Instabilities II

ACCELERATOR PHYSICS

HT 2015

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http://cas.web.cern.ch/cas/Loutraki-Proc/PDF-files/I-Schindl/paper2.pdf

Summary of last lecture – Instabilities I

- 1. General Comment on Instabilities
- 2. Negative Mass Instability
- 3. Driving terms (second cornerstone)
- 4. A cavity-like object is excited
- 5. Equivalent circuit
- 6. Above and below resonance
- 7. Laying the bricks in the wall (row 1)
- 8. Laying the bricks in the wall (row 2)
- 9. By analogy with the negative mass

Instabilities II

- 1. A short cut to solving the instability
- 2. An imaginative leap
- 3. The effect of frequency shift
- 4. Square root of a complex Z
- 5. Contours of constant growth
- 6. Landau damping
- 7. Stability diagram
- 8. Robinson instability
- 9. Coupled bunch modes
- 10 Microwave instability

A short cut to solving the instability

- **From theory of synchrotron motion:**
- **Recall the effect of a voltage of a cavity**

$$\frac{d}{dt}\left[\frac{E_0\beta^2\gamma\dot{\phi}}{2\pi\eta f^2}\right]+V_0\left(\sin\phi-\sin\phi_s\right)=0$$

- **Assume the particles have initially a small phase excursion about ϕs = 0**

$$\left[\frac{E_0\beta^2\gamma}{2\pi\eta f^2}\right]\ddot{\phi}+eV_0\phi=0$$

or

$$\ddot{\phi}+\Omega_s{}^2\phi=0$$

where $$\Omega_s^2=\left[\frac{\eta h V_0}{2\pi E_0\beta^2\gamma}\right]\omega_0^2$$

is the synchrotron frequency and ω_0 **is the revolution frequency.**

An imaginative leap

$$\boxed{\Omega_s^2 = \left[\frac{\eta h V_0}{2\pi E_0 \beta^2 \gamma}\right]\omega_0^2}$$

- Put in the volts induced by the beam in the cavity instad of the volts imposed from outside

$$V_o h \Rightarrow -inZI_0$$

$$h \Rightarrow n = \omega / \omega_0$$

- *i* reflects the fact that, unlike the RF wave the volts induced by a resistive load cross zero 90 degrees after the passage of the particle
- This bypasses much analysis and gives the right formula for the frequency shift.

$$\boxed{(\Delta\Omega)^2 = -i\left[\frac{\eta \omega_0^2 n I_0}{2\pi\beta^2 E}\right]Z == i\xi Z}$$

The effect of frequency shift

- Remember that a force driving an oscillator may be written on the right hand side:

$$\boxed{\ddot{\phi} + \Omega_0{}^2 \phi = F(t)}$$

- Alternatively it can be assimilated into the frequency

$$\boxed{\ddot{\phi} + (\Omega_0 + \Delta\Omega)^2 \phi = 0}$$

where:

$$\boxed{(\Delta\Omega)^2 = -i\left[\frac{\eta\,\omega_0^2 n I_0}{2\pi\beta^2 E}\right] Z == i\xi Z}$$

- if η is positive and Z pure imaginary (reactive) $\Delta(\Omega)^2$ is real and there is just a change in frequency.
- if Z has a resistive component this gives an imaginary part to $\sqrt{iZ}$
- Imaginary frequencies can signal exponential growth

Square root of a complex Z

- **Be careful to first multiply *Z* by *i* and then take the square root** $\sqrt{iZ} = \sqrt{i(X+iY)}$

- **There will be a locus in (X,Y) space where the imaginary part is constant which will be a contour of constant growth rate**
- **Suppose the solution to the differential equation is**

$$\phi = \phi_0 \; e^{-i\Omega t} = \phi_0 \; e^{-i(\alpha + i\beta)t}$$
$$= \phi_0 \; e^{+\beta t} \; e^{-i\alpha t}$$

- **We must solve for constant β**

$$(\Delta\Omega)^2 = (\alpha + i\beta)^2$$
$$(\Delta\Omega)^2 = (\alpha^2 - \beta^2) + 2i\alpha\beta$$
$$\Delta\Omega^2 = -i\xi Z = \xi Y - i\xi X$$

$$\xi X = -2\alpha\beta$$
$$\alpha = -\frac{\xi X}{2\beta}$$

- **Eliminate α**

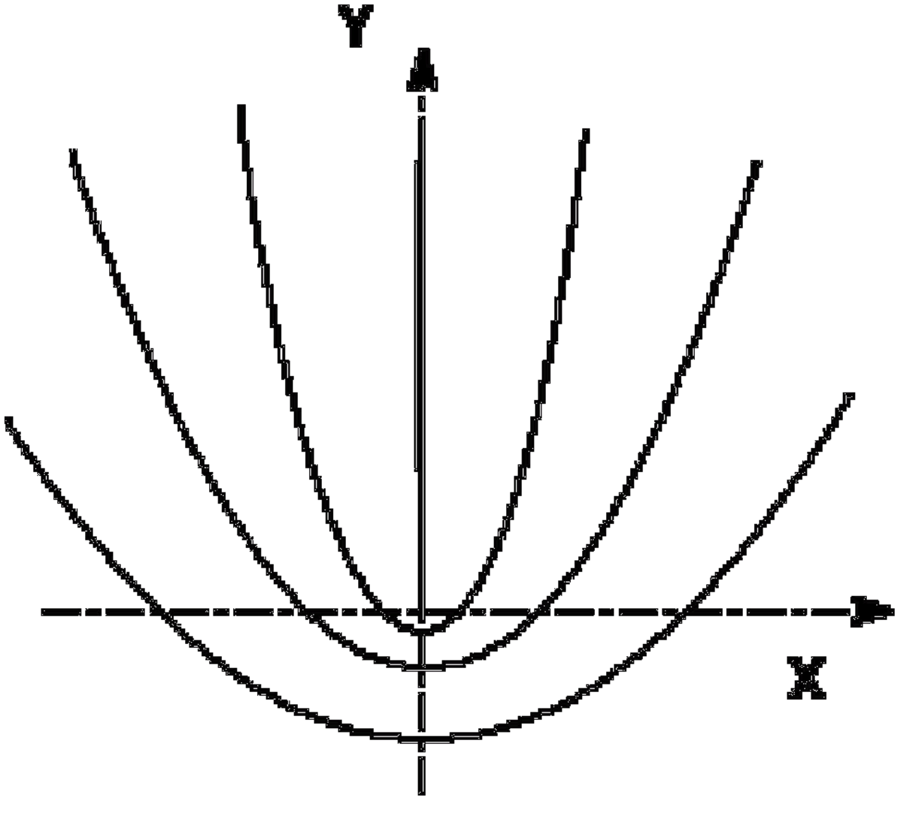
$$\xi Y = \frac{\xi^2 X^2}{(2\beta)^2} - \beta^2 \rightarrow \boxed{X = 2\beta\sqrt{Y/\xi + \beta^2/\xi^2}}$$

Contours of constant growth

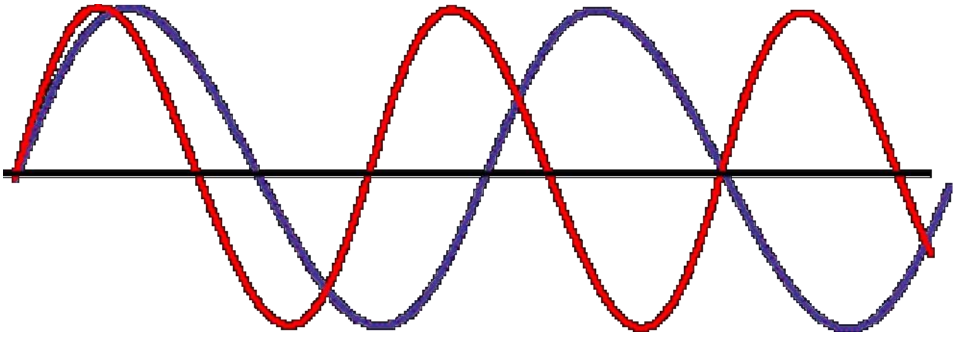
$$\boxed{X = 2\beta\sqrt{Y / \xi + \beta^2 / \xi^2}}$$

$$\text{growth rate: } \beta = 1/\tau_{rise}$$

- **Changing the growth rate parameter β we have a set of parabolas**

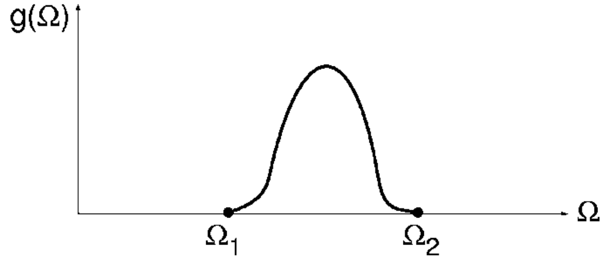


Landau damping – the idea



- **Two oscillators excited together become incoherent and give zero centre of charge motion after a number of turns comparable to the reciprocal of their frequency difference**

Landau Damping - the maths



normalization

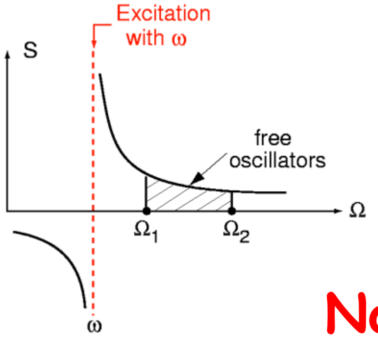
$$\int_{\Omega_1}^{\Omega_2} g(\Omega)\, d\Omega = 1$$

N particles (oscillators), each **resonating** at a frequency between Ω_1 and Ω_2 with a density $g(\Omega)$

$$X = \frac{1}{\Omega^2 - \omega^2}\, e^{i\omega t} = \frac{1}{(\Omega - \omega)\underbrace{(\Omega + \omega)}_{\sim 2\Omega_0}}\, e^{i\omega t}$$

Response X of an individual oscillator with frequency Ω to an external excitation with ω

Coherent response of the beam obtained by summing up the single-particle responses of the n oscillators

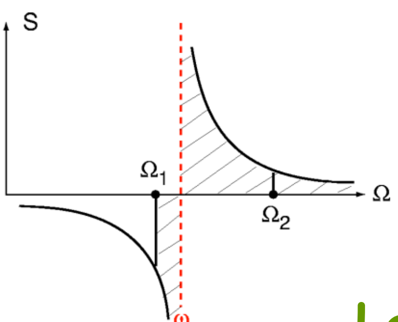


External excitation is **outside the frequency range** of the oscillators

No damping

$$S = \frac{N}{2\Omega_0} \int_{\Omega_1}^{\Omega_2} \frac{i \frac{dg(\Omega)}{d\Omega}}{\Omega - \omega}\, d\Omega \cdot e^{i\omega t}$$

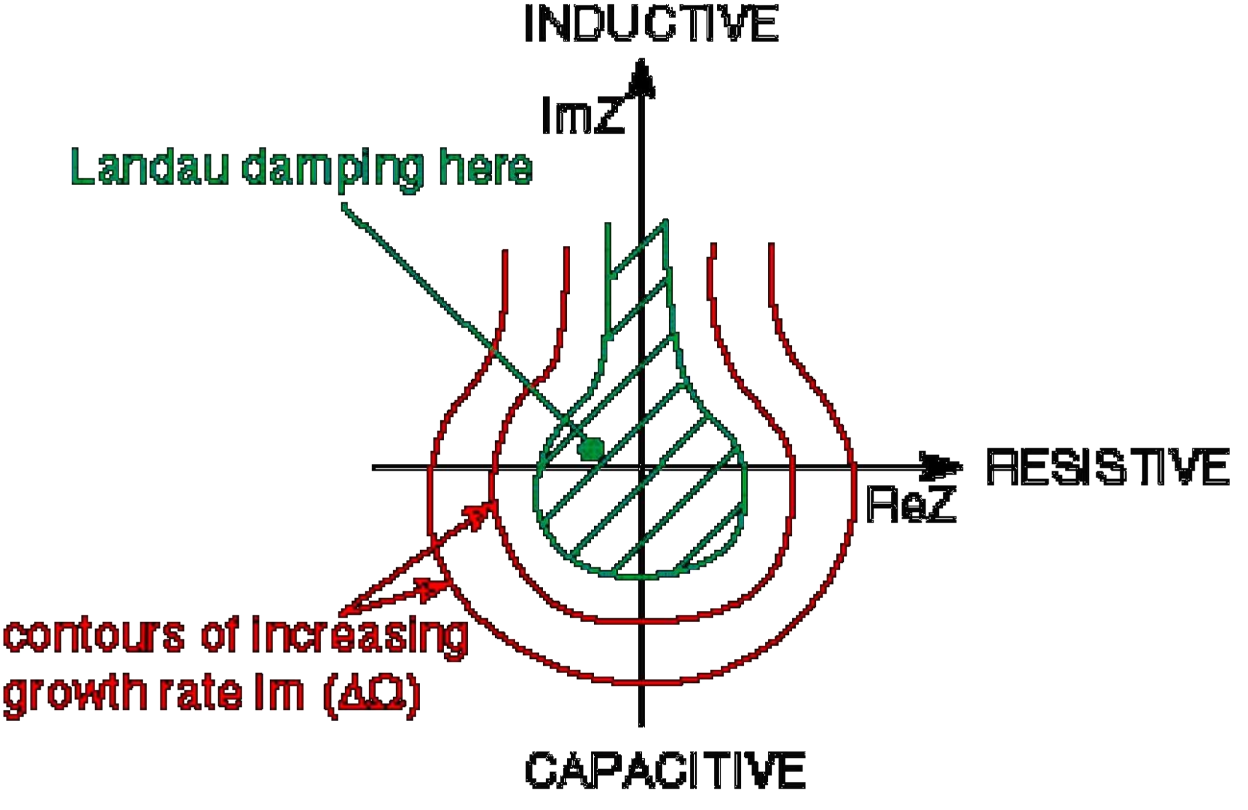
See Schindl p9 for more about this integration



External excitation is **inside the frequency range** of the oscillators
The integral has a **pole at $\Omega = \omega$**

Landau damping

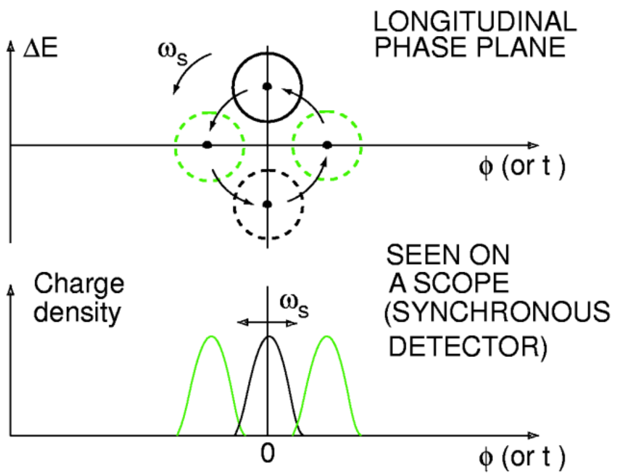
Stability diagram



- **Keil Schnell stability criterion:**

$$\left|\frac{Z}{n}\right| \leq \frac{F m_0 c^2 \beta^2 \gamma \eta}{I_o} \left(\frac{\Delta p}{p}\right)^2_{FWHH}$$

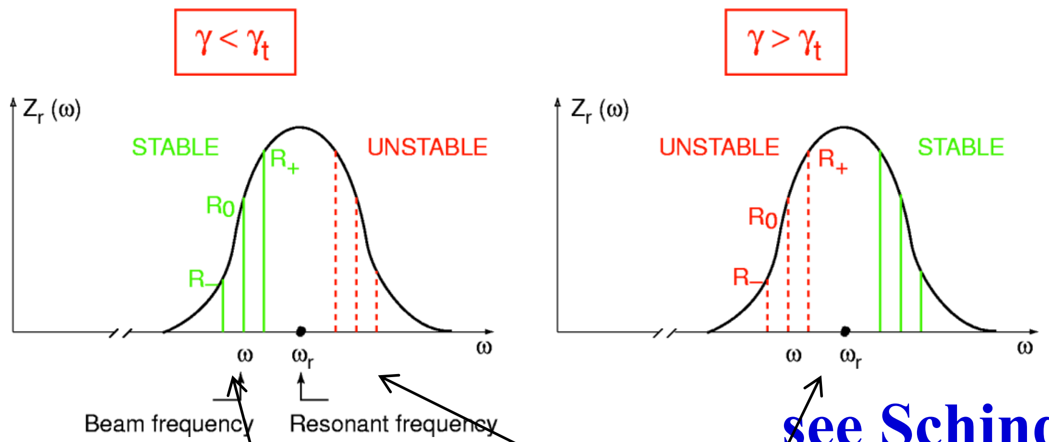
Single Bunch + Resonator: "Robinson" Instability



"Dipole" mode or "Rigid Bunch" mode

A single bunch rotates in longitudinal phase plane with ω_s: its **phase** ϕ and **energy** ΔE also vary with ω_s

Bunch sees **resonator impedance** at $\omega_r \cong \omega_0$



see Schindl p 10

$\omega < \omega_r$

Whenever $\Delta E>0$:
- ω **increases** (below transition)
- sees **larger** real impedance R_+
- **more** energy taken from beam

➢ STABILIZATION

Whenever $\Delta E>0$:
- ω **decreases** (above transition)
- sees **smaller** real impedance R_+
- **less** energy taken from beam

➢ INSTABILITY

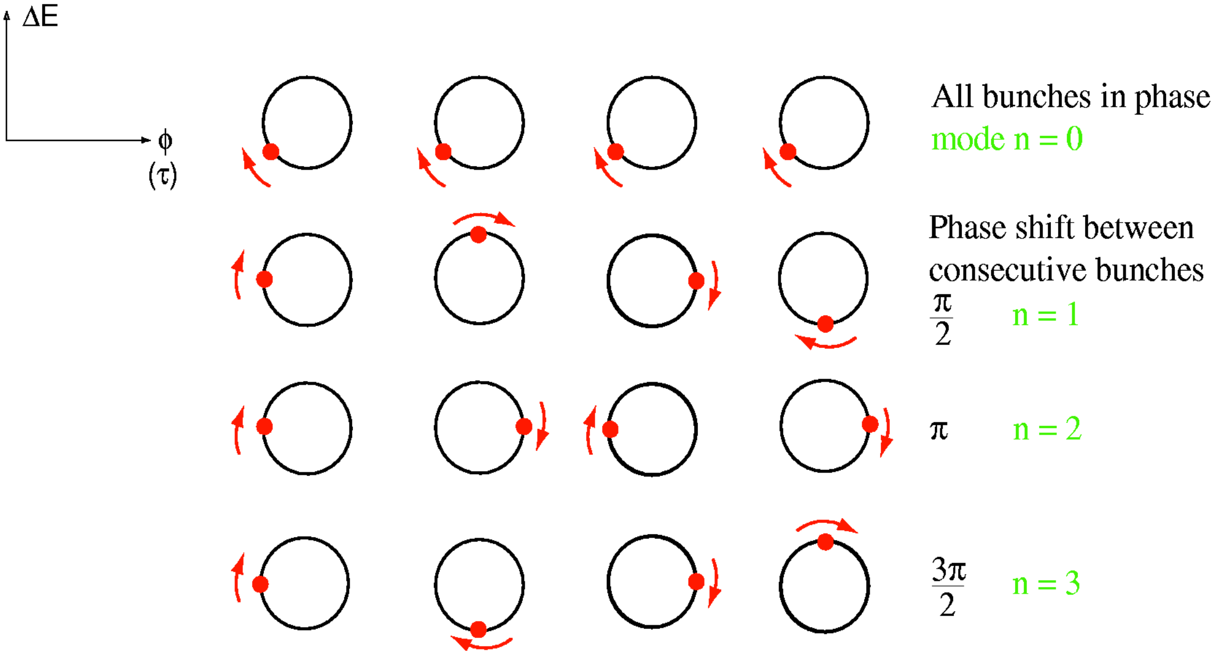
$\omega > \omega_r$

UNSTABLE STABLE

Longitudinal Instabilities with Many Bunches

- ❑ Fields induced in resonator remain long enough to influence subsequent bunches
- ❑ Assume M = 4 bunches performing synchrotron oscillations

Coupled-Bunch Modes n

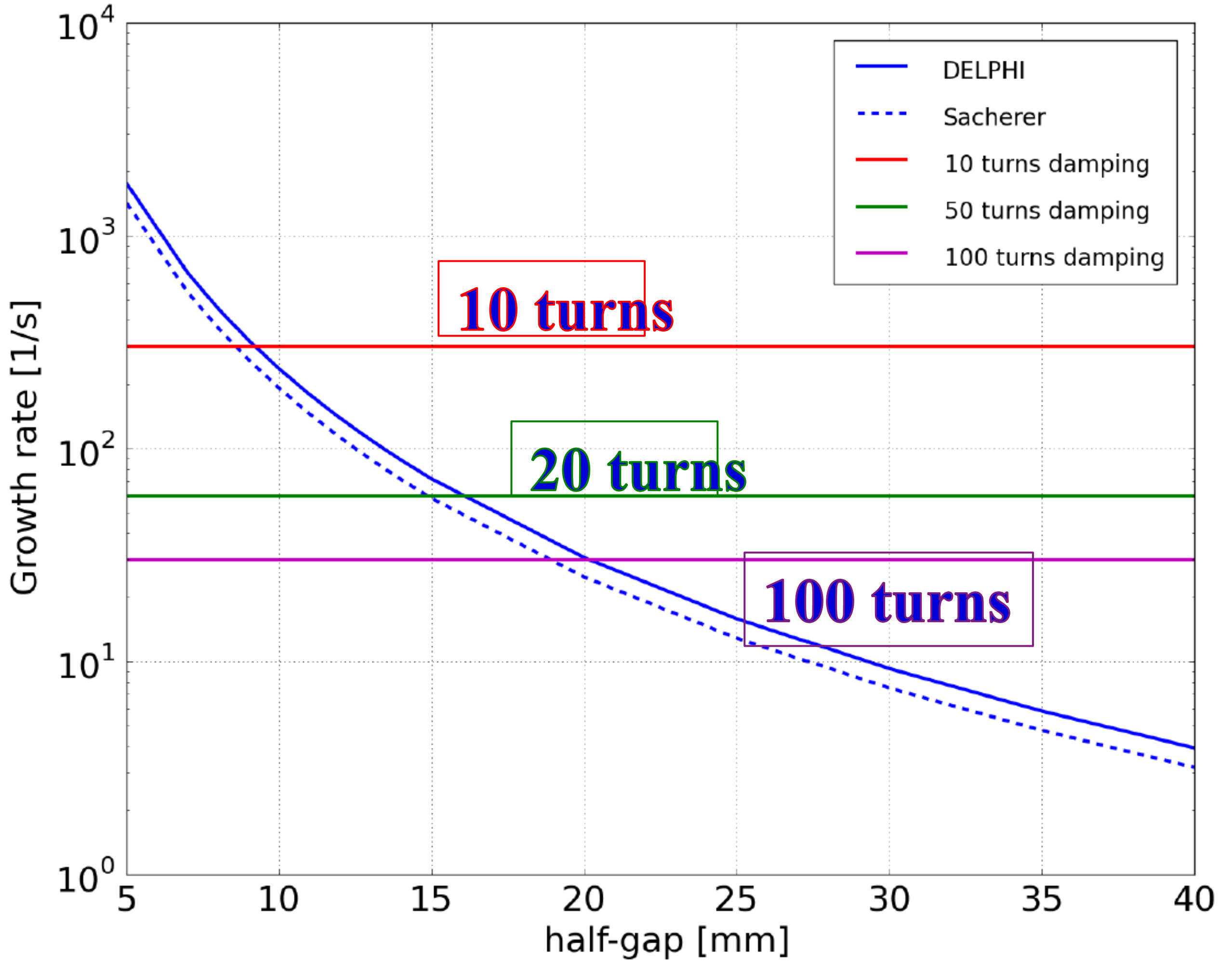


- ❑ **Four possible phase shifts** between **four bunches**
- ❑ **M bunches: phase shift** of coupled-bunch mode **n**:

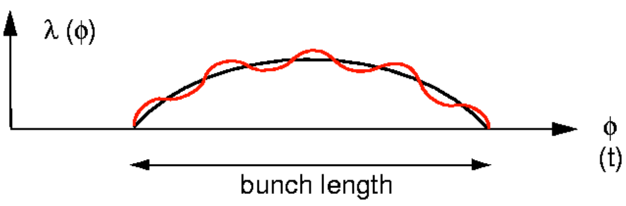
$$2\pi\frac{n}{M},\ 0 \le n \le M-1 \Rightarrow$$ M modes

More in Schindl pp. 14-17

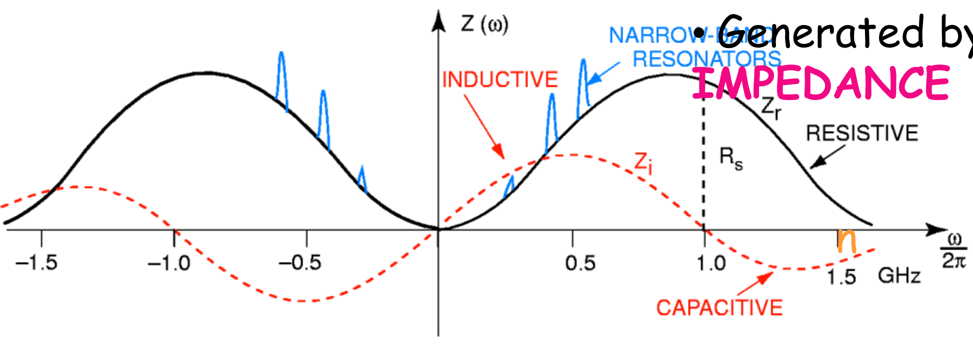
Growth rate of multi-bunch transverse instability in the FCC hh



Longitudinal Microwave Instability



- **High-frequency** density modulation along the bunch
- **wave length « bunch length** (frequencies **0.1-1 GHz**)
- **Fast growth rates** - even **leptons** concerned
- Generated by "**BROAD-BAND" IMPEDANCE**



All elements in a ring are "lumped" into a low-Q resonator yielding the impedance

$$Z(\omega) = R_s \frac{1 - iQ\dfrac{\omega^2 - \omega_r^2}{\omega\omega_r}}{1 + \left(Q\dfrac{\omega^2 - \omega_r^2}{\omega\omega_r}\right)^2} \qquad Q \approx 1 \qquad \omega_r \approx 1\ \text{GHz}$$

For small ω and $\quad Q = \dfrac{R_s}{\omega_r L}$

"Impedance" of a synchrotron in Ω

$$Z(\omega) \approx i\frac{R_s\omega}{Q\omega_r} = i\frac{R_s}{Q}\frac{\omega}{\omega_0}\frac{\omega_0}{\omega_r} = i\frac{R_s}{Q}\frac{\omega_0 n}{\omega_r}$$

$$\boxed{\left|\frac{Z}{n}\right|_0 = L\omega_0}$$

- This **inductive impedance** is caused mainly by **discontinuities** in the beam pipe
- If **high**, the machine is **prone to instabilities**
- Typically **20…50 Ω** for **old** machines
- **< 1 Ω** for **modern** synchrotrons

More in Schindl pp. 16-18

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