

John Adams Institute.

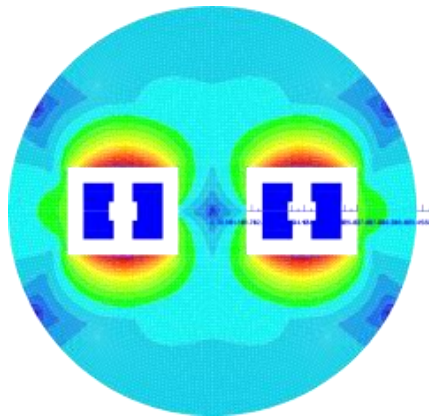
Magnet Technology Tutorial 29/1/2015

Neil Marks.

1. Objective.

To model and subsequently optimise a 'fit for purpose' two-dimensional cross section model of the main dipole ('M.B.') for a Future Circular Collider, present under study. The design needs to be a super-conducting dipole magnet, providing 16 T at the beam. This is believed to be possible using Nb₃Sb conductors at 1.9 K.

The current investigation at CERN envisages a dual beam magnet (as in the LHC):



However, for the purposes of this study, we shall only investigate a **single, fully symmetric** dipole assembly.

2. Supplied Parameters.

The FCC group at CERN are exploring a variety of designs but, for this study, have provided the parameters below:

Beam pipe diameter	mm	50
Operating current	A	16,260
Operating temperature	K	1.9
Operating flux density	T	16.0
Peak flux density	T	16.4
Operating point along load-line	%	80
Radial width of stainless-steel collar	mm	50
Circular yoke internal diameter	mm	240

3. Derived parameters.

i) Required excitation current (Amp-turns).

Assuming that the cylindrical yoke has a high permeability (an incorrect assumption but a reasonable approximation for an initial assessment as flux will also spread into the surrounding area), the current density required is given by the equation given by Wilson (overhead 81):

$$B_y = \mu_0 J_{op} t/2$$

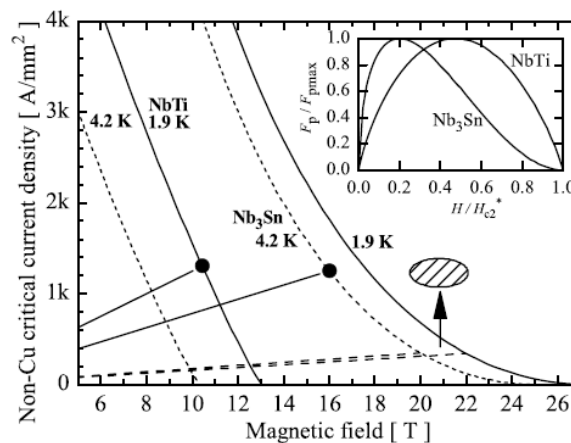
Where 't' is the separation between the two overlapping cylinders

Taking 't' as the beam pipe diameter, we get **509 A/mm²**.

This is the **approximate** operational current density required to generate the 16 T.

ii) The maximum Operating Current Density J_{op}

Shown below is the diagram for the inter-relationship between flux density and 'material current density' presented in the lectures:



As we seek to generate a dipole field that is uniform, it is certain that some parts of the coil will experience the full flux density of 16T. (Depending on the coil configuration, there may be parts which are subjected to higher flux densities; adjustments will be needed during the design iterations).

From the above diagram, the critical current density in Nb₃Sn, at 1.9K and 16T is

$$J_{crit} = 1,800 \text{ A/mm}^2;$$

To obtain the maximum possible 'engineering current density' the various packing factors of the s.c. in the coil must be considered:

- a) Ratio of s.c. material to Cu stabiliser is quoted as 1:1, giving a s.c. material packing factor in the cable of **0.5**;
- b) The circular cable is arranged in the coil in a rectangular lattice, giving a packing factor of $\pi/4 = \mathbf{0.785}$;
- c) Wilson (overhead 80) gives a figure of '0.7 to 0.8' for packing factor to account for the space occupied by insulation, cooling channels, mechanical reinforcement, etc. We take the mean of **0.75**.

Considering these three factors gives:

$$J_{\text{eng}} \leq 1,800 \times 0.5 \times 0.785 \times 0.75 = \mathbf{529.9 \text{ A/mm}^2}.$$

This is a reasonable estimate, but the correct figure for J_{eng} cannot be determined without a detailed design for the coil.

However, the provided parameters define an 80% operating point along the load-line, to allow for local perturbations and contingency; hence:

$$\text{the preferred operating current density } J_{\text{op}} \leq \mathbf{423.9 \text{ A/mm}^2}.$$

Note that the first estimate of the required current density is **less than** the **maximum engine current density** but **greater than** the **preferred operating current density**.

iii) Coil shape and dimensions.

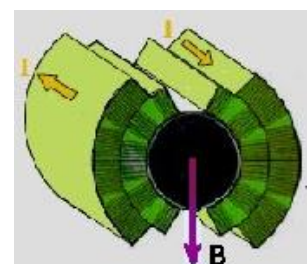
The coil is located between the beam pipe and the circular inner wall of the collar, which then is located inside the yoke; this gives the following radii:

inner radius of coil:	25 mm;
outer radius of coil (& inner radius of collar):	70 mm;
outer radius of collar (& inner radius of yoke):	120 mm;
outer radius of yoke:	325 mm.

The coil needs to approximate to two intersecting cylinders, as in the presentation and in the cross section of the LHC coil:

The two separate coils (inner and outer), of equal width, with the approximate angular placements about the vertical centre line:

inner coil: $\pm 70^\circ$;



outer coil: $\pm 55^\circ$.

These are to be adjusted empirically during the optimisation of the magnet.

4. Procedure.

Using OPERA 2D:

- i) Establish a model of the dipole cross section using the above parameters, with coils approximating to a pair of intersecting cylinders as shown and specified above. For the initial work, create a model of $\frac{1}{4}$ of the complete magnet ie the upper right quadrant. Depending on the quality of results you obtain, you may wish, later, to model a half magnet.
- ii) I would propose, for the first iteration of the magnet design, that you use the value of current density in the coils that was detailed in **3 (i)** above. At this stage ignore whether the J you are using is consistent with the maximum J_{eng} or J_{op} set above.
- iii) Examine you model; note the field quality across the $r = 25$ mm beam tube and B_y amplitude at the centre of the beam (0,0)
- iv) Carry out an initial optimisation, adjusting **the angular extent of the inner and outer coils**, (not their radii) and the magnitude of the **current density in the coils**. The optimisation is to achieve the following performance figures:

$$|\Delta B_y(x)/B_y(0)| \leq 0.01 \% \text{ between } 0 \leq x \leq 20 \text{ mm on the } y = 0 \text{ axis;}$$

$$B_y = 16.0 \text{ T at } x = 0, y = 0.$$

The two optimisations **may** not be linearly independent, as the steel yoke introduces some non-linearity which results in the field distribution being influenced by the amplitude; but this should be a very small effect..

If you are not able to achieve the field quality required, note the best quality that can be achieved.

- v) Examine the flux densities 'BMOD' **throughout the coil area** and determine whether the flux density at any point in the coil is significantly higher than 16T. If so, this maximum BMOD in the coil will be on a different load-line on the B vs J diagram. This will then define a different $J_{mat.}$; you will need to redefine the allowed maximum preferred J_{op} using the new figure for $J_{mat} \times 0.5 \times 0.785 \times 0.75 \times 0.8$.

However, if the applied J is higher than the new J_{op} but is **significantly** below the new maximum engineering J_{eng} ($J_{mat} \times 0.5 \times 0.785 \times 0.75$) in a small part of the coil, **this can be regarded as acceptable.**

Record all your results and the coil parameters that resulted from your optimisation.

- vi) If your model achieves the required flux density amplitude and distribution, and the B/J working point is acceptable according to the above criteria, **you have succeeded and you can terminate your investigation at this point.** You can complete your 'writing up'.
- vii) **If you have not been able to achieve the required 16 T** within the required limits of the B/J diagram, (either the preferred operational J or the higher engineering J) reduce the flux density in the coils until they are within the required B/J figures in all parts of the coil. Due to the non-linearity of the steel yoke it may be necessary to adjust the angular extent of the two coils to maintain the field quality.

Record all your results, particularly the flux density at $x = y = 0$ that you have achieved.

- viii) Subsequently, increase the coil outer radius by 5mm without changing its inner radius or the outer radius of the steel yoke (this corresponds to a reduction in the collar width to 45 mm and an increase in the total coil width to 50 mm), allowing a small reduction in the current density to achieve a given flux density. Explore whether the 16T can now be achieved with this new geometry within the bounds of the preferred J_{op} **as determined by the load line on the B/J diagram and the application of the packing factors.**

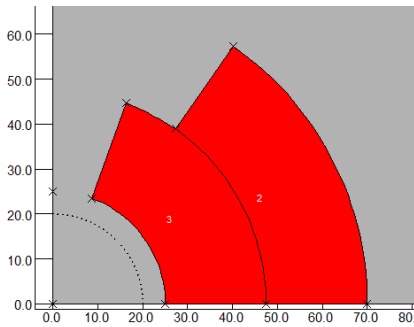
In all cases, record your results.

5. Interpreting Harmonic Series.

OPERA 2D provides output of the harmonic contents of the flux density by means of a **Fourier analysis** of the **vector potential** around a **circular arc**; this is part of the post-processor and will be demonstrated during the seminar. This gives information that will allow the determination of the coefficients (b_n) of the Taylor series:

$$B_y = \sum_{n=1}^{\infty} b_n x^{n-1}$$

An example of the output for the FCC dipole obtained from an r=20mm quarter arc,



The 1/4 period 30 point summation line at r = 20 mm inside the FCC coil assembly.

gives

Order	Sine term	Cosine term	Amplitude	Phase
n	A_n	B_n		
0	0.0	-1.9516E-18	1.95156E-18	179.9999950
1	3.26514E-18	0.330496269	0.330496269	-5.6605E-16
2	-2.1767E-17	-2.4720E-17	3.29376E-17	138.6339891
3	8.33175E-18	-2.5229E-03	2.52290E-03	-179.999995
4	-1.1009E-17	1.34441E-17	1.73763E-17	39.31236175
5	2.90651E-17	-2.5278E-04	2.52782E-04	-179.999995
6	1.25435E-17	4.72712E-17	4.89071E-17	-14.8611016
7	-1.5254E-17	1.49318E-04	1.49318E-04	5.85309E-12
8	4.69378E-17	2.16840E-18	4.69879E-17	-87.3549632
9	2.81624E-17	-5.4078E-05	5.40778E-05	-179.999995
10	3.21093E-17	2.47198E-17	4.05226E-17	-52.4086210
11	5.36662E-17	1.39898E-05	1.39898E-05	-2.1979E-10
12	-9.1573E-17	7.41594E-17	1.17836E-16	50.99810359
13	-5.7439E-17	-1.2168E-06	1.21678E-06	179.9999950
14	1.28607E-17	-1.3010E-18	1.29263E-17	-95.7766421

The harmonic output of OPERA 2D, for 30 points around the Fourier summation.

Interpreting these data is not straight forward. The 'upright' coefficients are in column B. As strict dipole symmetry was imposed by modelling a 1/4 magnet and defining appropriate conditions on all boundaries, the coefficients in the A columns and the even coefficients in B columns are artefacts and should be ignored. Of the remaining data, n=1 is dipole, n=3 is sextupole, n=5 is ten-pole, etc. – the 'allowed' error harmonics in a fully symmetric dipole.

To obtain the Taylor coefficients, normalisation is required; this is defined in the VF user manual (available electronically in the code). However, the Taylor coefficients b_n are defined below in terms of β_n , the coefficients in the 'B' column of the OPERA output:

$$b_n = \beta_n n \left(\frac{1}{r}\right)^n$$

where; n is the order of the term in the Taylor series;
 r is the radius of the curve used in the Fourier series.