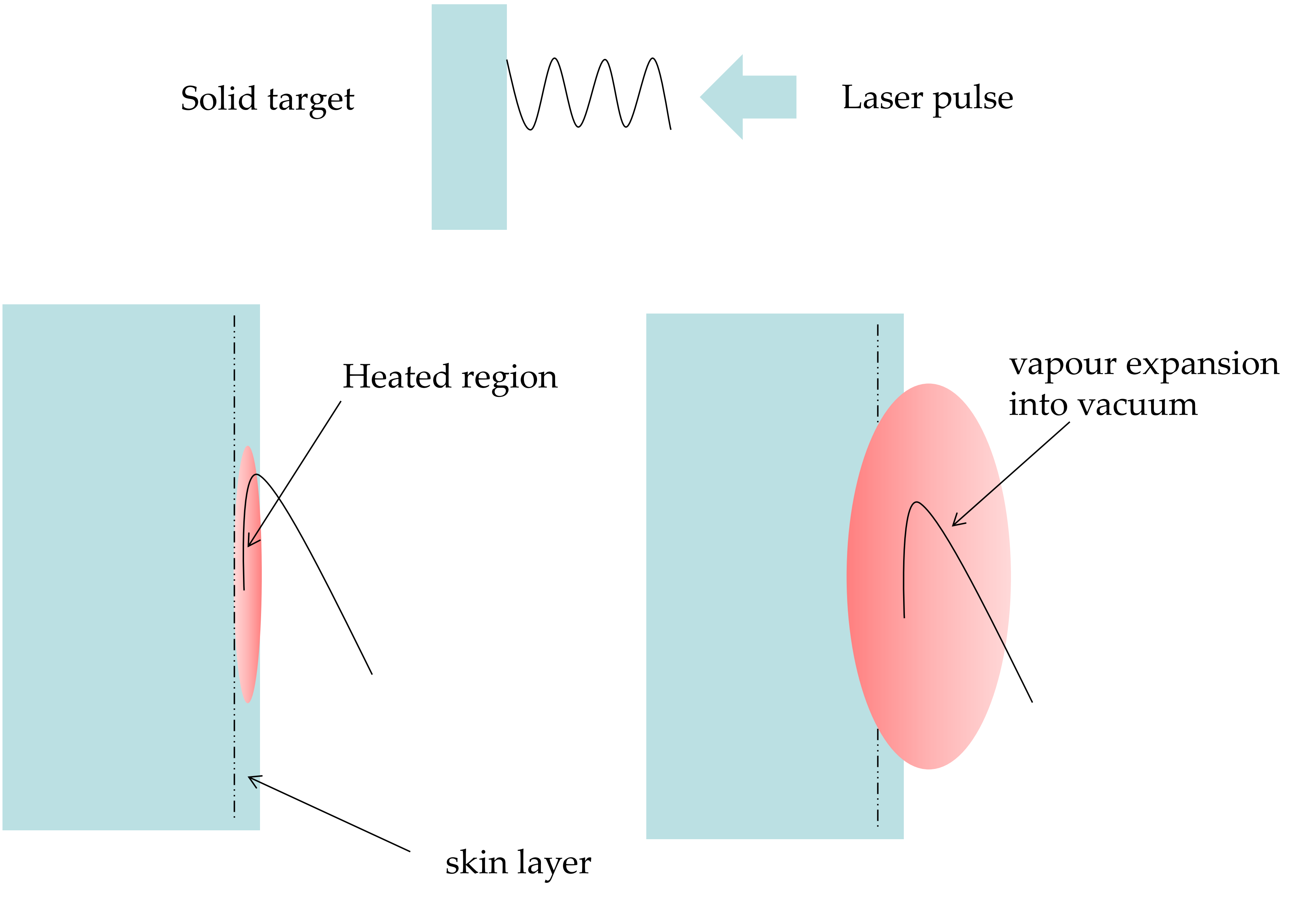


i.e.

if $\gamma = \frac{\omega}{\omega_t} << 1$ ⇨ **TUNNELLING**

if $\gamma >> 1$ ⇨ **MULTIPHOTON IONISATION**

[γ = 1, I =6×10^{13} Wcm^{-2}, λ 1μm, xenon]



Solid target
Laser pulse
Heated region
skin layer
vapour expansion
into vacuum

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EM wave propagation in plasma

Start with an electric field of the form $\underline{E} = \underline{E}(k)\exp(-i\omega\, t)$ (1)

The current density can be written $\underline{j} = \sigma \underline{E}$ where the conductivity σ is $\sigma = \dfrac{i\omega_{pe}^2}{4\pi\omega}$

and $\omega_{pe}^2 = 4\pi\ e^2 n_0 / m_e$

Substituting (1) into Faraday's law $\nabla \times \underline{E} = -\dfrac{1}{c}\dfrac{\partial \underline{B}}{\partial t}$ gives $\nabla \times \underline{E} = -\dfrac{i\omega}{c}\underline{B}$ (2)

Substituting (1) and **j** into Ampere's law $\nabla \times \underline{B} = \dfrac{4\pi}{c}\underline{J} + \dfrac{1}{c}\dfrac{\partial \underline{E}}{\partial t}$ gives

$$\nabla \times \underline{B} = \frac{4\pi}{c}\sigma \underline{E} - \frac{i\omega}{c}\underline{E} = -\frac{i\omega}{c}\varepsilon \underline{E} \qquad (3)$$

Where we have defined the dielectric function of the plasma $\varepsilon = \left(1 - \dfrac{\omega_{pe}^2}{\omega^2}\right)$

Taking the curl of Faraday's law (2) and substituting Ampere's law (3) with a standard vector identity gives

$$\nabla^2 \underline{E} - \nabla(\nabla.\underline{E}) + \frac{\omega^2}{c^2}\varepsilon \underline{E} = 0 \qquad \text{i.e.} \qquad \nabla^2 \underline{E} + \frac{\omega^2}{c^2}\varepsilon \underline{E} = 0 \qquad (4)$$

EM wave propagation in a plasma with a linear density gradient

Consider plane waves at normal incidence. Assume the density gradient is in the propagation direction Z. This

$$\begin{aligned} n_0 &= n_0(z) \\ \varepsilon &= \varepsilon(\omega, z) \\ \underline{E}(x) &= \underline{E}(z)\exp(-i\omega t) \end{aligned} \qquad (5)$$

Substituting (5) into (4) gives $\dfrac{d^2}{dz^2}E_{x,y} + \dfrac{\omega^2}{c^2}\varepsilon\, E_{x,y} = 0$ (6)

$$\varepsilon\, E_z = 0$$

We can rewrite the dielectric function as $\varepsilon \; = 1 - \dfrac{n(x)}{n_c} = 1 - \dfrac{\omega_{pe}^2}{\omega^2} = 1 - \dfrac{z}{L}$ (7)

when $n(x) = \dfrac{z}{L}n_c$ and the critical density is $n_c = \dfrac{m\omega^2}{4\pi\, e^2}$

Equation (6) can then be written as $\dfrac{d^2E}{dz^2} + \dfrac{\omega^2}{c^2}\left(1 - \dfrac{z}{L}\right)E = 0$ (8)

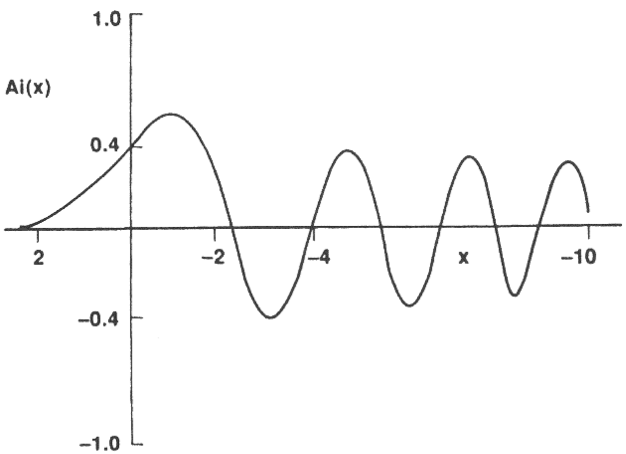
EM wave propagation in a plasma with a linear density gradient continued (1)

Change variables $\eta = \left(\dfrac{\omega^2}{c^2 L}\right)^{1/3}(z-L)$

Gives $\dfrac{d^2E}{d\eta^2} - \eta\, E = 0$ whose solution is an Airy function

$$E(\eta) = \alpha A_i(\eta) + \beta B_i(\eta) \qquad (9)$$

α and β are constants determined by the boundary condition matching.



Physically, E should represent a standing wave for $\eta < 0$ and to decay at $\eta \to \infty$. Since $B_i(\eta) \to \infty$ as $\eta \to \infty$, β is chosen to $\beta = 0$. α is chosen by matching the E-fields at the interface between the vacuum and the plasma at $z=0$. This gives $\eta = (\omega L/c)^{2/3}$. If we assume that ωL/c >>1 and represent $A_i(\eta)$ by the expression

$$A_i(-\eta) = \frac{1}{\sqrt{\pi}\,\eta^{1/4}}\cos\left(\frac{2}{3}\eta^{2/3} - \frac{\pi}{4}\right) \qquad (10)$$

EM wave propagation in a plasma with a linear density gradient continued (2)

Then $$E(z=0)=\frac{\alpha}{2\sqrt{\pi}(\omega\ L/c)^{1/6}}\left[\exp i\left(\frac{2}{3}\frac{\omega L}{c}-\frac{\pi}{4}\right)+\exp -i\left(\frac{2}{3}\frac{\omega L}{c}-\frac{\pi}{4}\right)\right] \quad (11)$$

Now $E(z=0)$ can be represented as the sum of the incident wave with amplitude E_{FS} with a reflected wave with the same amplitude but shifted in phase

$$E(z=0)=E_{FS}\left[1+\exp -i\left(\frac{4}{3}\frac{\omega L}{c}-\frac{\pi}{2}\right)\right] \quad (12)$$

Provided that $$\alpha=2\sqrt{\pi}\left(\frac{\omega L}{c}\right)^{1/6}E_{FS}e^{i\phi}$$

E_{FS} is the free space value of the incident light and ϕ is an arbitrary phase that does not affect $|E|$

$$\therefore E(\eta)=2\sqrt{\pi}\left(\frac{\omega L}{c}\right)^{1/6}E_{FS}e^{i\phi}A_i(\eta) \quad (13)$$

This function is maximised at $\eta = 1$ corresponding to $(z - L) = -(c^2 L / \omega^2)^{1/3}$

$$\therefore \left|\frac{E_{\max}}{E_{FS}}\right|^2 \approx 3.6\left(\frac{\omega L}{c}\right)^{1/3} \approx 3.6\left(\frac{2\pi L}{\lambda}\right)^{1/3}$$ (14)(valid for $L/\lambda >> 1$)

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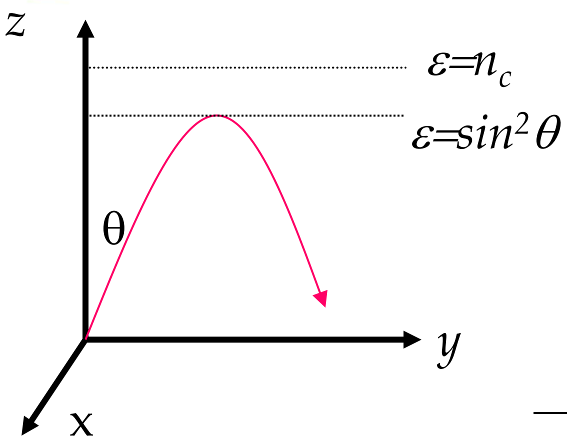
Oblique incidence



$$k_x = 0 \qquad \frac{\partial}{\partial x} = 0$$

$$k_y = \left(\frac{\omega}{c}\right)\sin\theta \qquad k_z = \left(\frac{\omega}{c}\right)\cos\theta$$

s-polarised

$\underline{E} = E_x \hat{x}$ Eqn (4) becomes $\dfrac{\partial^2 E_x}{\partial y^2} + \dfrac{\partial^2 E_x}{\partial z^2} + \dfrac{\omega^2}{c^2}\varepsilon(z)E_z = 0$ (15)

Since ε is a function only of *z*, k_y must be conserved and hence $k_y = (\omega / c)\sin\theta$ and

$$E_x = E(z)\exp\left(\frac{-i\omega y\sin\theta}{c}\right) \qquad (16)$$

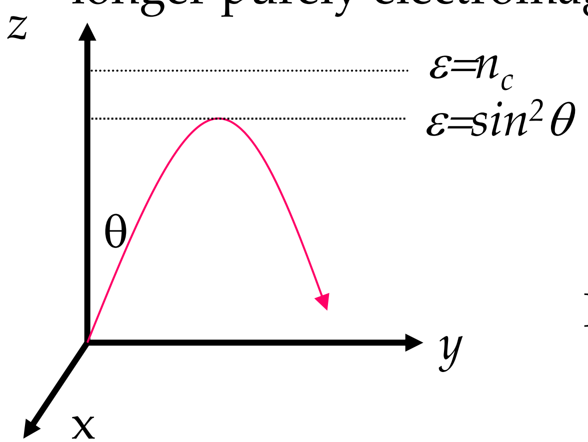
Substituting (16) into (15) gives $\dfrac{d^2E(z)}{dz^2} + \dfrac{\omega^2}{c^2}\left(\varepsilon(z) - \sin^2\theta\right)E(z) = 0$ (17)

Reflection of the light occurs when $\varepsilon(z) = \sin^2\theta$. Remember that $\varepsilon = (1 - \omega_{pe}^2(z) / \omega^2)$ and therefore reflection takes place when $\omega_{pe} = \omega\cos\theta$ at a density lower than critical given by $n_e = n_c\cos^2\theta$.

Oblique incidence – resonance absorption

p-polarised

In this case, there is a component of the electric field that oscillates along the density gradient direction, i.e. $\underline{E}.\nabla n_e \neq 0$ Part of the incident light is transferred to an electrostatic interaction (electron plasma wave) and is no longer purely electromagnetic.



$$\underline{E} = E_y \hat{y} + E_z \hat{z}$$
$$\nabla.(\varepsilon \underline{E}) = 0 \qquad (18)$$

Remember the vector identity $(\nabla.(\varepsilon \underline{E}) = \varepsilon \nabla.\underline{E} + \nabla\varepsilon.\underline{E})$

then $$\nabla.\underline{E} = -\frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial z} E_z \qquad (19)$$

This means that there is a resonant response when $\varepsilon = 0$, i.e. when $\omega = \omega_{pe}$

We seen previously that the light does not penetrate to n_c. Instead,the field penetrates through to $\omega = \omega_{pe}$ to excite the resonance.

Oblique incidence – resonance absorption continued

Note that the magnetic field $\underline{B} = B_x \hat{x}$ and use the conservation of $k_y = \left(\frac{\omega}{c}\right)\sin\theta$

The B-field can be expressed as $\underline{B} = B(z)\exp\left(-i\omega t + \frac{i\omega y}{c}\sin\theta\right)\hat{x}$ (20)

Substituting into Ampere's law (equation (3)) $\nabla\times\underline{B} = -\frac{i\omega}{c}\varepsilon\underline{E}$ gives

$$E_z = \frac{\sin\theta}{\varepsilon(z)}B_z \approx \frac{E_d}{\varepsilon(z)} \qquad (21)$$

This implies that E_z is strongly peaked at the critical density surface and the resonantly driven field E_d is evaluated there by calculating the B-field at $n_{c.}$

From Faraday's law $B = -\frac{ic}{\omega}\frac{\partial E}{\partial z}$ or $B = \frac{-ic}{\omega}\left(\frac{\partial\eta}{\partial z}\right)\left(\frac{\partial E}{\partial\eta}\right)$ (22)

$$E(\eta) = 2\sqrt{\pi}\left(\frac{\omega L}{c}\right)^{1/6}E_{FS}e^{i\phi}A_i(\eta) \qquad B(\eta) = -i2\sqrt{\pi}\left(\frac{c}{\omega L}\right)^{1/6}E_{FS}e^{i\phi}A_i(\eta) \qquad (23)$$

B decreases as E gets larger near the reflection point

At the reflection point, $|B(\eta=0)| \approx 0.9\left(\frac{c}{\omega L}\right)^{1/6} E_{FS}$ (24)

The B field decays like exp (-β) where $\beta = \int_{L\cos^2\theta}^{L} k(z)dz$.

$$\beta = \int_{L\cos^2\theta}^{L} \frac{1}{c}\left(\omega_p^2 - \omega^2\cos^2\theta\right)^{1/2} dz \qquad \frac{\omega_{pe}^2}{\omega^2} = \frac{z}{L} \tag{25}$$

Solution: $\beta = \left(\frac{2\omega L}{3c}\right)\sin^3\theta$ (26)

$$\therefore B(z=L) \approx 0.9 E_{FS}\left(\frac{c}{\omega L}\right)^{1/6} \exp\left(-\frac{2\omega L\sin^3\theta}{3c}\right) \tag{27}$$

Defining $\tau \equiv \left(\frac{\omega L}{c}\right)^{1/3} \sin\theta$ and remember that $E_z = \frac{E_d}{\varepsilon(z)} = \frac{B\sin\theta}{\varepsilon(z)}$

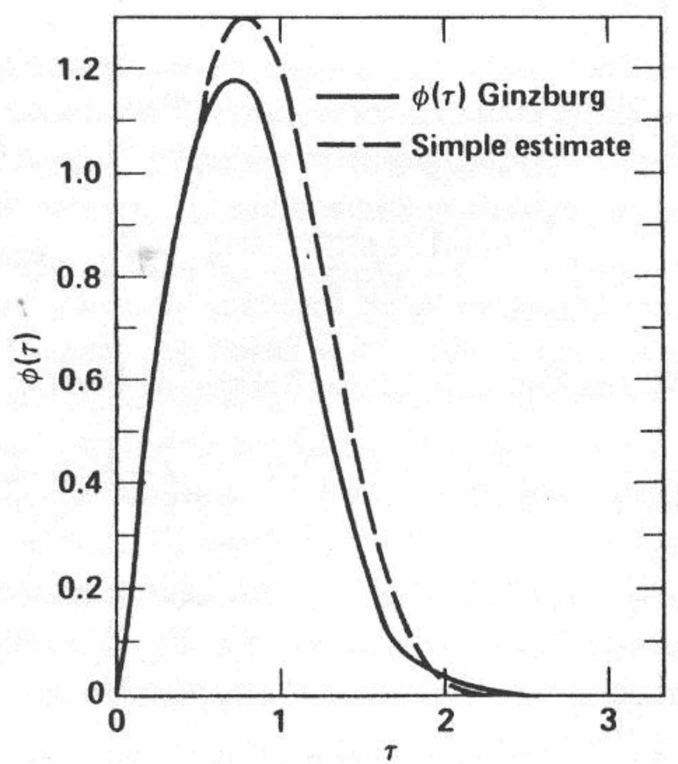
$$\therefore E_d = \frac{E_{FS}}{(2\pi\omega L/c)}\phi(\tau) \qquad\qquad \phi(\tau) \approx 2.3\tau\exp\left(-\frac{2}{3}\tau^3\right) \qquad (28)$$

The driver field E_d vanishes as $\tau \to 0$, as the component along z varies as $\sin\theta$.

Also, E_d becomes very small when $\tau \to \infty$, because the incident wave has to tunnel through too large a distance to reach n_c

The simple estimate for $\phi(\tau)$ from (28) is plotted here against an numerical solution of the wave equation.

Note that the maximum value of $\phi(\tau)$ is ~1.2 (needed to estimate the absorption fraction in the next section)



Resonance absorption – energy transfer

Start from the electrostatic component of the electric field. $E_z \approx \dfrac{E_d}{\varepsilon(z)}$

Now include a damping term ν in the dielectric function of the plasma that can arise form collisions, wave particle interactions etc. $\varepsilon(z) = 1 - \dfrac{\omega_p^2}{\omega(\omega - i\nu)}$

$$I_{abs} = \int_0^\infty \nu \frac{E_z^2}{8\pi} dz = \frac{\nu}{8\pi} \int_0^\infty \frac{{E_d}^2(z)}{|\varepsilon|^2} dz$$

ν can be considered as the rate of energy dissipation and $E_z^2/8\pi$ the incident energy density.

For a linear density profile, ($n_e = n_{cr}$ z / L) $|\varepsilon|^2 = \left(1 - \dfrac{z}{L}\right)^2 + \left(\dfrac{\nu}{\omega}\right)^2 \dfrac{z^2}{L^2}$

Substituting and approximating that E_d is constant over a narrow width of the resonance function gives $I_{abs} \approx \dfrac{\nu E_d^2(z = L)}{8\pi} \displaystyle\int_0^\infty \frac{dz}{(1 - z/L)^2 + (\nu/\omega)^2}$

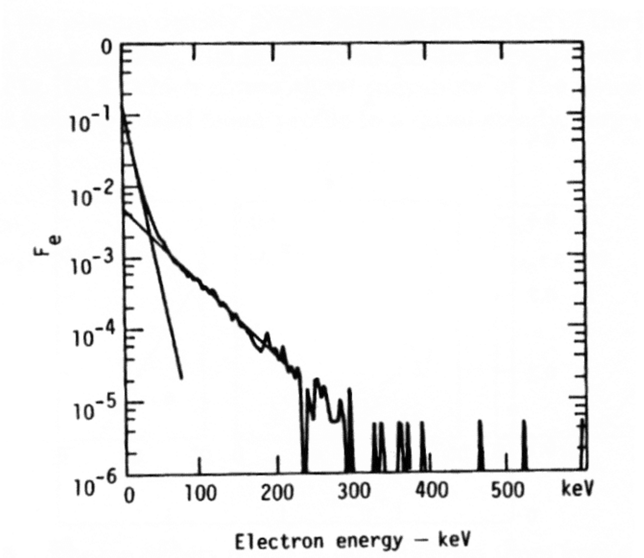
The integral gives π ω/ ν, $\therefore I_{abs} \approx \omega L E_d^2 / 8$ $I_{abs} = f_A \dfrac{c E_{FS}^2}{8\pi}$ $f_A \approx \phi^2(\tau)/2$

I_{abs} peaks at ~0.5

The absorption is maximised at an angle given by

$$\theta_{\max} \approx \sin^{-1}\left[0.8(c/\omega L)^{1/3}\right]$$

This is the characteristic signature of resonance absorption – the dependence on both angle of incidence and density scale-length.



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Collisional absorption

Principal energy source for direct drive inertial fusion plasmas

From the Vlasov equation, the linearized force equation for the electron fluid is

$$\frac{\partial \underline{u_e}}{\partial t} = -\frac{e}{m}\underline{E} - \nu_{ei}\underline{u_e}$$

where ν_{ei} is the damping term.

Since the field varies harmonically in time

$$u_e = \frac{-ie\underline{E}}{m(\omega + i\nu_{ei})}$$

The plasma current density is $$\underline{J} = -n_e e \underline{u_e} = \frac{i\omega_{pe}^2}{4\pi(\omega + i\nu_{ei})}\underline{E}$$

The conductivity σ (**J**=σ**E**) $$\sigma = \frac{i\omega_{pe}^2}{4\pi(\omega + i\nu_{ei})}$$ and dielectric function $$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_{ei})}$$

Collisional absorption

Faraday's law (cf (eqn (2))) $\nabla \times \underline{E} = -\frac{i\omega}{c}\underline{B}$

Ampere's law (cf eqn (3)) $\nabla \times \underline{B} = -\frac{i\omega}{c}\varepsilon \underline{E}$

Combining them gives $\nabla^2 \underline{E} + \frac{\omega^2}{c^2}\varepsilon \underline{E} = 0$

To derive the dispersion relation for EM waves in a spatially uniform plasma, take

$\underline{E}(x) \sim e^{i\underline{k}.\underline{x}}$ and substitute for ε and assume that $\nu_{ei}/\omega \ll 1$ gives

$$\omega^2 = k^2c^2 + \omega_{pe}^2\left(1 - \frac{i\nu_{ei}}{\omega}\right)$$

Expressing $\omega = \omega_r - i\nu/2$ gives $\boxed{\omega_r = \left(\omega_{pe}^2 + k^2c^2\right)^{1/2}}$

$$\nu = \frac{\omega_{pe}^2}{\omega_r^2}\nu_{ei}$$

where ν is the energy damping term.

collisional damping in linear density gradient

The rate of energy loss from the EM wave [= ($\nu E^2/8\pi$)] must balance the oscillatory velocity of the electrons is randomised by electron- ion scattering. Assume again that $n_e = n_{cr}\, z / L$ and write down equation (4) again

$$\frac{\partial^2 E}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon(z)\underline{E} = 0$$

The dielectric function $\varepsilon\,(z)$ is now $\varepsilon = 1 - \dfrac{z}{L\left(1 + i\,\nu_{e1}^{*}/\omega\right)}$ where ν^{*}_{ei} is its value at n_{cr}

$$\frac{d^2 E}{dz^2} + \frac{\omega^2}{c^2}\left(1 - \frac{z}{L\left(1 + i\,\nu_{ei}^{*}/\omega\right)}\right)\underline{E} = 0$$

Changing variables again, gives Airy's equation $\dfrac{d^2\underline{E}}{d\eta^2} - \eta\underline{E} = 0$

$$\eta = \left(\frac{\omega^2}{c^2 L\left(1 + i\,\nu_{ei}^{*}/\omega\right)}\right)^{1/3}\left(z - L\left(1 + \frac{i\,\nu_{ei}^{*}}{\omega}\right)\right)$$

$$E(\eta) = \alpha\ A_i(\eta)$$

Again, match the incident light wave to the vacuum plasma boundary interface at z=0

$$A_i(-\eta) = \frac{1}{\sqrt{\pi}\,\eta^{1/4}}\cos\left(\frac{2}{3}\eta^{2/3} - \frac{\pi}{4}\right)$$

Energy absorption

Thus at *z=0*, *E* can be represented as an incident and reflected wave whose amplitude is multiplied by $e^{i\phi}$

$$\phi = \frac{4}{3}\left(-\eta(z=0)\right) - \frac{\pi}{2}$$

As η is now complex, there is both a phase shift and a damping term for the reflected wave

$$\eta(0) = -\left(\left(\frac{\omega L}{c}\right)\left(1 + \frac{i\nu_{ei}^*}{\omega}\right)\right)^{2/3}$$

For $\nu^*_{ei}/\omega \ll 1$ $\qquad \phi_{real} = \frac{4\omega L}{3c} - \frac{\pi}{2} \qquad \phi_{imaginary} = \frac{4\nu_{ei}^* L}{3c}$

$$Energy \sim (amplitude)^2 \sim \exp\left(-\frac{8}{3}\frac{\nu_{ei} L}{c}\right)$$

$$\therefore f_A = 1 - \exp\left(-\frac{8\nu_{ei}^* L}{3c}\right) \qquad \nu_{ei} \approx 3\times 10^{-6} \ln\Lambda \frac{n_e Z}{T_{eV}^{3/2}} \sec^{-1}$$

Experiment shows $$f_A \propto \frac{Z^{3/2}\tau_L^{0.6}}{I_L^{0.4}\lambda_L^2}$$

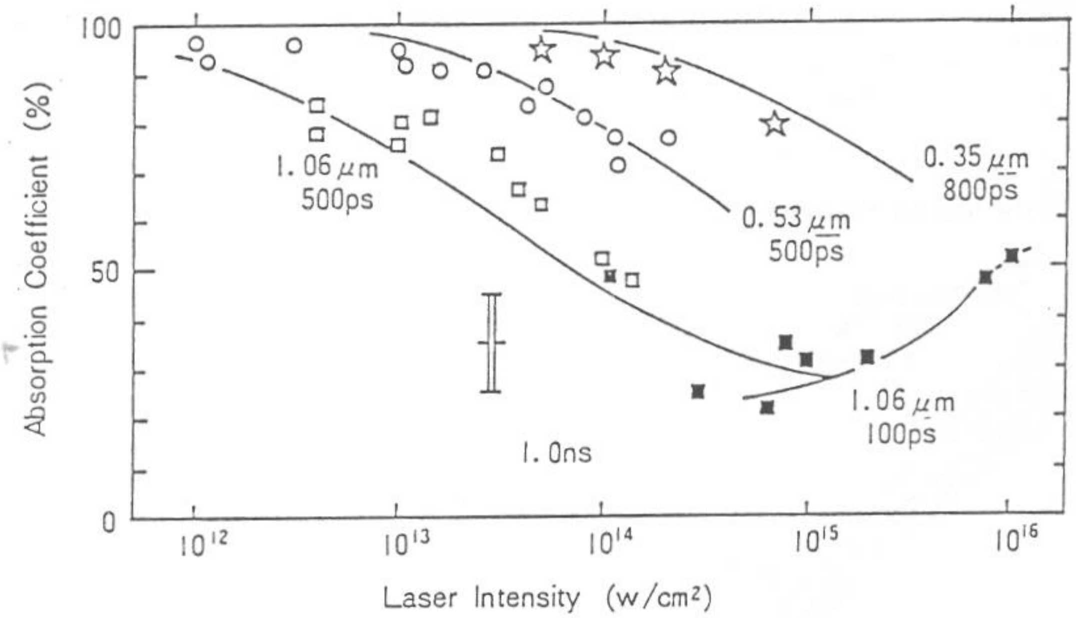

Fig. II-2 Absorption coefficient as a function of laser intensity and laser wavelength

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Collisionless "Brunel" absorption

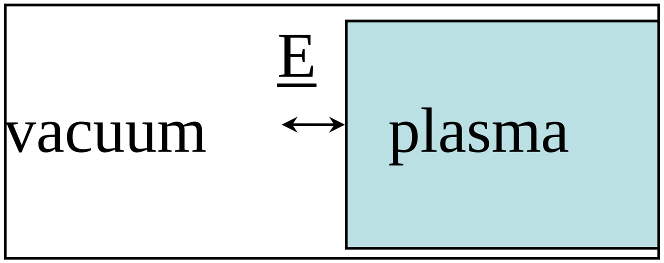
•Dominant mechanism is $L/\lambda < 1$

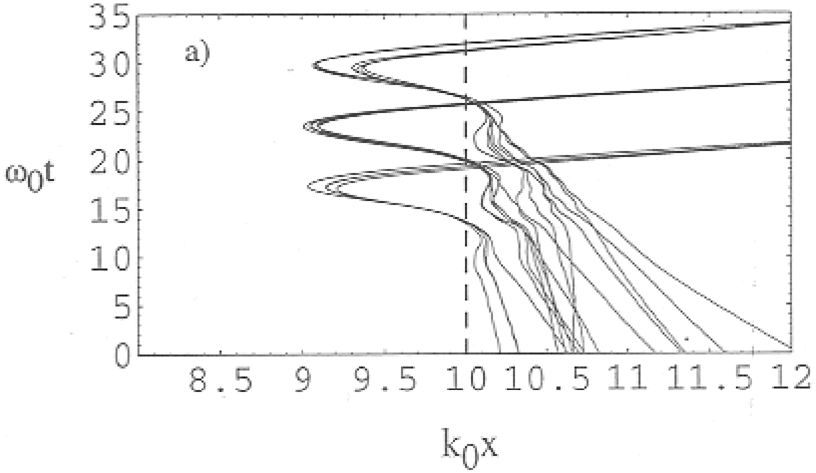
•Quiver motion of the electrons in the field of a p-polarised laser pulse

• Laser energy is absorbed on each cycle

• Can result in higher absorption than classical resonance absorption (up to 0.8)

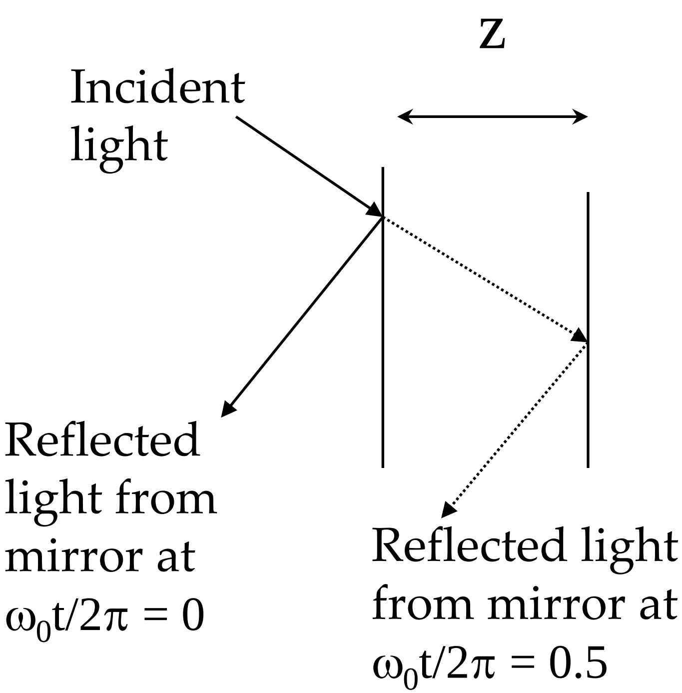
•Not as dependent on angle

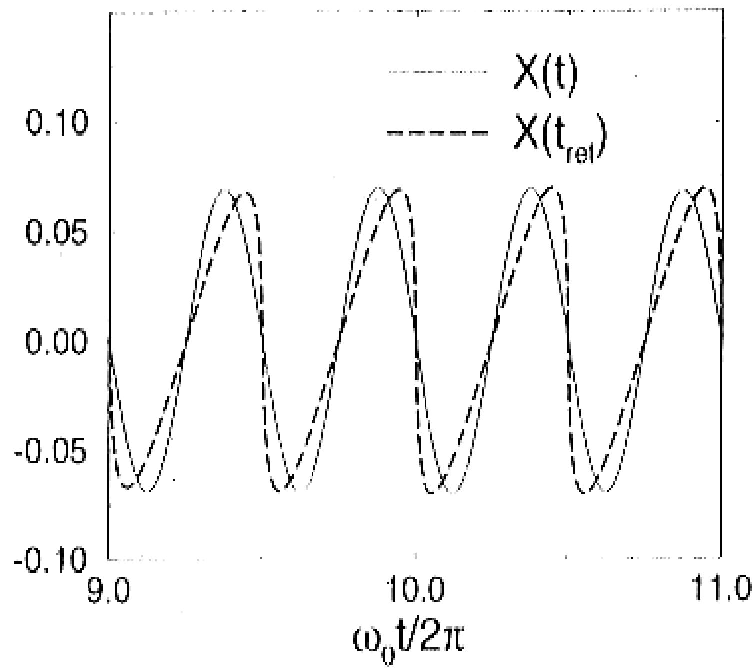




From P.Gibbon *Phys. Rev. Lett.* **76**, 50 (1996)

Harmonic generation: the moving mirror model





see references:

D. von der Linde and K. Rzazewski: *Applied Physics B* **63,** 499 (1996).

R. Lichters, J. Meyer-ter-Vehn and A. Pukhov *Phys. Plasmas* **3**, 3425 (1996).

The moving mirror model

Ignoring retardation effects, the phase shift of the reflected wave resulting from a sinusoidal displacement of the reflecting surface in the z direction $s(t)=s_0 \sin w_m t$ is

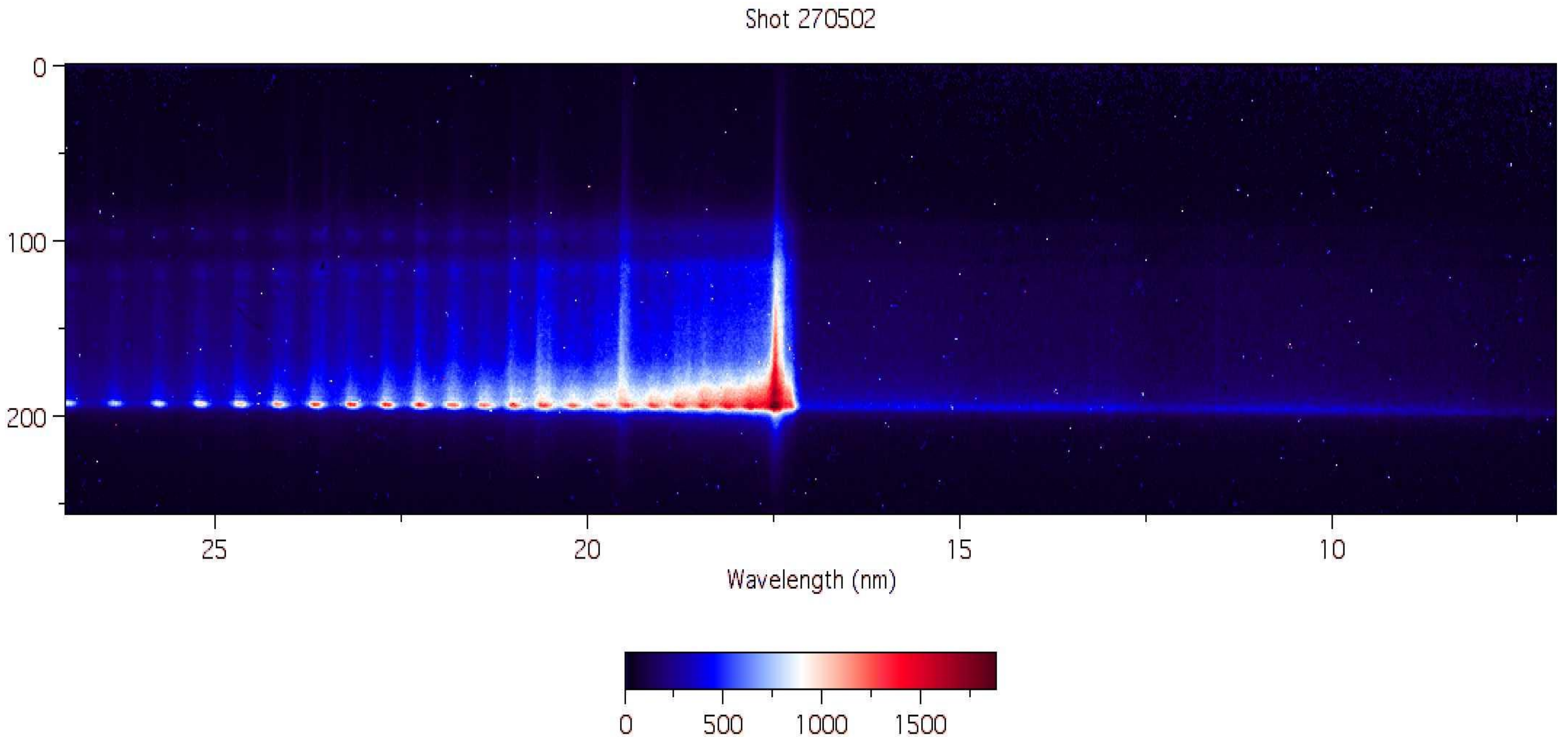
$$\phi(t) = (2\omega_0 s_0)\cos\theta \sin\omega_m t$$

θ is the angle of incidence, ω_m is the modulation frequency. The electric field of the reflected wave is

$$E_R \propto e^{-i\omega_0 t} e^{i\phi(t)} = e^{-i\omega_0 t} \sum_{n=-\infty}^{n=\infty} J_n(\chi) e^{-in\omega_m t}$$

$J_n(\chi)$ is the Bessel function of order n and $\chi = (2\omega_0 s_0 / c)\cos\vartheta$

where we've made use of the Jacobi expansion $e^{-iz\sin\omega_m t} = \sum_{n=-\infty}^{n=\infty} J(\chi) e^{-in\omega_m t}$

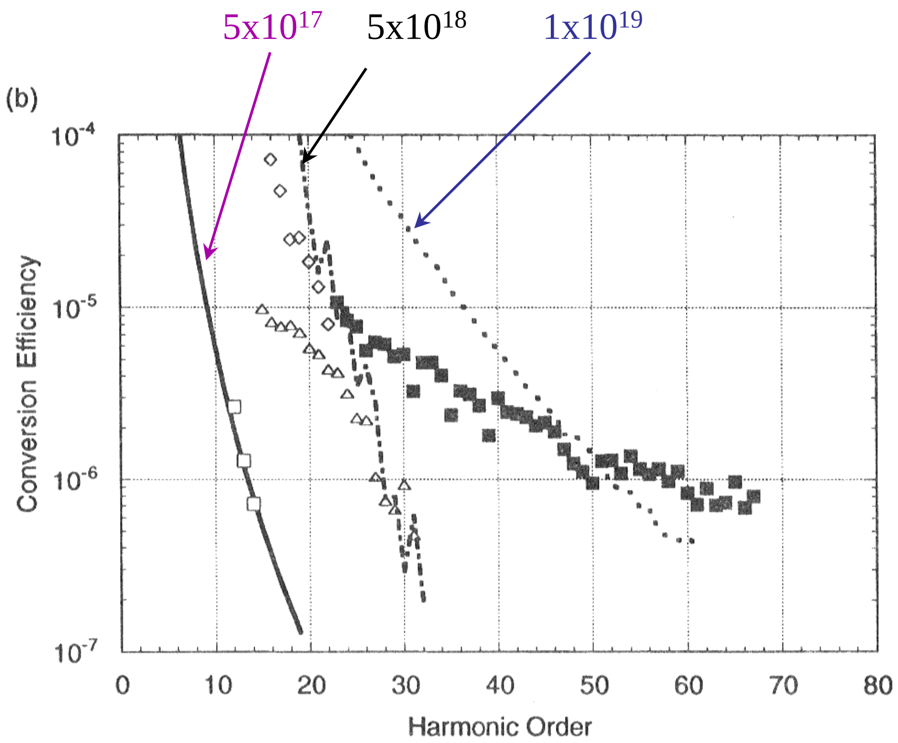


Shot 270502
0
100
200
25
20
15
10
Wavelength (nm)
0
500
1000
1500

Conversion efficiency dependence on intensity on target

Energy conversion efficiencies of 10^{-6} in X-UV harmonics up to the 68th order were measured and scaled as $E(\omega) = E_L(\omega/\omega_0)^{-q}$

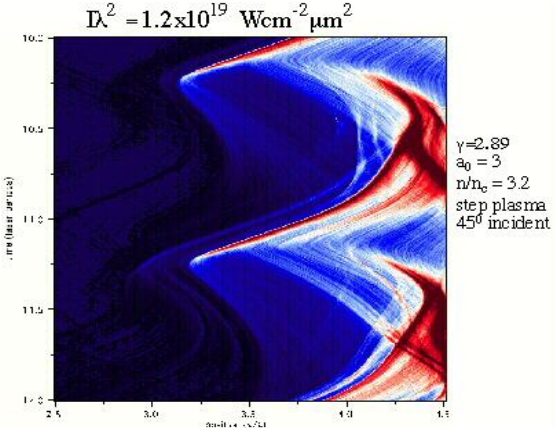
Good agreement was found with particle in cell simulations

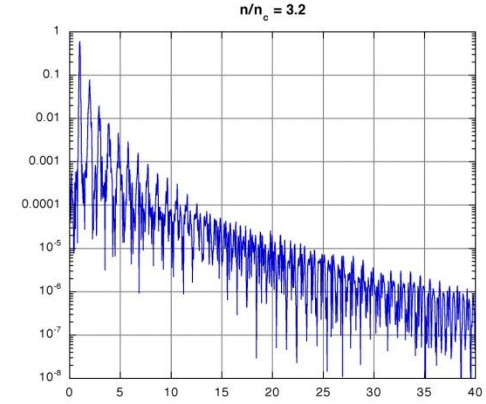


P.A.Norreys *et al.* Physical Review Letters **76**, 1832 (1996)

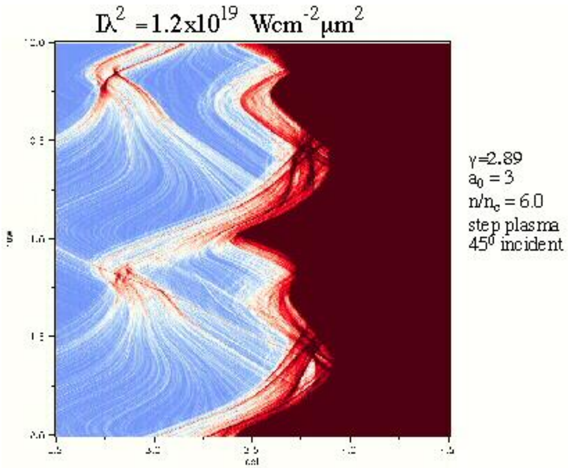
Complex dynamics of n_c motion

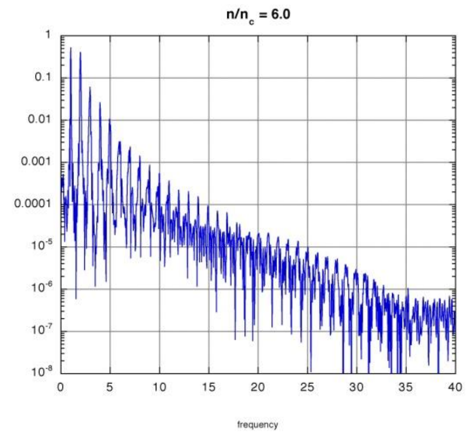
For $I\lambda^2 = 1.2\times10^{19}$ $Wcm^{-2}\mu m^2$ and $n/n_c = 3.2$ only ω_0 oscillations are observed. The power spectrum extends to ~35 and still shows modulations. Over dense regions of plasma are observed at the peak of the pulse.





$I\lambda^2 = 1.2 \times 10^{19}$ $Wcm^{-2}\mu m^2$ and $n/n_c = 6.0$, both ω_0 and $2\omega_0$ oscillations appear.





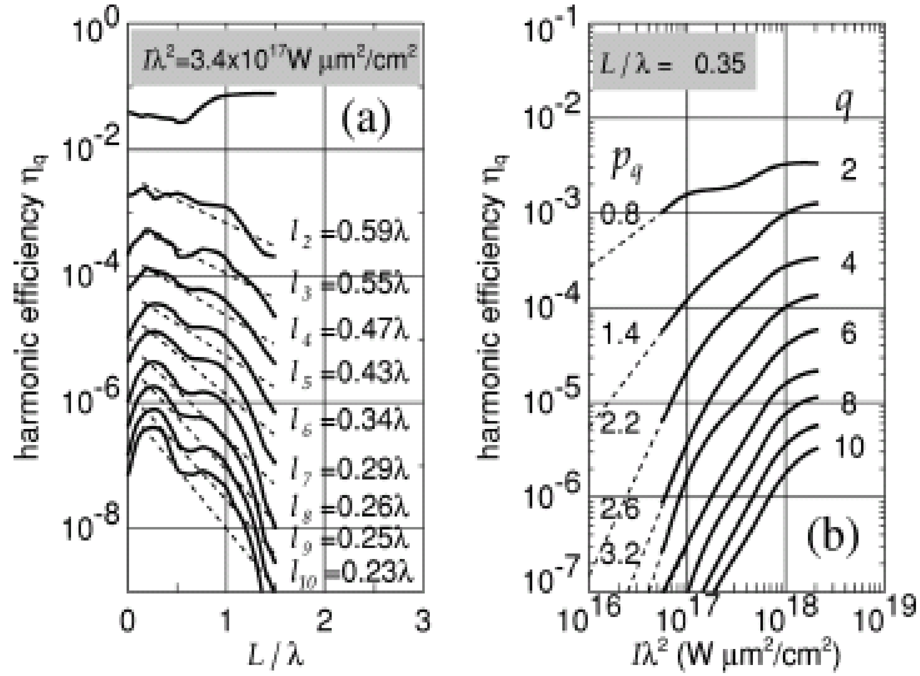
Additional order results in the modulation of the harmonic spectrum

I.Watts *et al*., Physical Review Letters **88**, 155001 (2002).

• Harmonic efficiency changes with increasing density scale-length and mirrors the change in the absorption process

• As the density scale-length increases, the absorption process changes from Brunel-type to classical resonance absorption.

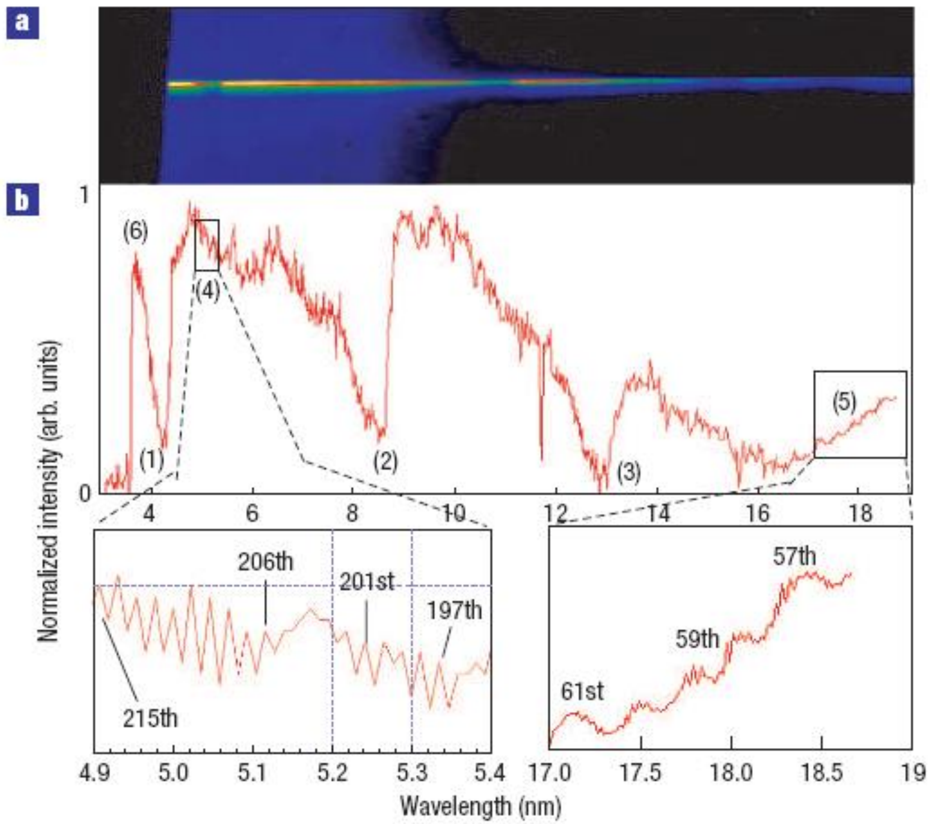
• Maximum absorption at L/λ ~0.2

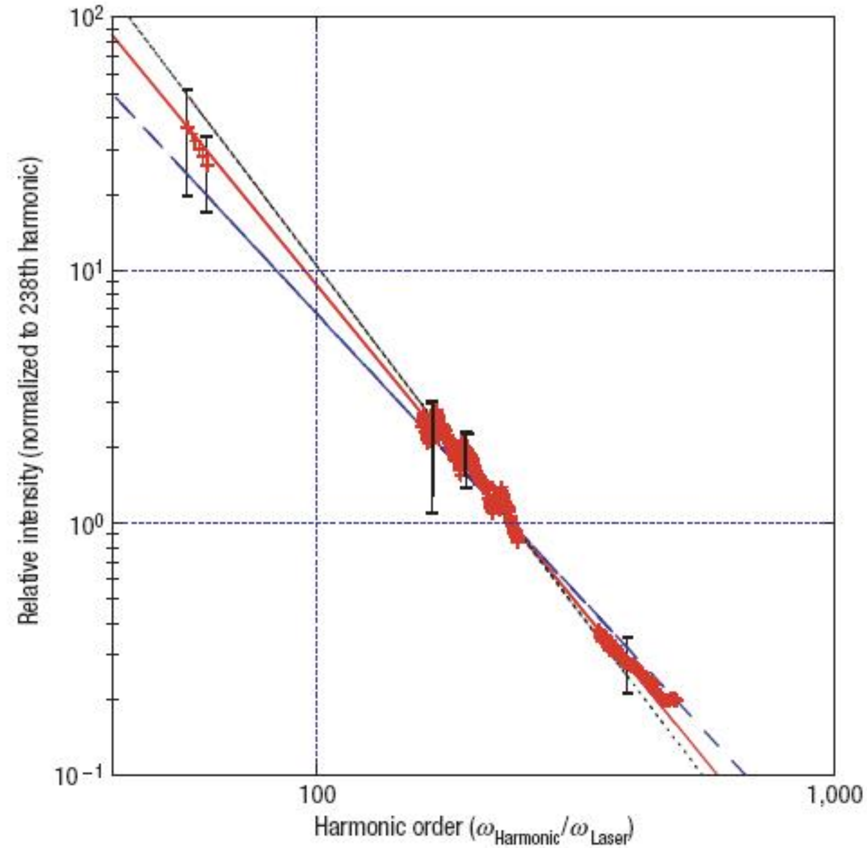


M. Zepf et al., Physical Review E 58, R5253 (1998).

Intense laser-plasma interaction physics

High harmonic generation – towards attosecond interactions





B.Dromey, M.Zepf….P.A.Norreys *Nature Physics* **2** 546 (2006)

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Ponderomotive force

Neglecting electron pressure, the force equation on the electron fluid is

$$\frac{\partial \underline{u_e}}{\partial t} + \underline{u_e} \cdot \nabla \underline{u_e} = -\frac{e}{m} E(x) \sin \omega t$$

To a first approximation $\underline{u_e} = \underline{u}^h$ where $\underline{u}^h = \frac{e\underline{E}}{m\omega} \cos \omega t$

$$\frac{\partial \underline{u}^h}{\partial t} - \frac{e}{m} \underline{E}(x) \sin \omega t$$

Averaging the force equation over the rapid oscillations of the electric field,

$$m \frac{\partial \underline{u}^s}{\partial t} = -e\underline{E}^s - m < \underline{u}^h . \nabla \underline{u}^h > t$$

< > denotes the average of the high frequency oscillation, $\underline{u}^s = < u_e >_t$ $\underline{E}^s = < \underline{E} >_t$

$$m \frac{\partial \underline{u}^s}{\partial t} = -e\underline{E}^s - \frac{1}{4} \frac{e^2}{m\omega^2} \nabla \underline{E}^2(x)$$

Ponderomotive force

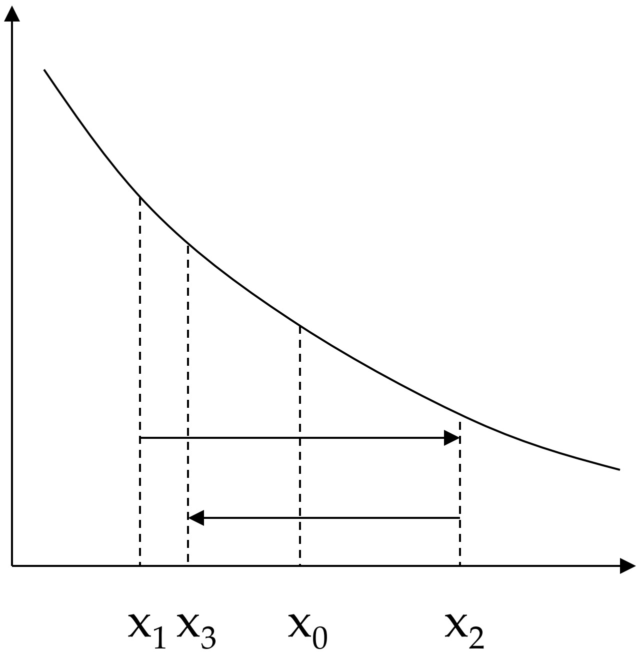
- An electron is accelerated from its initial position x_0 to the left and stops at position x_1.

- It is accelerated to the right until its has passed its initial position x_0.

- From that moment, it is decelerated by the reversed electric field and stops at x_2

- If x_0' is the position at which the electric field is reversed, then the deceleration interval x_2- x_0' is larger than that of the acceleration since the E field is weaker and a longer distance is needed to take away the energy gained in the former ¼ period.

- On its way back, the electron is stopped at position x_3



The result is a drift in the direction of decreasing wave amplitude.

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Summary of laser-plasma instabilities

Stimulated Brillouin Scatter - SBS.	Scattered light is downshifted by $\omega_{ia} \sim \omega_L$ Source of large losses for ICF experiments Very dangerous for laser systems
Stimulated Raman Scatter – SRS.	Scattered light downshifted by ω_p Produces large amplitude plasma waves Hot electrons (Landau damping) Preheating of fuel
Two Plasmon Decay - TPD.	Decays into two plasma waves Not as serious Localised near n_{cr} / 4.
Filamentation	Can cause beam non-uniformities problematic for direct drive ICF

Regimes of applicability

Dispersion relation

$$\omega^2 = c^2 k^2 + \omega_{pe}^{\ 2} \Rightarrow n_{cr} = \frac{10^{21} cm^{-3}}{\lambda_{\mu m}^{\ 2}}$$

SRS

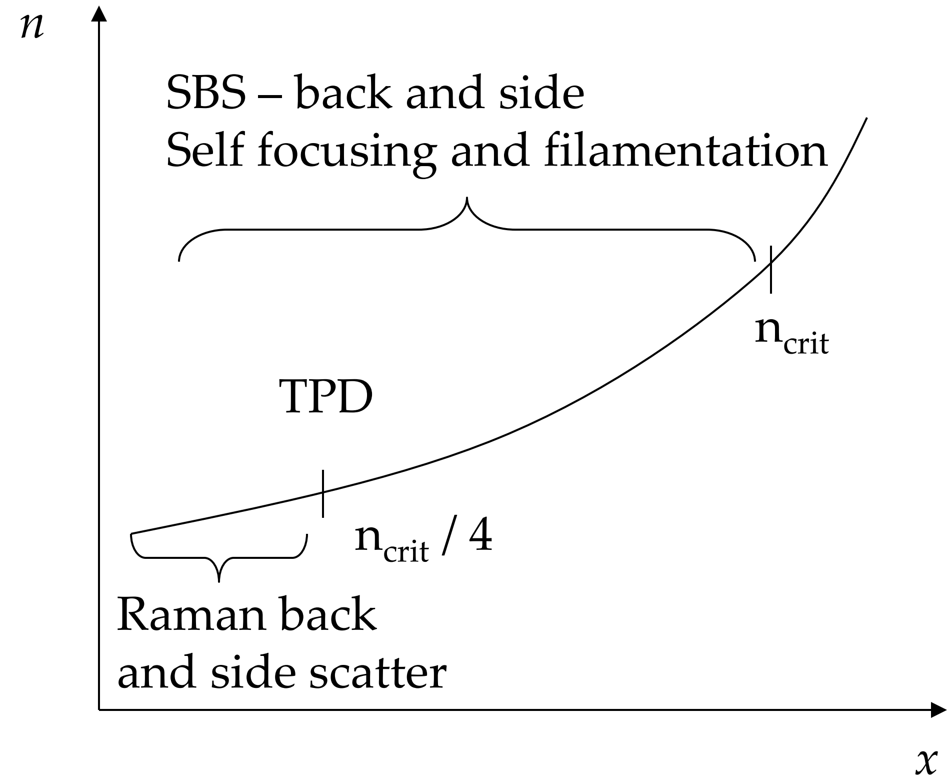
$$\omega \rightarrow \omega_{sc} + \omega_p$$

SBS

$$\omega \rightarrow \omega_{sc} + \omega_{ia}$$

TPD

$$\omega \rightarrow \omega_p + \omega_p$$



Parametric instability growth

Large amplitude light wave couples to a density fluctuation

$E_L \delta n$

Drives an electric field associated with an electron plasma wave

E_p

$E_L E_p$

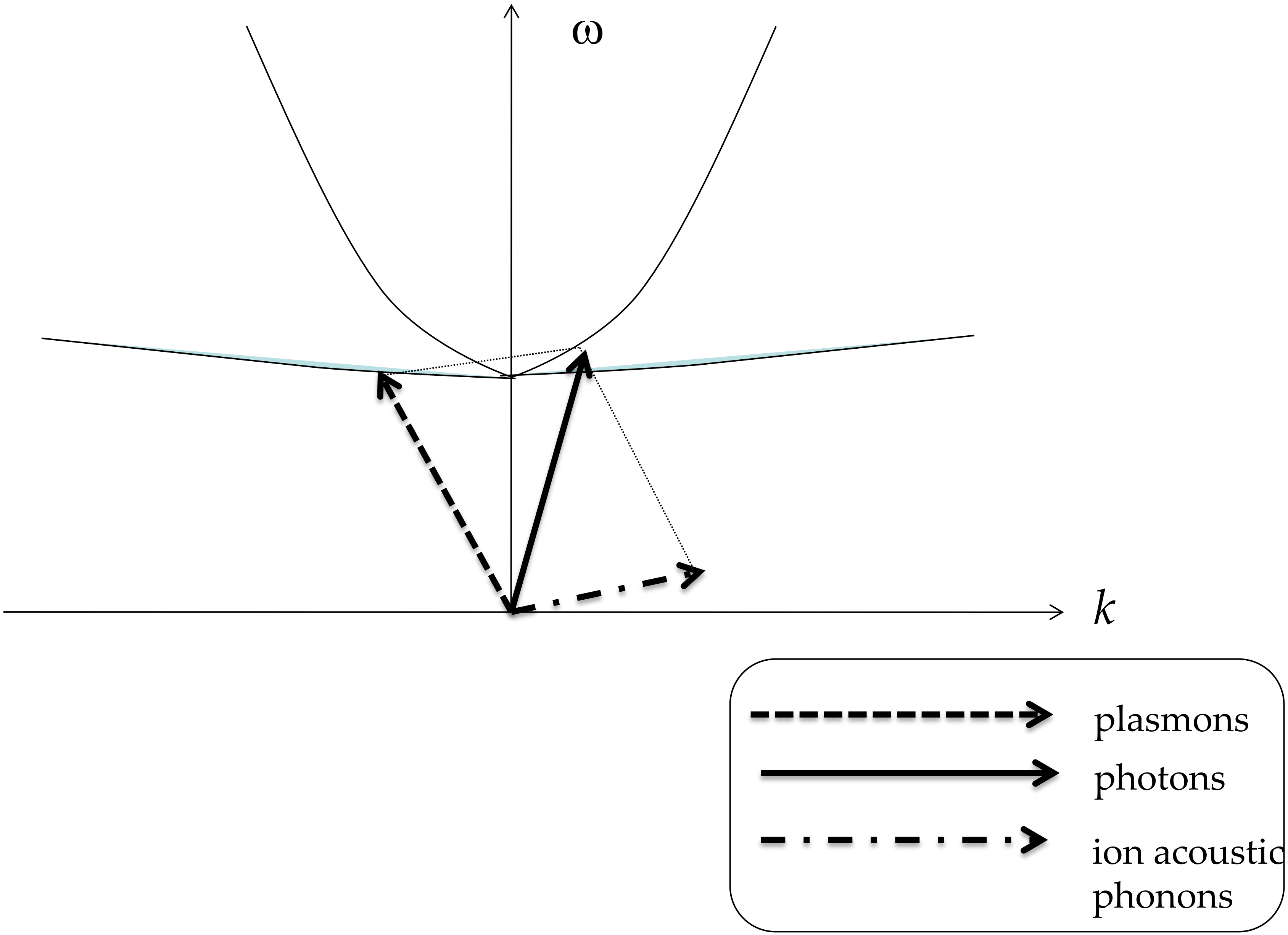
The electric fields combine to generate a fluctuation in the field pressure

δn

Enhances the density perturbation

The threshold for the instabilities is determined by spatial inhomogeneities and damping of plasma waves – this reduces the regions where the waves can resonantly interact

Parametric decay



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Stimulated Raman Scatter

$$\omega_0 = \omega_s + \omega_{pe}$$

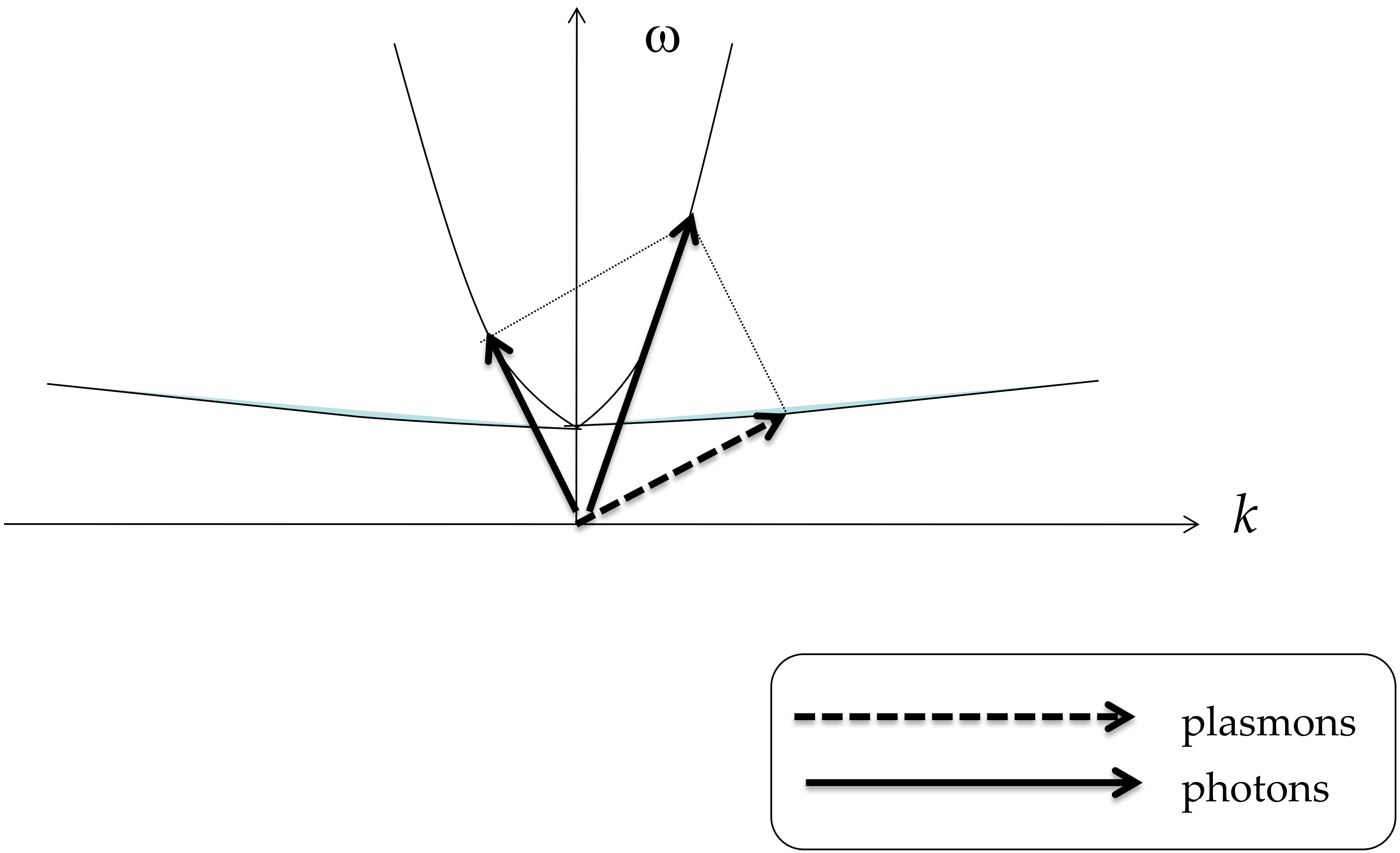
$$\underline{k}_0 = \underline{k}_s + k_{pe}$$

The instability requires $\omega_0 \geq 2\omega_{pe}$

i.e $n \leq \frac{n_{cr}}{4}$

- 10 % in ICF plasmas. The instability causes heating of the plasma, due to damping
- Can have high phase velocity – produces high energy electrons

Stimulated Raman scattering



General instability analysis

Derive an expression that relates the creation of the scattered EM waves (by coupling the density perturbations generated by the laser field) to the transverse current that produces the scattered light wave.

Start from Ampere's law $$\nabla\times\underline{B}=4\pi\underline{J}+\frac{1}{c}\frac{\partial\underline{E}}{\partial t}\qquad(1)$$

Remember that $\nabla\cdot\underline{A}=0$ $\qquad \underline{B}=\nabla\times\underline{A}$

$$\underline{E}=\frac{1}{c}\frac{\partial\underline{A}}{\partial t}-\nabla\phi \qquad \nabla\times(\nabla\times\underline{A})=\nabla(\nabla.\underline{A})-\nabla^2\underline{A}$$

$$\Rightarrow\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\underline{A}=\frac{4\pi}{c}\underline{j}-\frac{1}{c}\frac{\partial}{\partial t}\nabla\phi\qquad(2)$$

General instability analysis continued…1

Separate J into a EM part $\boldsymbol{j}_t$ and a longitundinal part $\boldsymbol{j}_l$ $\quad \underline{j} = \underline{j}_t + \underline{j}_l$ (3)

Poisson's equation $\quad \nabla^2\phi = -4\pi\rho$ (4) $\quad \rho$ is the charge density

Conservation of charge $\quad \dfrac{\partial\rho}{\partial t} + \nabla.\underline{j} = 0$ (5)

Taking ∂/∂t of (4) and substituting for ∂ρ/∂t from (5)

$$\nabla\cdot\left(\frac{\partial}{\partial t}\nabla\phi - 4\pi\,\underline{j}\right) = 0 \qquad (6)$$

$$\therefore \frac{\partial}{\partial t}\nabla\phi = 4\pi\,\underline{j}_l \qquad \text{because} \qquad \nabla.\underline{j}_t = 0 \qquad (7)$$

Equation (2) becomes

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\underline{A} = \frac{4\pi}{c}\underline{j}_t \qquad (8)$$

If we restrict the problem to $\boldsymbol{A}.\nabla n_e = 0$, the transverse current becomes $\boldsymbol{j}_t = -\, n_e\, e\, \boldsymbol{u}_t$ where $\boldsymbol{u}_t$ is v_{osc}. For $\boldsymbol{u}_t \ll$c, $\boldsymbol{u}_t = e\boldsymbol{A}/mc$ because

$$\frac{\partial \underline{u}_t}{\partial t} = -\frac{e}{m}\underline{E}_t = \frac{e}{mc}\frac{\partial \underline{A}}{\partial t} \qquad (9)$$

This gives eqn for propagation of a light wave in a plasma

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\underline{A} = -\frac{4\pi e^2}{m} n_e \underline{A} \qquad (10)$$

Substitute for $\underline{A} = \underline{A}_L + \underline{A}_1$ and $n_e = n_0 + n_1$

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_{pe}^2\right)\underline{A}_1 = -\frac{4\pi e^2}{m} n_1 \underline{A}_L \qquad (11)$$

The right hand side is simply the transverse current ($\propto n_1 \boldsymbol{v}_L$) that produces the light wave.

Now we need an equation for the density perturbation

Continuity equation $$\frac{\partial n_e}{\partial t} + \nabla\cdot\left(n_e \underline{u}_e\right) = 0 \qquad (12)$$

Force equation $$\frac{\partial \underline{u}_e}{\partial t} + \underline{u}_e \cdot \nabla \underline{u}_e = \frac{-e}{m}\left(E + \frac{\underline{u}_e \times \underline{B}}{c}\right) - \frac{\nabla p_e}{n_e m} \qquad (13)$$

Here the ions are fixed and provide a neutralising background

General instability analysis continued…2

$$\underline{u}_e = \underline{u}_L + \frac{e\underline{A}}{mc} \qquad (14)$$

$\underline{u}_L$: longitudinal; $\frac{e\underline{A}}{mc}$: transverse

$$\frac{\partial}{\partial t}\left(\underline{u}_L + \frac{e\underline{A}}{mc}\right) + \left(\underline{u}_L + \frac{e\underline{A}}{mc}\right)\nabla\cdot\left(\underline{u}_L + \frac{e\underline{A}}{mc}\right) = -\frac{\nabla p_e}{n_e m} - \frac{e}{m}\left(\underline{E} + \frac{1}{c}\left(\underline{u}_L + \frac{e\underline{A}}{mc}\right)\times B\right)$$

substituting

$$\underline{B} = \nabla\times\underline{A} \qquad \underline{E} = \frac{1}{c}\frac{\partial\underline{A}}{\partial t} - \nabla\phi \qquad (\underline{A}\cdot\nabla)\underline{A} + \underline{A}\times(\nabla\times\underline{A}) = \frac{1}{2}\nabla(\underline{A})^2$$

gives

$$\frac{\partial \underline{u}_L}{\partial t} = \underbrace{\frac{e}{m}\nabla\phi}_{\text{Electric field}} - \underbrace{\frac{1}{2}\nabla\left(\underline{u}_L + \frac{e\underline{A}}{mc}\right)^2}_{\text{Ponderomotive force}} - \underbrace{\frac{\nabla p_e}{n_e m}}_{\text{Plasma pressure}} \qquad (15)$$

General instability analysis continued…3

Use adiabatic equation of state $p_e/n_e{}^3$ = constant (16)

Linearise (12) and (15). Do this by taking $\underline{u}_L = \underline{u}, n_e = n_o + n_1, \underline{A} = \underline{A}_L + \underline{A}_1$

$$\left.\begin{aligned} &\nabla\left(\frac{p_e}{n_e^3}\right) = 0 \\ &\nabla p_e = \frac{3p_e}{n_e}\nabla n_1 \\ &\frac{\nabla p_e}{n_e m_e} = \frac{3v_{th}^2}{n_0}\nabla n_1 \end{aligned}\right\} \quad (17)$$

$$\frac{\partial \underline{u}}{\partial t} = \frac{e}{m}\nabla\phi - \frac{1}{2}\nabla\left(\underline{u}_1 + \frac{e}{mc}\left(\underline{A}_L + \underline{A}_1\right)\right)^2 - \frac{3v_{th}{}^2}{n_0}\nabla n_1 \quad (18)$$

General instability analysis finalised

To 1st order ignore A_1^2, u_1^2 and $\underline{u}_1 \cdot \underline{A} = 0$

$$\frac{\partial \underline{u}}{\partial t} = \frac{e}{m}\nabla\phi - \frac{e^2}{m^2c^2}\nabla\left(\underline{A}_L \cdot \underline{A}_1\right) - \frac{3{v_{th}}^2}{n_0}\nabla n_1 \qquad (19)$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla . \underline{u}_1 = 0 \qquad (20)$$

Combining by taking time derivative of (20), a divergence of (19) gives

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - 3v_e^2\nabla^2\right)n_1 = \frac{n_0 e^2}{m^2c^2}\nabla^2\left(\underline{A}_L \cdot \underline{A}_1\right) \qquad (21)$$

Where $\nabla^2\phi = 4\pi n_1 e$

This is now the equation for the density perturbations generated by fluctuations in the intensity of the EM wave

Dispersion relationship for SRS

Start with $\underline{A}_L = \underline{A}_0 \cos(\underline{k}_0 \cdot \underline{x} - \omega_0 t)$

Fourier analyse

$$\left(\omega^2 - k^2c^2 - \omega_{pe}^2\right)\underline{A}_1(\underline{k},\omega) = \frac{4\pi\ e^2}{2m}\underline{A}_0\left[n_1(k-k_0,\omega-\omega_0) + n_1\left(k+k_0,\omega+\omega_0\right)\right] \tag{22}$$

$$\left(\omega^2 - \omega_{ek}^2\right)n_1(\underline{k},\omega) = \frac{k^2e^2n_0}{2m^2c^2}\underline{A}_0 \cdot \left[\underline{A}_1(k-k_0,\omega-\omega_0) + \underline{A}_1\left(k+k_0,\omega+\omega_0\right)\right] \tag{23}$$

Where $\omega_{ek} = \left(\omega_{pe}^2 + 3k^2v_e^2\right)^{1/2}$ is the Bohm-Gross frequency and ω_0 and k_0 are the frequency and wave number of the laser wave. Also

$$\left.\begin{aligned}
\underline{A}_1 &= A_1 \exp(ikx - iwt) \\
n_1 &= n_1 \exp(ikx - i\omega t) \\
\underline{A}_0 &= A_0 \cos(k_0x - w_0t) \\
\underline{A}_0 &= A_0[\exp i(k_0x - w_0t) + \exp - i(k_0x - w_0t)]
\end{aligned}\right\} \tag{24}$$

Growth rates

Use (19) to eliminate A_1

$$\left(\omega^2-\omega_{ek}^2\right)=\frac{\omega_p^2 k^2 v_{os}^2}{4}\cdot\left[\frac{1}{D\left(\omega-\omega_0,k-k_0\right)}+\frac{1}{D\left(\omega+\omega_0,k+k_0\right)}\right] \qquad (25)$$

$$D(\omega,k)=\omega^2-k^2c^2-\omega_{pe}^2$$

v_{os} is the oscillatory velocity (eA_0/mc)

Note: in deriving (25), it was assumed that $\omega \sim \omega_{pe}$ and terms $n_1\ (k-2k_0,\ \omega-2\omega_0)$ and $n_1\ (k+2k_0,\ \omega+2\omega_0)$ were neglected as non-resonant.

From this dispersion relation, the growth rates can be found. Neglecting the up-shifted light wave as non-resonant gives

$$\left(\omega^2-\omega_{ek}^2\right)\left[\left(\omega-\omega_0\right)^2-\left(\underline{k}-\underline{k}_0\right)^2c^2-\omega_{pe}^2\right]=\frac{\omega_{pe}^2 k^2 v_{os}^2}{4} \qquad (26)$$

Maximum growth rate

Take $\omega_{ek} + \delta\omega$ where $\delta\omega << \omega_{ek}$. Note that maximum growth rate occurs when the scattered light is resonant, i.e. when

$$(\omega_{ek} - \omega_0)^2 - (\underline{k} - \underline{k}_0)^2 c^2 - \omega_{pe}^2 = 0 \qquad (27)$$

Then take $\delta\omega = i\gamma$ where $\gamma = \frac{k v_{os}}{4}\left[\frac{\omega_{pe}^2}{\omega_{ek}(\omega_0 - \omega_{ek})}\right]^{1/2}$ (28)

The wave number k is given by (27). For backscattered light the growth rate maximises for

$$\boxed{k = k_0 + \frac{\omega_0}{c}\left(1 - \frac{2\omega_{pe}}{\omega_0}\right)^{1/2}} \qquad (29)$$

The wave numbers start at $k = 2k_0$ for $n << n_c / 4$ and goes to $k = k_0$ for $n \sim n_{cr}/4$

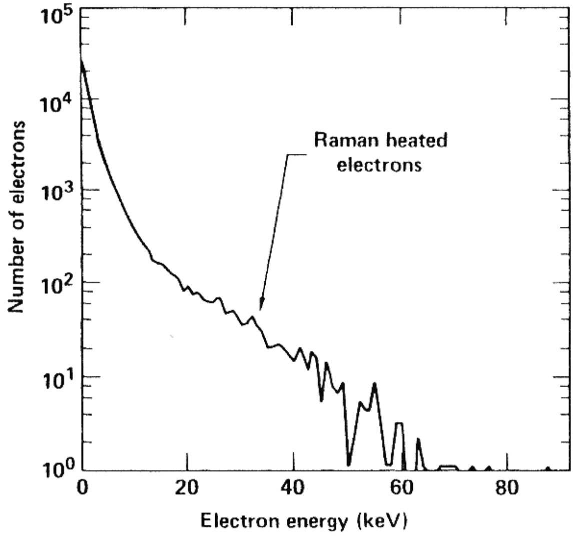
Growth rate in low density plasmas

For forward scattered light at low density $k \ll \omega_0/c$, both the up-shifted and down-shifted light waves can be nearly resonant

$$D\left(\omega \pm \omega_0, \underline{k} \pm \underline{k}_0\right) \approx 2\left(\omega_{pe} \pm \omega_0\right)\delta\omega \qquad (30)$$

Substitute into (25), the maximum growth rate $\delta\omega = i\gamma$

$$\gamma \approx \frac{\omega_{pe}^2}{2\sqrt{2}\omega_0}\frac{v_{os}}{c} \qquad (31)$$



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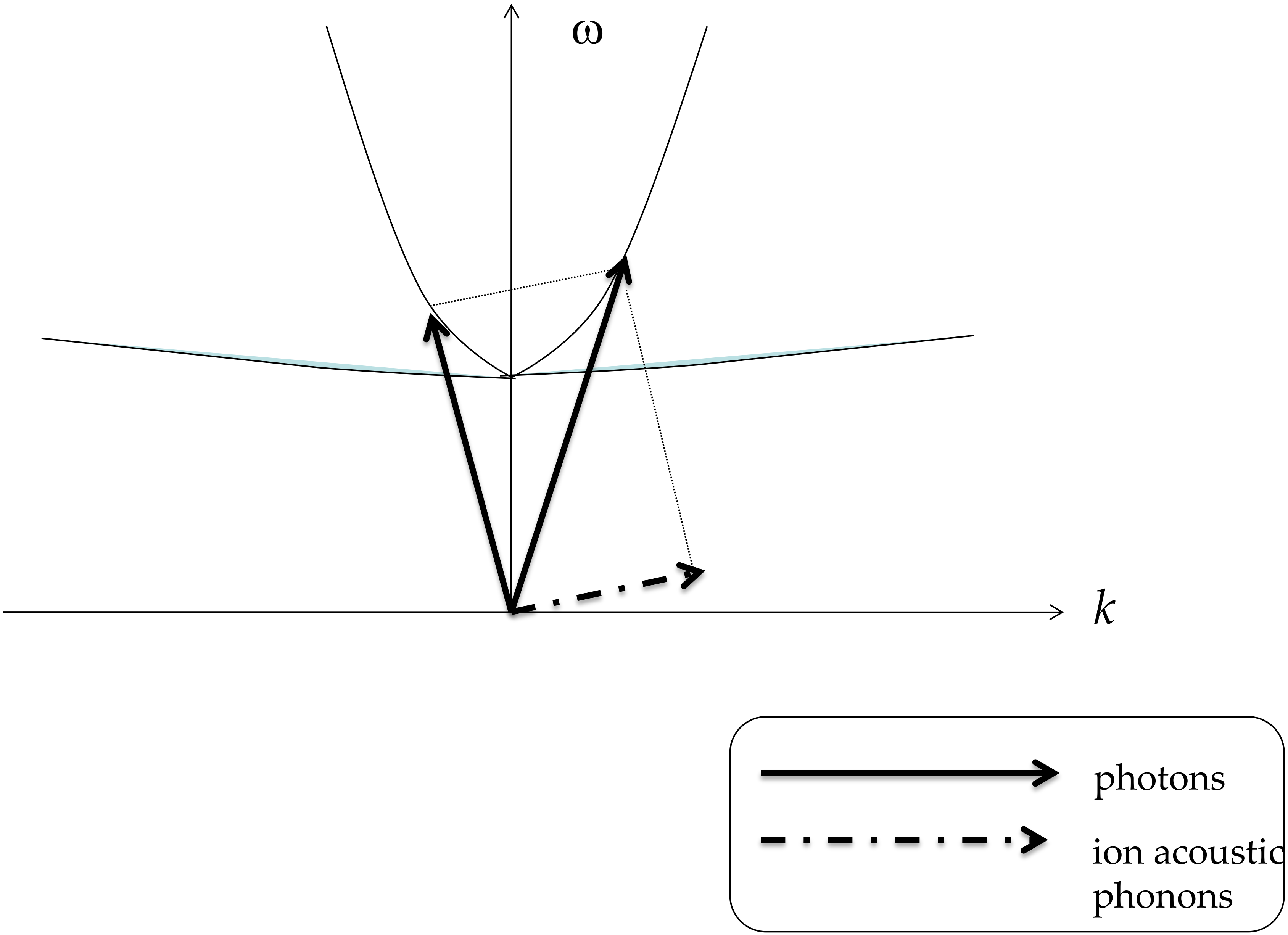
1. Introduction
   - Ionisation processes
2. Laser propagation in plasmas
   - Resonance absorption
   - Inverse bremsstrahlung
   - Not-so-resonant resonance absorption (Brunel effect)
3. Ponderomotive force
4. Instabilities in laser-produced plasmas
   - Stimulated Raman scatter (SRS) instability
   - **Stimulated Brillouin scatter (SBS) instability**
   - Two plasmon decay
   - Filamentation

Stimulated Brillouin Scatter



Stimulated Brillouin Scattering (SBS)

- density fluctuations associated with high frequency ion acoustic waves causes scattering of light

- > 60 % has been observed in experiment

$$\omega_0 \rightarrow \omega_s + \omega_{ia}$$

$$\underline{k}_0 \rightarrow \underline{k}_s + \underline{k}_{ia}$$

- Similar analysis to SRS, except n_1 is density fluctuation associated with an ion acoustic wave. We write

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_{pe}^2\right)\underline{A}_1 = -\frac{4\pi\, e^2}{m} n_1 \underline{A}_L \qquad (32)$$

- Force equation gives $\dfrac{\partial^2 n_1}{\partial t^2} - c_s{}^2\nabla n_1 = \dfrac{Zn_0e^2}{mMc^2}\nabla^2\left(\underline{A}_L \cdot \underline{A}_1\right)$ (33)

$c_s = (ZkT_e / M)^{1/2}$ ion acoustic velocity

Growth rate

Fourier analyse (32) and (33)

$$\omega = kc_s + i\gamma$$

$$\gamma << kc_s$$

gives

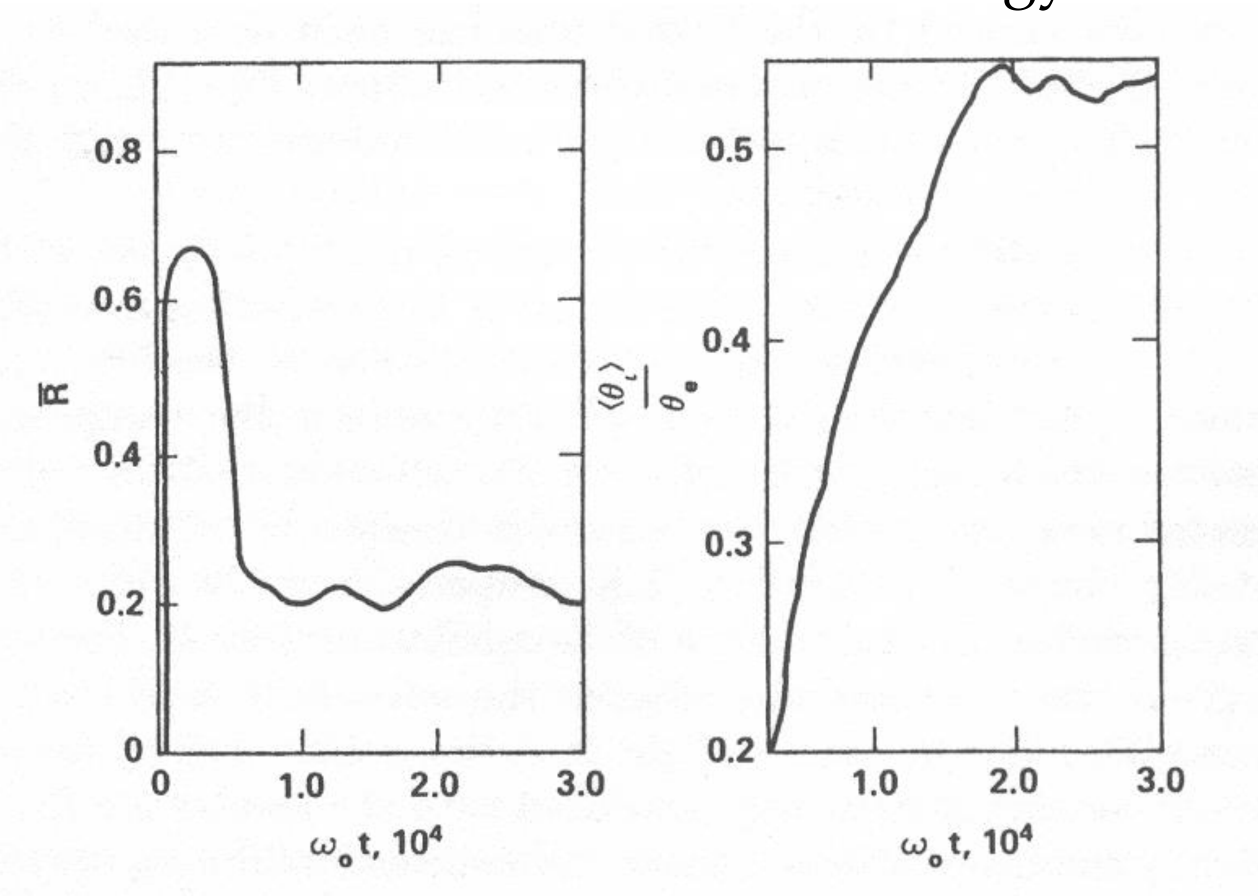
$$\underline{k} = 2\underline{k}_0 - \frac{2\omega_0 c_c}{c^2} \qquad (34)$$

$$\gamma = \frac{1}{2\sqrt{2}} \frac{k_0 v_{os} \omega_{pi}}{(\omega_0 k_0 c_s)^{1/2}} \qquad (35)$$

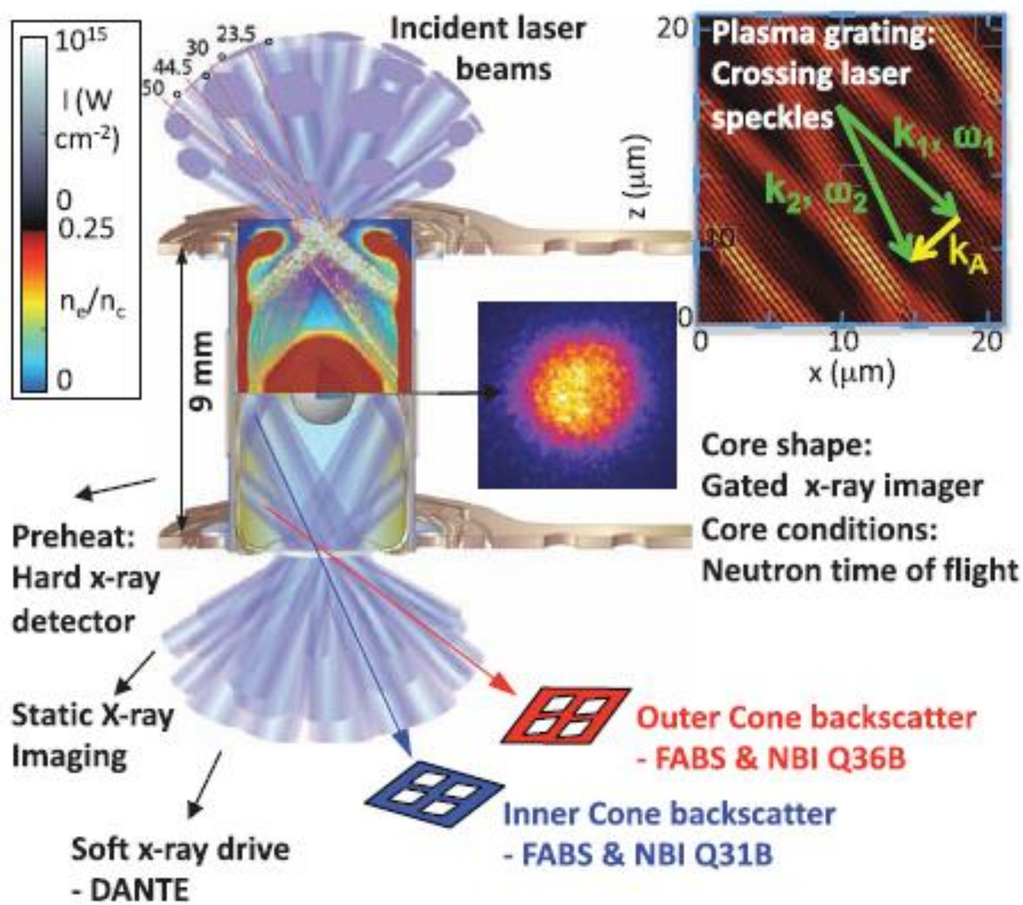
for backscattered light

reflectivity

average ion energy



Holhraum irradiation uniformity by SBS



S. Glenzer et al., Science **327**, 1228 (2010)
P. Michel et al., Phys. Rev. Lett. **102**, 025004 (2009)

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Two plasmon decay instability

- Occurs in a narrow density region at n_{cr} / 4

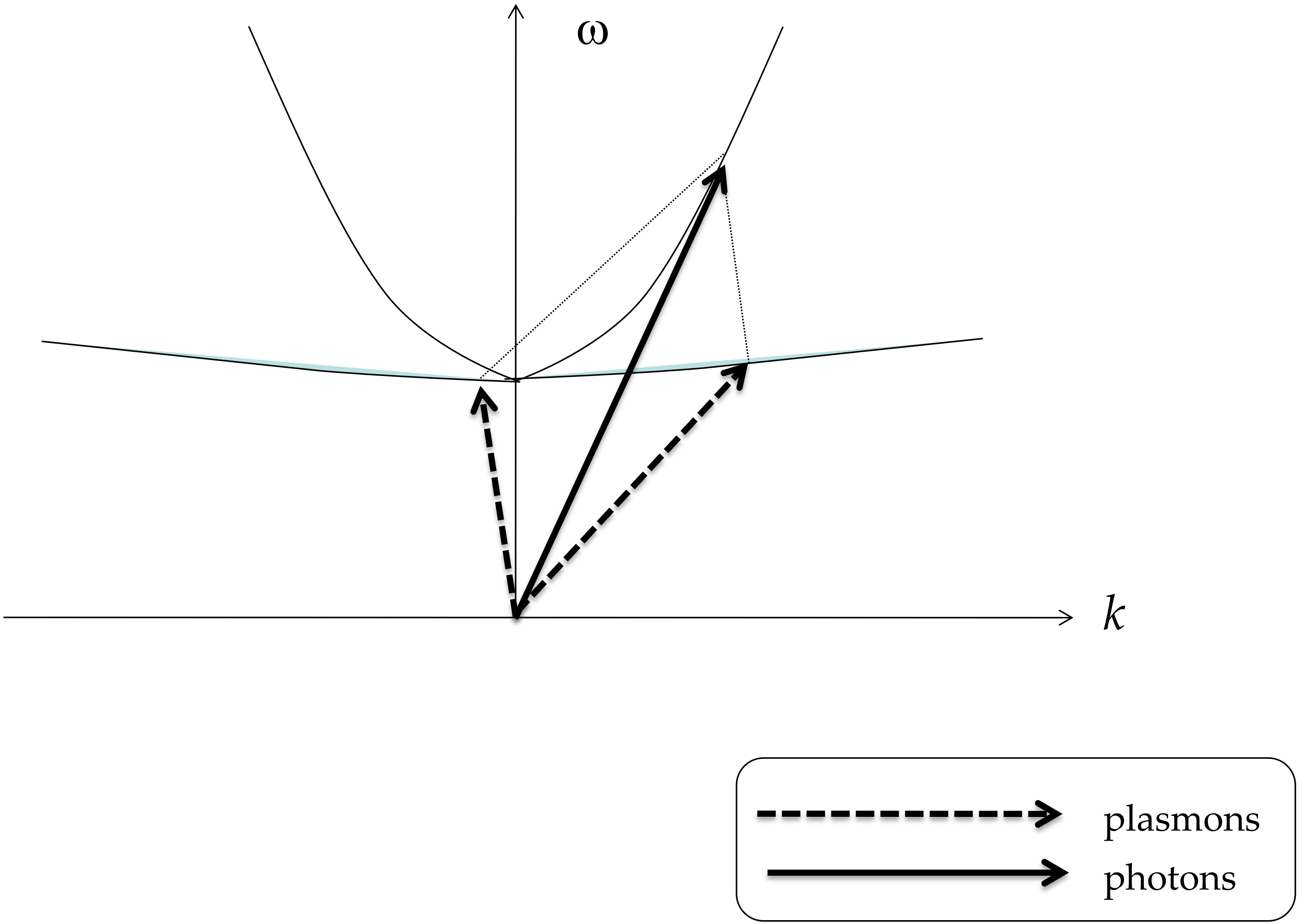
$$\omega \rightarrow \omega_{pe} + \omega_{pe}$$

$$\underline{k} \rightarrow \underline{k}_1 + \underline{k}_2$$

- Similar analysis as SRS except the equations describe the coupling of the electron plasma waves with wave number $\underline{k}$ and $\underline{k} - \underline{k}_0$ with the large light wave

- For k>>k$_0$, $\gamma = \dfrac{k_0 v_{os}}{4}$ for plasma waves at 45° to both $\underline{k}$ and $\underline{v}_{os}$
- Threshold is much less than SRS at n_{cr} / 4 unless the plasma is hot

Two plasmon decay



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Filamentation instability

- Same dispersion relation as SBS also describes filamentation

- Zero frequency density perturbation corresponding to modulations of the light intensity

- Occurs in a plane orthogonal to propagation vector of light wave $\underline{k} \cdot \underline{k}_o = 0$

$$\omega = i\gamma << \omega_0$$

$$\gamma = \frac{1}{8}\left(\frac{v_{os}}{v_t}\right)^2 \frac{\omega_p^2}{\omega_0}$$

- Filamentation can also be caused by thermal or relativistic effects:
  1. enhanced temperature changes refractive index locally
  2. relativistic electron mass increase has same effect as reducing the density

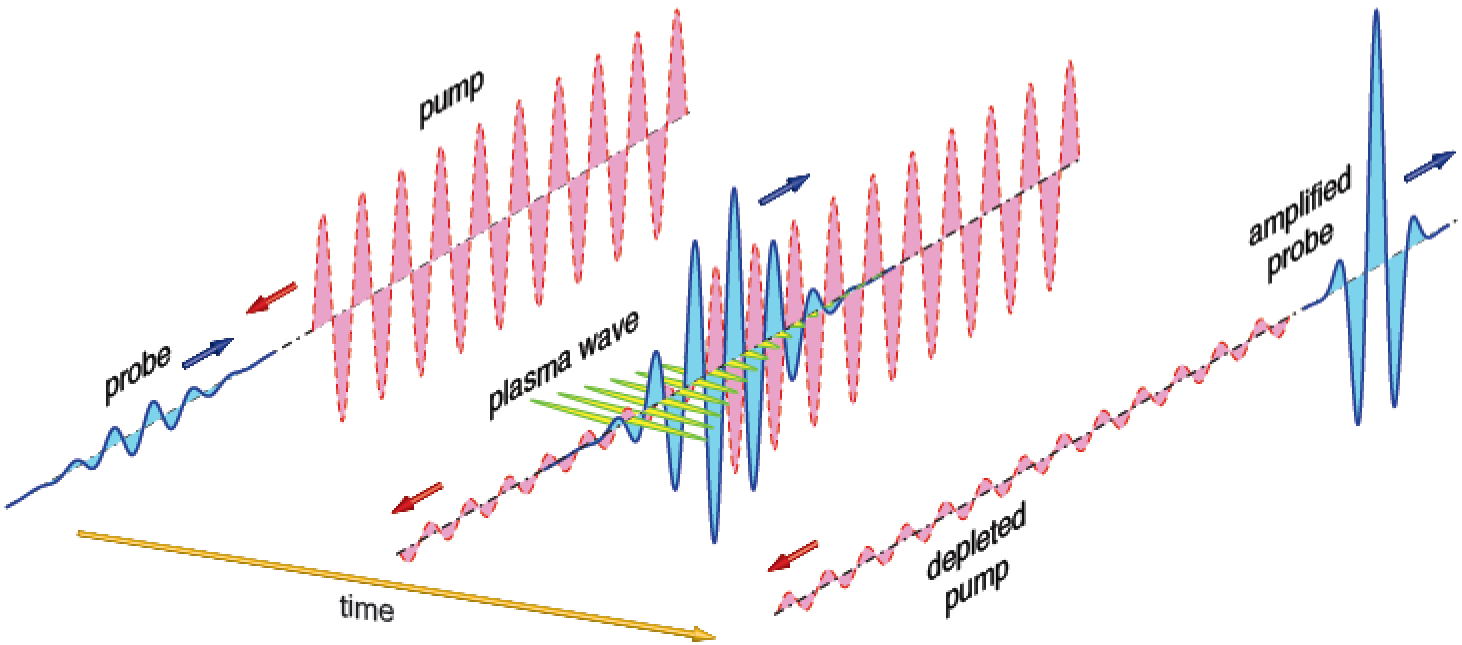
$$P_{critical} = 20\frac{\omega^2}{\omega_{pe}^2} GW \quad \text{for self-focusing}$$

Efficient Raman amplification into the petawatt regime

R. Trines , F. Fiuza, R. Bingham, R.A. Fonseca,
L.O.Silva, R.A. Cairns and P.A. Norreys

Nature Physics **7**, 87 (2011)

Raman amplification of laser pulses



- A long laser pulse (pump) in plasma will spontaneously scatter off Langmuir waves: Raman scattering

Stimulate this scattering by sending in a short, counter propagating pulse at the frequency of the scattered light (probe pulse)

Because scattering happens mainly at the location of the probe, most of the energy of the long pump will go into the short probe: efficient pulse compression

Miniature pulse compressor

Solid state compressor (Vulcan)

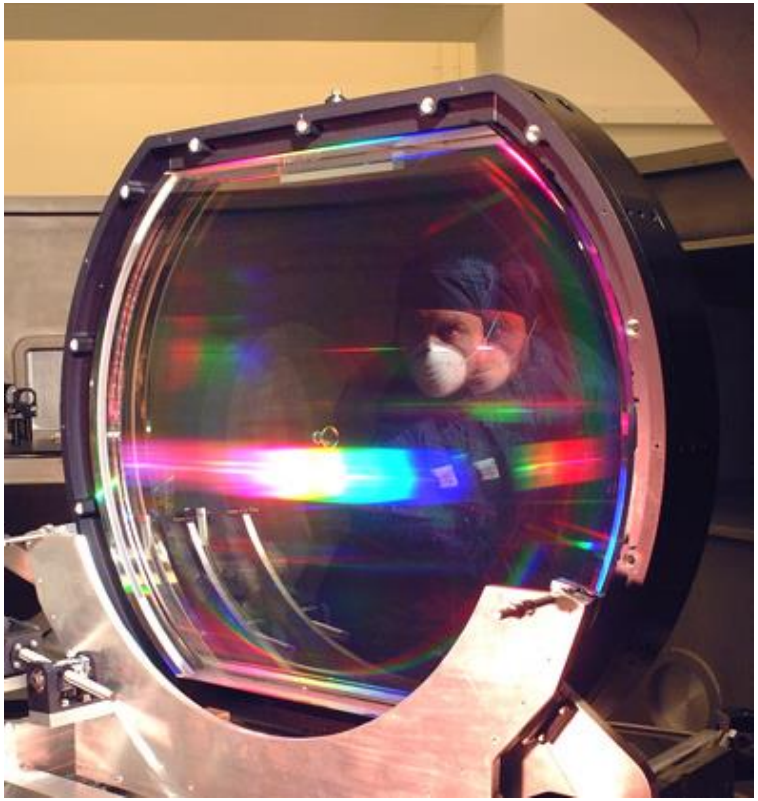
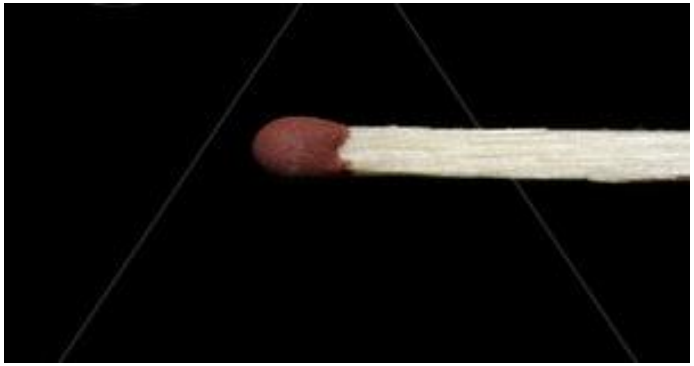


Image: STFC Media Services

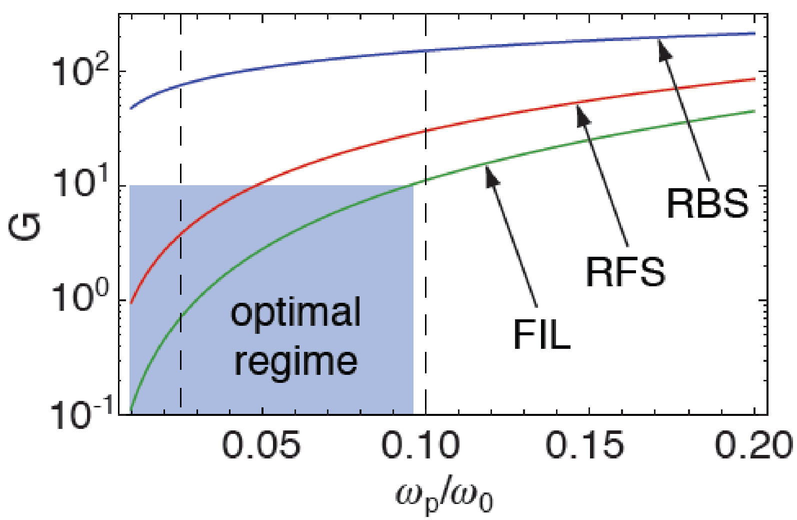
Volume of a plasma-based compressor



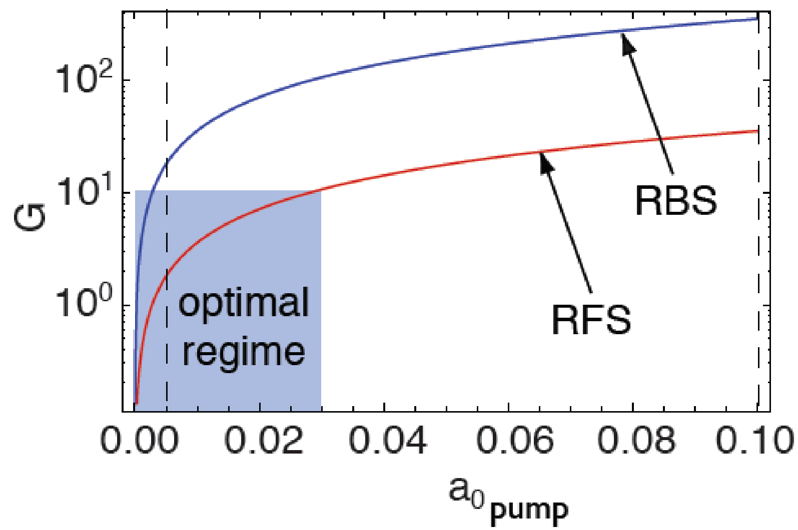
Simulations versus theory

- RBS growth increases with pump amplitude and plasma density, but so do pump RFS and probe filamentation
- Optimal simulation regime corresponds to at most 10 e-foldings for pump RFS and probe filamentation

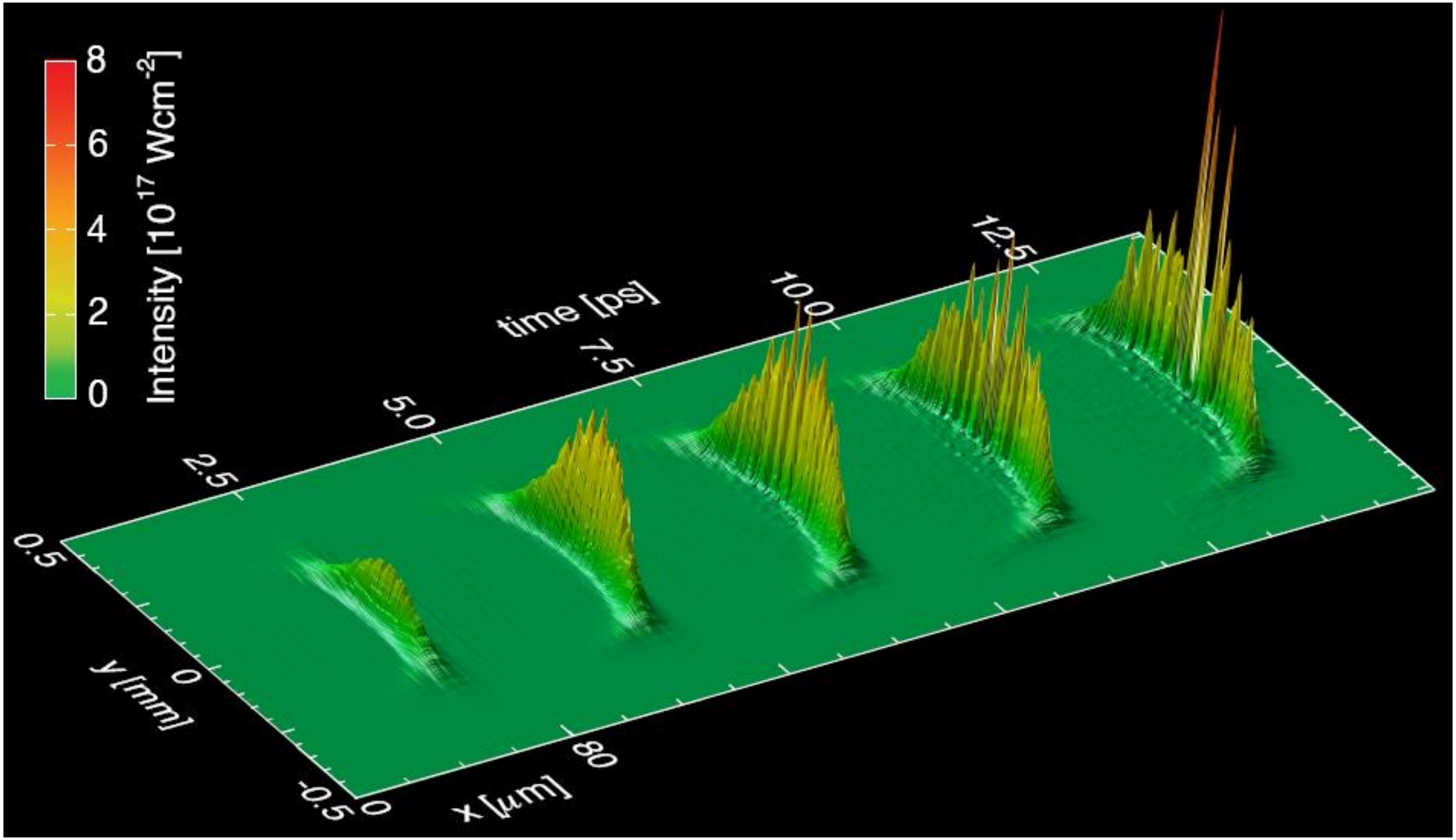
Growth versus plasma density



Growth versus pump amplitude

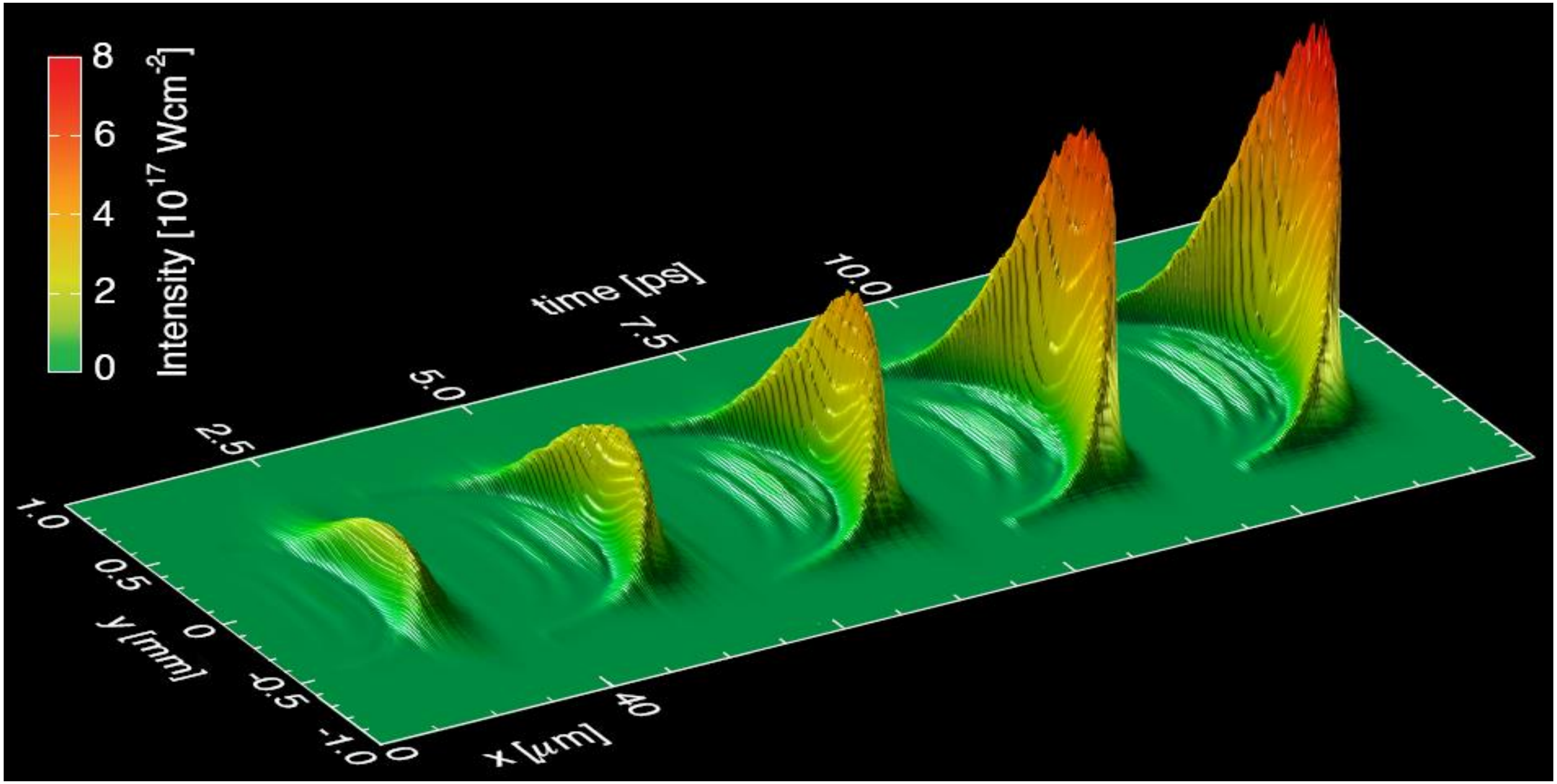


A bad result



For a $2*10^{15}$ W/cm^2 pump and $\omega_0/\omega_p = 10$, the probe is still amplified, but also destroyed by filamentation

A good result



For a $2*10^{15}$ W/cm^2 pump and $\omega_0/\omega_p = 20$, the probe is amplified to $8*10^{17}$ W/cm^2 after 4 mm of propagation, with limited filamentation

10 TW → 2 PW and transversely extensible!