

# ***Beam-Beam***

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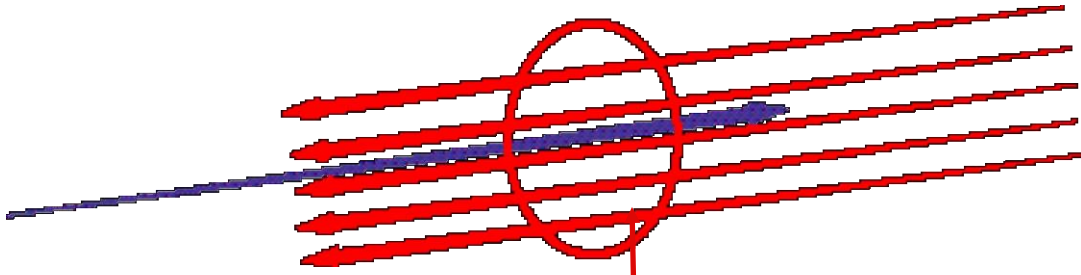
## **ACCELERATOR PHYSICS**

**HT 2015**

***E. J. N. Wilson***



# Luminosity (recall)



- ◆ Imagine a blue particle colliding with a beam of cross section area -  $A$
- ◆ Probability of collision is  $\frac{\sigma}{A} \cdot N$
- ◆ For  $N$  particles in both beams  $\frac{\sigma}{A} \cdot N^2$
- ◆ Suppose they meet  $f$  times per second at the revolution frequency

- ◆ Event rate  $f_{rev} = \frac{\beta c}{2\pi R}$

$$\frac{f_{rev} N^2}{A} \cdot \sigma$$

Make big

e.g.  $10^{-25}$

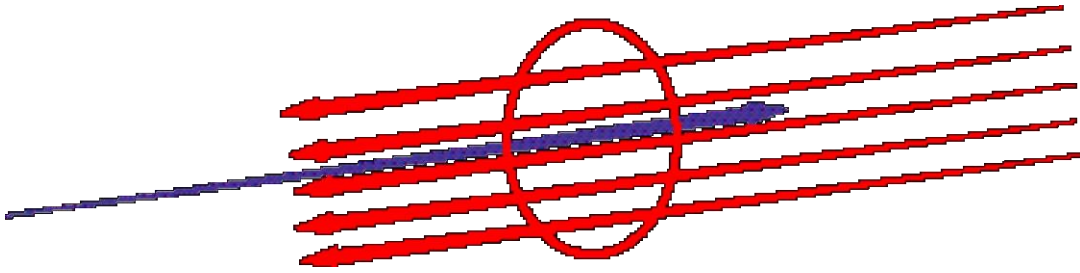
Make small

**LUMINOSITY**

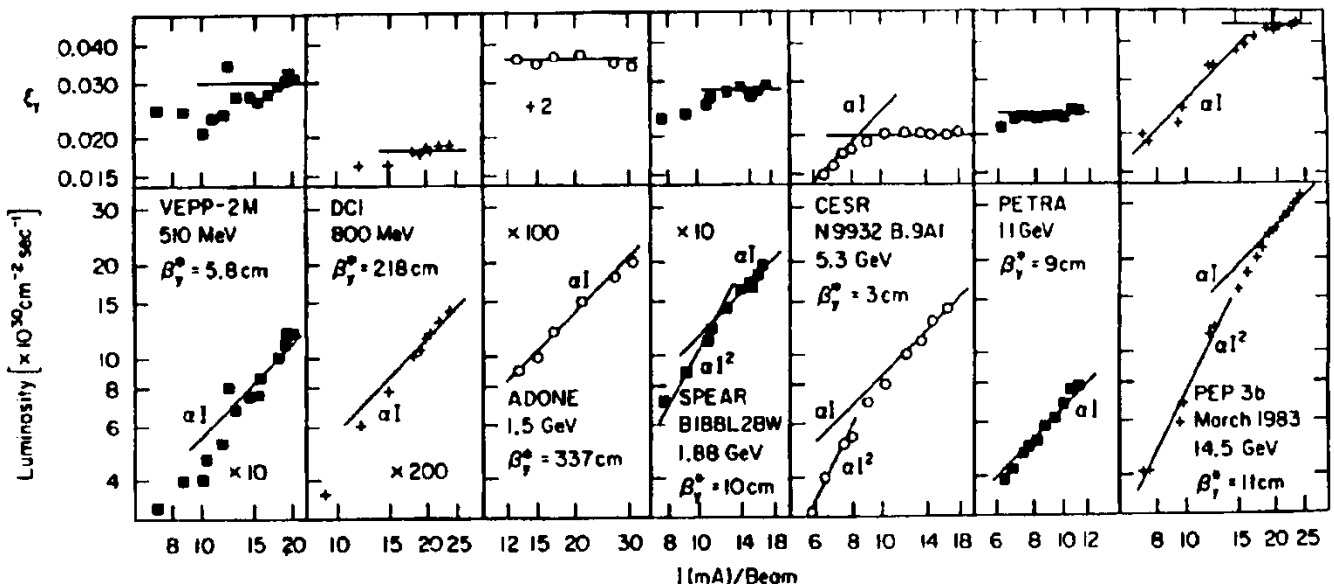
$$\approx 10^{30} \text{ to } 10^{34} \text{ [cm}^{-2} \text{ s}^{-1}\text{]}$$

# Beam-beam interaction Limits

## Luminosity



- ◆ Blue particle sees electromagnetic force from blue charges and currents



- ◆ Luminosity “saturates” when beam-beam tune shift is about 0.03 (for lepton colliders)
- ◆ About 0.003 for hadron colliders

# The Beam-beam effect - E. Wilson

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## ◆ Luminosity

$$L = \frac{N^2 f k}{4\pi\sigma_x \sigma_y}$$

**f = the rev frequency**

**k = number of bunches**

**$\sigma$  = rms beam width or height**

## ◆ We shall show that the beam beam tune shift (linear effect on Q)

$$\xi_y = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)}$$

## ◆ We can solve to fine dependence of one upon the other

$$L = \frac{N f k \gamma}{2 r_0 \beta_y^*} \xi_y \left( \frac{\sigma_x + \sigma_y}{\sigma_x} \right)$$

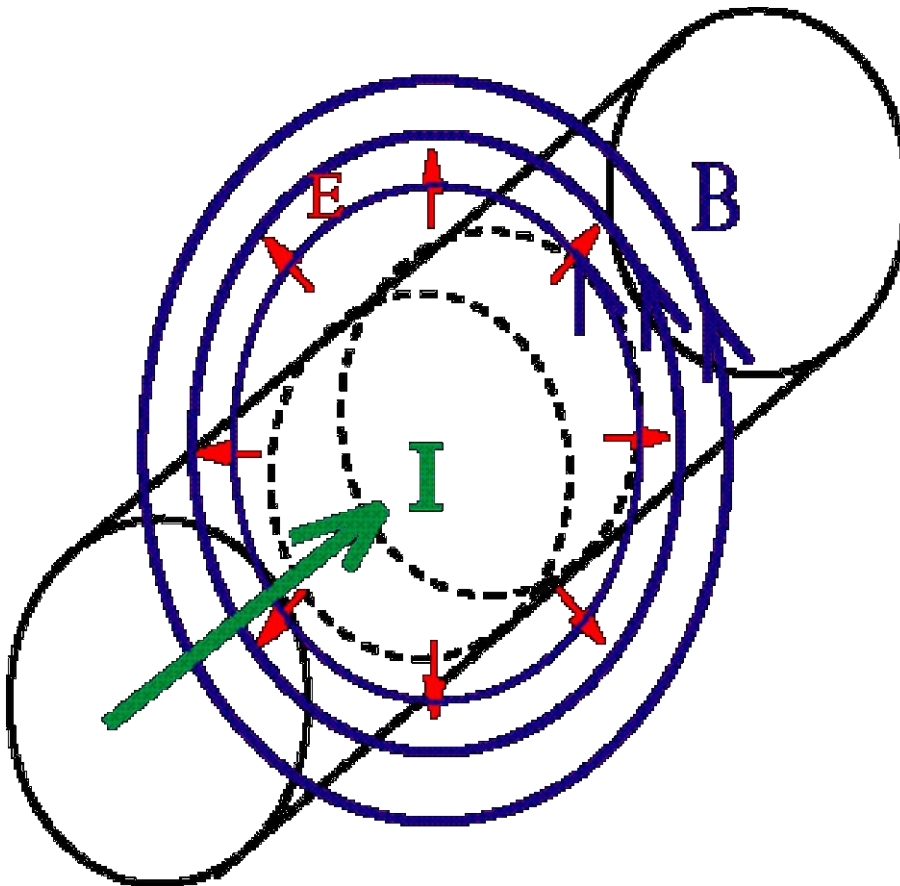
## ◆ Low beta, large number of bunches and high intensity helps --- but instabilities limit N

# Field around a moving cylinder of charge

- ◆ Take unit length of the bunch

$$2\pi r E_r = (1/\epsilon_0) \int_0^r 2\pi r' \rho(r') dr'$$

$$2\pi r B_\phi = \mu_0 \int_0^r 2\pi r' \beta c \rho(r') dr'$$



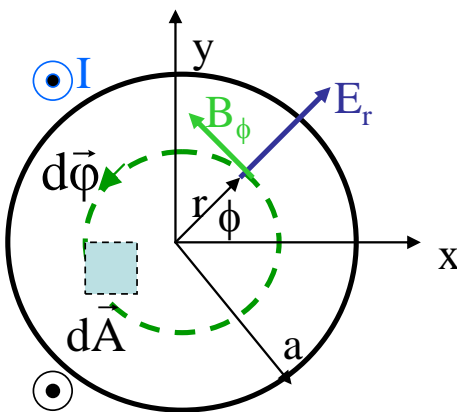
# Electric and Magnetic field within a beam

$\eta$ ...charge density in Cb/m<sup>3</sup>  
 $\lambda$ ... constant line charge  $\pi a^2\eta$   
 $I$ ...constant current  $\lambda\beta c = \pi a^2\eta\beta c$   
 $a$ ...beam radius

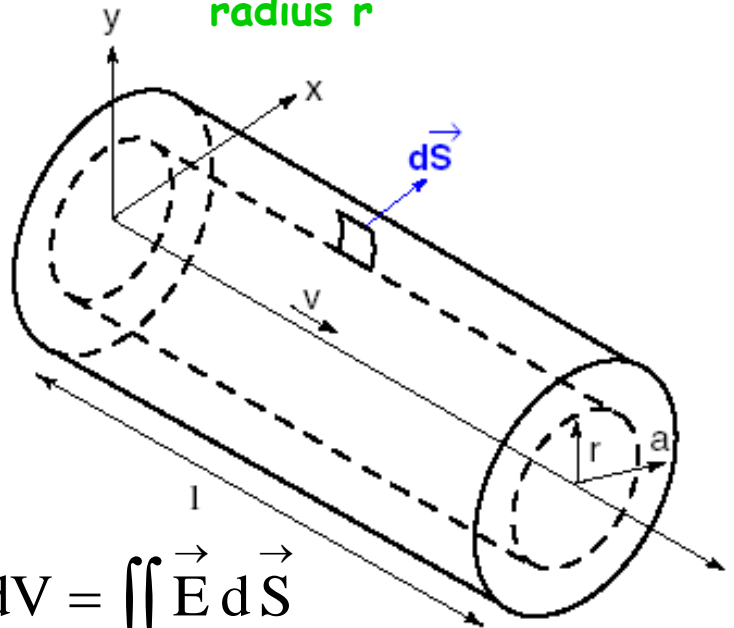
$$\vec{E} = E_r$$

$$\text{div } \vec{E} = \frac{\eta}{\epsilon_0}$$

cylinder radius  $r$   
length  $l$



cross section radius  $r$



$$\iiint \text{div } \vec{E} dV = \iint \vec{E} d\vec{S}$$

$$r^2 \pi l \frac{\eta}{\epsilon_0} = E_r 2r\pi l$$

$$E_r = \frac{I}{2\pi\epsilon_0\beta c} \frac{r}{a^2}$$

$$B_\phi = \frac{I}{2\pi\epsilon_0 c^2} \frac{r}{a^2}$$

**Within the beam at radius  $r'$**

$$2\pi r E_r = \left(1/\epsilon_0\right) \int_0^r 2\pi r' \rho(r') dr'$$

# E field and force on a test particle

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## ◆ From previous slide

$$2\pi r E_r = (1/\epsilon_0) \int_0^r 2\pi r' \rho(r') dr'$$

## ◆ Assume the beam is Gaussian

$$\begin{aligned} 2\pi r E_r &= (1/\epsilon_0) \int_0^r \frac{2\pi n q r'}{2\pi\sigma^2} e^{-r'^2/2\sigma^2} dr' \\ &= (nq/\epsilon_0) \left[ e^{-r'^2/2\sigma^2} \right]_0^r = (nq/\epsilon_0) \left[ 1 - e^{-r^2/2\sigma^2} \right] \end{aligned}$$

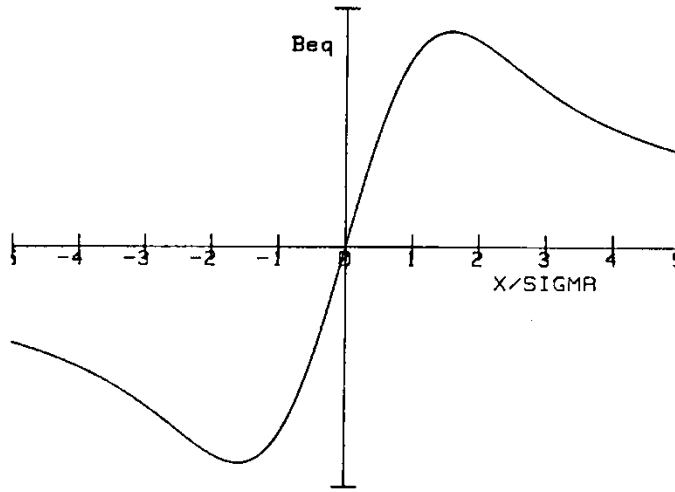
## Force on test particle including magnetic field:

$$F_r(r) = -\frac{nq^2(1+\beta^2)}{4\pi\epsilon_0\sigma^2} \left[ 1 - e^{-r^2/2\sigma^2} \right]$$

$$F_r(r) = -\frac{nq^2(1+\beta^2)r}{4\pi\epsilon_0\sigma^2} \quad \text{for } r \ll \sigma$$

$$F_r(r) = -\frac{nq^2(1+\beta^2)}{2\pi\epsilon_0 r} \quad \text{for } r \gg \sigma$$

# Beam Beam Force is non linear



- ◆ A linear focusing effect at beam center
- ◆ But elsewhere it is non linear

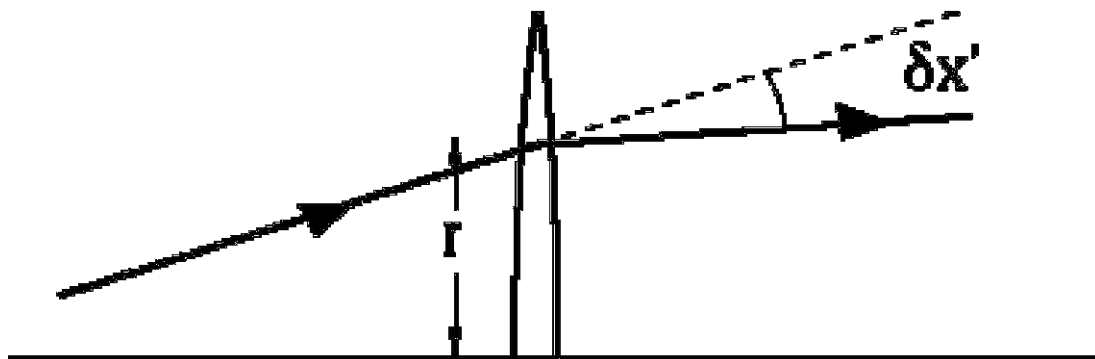
$$F_r(r) = -\frac{nq^2(1+\beta^2)}{4\pi\epsilon_0\sigma^2} \left[ 1 - e^{-r^2/2\sigma^2} \right]$$

$$F_r(r) = -\frac{nq^2(1+\beta^2)r}{4\pi\epsilon_0\sigma^2} \quad \text{for } r \ll \sigma$$

$$F_r(r) = -\frac{nq^2(1+\beta^2)}{2\pi\epsilon_0 r} \quad \text{for } r \gg \sigma$$



# Remember Q-shift from quadrupole



$$\delta x' = \frac{r}{f} \quad \text{where} \quad \frac{1}{f} = kdl \quad k = \frac{1}{(B\rho)} \frac{\partial B_y}{\partial x}$$

$$\Delta Q = \frac{1}{4\pi} \oint \beta(s)k(s)ds = \frac{1}{4\pi} \beta^* \frac{\delta x'}{r}$$

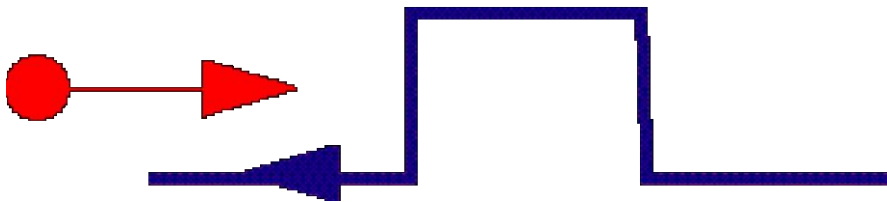
**relates to radial impulse**

$$\delta x' = \frac{\delta p_{\uparrow}}{p_{\rightarrow}} = \frac{F(r)dt}{p_{\rightarrow}}$$

$$\Delta Q = \frac{1}{4\pi} \beta^* \frac{F(r)dt}{p_{\rightarrow}}$$

# Linear beam beam Q shift (relativistic)

$$\Delta Q = \frac{1}{4\pi} \beta^* \frac{F(r) dt}{p_{\Rightarrow}}$$



$$dt = \frac{ds}{2\beta c}$$

$$\Delta Q = \frac{1}{4\pi} \beta^* \frac{F(r) ds}{2\beta c p_{\Rightarrow}}$$

$$= \frac{1}{4\pi} \beta^* \frac{F(r) ds}{2\beta\beta\gamma m_0 c^2}$$

$$p_{\Rightarrow} c = \beta\gamma m_0 c^2$$

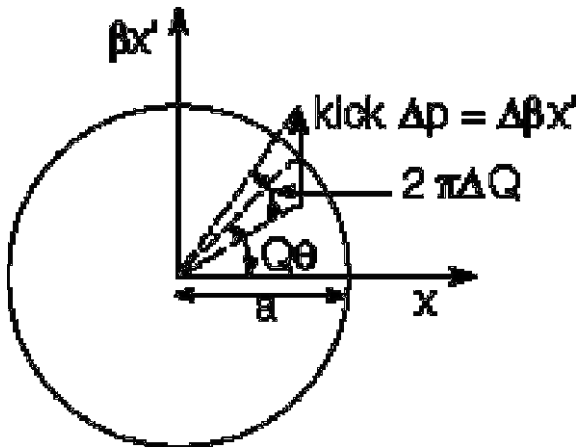
$$= \beta^* \frac{n(1 + \beta^2) ds}{\gamma(2\beta^2)\sigma^2} \left( \frac{q^2}{4\pi\epsilon_0 m_0 c^2} \right)$$

$$\Delta Q = \frac{\beta^* N r_0}{\gamma \sigma^2}$$

$$\int n ds = N$$

**RESULT**

# Fourth integer resonance



**Substituting**

$$x = a \cos Q\theta$$

$$\Delta x' = \Delta(B\ell) / B\rho = \Delta(B''' \ell) x^2 / 3! B\rho$$

$$\Delta p = \hat{\beta} \Delta x'$$

$$2\pi\Delta Q = \frac{\Delta p}{a} \cos Q\theta$$

**Incidentally**

$$(\Delta a = \Delta p \sin Q\theta)$$

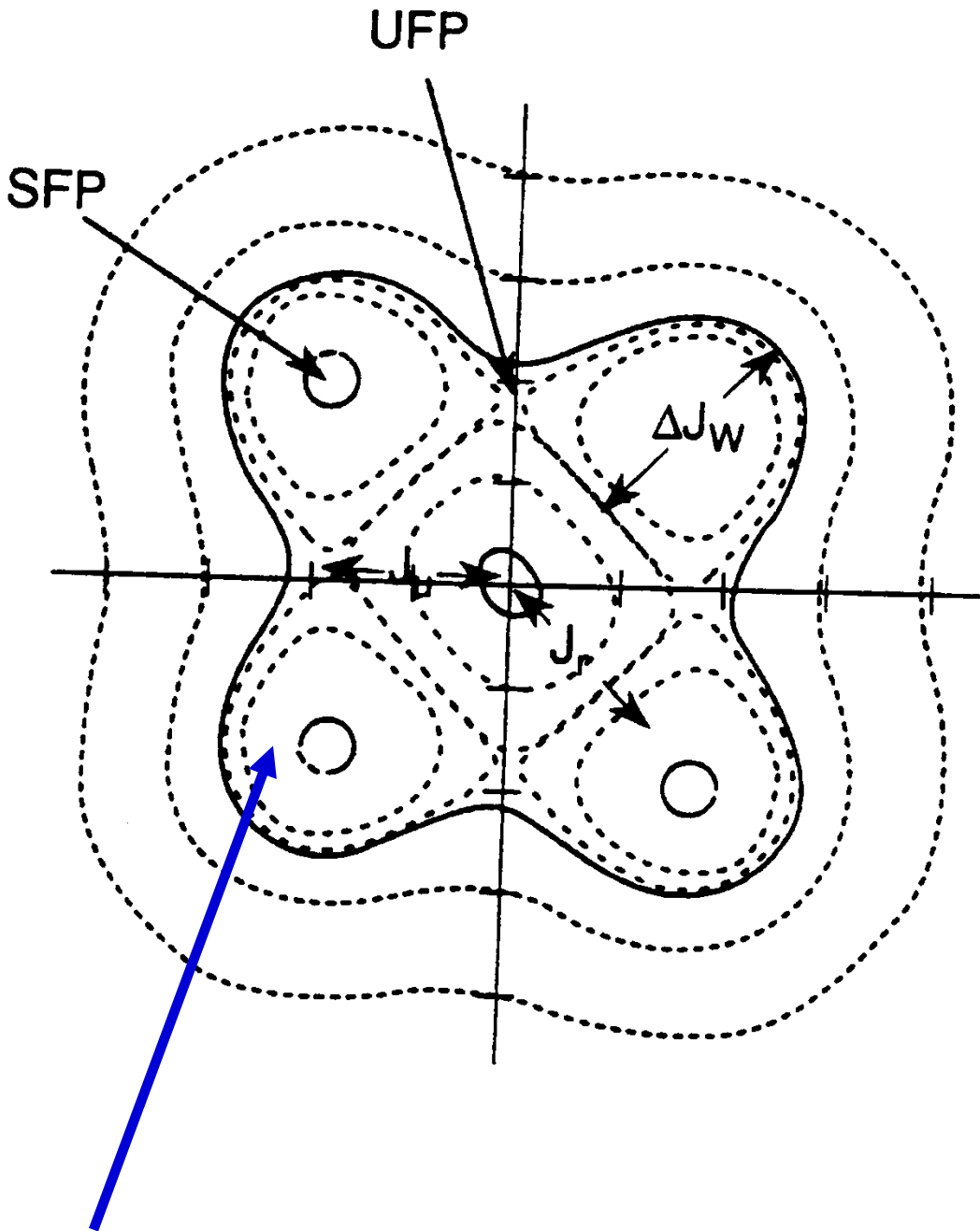
$$2\pi\Delta Q = \hat{\beta} \frac{\Delta(B''' \ell)}{6(B\rho)} a^2 \cos^4 Q\theta$$

$$\Delta Q = \frac{\hat{\beta} \Delta(B''' \ell)}{96\pi(B\rho)} a^2 (1 - 4 \cos 2Q\theta + \cos 4Q\theta)$$

**Amplitude Q dependence  
brings chains close**

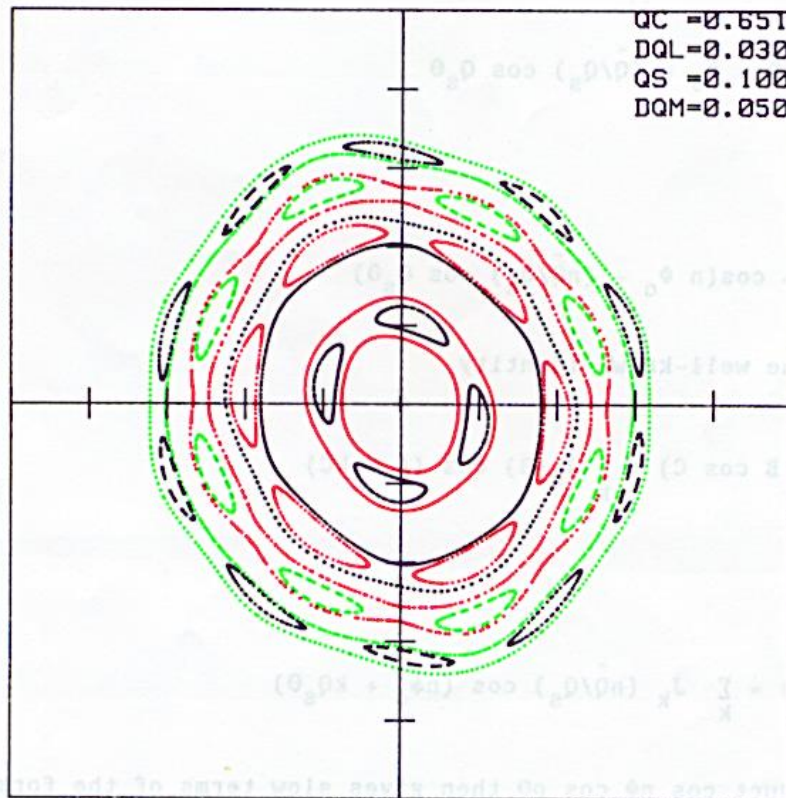
**Excites resonances  
making islands**

# Phase space for fourth order resonance



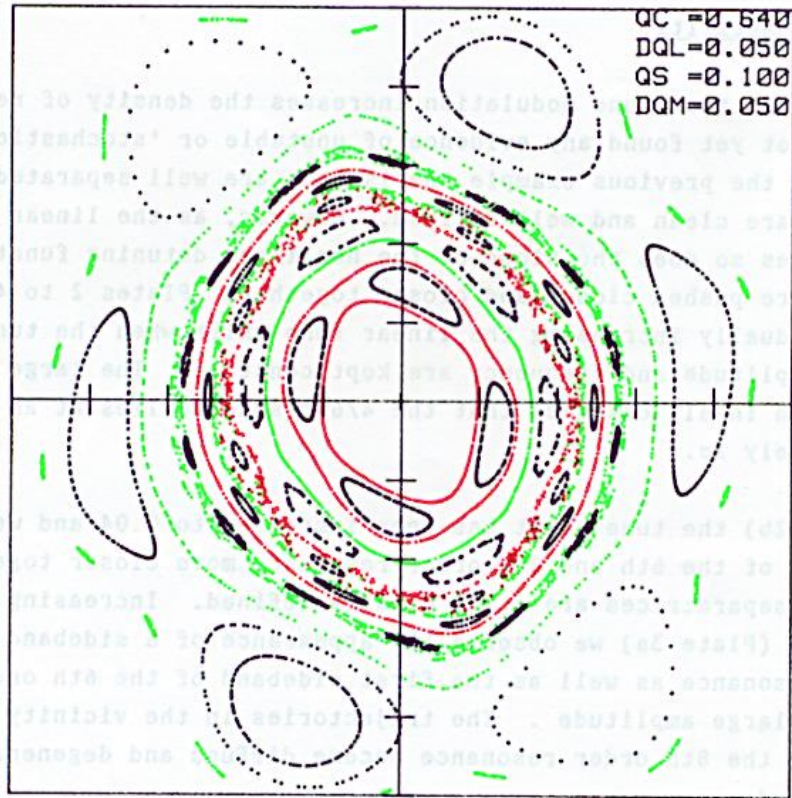
**Island of stability**

# Example of tracking

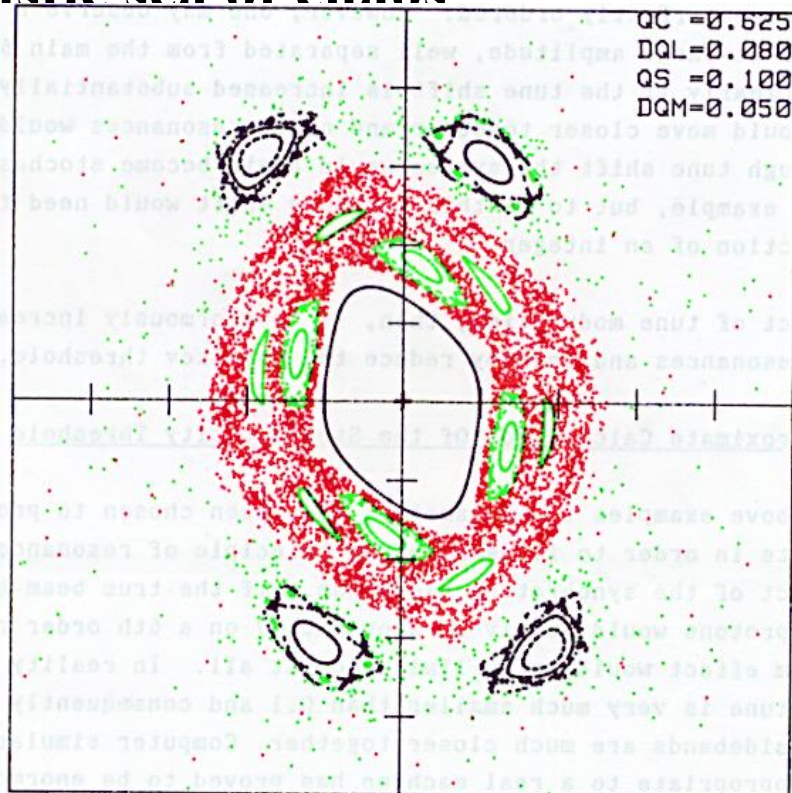


- ◆ Below the beam-beam limit and without tune modulation we see the many archipelagos of islands due to different multipole components in the beam-beam potential.
- ◆ Increasing the beam-beam interaction will enlarge the islands so that they overlap in stochastic regions where particle may diffuse out in 4 dimensional phase space.
- ◆ Increasing amplitude tune slope will merge them too
- ◆ Hadron collider luminosity limitations Author(s) [Evans, Lyndon R](#) In: [4th US-CERN School on Particle Accelerators](#), Hilton Head Island, South CA, USA, 7 - 14 Nov 1990, pp.592-599

# Crossing the stochastic limit

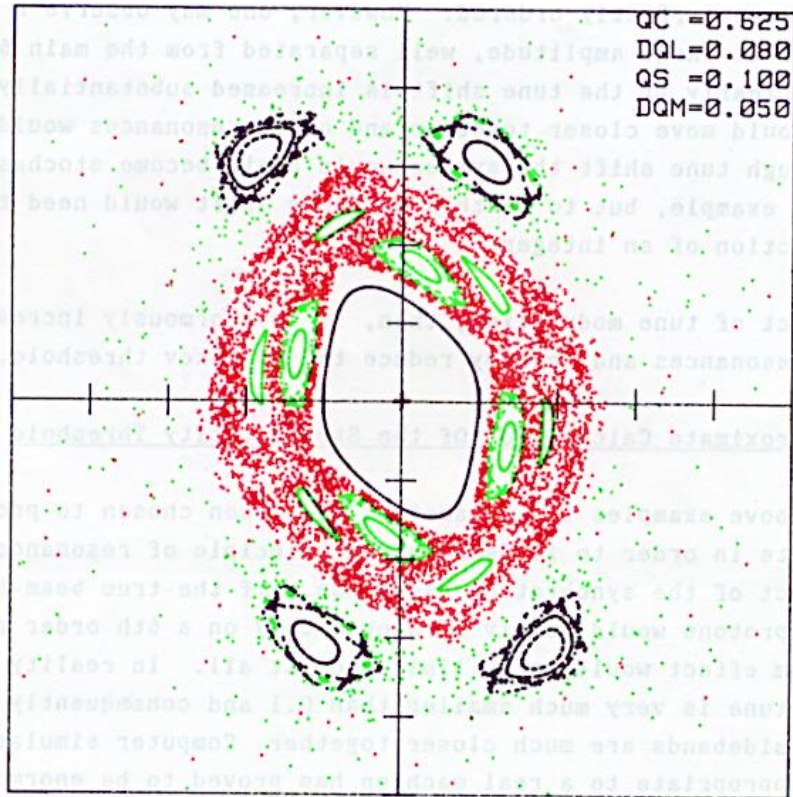


- ◆ Increasing  $Q$  shift causes islands to merge in a stochastic sea of chaos

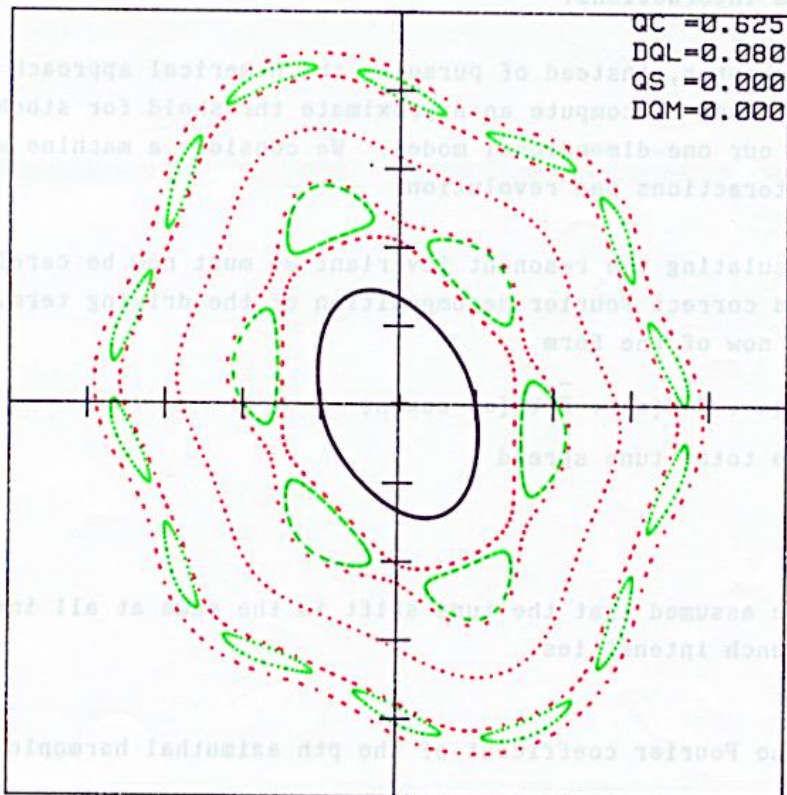




# Importance of tune modulation



◆ Above is with – below is without modulation



# Summary Beam-Beam Tune Shift

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- ◆ **The Beam-beam effect**
- ◆ **Examples of the limit**
- ◆ **Field around a moving cylinder of charge**
- ◆ **Force on test particle**
- ◆ **Beam Beam Force**
- ◆ **Remember Q-shift from quadrupole**
- ◆ **Beam beam Q – shift**
- ◆ **Fourth integer resonance**
- ◆ **Phase space for fourth order resonance**
- ◆ **Example of tracking**
  
- ◆ **FURTHER REFINEMENTS IN FOLLOWING SLIDES**



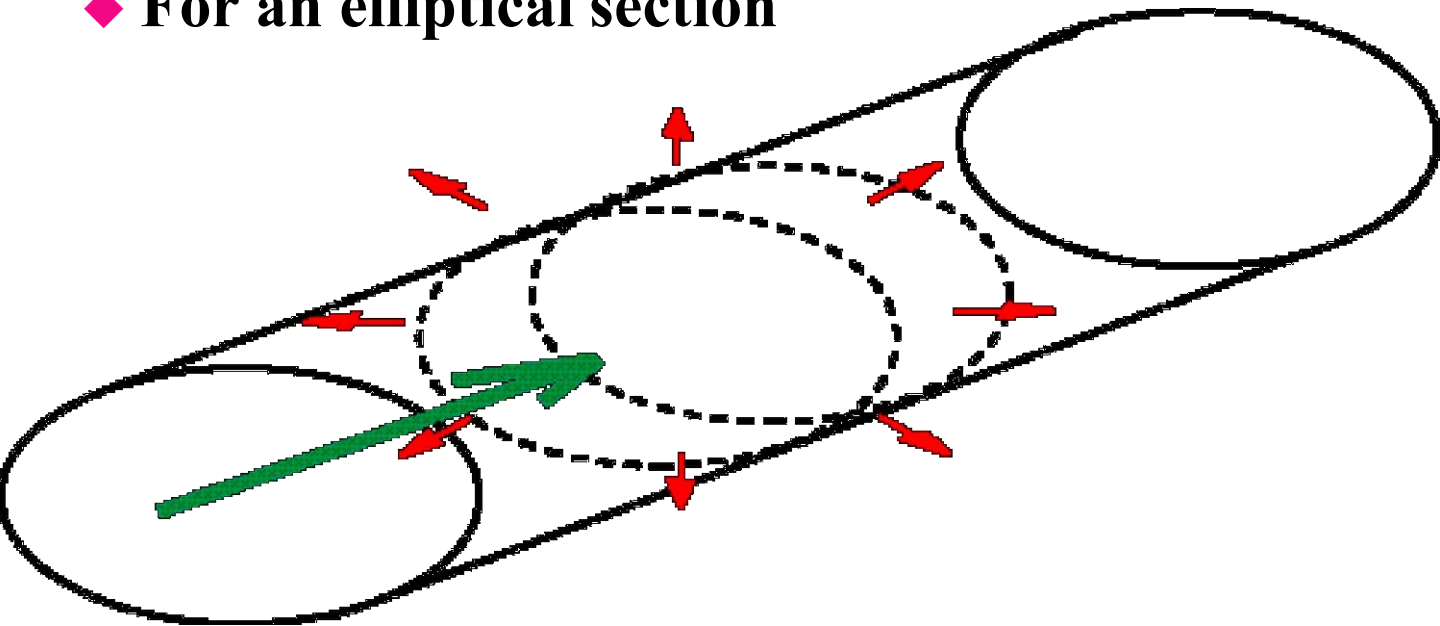


# Elliptical beam section

- ◆ In the round beam case we calculated E

$$2\pi r E_r = (1/\epsilon_0) \int_0^r \frac{2\pi n q r'}{2\pi\sigma^2} e^{-r'^2/2\sigma^2} dr'$$

- ◆ For an elliptical section



- ◆ The integrals are different

$$\int ds = \pi(\sigma_x + \sigma_y)$$

$$\iint da = \pi\sigma_x\sigma_y$$

$$\xi_y = \frac{Nr_0\beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)}$$

- ◆ Leading to

# Choosing lattice parameters

$$L = \frac{Nfk\gamma}{2r_0\beta_y^*} \xi_y \left( \frac{\sigma_x + \sigma_y}{\sigma_x} \right)$$

## ◆ Invert

$$\xi_y = \frac{2r_0\beta_y^*}{Nfk\gamma} \left( \frac{\sigma_x}{\sigma_x + \sigma_y} \right) L \quad \xi_x = \frac{2r_0\beta_x^*}{Nfk\gamma} \left( \frac{\sigma_y}{\sigma_x + \sigma_y} \right) L$$

$$\frac{\xi_y}{\xi_x} = \frac{\beta_y^*}{\beta_x^*} \left( \frac{\sigma_x}{\sigma_y} \right) = \frac{\beta_y^*}{\beta_x^*} \left( \frac{\beta_x^* \varepsilon_x}{\beta_y^* \varepsilon_y} \right)^{1/2} = \left( \frac{\beta_y^*}{\beta_x^*} \right)^{1/2} \left( \frac{\varepsilon_x}{\varepsilon_y} \right)^{1/2} = 1$$

◆ It always pays to reduce  $\beta^*$  s

◆ A narrow beam goes with a small beta

(strong kick within large divergence beam)

◆ In electron machines (coupling)  $\varepsilon_x \gg \varepsilon_y$

◆ We should put lowest  $\beta^*$  to be  $\beta_y^*$

$$\sigma_y \ll \sigma_x$$

◆ This enhances!

◆ Choose ratio to match coupling  $\frac{\beta_x^*}{\beta_y^*} = \frac{\varepsilon_x}{\varepsilon_y}$

◆ Adjust coupling so limits balance  $\frac{\varepsilon_x}{\varepsilon_y} = \frac{\beta_y^*}{\beta_x^*}$



# Estimating field tolerances for growth at a resonance

◆ Deflection in a dipole

$$\Delta\theta = \frac{\Delta(B\ell)}{(B\rho)}$$

◆ Total ring

$$\ell = 2\pi\rho$$

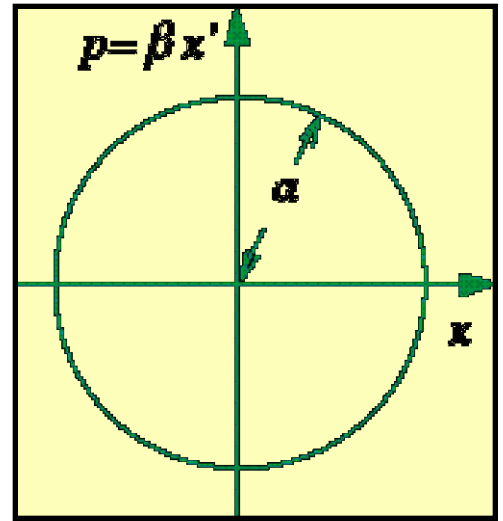
$$\frac{1}{\sqrt{N}}$$

◆ Reduced by

$$\Delta x' = \Delta\theta \approx \frac{2\pi}{\sqrt{N}} \left\langle \frac{\Delta B}{B} \right\rangle_{rms}$$

$$\beta = \lambda / 2\pi \text{ of betatron oscillations} = \rho / Q$$

$$2\pi\Delta Q = \frac{\Delta a}{a} = \frac{\beta\Delta\theta}{a} = \frac{\rho}{Qa} \frac{2\pi}{\sqrt{N}} \left\langle \frac{\Delta B}{B} \right\rangle_{rms}$$

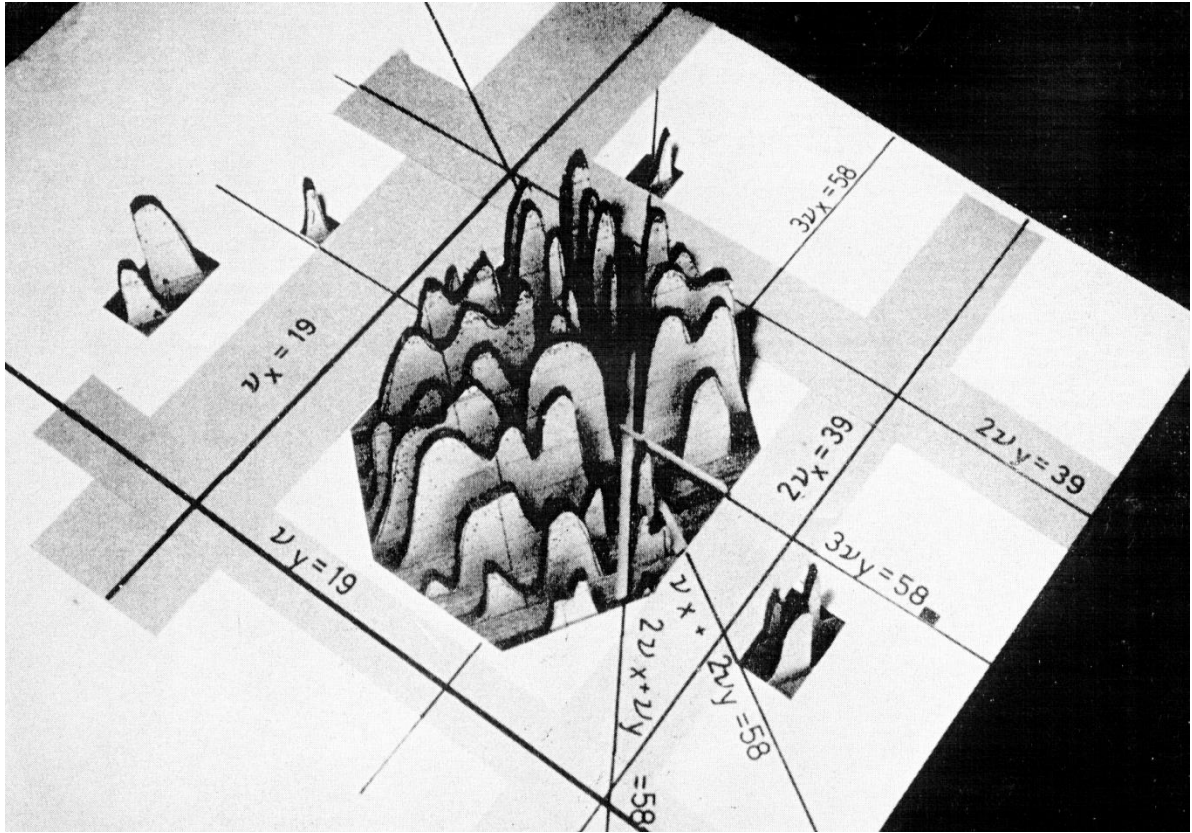


$$Q = 0.0038 = \frac{\rho(=1000)}{Q(=25) \times a(=0.01)} \frac{1}{\sqrt{1000}} 0.3 \times 10^{-4}$$

**42 turns = 1ms exit time**

# INJECTION STUDIES AT FNAL

(From lecture 24)



- ◆ **Remanent sextupole in the FNAL main ring caused serious beam loss due to non-linear resonances.**
- ◆ **This was exacerbated by magnet ripple.**
- ◆ **A three dimensional hill and dale model spanning the Q ( or  $\nu$  ) diagram**
- ◆ **The vertical co-ordinate of this three dimensional model is the fraction of the beam which survives to be accelerated (always < 70%)**

# Satellite stop bands

$$2\pi\Delta Q = \frac{\hat{\beta}\Delta(B'''\ell)}{48(B\rho)} a^2 (1 - 4\cos 2Q\theta + \cos 4Q\theta) = \frac{\Delta a}{a}$$

◆ **Fourier analysis of octupole pattern**

$$\sum_p f_p \cos(p\theta)$$

**Resonance when  $p = 4Q$**

◆ **Ripple or rf modulation of Q**

$$Q = Q_0 + \hat{Q} \sin Q_S \theta$$

◆ **Makes sidebands at**

$$Q_0 \pm nQ_S$$

$$p = 4(Q \pm nQ_S)$$

◆ **Resonance when:**

# Effect of satellites in phase space

## HAMILTONIAN TREATMENT OF SYNCHROBETATRON RESONANCES

*G. Guignard*  
CERN, Geneva, Switzerland

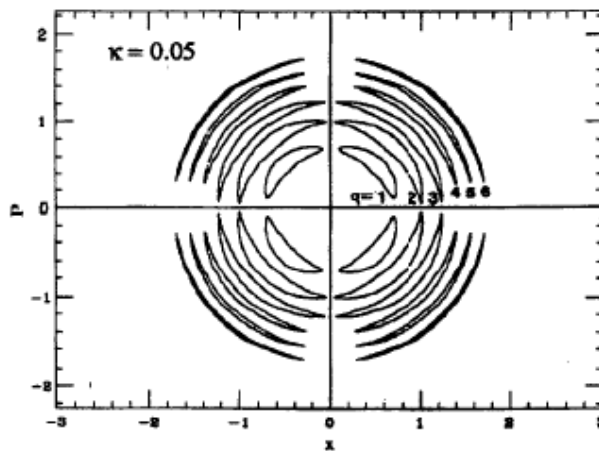


Fig. 3: Typical island contours of six synchrotron sidebands of a fourth-order resonance, in a stable case.

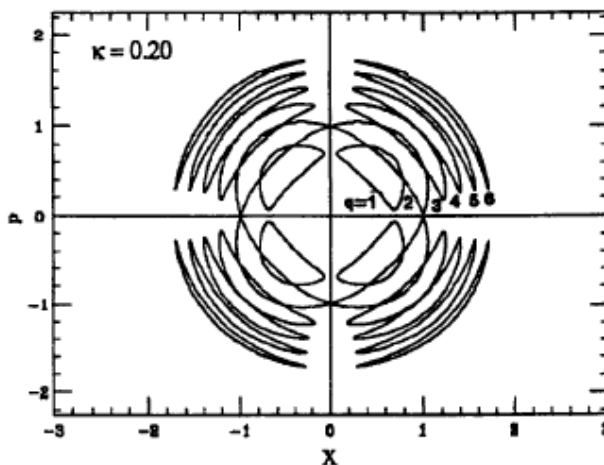


Fig. 4: Typical island contours of six synchrotron sidebands of a fourth-order resonance, with limited amplitude growth.

# Experimental results of satellites

$$NQ_0 + qQ_s - p = 0 .$$

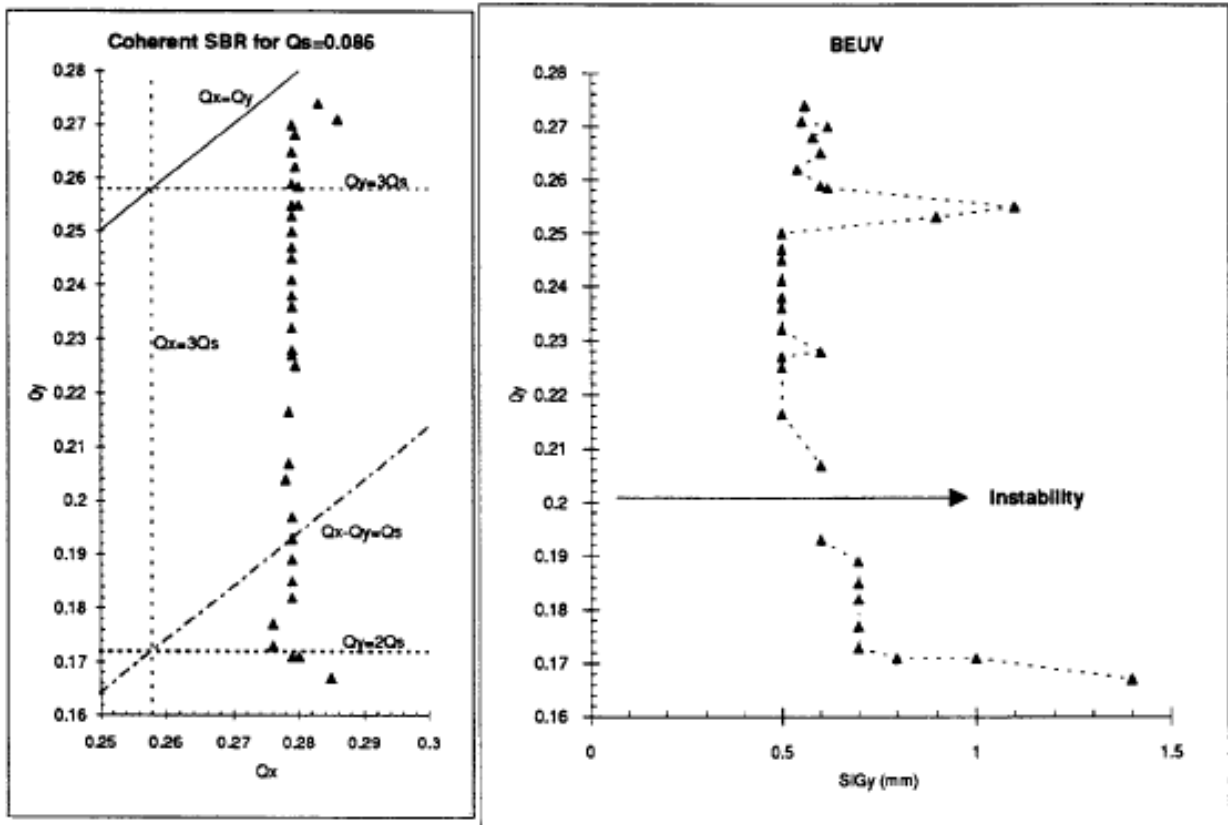


Fig. 7: Synchrotron resonances in the tune diagram and observed vertical emittance growth in LEP.