Instabilities I

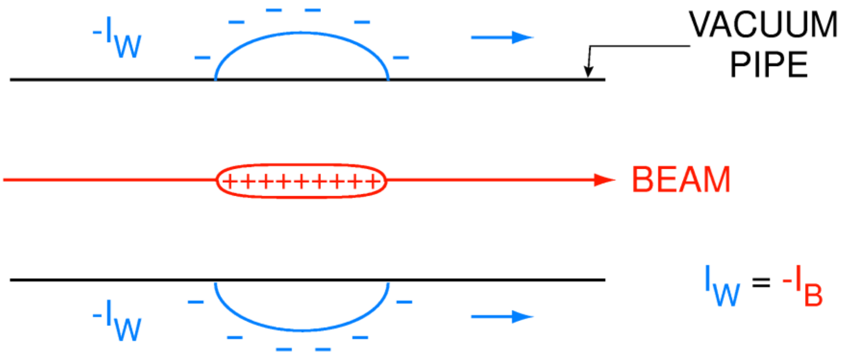
ACCELERATOR PHYSICS

HT 2015

E. J. N. Wilson

http://cas.web.cern.ch/cas/Loutraki-Proc/PDF-files/I-Schindl/paper2.pdf

Impedance of the wall



Wall current I_w due to circulating bunch
Vacuum pipe not smooth, I_w sees an
IMPEDANCE :

Resistive = in phase.
capacitive lags,
inductive leads

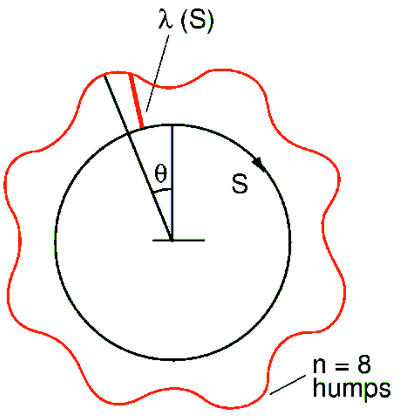
Impedance $\mathbf{Z} = \mathbf{Z_r} + \mathbf{iZ_i}$
Induced voltage $\mathbf{V} \sim \mathbf{I_w}\,\mathbf{Z} = -\mathbf{I_B}\,\mathbf{Z}$

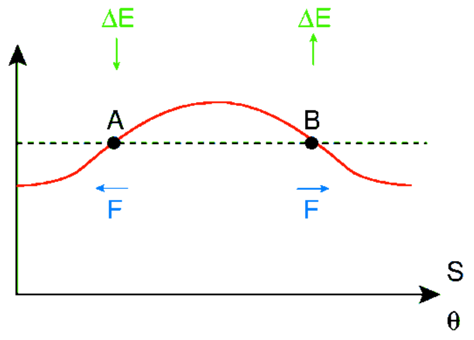
V acts back on the beam
⇨ INSTABILITIES
INTENSITY DEPENDENT

Test: If Initial Small Perturbation is :

INCREASED? INSTABILITY
DECREASED? STABILITY

"Negative Mass" Instability
Qualitative





WILL THE HUMPS INCREASE OR ERODE?

The **self-force F** (proportional to $-\partial\lambda/\partial s$)

$$E = -\left[\frac{e}{4\pi\varepsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}$$

Increases energy of particles in B

Decreases energy of particles in A

STABLE

$\gamma < \gamma_t$: if $\Delta E\uparrow$ then $\Delta\omega_0\uparrow$ A and B move away from the hump eroding the mountain

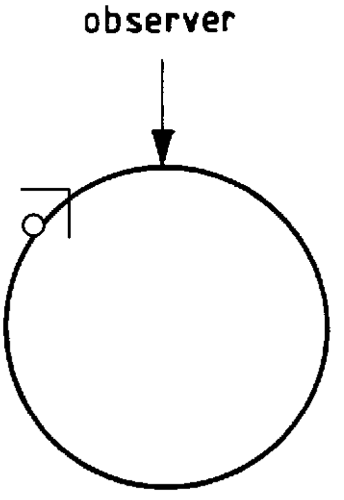
UNSTABLE

$\gamma > \gamma_t$: if $\Delta E\uparrow$ then $\Delta\omega_0\downarrow$ A and B move towards the hump enhancing the mountain

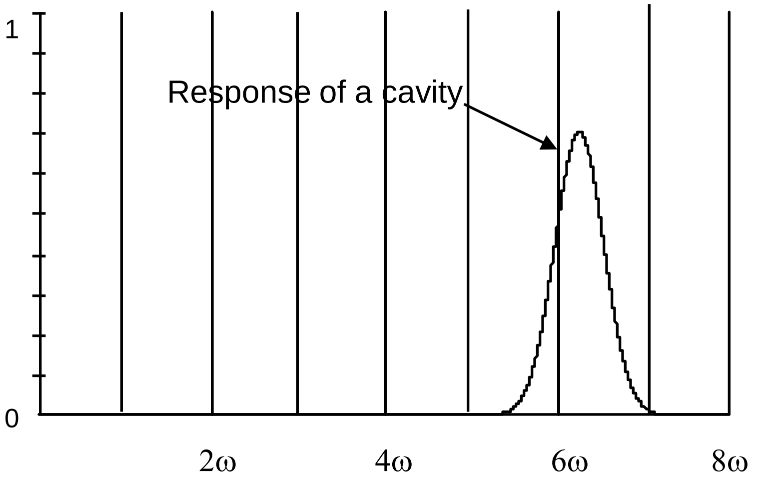
Driving terms (second cornerstone)

- **Fourier analysis of a circulating delta function bunch of charge passing an observer.**

$$I = \sum I_n \;\; e^{in\omega_0 t}$$



- **Produces a fundamental at the revolution frequency plus all higher harmonics are in equal strength**



- **Spectrum from a bunch showing response of an r.f. cavity**

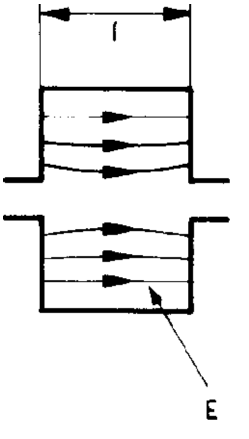
J.A.I.

A cavity-like object is excited

- **Local enlargement in the beam tube which can resonate like a cavity**
- **Voltage experienced has same form as the current which excites it**

$$I = \hat{I}\ e^{-i\omega t}$$

$$U = \hat{U}\ e^{-i\omega t}$$



- **Impedance** $$Z = X + iY = \frac{U}{I}$$
- **Relates force on particles to the Fourier component of the beam current which excites the force.**
- **A complex quantity**
  - **- REAL if the voltage and current are in phase**
  - **- IMAGINARY if 90 degrees or "i" between voltage and current (L = +, C = –)**
  - **- different from r.f. wave by 90 degrees!**

Laying the bricks in the wall (row 1)

- **Instead of the line density λ, the beam current I which we write as**

$$I = I_0 + I_1 \; e^{i(n\theta - \Omega t)}$$

n is the number of humps

ω , angular frequency felt by an antenna in the wall

dI/ds drives instability (like resistive wall)

- **we can ignore constant current and write:**

$$\frac{dI}{ds} = \frac{1}{R} \cdot \frac{dI}{d\theta} = \frac{in}{R} I_1 e^{i(n\theta - \Omega t)}$$

Rearranging this:

$$I_1 e^{i(n\theta - \Omega t)} = \frac{R}{in} \frac{dI}{ds}$$

A cavity-like object presents an impedance, *X*+ *iY*, at this frequency- hence the V/turn (at zero azimuth):

$$(X + iY) I_1 \; e^{-\Omega t} = R\left(\frac{X + iY}{in}\right)\frac{dI}{ds}$$
$$= -\frac{iR}{n}(X + iY)\frac{dI}{ds} = \frac{R}{n}(Y - iX)\frac{dI}{ds} \; .$$

Laying the bricks in the wall (row 2)

- **Accelerating voltage:**

$$V = \frac{R}{n}(Y - iX)\frac{dI}{ds} \quad .$$

Compare with the acceleration for negative mass instability which was:

$$-\left(\frac{e}{4\pi\varepsilon_0\gamma^2}\right)\frac{d\lambda}{ds}$$

- **Thus we have the same effect as the negative mass**

Above transition.- Instability when Y is negative, i.e. capacitive.

Inductive impedance causes instability below transition

SUMMARY

We have calculated the slope of a harmonic of I

Multiplying by Z gives a voltage

Effect is like negative mass