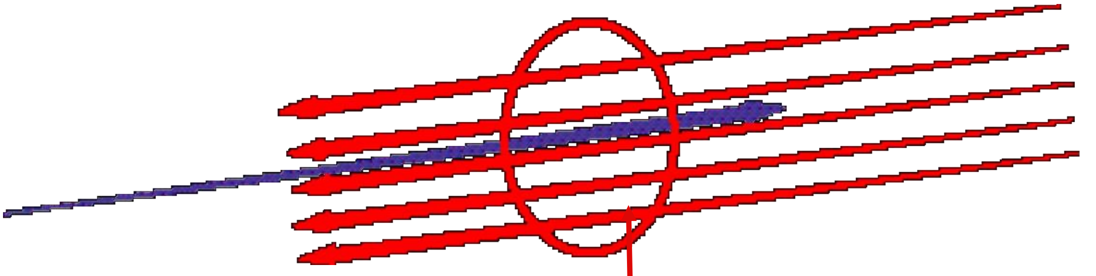
Beam-Beam

ACCELERATOR PHYSICS

HT 2015

E. J. N. Wilson

Luminosity (recall)



- Imagine a blue particle colliding with a beam of cross section area - A
- Probability of collision is $\frac{\sigma}{A} \cdot N$
- For N particles in both beams $\frac{\sigma}{A} \cdot N^2$
- Suppose they meet *f* times per second at the revolution frequency

$$f_{rev} = \frac{\beta c}{2\pi R}$$

- Event rate

$$\frac{f_{rev} N^2}{A} \cdot \sigma$$

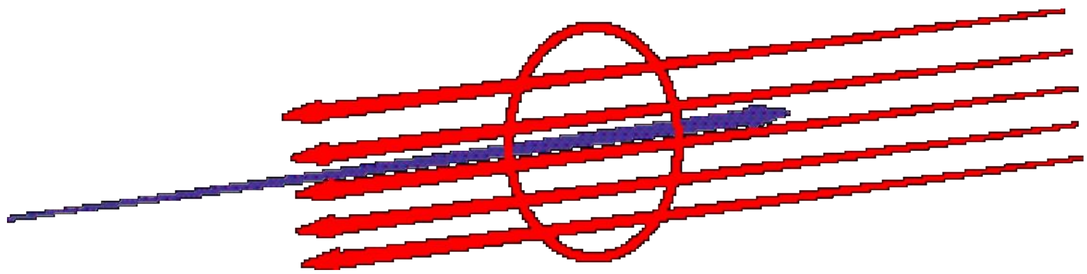
Make big

e.g. 10^{-25}

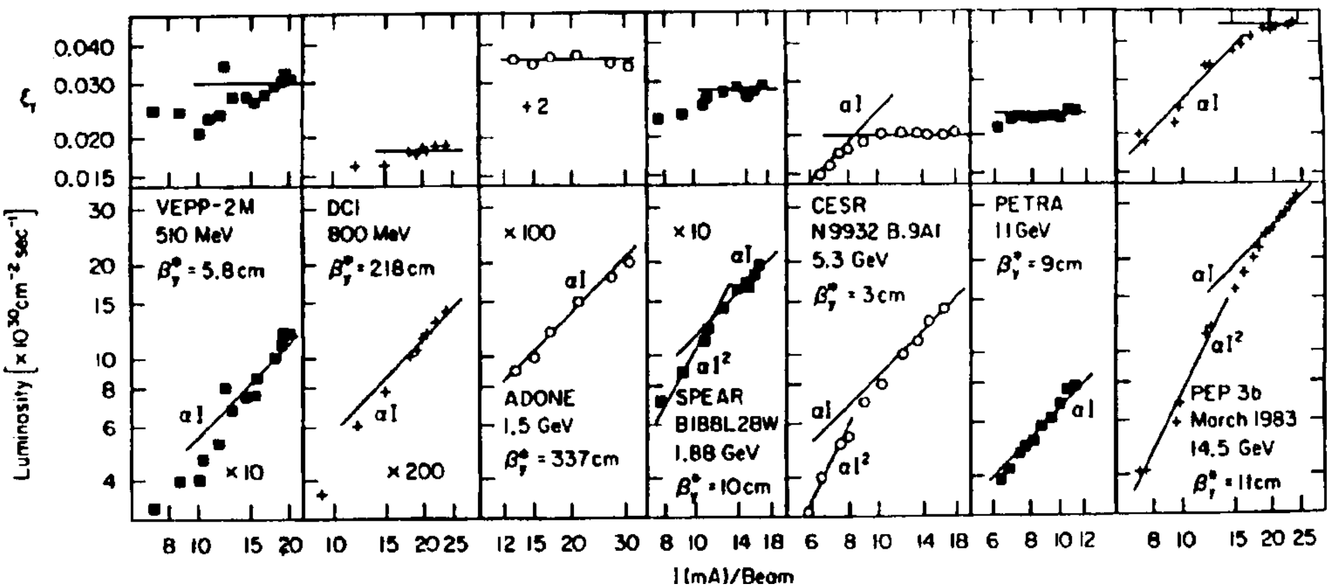
Make small

LUMINOSITY $\approx 10^{30}$ to 10^{34} $[\mathrm{cm}^{-2}\ \mathrm{s}^{-1}]$

Beam-beam interaction Limits Luminosity



- Blue particle sees electromagnetic force from blue charges and currents



- Luminosity "saturates" when beam-beam tune shift is about 0.03 (for lepton colliders)
- About 0.003 for hadron colliders

The Beam-beam effect - E. Wilson

- **Luminosity**

$$L = \frac{N^2 fk}{4\pi\sigma_x\sigma_y}$$

f = the rev frequency

k = number of bunches

σ = rms beam width or height

- **We shall show that the beam beam tune shift (linear effect on Q)**

$$\xi_y = \frac{Nr_0\beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)}$$

- **We can solve to fine dependence of one upon the other**

$$L = \frac{Nfk\gamma}{2r_0\beta_y^*}\xi_y\left(\frac{\sigma_x + \sigma_y}{\sigma_x}\right)$$

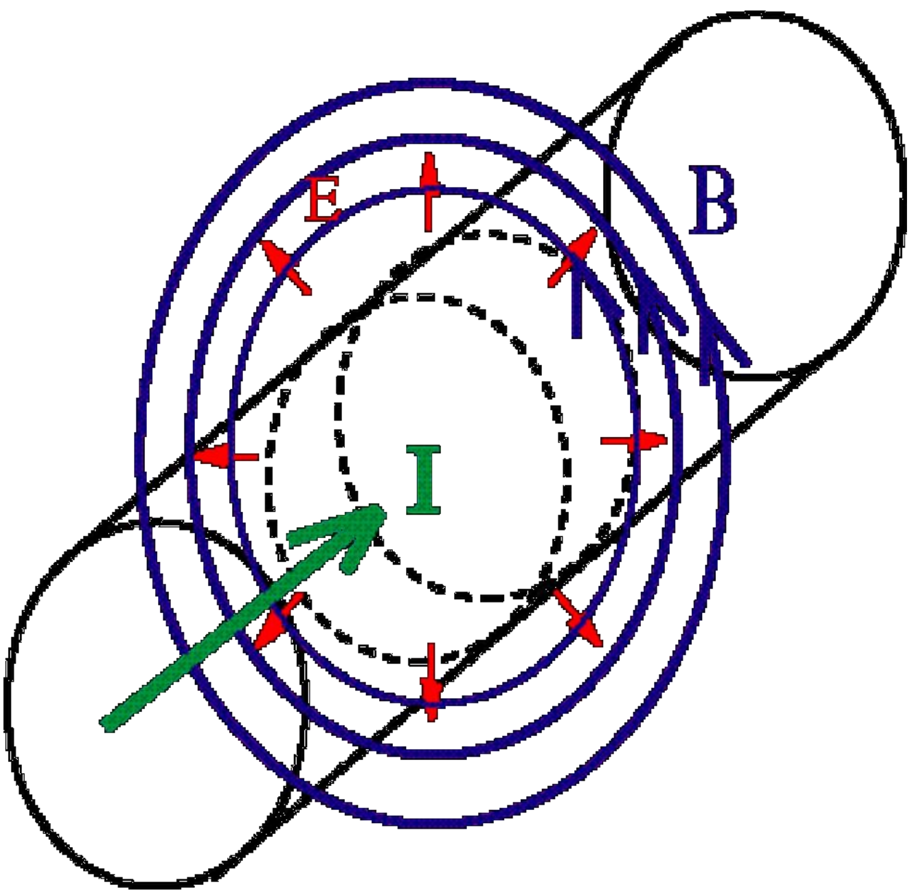
- **Low beta, large number of bunches and high intensity helps --- but instabilities limit N**

Field around a moving cylinder of charge

- Take unit length of the bunch

$$2\pi r E_r = (1/\varepsilon_0)\int_0^r 2\pi r' \rho(r')dr'$$

$$2\pi r B_\phi = \mu_0 \int_0^r 2\pi r' \beta c \rho(r')dr'$$



Electric and Magnetic field within a beam

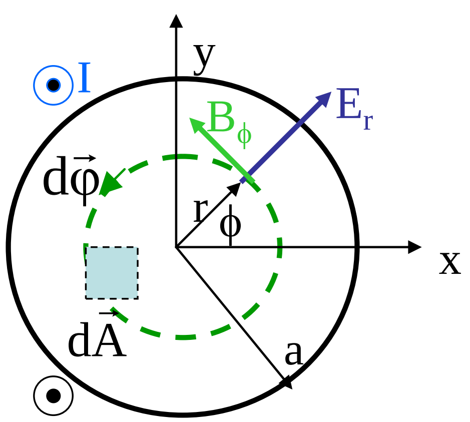
η…charge density in Cb/m³
λ… constant line charge $\pi a^2\eta$
I…constant current $\lambda\beta c = \pi a^2\eta\beta c$
a…beam radius

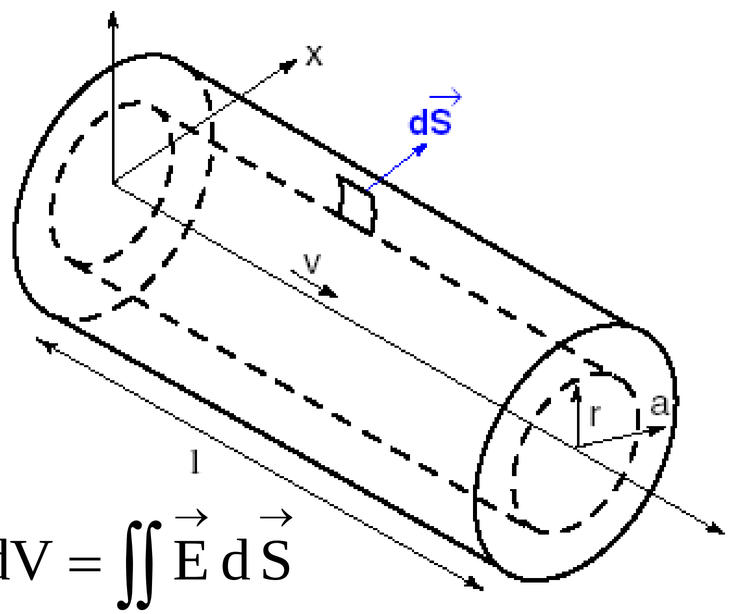
$$\vec{E} = E_r$$

$$\operatorname{div}\vec{E} = \frac{\eta}{\varepsilon_0}$$

cylinder radius r
length l

cross section
radius r





$$\iiint \operatorname{div}\vec{E}\, dV = \iint \vec{E}\, d\vec{S}$$

$$r^2\pi l\frac{\eta}{\varepsilon_0} = E_r 2r\pi l$$

$$E_r = \frac{I}{2\pi\varepsilon_0\beta c}\frac{r}{a^2} \qquad B_\varphi = \frac{I}{2\pi\varepsilon_0 c^2}\frac{r}{a^2}$$

Within the beam at radius r’

$$2\pi r E_r = (1/\varepsilon_0)\int_0^r 2\pi r'\,\rho(r')dr'$$

E field and force on a test particle

- **From previous slide**

$$2\pi r E_r = (1/\varepsilon_0)\int_0^r 2\pi r' \rho(r')dr'$$

- **Assume the beam is Gaussian**

$$2\pi r E_r = \quad (1/\varepsilon_0)\int_0^r \frac{2\pi n q r'}{2\pi\sigma^2} e^{-r'^2/2\sigma^2} dr'$$

$$= (nq/\varepsilon_0)\left[e^{-r'^2/2\sigma^2}\right]_0^r = (nq/\varepsilon_0)\left[1 - e^{-r^2/2\sigma^2}\right]$$

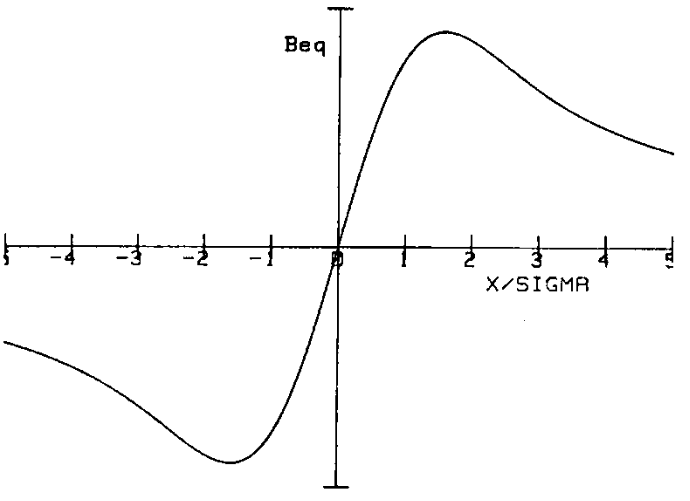
Force on test particle including magnetic field:

$$F_r(r) = -\frac{nq^2(1+\beta^2)}{4\pi\varepsilon_0\sigma^2}\left[1 - e^{-r^2/2\sigma^2}\right]$$

$$F_r(r) = -\frac{nq^2(1+\beta^2)r}{4\pi\varepsilon_0\sigma^2} \qquad \text{for} \quad r \ll \sigma$$

$$F_r(r) = -\frac{nq^2(1+\beta^2)}{2\pi\varepsilon_0 r} \qquad \text{for} \quad r \gg \sigma$$

Beam Beam Force is non linear



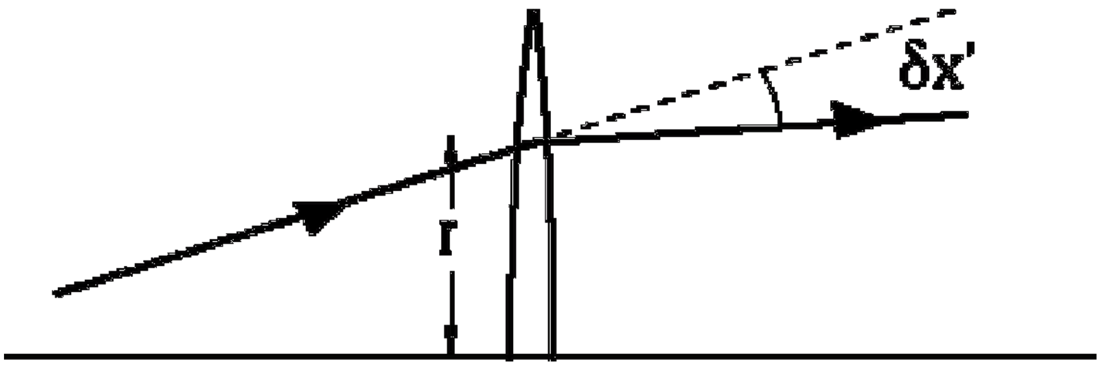
- **A linear focusing effect at beam center**
- **But elsewhere it is non linear**

$$F_r(r) = -\frac{nq^2\left(1+\beta^2\right)}{4\pi\varepsilon_0\sigma^2}\left[1-e^{-r^2/2\sigma^2}\right]$$

$$F_r(r) = -\frac{nq^2\left(1+\beta^2\right)r}{4\pi\varepsilon_0\sigma^2} \quad \text{for} \quad r \ll \sigma$$

$$F_r(r) = -\frac{nq^2\left(1+\beta^2\right)}{2\pi\varepsilon_0 r} \quad \text{for} \quad r \gg \sigma$$

Remember Q-shift from quadrupole



$$\delta x' = \frac{r}{f} \quad \textbf{where} \quad \frac{1}{f} = kdl \quad k = \frac{1}{(B\rho)} \frac{\partial B_y}{\partial x}$$

$$\Delta Q = \frac{1}{4\pi} \oint \beta(s) k(s) ds = \frac{1}{4\pi} \beta^* \frac{\delta x'}{r}$$

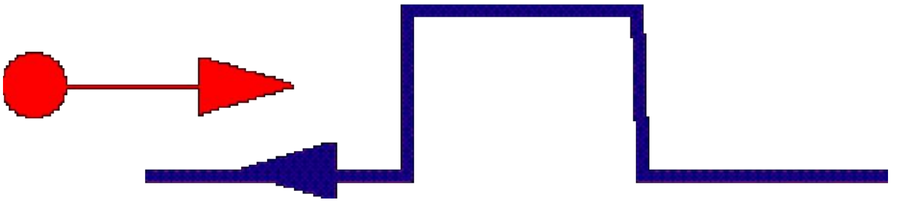
relates to radial impulse

$$\delta x' = \frac{\delta p_{\Uparrow}}{p_{\Rightarrow}} = \frac{F(r)dt}{p_{\Rightarrow}}$$

$$\boxed{\Delta Q = \frac{1}{4\pi} \beta^* \frac{F(r)dt}{p_{\Rightarrow}}}$$

Linear beam beam Q shift (relativistic)

$$\Delta Q = \frac{1}{4\pi}\beta^* \frac{F(r)dt}{p_{\Rightarrow}}$$



$$dt = \frac{ds}{2\beta c}$$

$$\Delta Q = \frac{1}{4\pi}\beta^* \frac{F(r)ds}{2\beta c p_{\Rightarrow}}$$

$$= \frac{1}{4\pi}\beta^* \frac{F(r)ds}{2\beta\beta\gamma m_0 c^2}$$

$$p_{\Rightarrow}c = \beta\gamma m_0 c^2$$

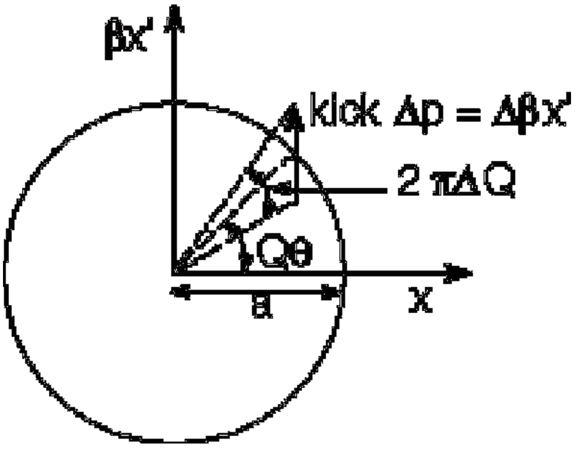
$$= \beta^* \frac{n\left(1+\beta^2\right)ds}{\gamma\left(2\beta^2\right)\sigma^2}\left(\frac{q^2}{4\pi\varepsilon_0 m_0 c^2}\right)$$

$$\int n ds = N$$

$$\Delta Q = \frac{\beta^* N r_0}{\gamma\sigma^2}$$

RESULT

Fourth integer resonance



Substituting

$$x = a \cos Q\theta$$

$$\Delta x' = \Delta(B\ell)/B\rho = \Delta(B'''\ell)x^2/3!B\rho$$

$$\Delta p = \hat{\beta}\Delta x'$$

$$2\pi\Delta Q = \frac{\Delta p}{a}\cos Q\theta$$

Incidentally

$$(\Delta a = \Delta p \ \sin Q\theta)$$

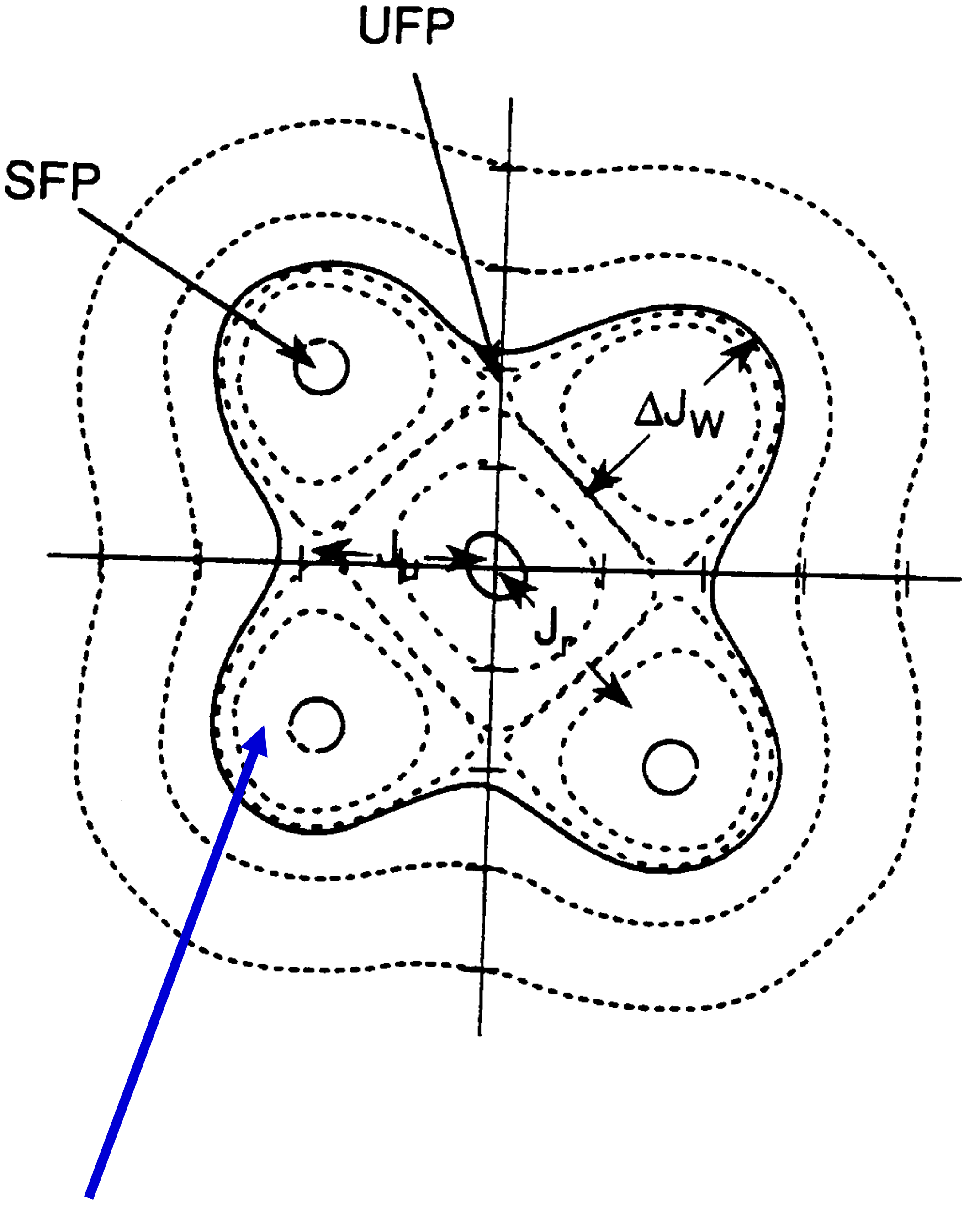
$$2\pi\Delta Q = \hat{\beta}\frac{\Delta(B'''\ell)}{6(B\rho)}a^2 \cos^4 Q\theta$$

$$\Delta Q = \frac{\hat{\beta}\Delta(B'''\ell)}{96\pi(B\rho)}a^2(1 - 4\cos 2Q\theta + \cos 4Q\theta)$$

Amplitude Q dependence brings chains close

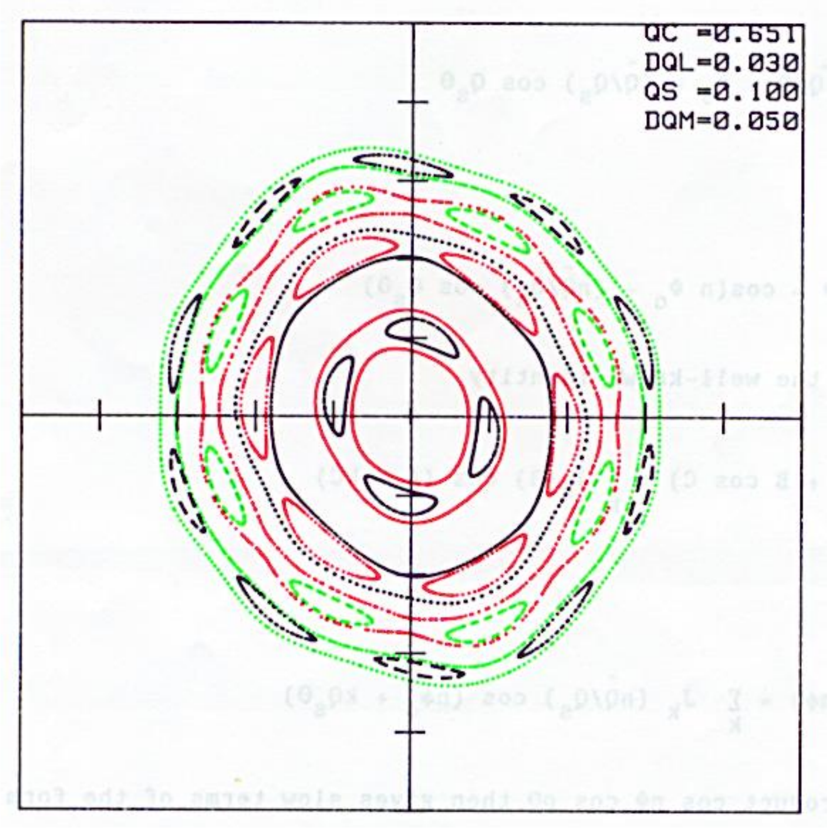
Excites resonances making islands

Phase space for fourth order resonance



Island of stability

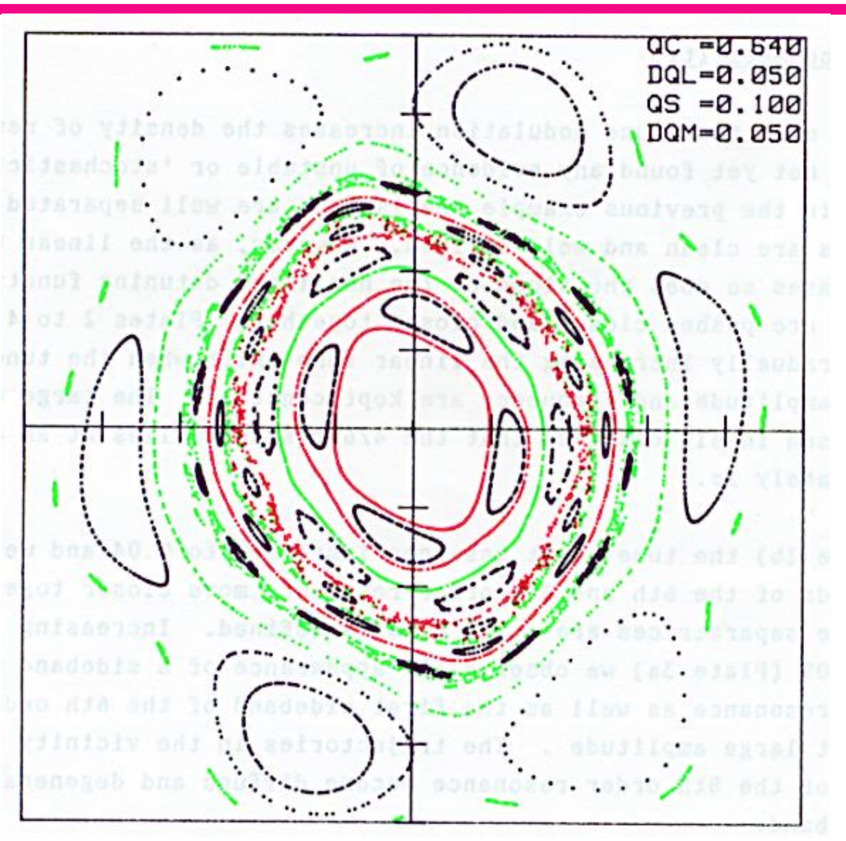
Example of tracking



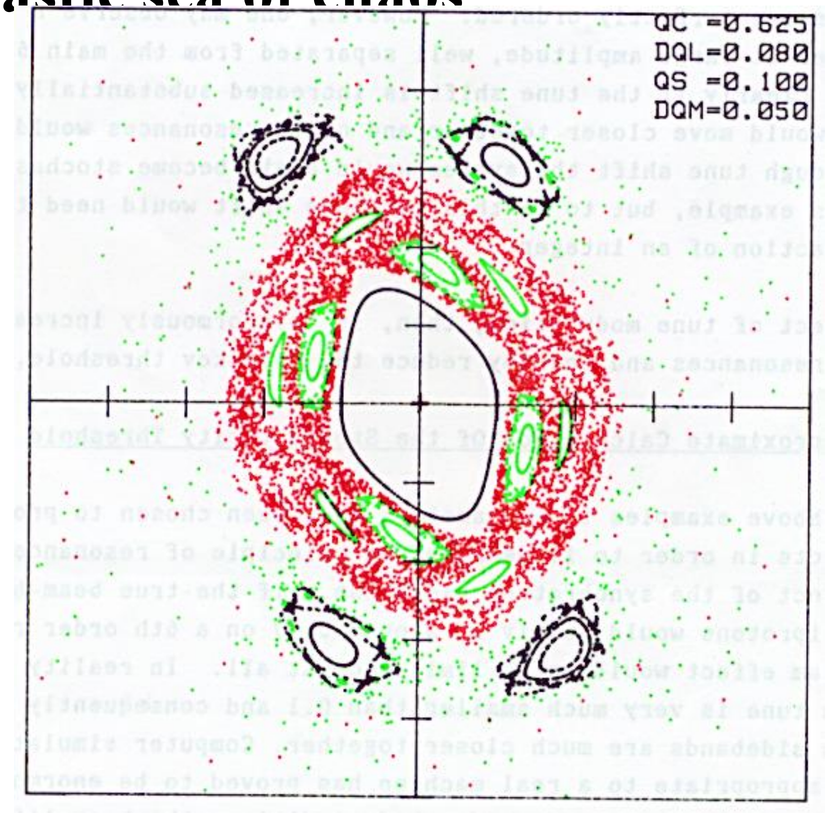
- Below the beam-beam limit and without tune modulation we see the many archipelagos of islands due to different multipole components in the beam-beam potential.
- Increasing the beam-beam interaction will enlarge the islands so that they overlap in stochastic regions where particle may diffuse out in 4 dimensional phase space.
- Increasing amplitude tune slope will merge them too
- Hadron collider luminosity limitations Author(s) Evans, Lyndon R In: 4th US-CERN School on Particle Accelerators, Hilton Head Island, South CA, USA, 7 - 14 Nov 1990, pp.592-599

TED0.PCT

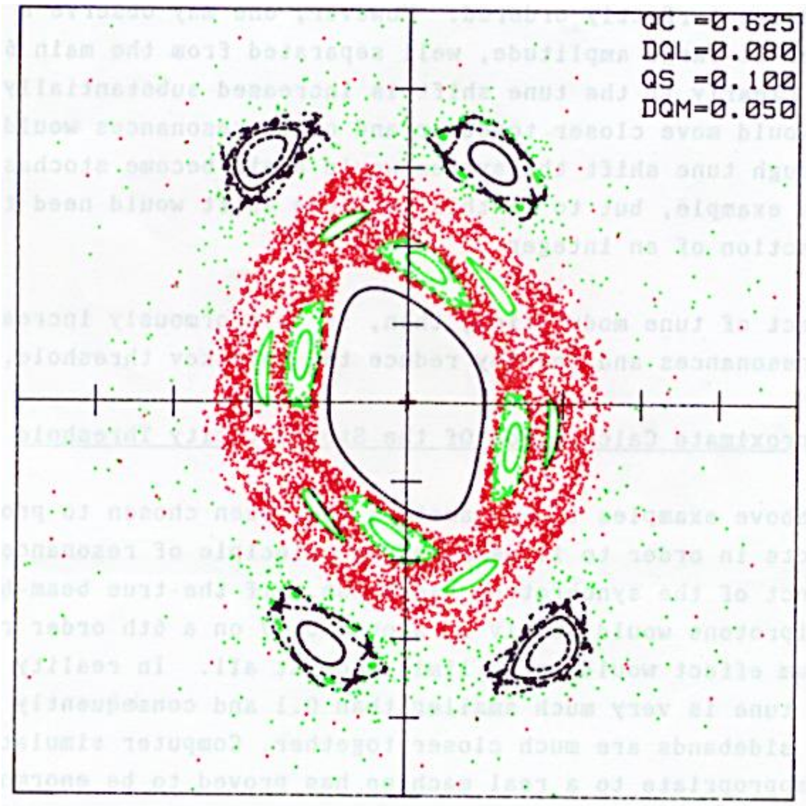
Crossing the stochastic limit



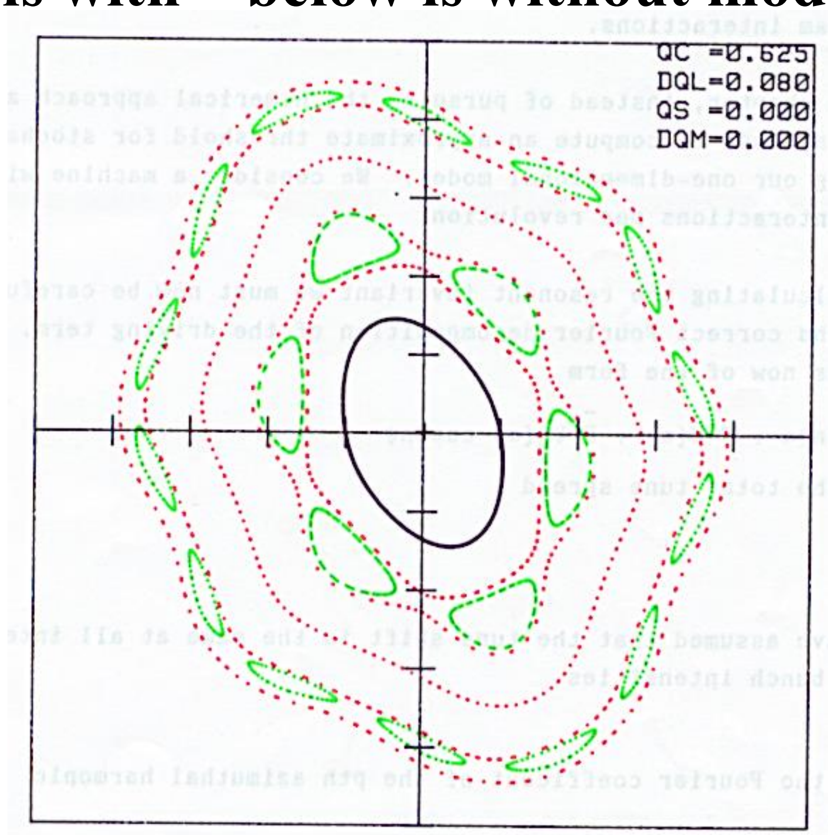
- Increasing Q shift causes islands to merge in a stochastic sea of chaos



Importance of tune modulation



- **Above is with – below is without modulation**



Summary Beam-Beam Tune Shift

- The Beam-beam effect
- Examples of the limit
- Field around a moving cylinder of charge
- Force on test particle
- Beam Beam Force
- Remember Q-shift from quadrupole
- Beam beam Q – shift
- Fourth integer resonance
- Phase space for fourth order resonance
- Example of tracking

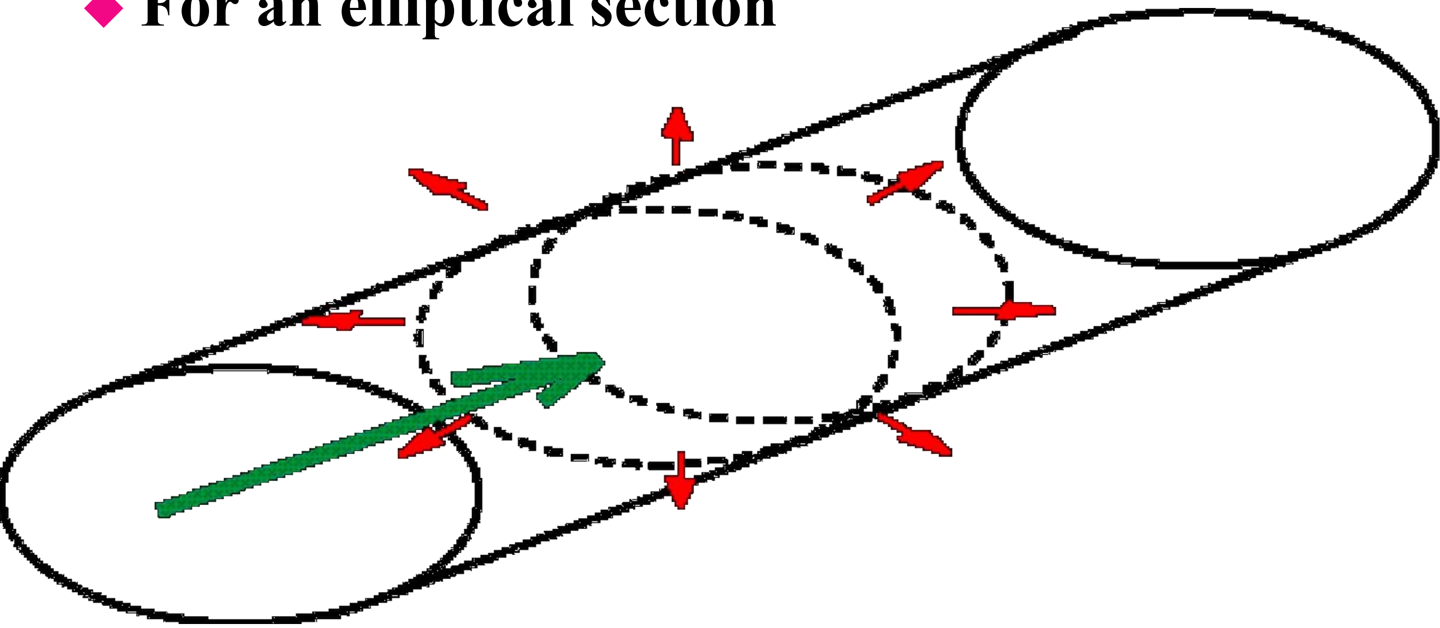
- FURTHER REFINEMENTS IN FOLLOWING SLIDES

Elliptical beam section

- **In the round beam case we calculated E**

$$2\pi r E_r = \left(1/\varepsilon_0\right)\int_0^r \frac{2\pi n q r'}{2\pi\sigma^2} e^{-r'^2/2\sigma^2} dr'$$

- **For an elliptical section**



- **The integrals are different**

$$\int ds = \pi(\sigma_x + \sigma_y)$$

$$\iint da = \pi\sigma_x\sigma_y$$

- **Leading to**

$$\xi_y = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y\left(\sigma_x + \sigma_y\right)}$$

Choosing lattice parameters

$$L = \frac{Nfk\gamma}{2r_0\beta_y^*}\xi_y\left(\frac{\sigma_x+\sigma_y}{\sigma_x}\right)$$

- **Invert**

$$\xi_y = \frac{2r_0\beta_y^*}{Nfk\gamma}\left(\frac{\sigma_x}{\sigma_x+\sigma_y}\right)L \qquad \xi_x = \frac{2r_0\beta_x^*}{Nfk\gamma}\left(\frac{\sigma_y}{\sigma_x+\sigma_y}\right)L$$

$$\frac{\xi_y}{\xi_x} = \frac{\beta_y^*}{\beta_x^*}\left(\frac{\sigma_x}{\sigma_y}\right) = \frac{\beta_y^*}{\beta_x^*}\left(\frac{\beta_x^*\varepsilon_x}{\beta_y^*\varepsilon_y}\right)^{1/2} = \left(\frac{\beta_y^*}{\beta_x^*}\right)^{1/2}\left(\frac{\varepsilon_x}{\varepsilon_y}\right)^{1/2} = 1$$

- **It always pays to reduce β^* s**
- **A narrow beam goes with a small beta (strong kick within large divergence beam)**
- **In electron machines (coupling)** $\varepsilon_x >> \varepsilon_y$
- **We should put lowest β^* to be β_y^***

$$\sigma_y << \sigma_x$$

- **This enhances!**
- **Choose ratio to match coupling** $\frac{\beta_x^*}{\beta_y^*} = \frac{\varepsilon_x}{\varepsilon_y}$
- **Adjust coupling so limits balance** $\frac{\varepsilon_x}{\varepsilon_y} = \frac{\beta_x^*}{\beta_y^*}$

Estimating field tolerances for growth at a resonance

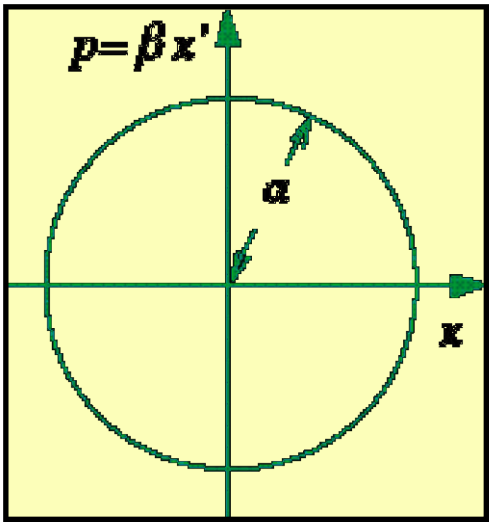
- **Deflection in a dipole**

$$\Delta\theta = \frac{\Delta(B\ell)}{(B\rho)}$$

- **Total ring** $\ell = 2\pi\rho$

$$\frac{1}{\sqrt{N}}$$

- **Reduced by**



$$\Delta x' = \Delta\theta \approx \frac{2\pi}{\sqrt{N}}\left\langle\frac{\Delta B}{B}\right\rangle_{rms}$$

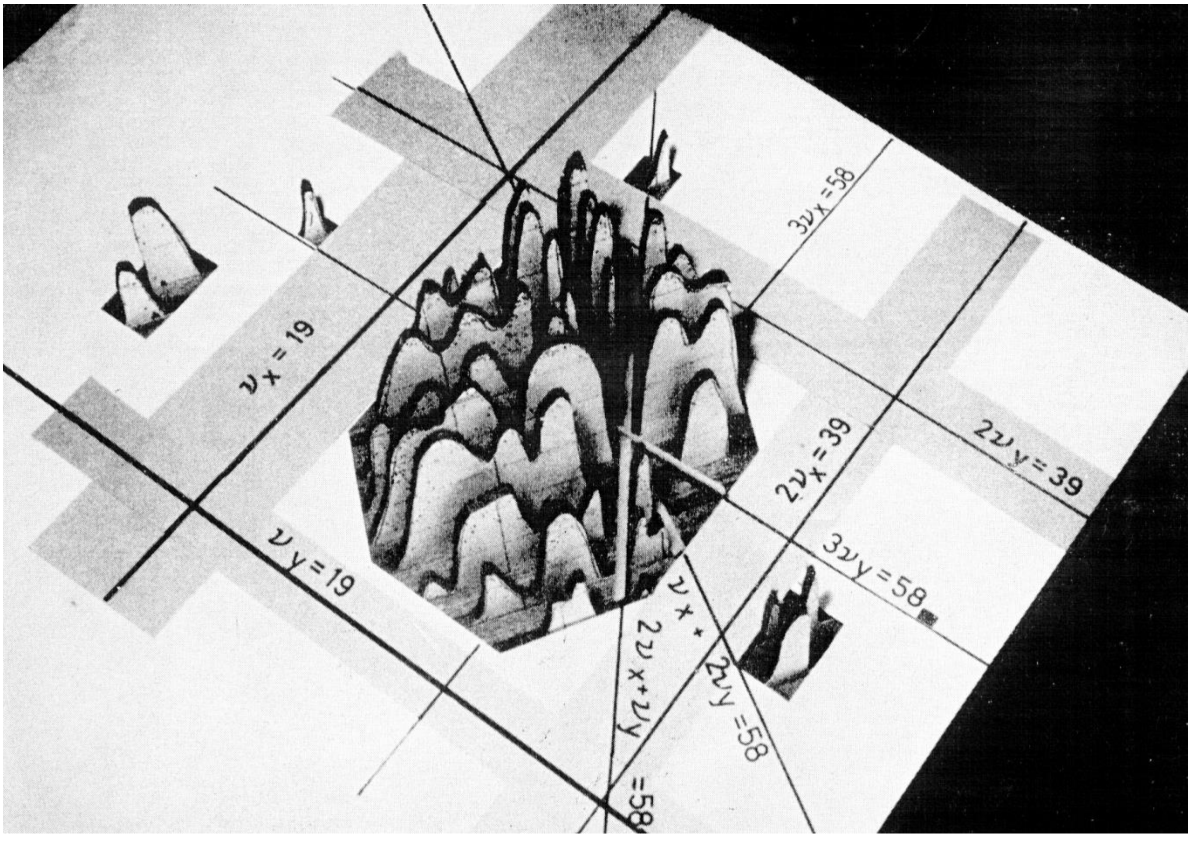
$\beta = \lambda / 2\pi$ of betatron oscillations $= \rho / Q$

$$2\pi\Delta Q = \frac{\Delta a}{a} = \frac{\beta\Delta\theta}{a} = \frac{\rho}{Qa}\frac{2\pi}{\sqrt{N}}\left\langle\frac{\Delta B}{B}\right\rangle_{rms}$$

$$Q = 0.0038 = \frac{\rho(=1000)}{Q(=25)\times a(=0.01)}\frac{1}{\sqrt{1000}}0.3\times 10^{-4}$$

42 turns = 1ms exit time

INJECTION STUDIES AT FNAL
(From lecture 24)



- **Remanent sextupole in the FNAL main ring caused serious beam loss due to non-linear resonances.**
- **This was exacerbated by magnet ripple.**
- **A three dimensional hill and dale model spanning the Q (or ν) diagram**
- **The vertical co-ordinate of this three dimensional model is the fraction of the beam which survives to be accelerated (always < 70%)**

FNAL-MODELL.PCT

Satellite stop bands

$$2\pi\Delta Q = \frac{\hat{\beta}\Delta(B'''\ell)}{48(B\rho)} a^2 (1 - 4\cos 2Q\theta + \cos 4Q\theta) = \frac{\Delta a}{a}$$

- **Fourier analysis of octupole pattern**

$$\sum_p f_p \cos(p\theta)$$

Resonance when $p = 4Q$

- **Ripple or rf modulation of Q**

$$Q = Q_o + \hat{Q}\sin Q_s\theta$$

- **Makes sidebands at** $Q_o \pm nQ_s$

- **Resonance when:** $p = 4(Q \pm nQ_s)$

Effect of satellites in phase space

HAMILTONIAN TREATMENT OF SYNCHROBETATRON RESONANCES

G. Guignard
CERN, Geneva, Switzerland

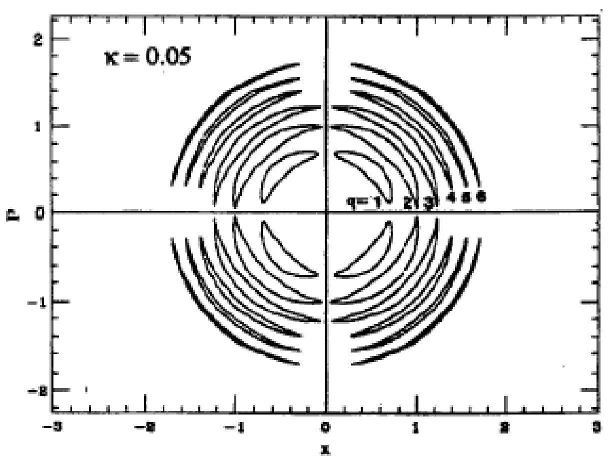


Fig. 3: Typical island contours of six synchrotron sidebands of a fourth-order resonance, in a stable case.

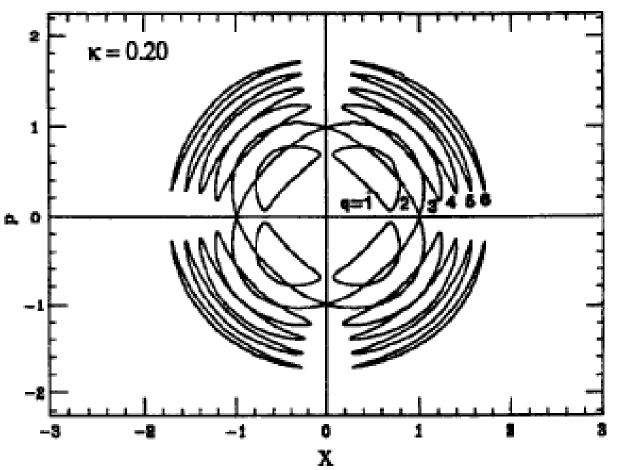


Fig. 4: Typical island contours of six synchrotron sidebands of a fourth-order resonance, with limited amplitude growth.

Experimental results of satellites

$$NQ_0 + qQ_s - p = 0\,.$$

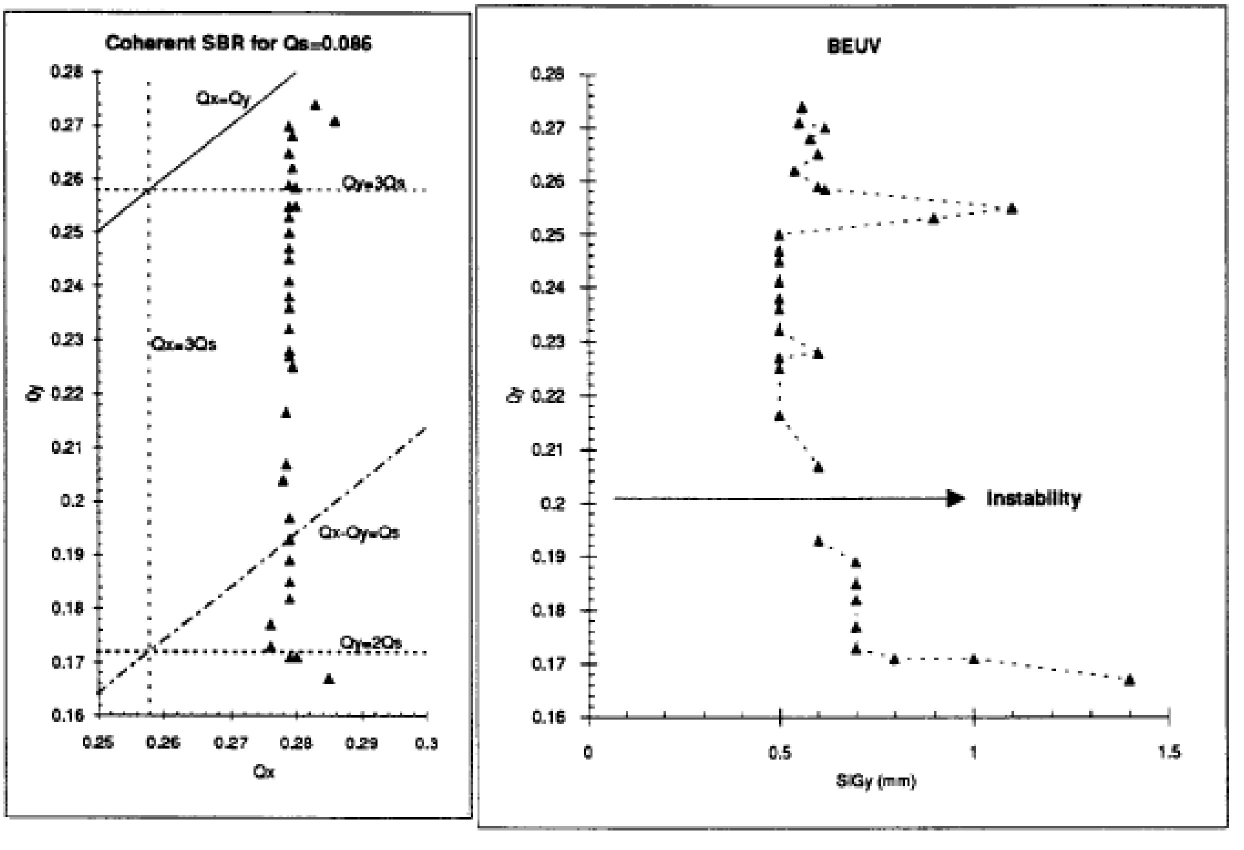


Fig. 7: Synchrobetatron resonances in the tune diagram and observed vertical emittance growth in LEP.