Instabilities I

ACCELERATOR PHYSICS

HT 2015

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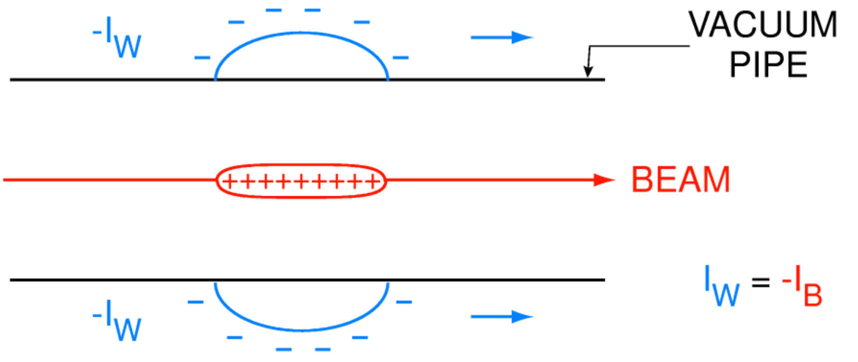
http://cas.web.cern.ch/cas/Loutraki-Proc/PDF-files/I-Schindl/paper2.pdf

Instabilities I

- 1. General Comment on Instabilities
- 2. Negative Mass Instability
- 3. Driving terms (second cornerstone)
- 4. A cavity-like object is excited
- 5. Equivalent circuit
- 6. Above and below resonance
- 7. Laying the bricks in the wall (row 1)
- 8. Laying the bricks in the wall (row 2)
- 9. By analogy with the negative mass

Impedance of the wall



Wall current I_w due to circulating bunch
Vacuum pipe not smooth, I_w sees an
IMPEDANCE :

Resistive = in phase.
capacitive lags,
inductive leads

Impedance $Z = Z_r + iZ_i$
Induced voltage $V \sim I_w Z = -I_B Z$

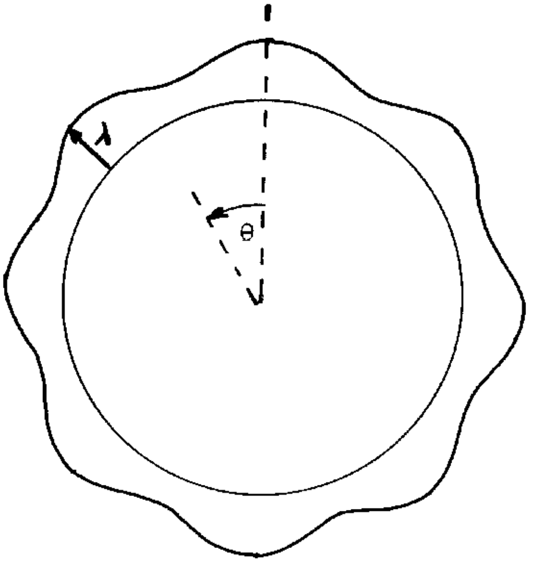
V acts back on the beam
⇨ INSTABILITIES
INTENSITY DEPENDENT

Test: If Initial Small Perturbation is :

INCREASED? INSTABILITY
DECREASED? STABILITY

Test on instability

- **Postulate a simple form for its effect**
- **Test whether such a pattern is self sustaining.**
  - » **examine forces set up by such a pattern**
  - » **if they reinforce its shape – it is sure to grow exponentially**
  - » **it starts from some random component of noise.**

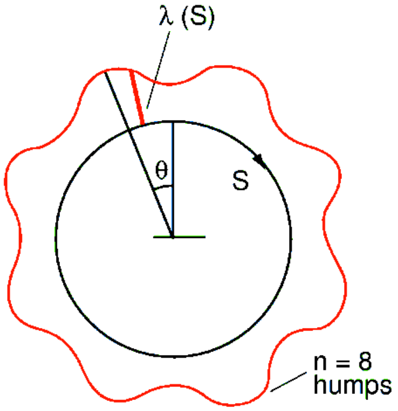


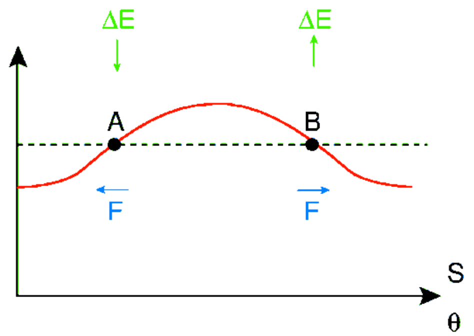
Typical perturbation
Local line density of charge around a synchrotron

Negative Mass Instability

Un-bunched (=coasting) beam in a proton/ion ring, travels around ring with angular frequency ω_0

Line density $\lambda(s)$ [particles/m] is modulated around the synchrotron





Line density modulation **"mode"** with n=8 humps

Zooming in one modulation

- **Azimuthal modulation replotted in Cartesian form.**
- **A finds itself with a larger charge density behind it than in front of it, pushing it forward.**
- **Conversely particle B will be decelerated by the mountain of charge in front of it.**

$$E = -\left[\frac{e}{4\pi\varepsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}$$

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Field in a pipe as the beeam passes

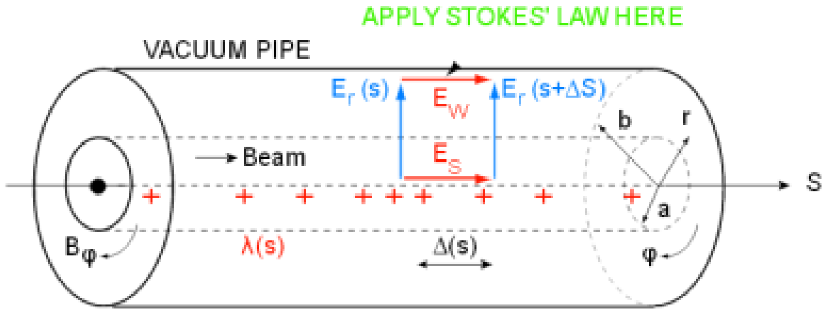


Fig. 3: Electrical (E) and magnetic (B) fields induced into a vacuum pipe (radius b) by a beam (radius a) with longitudinal density modulation $\lambda(s)$

$$E_r = \frac{e\lambda}{2\pi\varepsilon_0}\frac{1}{r} \qquad B_\phi = \frac{\mu_0 e\lambda\beta c}{2\pi}\frac{1}{r} \qquad r \geq a\,,$$
$$E_r = \frac{e\lambda}{2\pi\varepsilon_0}\frac{r}{a^2} \qquad B_\phi = \frac{\mu_0 e\lambda\beta c}{2\pi}\frac{r}{a^2} \qquad r \leq a\,. \tag{1}$$

Applying Stokes' law which relates a line integral to an integral over the surface enclosed by the line

$$\oint_{LINE} \vec{E}d\vec{\ell} = -\frac{\partial}{\partial t}\int_{SURFACE}\vec{B}d\vec{\sigma} = -\frac{\partial}{\partial t}\Delta s\int_0^b B_\phi\, dr \tag{2}$$

and using

$$\frac{\partial\lambda}{\partial t} = \frac{\partial\lambda}{\partial s}\frac{ds}{dt} = \beta c\frac{\partial\lambda}{\partial s} \tag{3}$$

and

$$g_0 = 1 + 2\ln\frac{b}{a}\,,$$

one gets for the longitudinal electrical field E_s on the beam axis

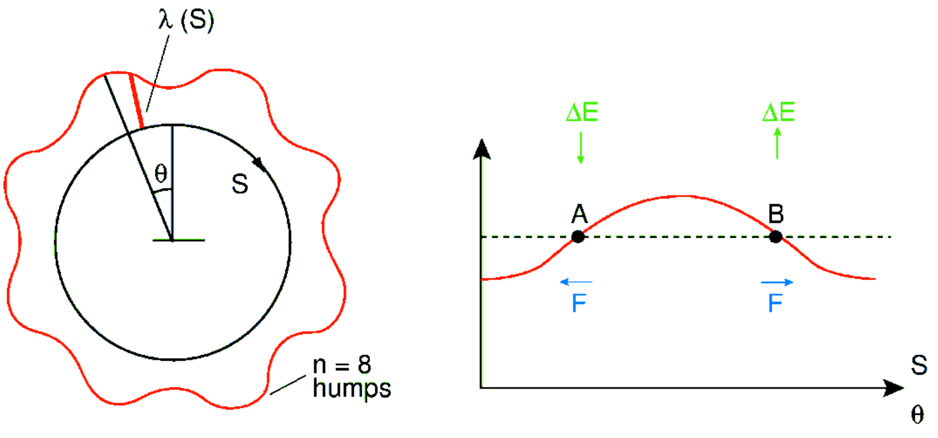
$$E_s = -\frac{eg_0}{4\pi\varepsilon_0}(1-\beta^2)\frac{\partial\lambda}{\partial s} + E_w = -\frac{eg_0}{4\pi\varepsilon_0}\frac{1}{\gamma^2}\frac{\partial\lambda}{\partial s} + E_w\,, \tag{4}$$

where E_w is the electrical field parallel to the vacuum pipe surface.

For a *perfectly conducting* wall, E_w vanishes, and one is left with

$$E_s = -\frac{eg_0}{4\pi\varepsilon_0}\frac{1}{\gamma^2}\frac{\partial\lambda}{\partial s}\,, \tag{5}$$

"Negative Mass" Instability Qualitative



WILL THE HUMPS INCREASE OR ERODE?

The self-force F (proportional to $-\partial\lambda/\partial s$)

$$E = -\left[\frac{e}{4\pi\varepsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}$$

Increases energy of particles in B

Decreases energy of particles in A

STABLE

$\gamma < \gamma_t$: if $\Delta E \uparrow$ then $\Delta\omega_0 \uparrow$ A and B move away from the hump eroding the mountain

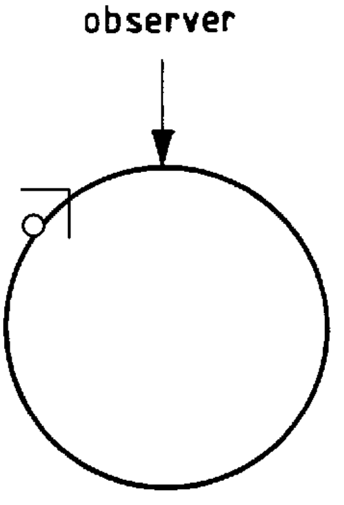
UNSTABLE

$\gamma > \gamma_t$: if $\Delta E \uparrow$ then $\Delta\omega_0 \downarrow$ A and B move towards the hump enhancing the mountain

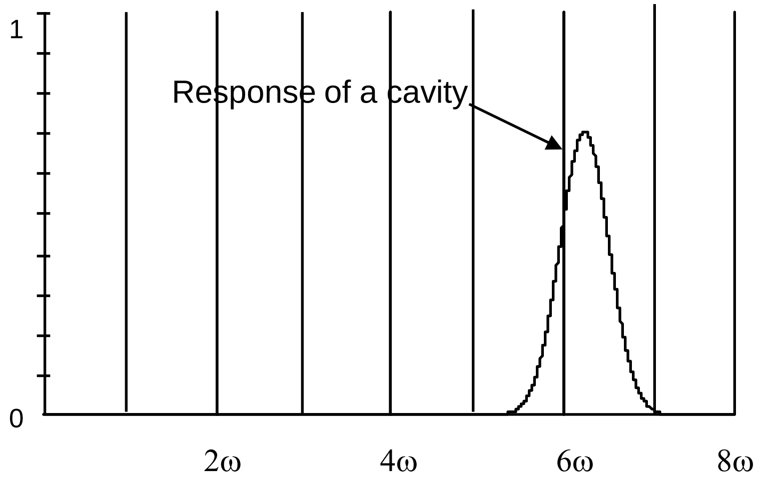
Driving terms (second cornerstone)

- Fourier analysis of a circulating delta function bunch of charge passing an observer.

$$I = \sum I_n \;\; e^{in\omega_0 t}$$



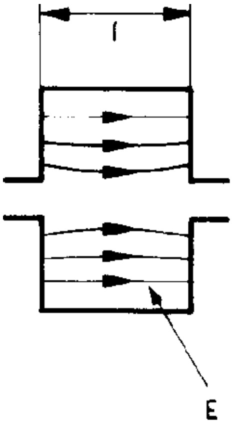
- Produces a fundamental at the revolution frequency plus all higher harmonics are in equal strength



- Spectrum from a bunch showing response of an r.f. cavity

A cavity-like object is excited

- Local enlargement in the beam tube which can resonate like a cavity
- Voltage experienced has same form as the current which excites it



$$I = \hat{I} \; e^{-i\omega t}$$

$$U = \hat{U} \; e^{-i\omega t}$$

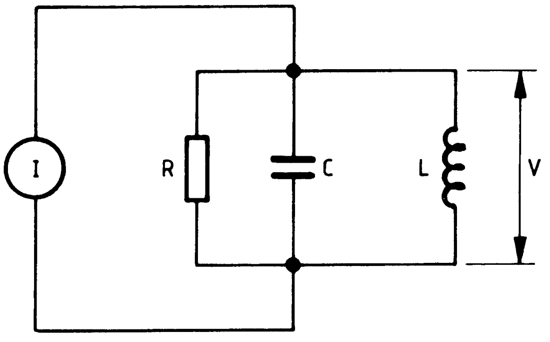
- Impedance

$$Z = X + iY = \frac{U}{I}$$

- Relates force on particles to the Fourier component of the beam current which excites the force.
- A complex quantity
  - REAL if the voltage and current are in phase
  - IMAGINARY if 90 degrees or "i" between voltage and current (L = +, C = –)
  - different from r.f. wave by 90 degrees!

Equivalent circuit

- **AC resonant circuit**



- **Elementary relations**

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}} = \frac{R}{L\omega_r} = RC\omega_r$$

- **Differential equation for voltage and current**

$$\ddot{V} + \frac{\omega_r}{Q}\dot{V} + \omega_r^{\,2}V = \omega_r \frac{R}{Q}\dot{I}$$

- **Solution:**

$$V = V_0 \; e^{-\alpha t} \sin\left[\omega_r \sqrt{1 - \frac{1}{4Q^2}}\,t + \mu\right]$$

Above and below resonance

$$V = V_0 \; e^{-\alpha t} \omega s \left[\omega_r \sqrt{1 - \frac{1}{4Q^2}} t + \mu \right]$$

- **Damping rate** $$\alpha = \frac{\omega_r}{2Q}$$

- **Books on AC circuit theory will show that if**

$$I = \hat{I} \; e^{i \omega t}$$

- **Impedance seen by the generator is:**

$$Z(\omega) = R \left[\frac{1 - iQ\left(\frac{\omega^2 - \omega_r^2}{\omega \omega_r}\right)}{1 + Q^2 \left(\frac{\omega^2 - \omega_r^2}{\omega \omega_r}\right)^2} \right] .$$

- **When ω is below resonance, the reactive component is inductive or positive**
- **After crossing zero it becomes negative and capacitive when the driving frequency is above the natural resonance of the oscillator.**

Laying the bricks in the wall (row 1)

- **Instead of the line density λ, the beam current I which we write as**

$$I = I_0 + I_1 \ e^{i(n\theta - \Omega t)}$$

n is the number of humps

ω , angular frequency felt by an antenna in the wall

dI/ds drives instability (like resistive wall)

- **we can ignore constant current and write:**

$$\frac{dI}{ds} = \frac{1}{R} \cdot \frac{dI}{d\theta} = \frac{in}{R} I_1 e^{i(n\theta - \Omega t)}$$

Rearranging this:

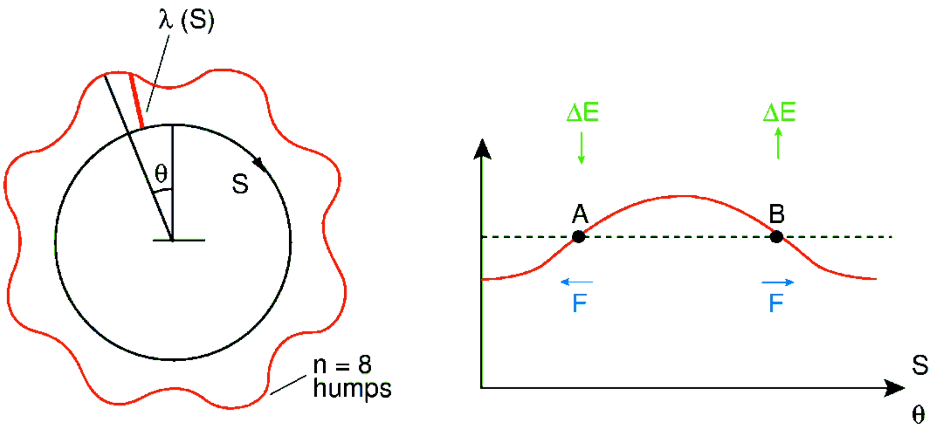
$$I_1 e^{i(n\theta - \Omega t)} = \frac{R}{in}\frac{dI}{ds}$$

A cavity-like object presents an impedance, *X*+ *iY*, at this frequency- hence the V/turn (at zero azimuth):

$$(X + iY) I_1 \ e^{-\Omega t} = R\left(\frac{X + iY}{in}\right)\frac{dI}{ds}$$

$$= -\frac{iR}{n}(X + iY)\frac{dI}{ds} = \frac{R}{n}(Y - iX)\frac{dI}{ds} \ .$$

Remember the negative mass



- **Where the field from the beam itself was**

$$E = -\left[\frac{e}{4\pi\varepsilon_0\gamma^2}\right]\frac{\partial\lambda}{\partial s}$$

Laying the bricks in the wall (row 2)

- Accelerating voltage:

$$V = \frac{R}{n}(Y - iX)\frac{dI}{ds} \quad .$$

Compare with the acceleration for negative mass instability which was:

$$-\left(\frac{e}{4\pi\varepsilon_0\gamma^2}\right)\frac{d\lambda}{ds}$$

- Thus we have the same effect as the negative mass

Above transition.- Instability when Y is negative, i.e. capacitive.

Inductive impedance causes instability below transition

SUMMARY

We have calculated the slope of a harmonic of I

Multiplying by Z gives a voltage

Effect is like negative mass

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