Phenomenology of $B \rightarrow K \pi \pi$ modes and Prospects with LHCb and Belle-II data

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On behalf of the CKMfitter Group
Outline

- Amplitude analyses
- The phenomenological framework
- Some theoretical scenarios for constraining CKM
- Constraints on hadronic amplitudes using the latest $B \rightarrow K^*\pi$ measurements
- Prospects for future LHCb and Belle-II data
- Summary and outlook
Amplitude Analyses
Three body decays described by two parameters

Mandelstam variables $m_{ij}^2 = (p_i + p_j)^2$
Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

\[
\begin{align*}
A(DP) &= \sum a_j F_j(DP) \\
\bar{A}(DP) &= \sum \bar{a}_j \bar{F}_j(DP)
\end{align*}
\]
Amplitude Analyses: Parametrization

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\]

\[
\bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP)
\]

Isobar amplitudes:
Weak phases information

Shapes of intermediate states over DP

Dalitz Plot
Isobar Model
Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

\[ A(DP) = \sum a_j F_j(DP) \]
\[ \overline{A}(DP) = \sum \overline{a}_j \overline{F}_j(DP) \]

\[ F_j^L(DP) = R_j(m) \times X_L(|\vec{p}^*| r) \times X_L(|\vec{q}| r) \times T_j(L, \vec{p}, \vec{q}) \]

- Line-shape
- Kinematic part

Shapes of intermediate states over DP

Dalitz Plot
Isobar Model

Relativistic Breit-Wigner: \( K^*(892)\pi \)
Flatté: \( f_0(980)K \)
Gounaris-Sakurai: \( \rho(770)K \)
S-wave \( K\pi: \) LASS
Non-resonant: Different parameterizations
Other contributions:

For \( B \rightarrow K\pi\pi \)
Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

\[
\begin{align*}
A(DP) &= \sum a_j F_j(DP) \\
\bar{A}(DP) &= \sum \bar{a}_j \bar{F}_j(DP)
\end{align*}
\]

**Time-dependent DP PDF** \(||q/p|| = 1\)

\[
f(\Delta t, DP, q_{\text{tag}}) \propto (|A|^2 + |\bar{A}|^2) \frac{e^{-|\Delta t|/\tau}}{4\tau} \\
\left(1 + q_{\text{tag}} \frac{2\text{Im}[\overline{(q/p)AA^*}]}{|A|^2 + |\bar{A}|^2} \sin(\Delta m_d \Delta t) - q_{\text{tag}} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos(\Delta m_d \Delta t)\right)
\]

- **Mixing and decay CPV**
- **Direct CPV**

Only different from zero for final states accessible to both \(B^0\) and \(\bar{B}^0\)

(e.g. \(B^0 \rightarrow K^0_s \pi^+\pi^-\))
Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

\[
\begin{align*}
A(DP) &= \sum a_j F_j(DP) \\
\overline{A}(DP) &= \sum \overline{a}_j F_j(DP)
\end{align*}
\]

**Dalitz Plot**

**Isobar Model**

Time-dependent DP PDF \(|q/p| = 1\)

\[
f(\Delta t, DP, q_{tag}) \propto \left( |A|^2 + |\overline{A}|^2 \right) \frac{e^{-|\Delta t|/\tau}}{4\tau} \left( 1 + q_{tag} \frac{2 \text{Im} [(q/p) \overline{A} A^*]}{|A|^2 + |\overline{A}|^2} \right) \sin(\Delta m_d \Delta t) - q_{tag} \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} \cos(\Delta m_d \Delta t)
\]

Sensitivity to phase difference between amplitudes in the same DP plane (B or \(\overline{B}\))

- **Mixing and decay CPV**
- **Direct CPV**

Sensitivity to phase differences between \(a_j\) and \(\overline{a}_j\) amplitudes

Includes \(q/p\) mixing phase
Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot

\[ A(DP) = \sum a_j F_j(DP) \]

\[ \bar{A}(DP) = \sum \bar{a}_j F_j(DP) \]

Isobar Model

**Time-dependent DP PDF**  \(|q/p| = 1\)

\[
f(\Delta t, DP, q_{tag}) \propto \left( |A|^2 + |\bar{A}|^2 \right) \frac{e^{-|\Delta t|/\tau}}{4\tau} \\
\left( 1 + q_{tag} \frac{2\text{Im}[(q/p)\bar{A}A^*]}{|A|^2 + |\bar{A}|^2} \sin(\Delta m_d \Delta t) - q_{tag} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos(\Delta m_d \Delta t) \right)
\]

Complex amplitudes \( a_j \) and \( \bar{a}_j \) determine DP interference pattern. Modules and phases can be directly fitted on data.

Direct CPV

mixing and decay CPV

Alejandro Pérez Pérez, New Physics at Belle II, Feb. 25th 2015
Amplitude Analyses: What can be measured?

- Any function of the isobar parameters which does not depend on conventions is a physical observable

**Examples**

- Direct CP-asymmetries:
  \[ A_{CP}^j = \frac{|\bar{a}_j|^2 - |a_j|^2}{|\bar{a}_j|^2 + |a_j|^2} \]

- Branching Fractions:
  \[ B_j \propto \int \int \left( |a_j|^2 + |\bar{a}_j|^2 \right) F_j(DP) dDP \]

- Phase differences in the same B or \( \bar{B} \) DP:
  \[ \varphi_{ij} = \arg \left( \frac{a_i}{a_j} \right) \quad \bar{\varphi}_{ij} = \arg \left( \frac{\bar{a}_i}{\bar{a}_j} \right) \]

- Phase differences between B and \( \bar{B} \) DP:
  \[ \Delta \varphi_j = \arg \left( \frac{\bar{a}_j}{a_j} \right) \]

- All amplitude analyses should provide the complete set of isobar parameters together with the full statistical and systematic covariance matrices
- This allows to properly use all the available experimental information and to correctly interpret the results
Amplitude Analyses: the signal model

- Isobar model needs predefined list of components with their lineshapes: **signal model**
- No straightforward way of determining the signal model from theory
- **The signal model is mainly determined from data**
  - Use previous experimental results to come out with a smart guess of this predefined list
    ⇒ **Raw Signal Model (RSM)**
  - Use the data to test for additional contributions which could eventually be added to RSM
    ⇒ **building of “Nominal Signal Model”**
  - Minor contributions treated as systematics ⇒ **Model uncertainties**
  - Additional model errors: uncertainties on line-shapes (e.g. non-resonant and $K\pi$ S-wave)
- **SU(3) prediction: same components should contribute to SU(3) related final states**
  - Final states with high efficiency and low background can be used to build the signal model
  - This model can then be used coherently among SU(3) related final states
  - This implies correlations of the model uncertainties of the SU(3) related final states which need to be evaluated ⇒ **currently it is assumed no correlation**

- **We strongly recommend to analyst of all $B\to hhh$ ($h = \pi, K$) modes to work in coordination, ideally the same set of conventions should be used by all experiments**
Phenomenological Framework
Due to CKM unitarity the hadronic amplitudes receive contributions of different topologies. In the above convention they are referred by the main contributions

- $T^+$ and $P^+$: colour allowed three and penguin
- $N^{0+}$: annihilation contributions
- $T^{00}$: colour suppressed tree
- $P_{EW}^-$ and $P_{EW}^C$: colour allowed and colour suppressed electroweak penguins
\[ A(B^0 \rightarrow K^{*+}\pi^-) = V^*_{us} V_{ub} T^{+} + V^*_{ts} V_{tb} P^{+} \]

\[ A(B^+ \rightarrow K^{*0+}) = V^*_{us} V_{ub} N^{0+} + V^*_{ts} V_{tb} (-P^{+}+P_{EW}^C) \]

\[ \sqrt{2} A(B^+ \rightarrow K^{*+}\pi^0) = V^*_{us} V_{ub} (T^{+}+T^{00}_C - N^{0+}) + V^*_{ts} V_{tb} (P^{+}-P_{EW}^C + P_{EW}) \]

\[ \sqrt{2} A(B^0 \rightarrow K^{*0}\pi^0) = V^*_{us} V_{ub} T^{00}_C + V^*_{ts} V_{tb} (-P^{+}+P_{EW}) \]

Neglecting \( P_{EW} \), the amplitude combinations:

\[ 3A_{3/2} = A(B^0 \rightarrow K^{*+}\pi^-) + \sqrt{2} A(B^0 \rightarrow K^{*0}\pi^0) = V^*_{us} V_{ub} (T^{+}+T^{00}) \]

\[ \overline{3A}_{3/2} = \overline{A}(B^0 \rightarrow K^{*+}\pi^+) + \sqrt{2} \overline{A}(B^0 \rightarrow K^{*0}\pi^0) = V^*_{us} V_{ub} (T^{+}+T^{00}) \]

which gives:

\[ R'_{3/2} = (3A_{3/2})/(\overline{3A}_{3/2}) = e^{-2i\gamma} \]

The actually physical observable is

(invariant under phase redefinitions)

\[ R_{3/2} = (q/p)(3A_{3/2})/(\overline{3A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha} \]

**CPS PRD74:051301**

**GPSZ PRD75:014002**

Alejandro Pérez Pérez, New Physics at Belle II, Feb. 25th 2015
B→K*π System: unknowns and observables count

\[
A(B^0 \to K^{*+}\pi^-) = V_{us}^* V_{ub} T^{+-} + V_{ts}^* V_{tb} P^{+-}
\]

\[
A(B^+ \to K^{*0}\pi^+) = V_{us}^* N^{0+} + V_{ts}^* (-P^{+-} + P_{EW}^C)
\]

\[
\sqrt{2} A(B^+ \to K^{*+}\pi^0) = V_{us}^* (T^{+-} + T^{00} C - N^{0+}) + V_{ts}^* (P^{+-} - P_{EW}^C + P_{EW})
\]

\[
\sqrt{2} A(B^0 \to K^{*0}\pi^0) = V_{us}^* T^{00} C + V_{ts}^* (-P^{+-} + P_{EW})
\]

11 QCD and 2 CKM = 13 unknowns

Event if N(unknowns) = N(obs), reparametrization invariance prevents the simultaneous extraction of all CKM and hadronic parameters without additional information

PRD71:094008 (2005)

Observables:
- 4 BFs and 4 A_{CP} from DP and Q2B analyses.
- 5 phase differences:
  - \( \Delta \phi = \text{arg}((q/p)\overline{A}(B^0 \to K^{*+}\pi^-)A^*(B^0 \to K^{*+}\pi^-)) : B^0 \to K^0_s \pi^+\pi^- \)
  - \( \phi = \text{arg}(A(B^0 \to K^{*0}\pi^0)A^*(B^0 \to K^{*+}\pi^-)) \) and \( \bar{\phi} = \text{arg}(\overline{A}(B^0 \to K^{*0}\pi^0)\overline{A}^*(B^0 \to K^{*+}\pi^-)) \) from \( B^0 \to K^+ \pi^-\pi^0 \)
  - \( \phi = \text{arg}(A(B^+ \to K^{*0}\pi^+)A^*(B^+ \to K^{*+}\pi^+)) \) and \( \bar{\phi} = \text{arg}(\overline{A}(B^+ \to K^{*0}\pi^-)\overline{A}^*(B^+ \to K^{*-}\pi^-)) \) from \( B^+ \to K^0 \pi^+\pi^0 \)

A total of 13 observables
# $B \to K^{*}\pi$ System: two strategies

Scenario 1: *set some constraints on hadronic parameters:*

- If $\text{Had} \to \text{Had} + \delta\text{Had}$ gives $\text{CKM} \to \text{CKM} + \delta\text{CKM}$
  
  *CPS/GPSZ method*

  **Goal:** test CPS/GPSZ method

- If $\text{Had} \to \text{Had} + \delta\text{Had}$ gives $\text{CKM} \to \text{CKM} + \Delta\text{CKM}$

**Scenario 2:** *CKM from external input (global fit) and fit hadronic parameters:*

- Uncontroversial: only assumes CKM unitarity

- **inputs:**
  - Fix CKM parameters from global fit
  - $B \to K\pi\pi$ experimental measurements

- **output:**
  - Prediction of unavailable observables
  - Exploration of hadronic amplitudes $\Rightarrow$ test of QCD predictions
**B → K^*\pi System: CPS/GPSZ theoretical prediction**

GPS/CPSZ: relation between the $P_{EW}$ and $T_{3/2} = T^{+} + T^{00}_{C}$

- $B \to \pi\pi$: $P_{EW} = RT_{3/2}$, $R = 1.35\%$ and real. (SU(2) and Wilson coeff. $|c_{8,9}|$ small). P and T CKM of same order $\rightarrow P_{EW}$ negligible

- $B \to K\pi$: $P_{EW} = RT_{3/2}$ (same as $\pi\pi$ and SU(3)) P amplified CKM wrt. T ($|V_{ts}^{\prime} V^{*}_{tb} / V_{us}^{\prime} V^{*}_{ub}| \sim 55$) $\rightarrow P_{EW}$ non-negligible

- $B \to K^{*}\pi$: $P_{EW} = R_{eff}T_{3/2}$
  - $R_{eff} = R(1-r_{VP})/(1+r_{VP})$
  - $r_{VP}$ complex $\rightarrow$ vector-pseudoscalar phase space
  - GPSZ estimation $|r_{VP}| < 5\%$

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CPS PRD74:051301
GPSZ PRD75:014002
B\rightarrow K^*\pi\text{ System: proposed parametrization of observables}

\begin{align*}
B^0\rightarrow & K^0_s\pi^+\pi^- \\
& B(B^0\rightarrow K^{*+}\pi^-) \\
& A_{CP}(B^0\rightarrow K^{*+}\pi^-) \\
& \Delta\phi(B^0\rightarrow K^{*+}\pi^-) \\
& \text{Re}(A(K^*-\pi^+)/A(K^{*+}\pi^-)) \\
& \text{Im}(A(K^*-\pi^+)/A(K^{*+}\pi^-)) \\
& B(B^0\rightarrow K^{*+}\pi^-) \\
B^0\rightarrow & K^+\pi^-\pi^0 \\
& B(B^0\rightarrow K^{*+}\pi^-) \\
& A_{CP}(B^0\rightarrow K^{*+}\pi^-) \\
& B(B^0\rightarrow K^{*0}\pi^0) \\
& A_{CP}(B^0\rightarrow K^{*0}\pi^0) \\
& \phi(K^{*0}\pi^0/K^{*+}\pi^-) \\
& \phi(K^{*0}\pi^0/K^{*+}\pi^-) \\
& |A(K^*-\pi^+)/A(K^{*+}\pi^-)| \\
& B(B^0\rightarrow K^{*+}\pi^-) \\
& \text{Re}(A(K^*-\pi^+)/A(K^{*+}\pi^-)) \\
& \text{Im}(A(K^*-\pi^+)/A(K^{*+}\pi^-)) \\
& B(B^0\rightarrow K^{*0}\pi^0) \\
B^+\rightarrow & K^+\pi^-\pi^+ \\
& B(B^+\rightarrow K^{*0}\pi^+) \\
& A_{CP}(B^+\rightarrow K^{*0}\pi^+) \\
& B(B^+\rightarrow K^{*+}\pi^0) \\
& A_{CP}(B^+\rightarrow K^{*+}\pi^0) \\
& \phi(K^{*+}\pi^0/K^{*0}\pi^+) \\
& \phi(K^{*+}\pi^0/K^{*0}\pi^-) \\
& |A(K^*\pi^-)/A(K^{*0}\pi^-)| \\
& B(B^+\rightarrow K^{*0}\pi^+) \\
& \text{Re}(A(K^*\pi^-)/A(K^{*0}\pi^-)) \\
& \text{Im}(A(K^*\pi^-)/A(K^{*0}\pi^-)) \\
& B(B^+\rightarrow K^{*+}\pi^0) \\
B^+\rightarrow & K^0_s\pi^+\pi^0 \\
& B(B^+\rightarrow K^{*0}\pi^+) \\
& A_{CP}(B^+\rightarrow K^{*0}\pi^+) \\
& B(B^+\rightarrow K^{*+}\pi^0) \\
& A_{CP}(B^+\rightarrow K^{*+}\pi^0) \\
& \phi(K^{*0}\pi^0/K^{*+}\pi^-) \\
& \phi(K^{*0}\pi^0/K^{*+}\pi^-) \\
& |A(K^*\pi^-)/A(K^{*0}\pi^-)| \\
& B(B^+\rightarrow K^{*0}\pi^+) \\
& \text{Re}(A(K^*\pi^-)/A(K^{*0}\pi^-)) \\
& \text{Im}(A(K^*\pi^-)/A(K^{*0}\pi^-)) \\
& B(B^+\rightarrow K^{*+}\pi^0)
\end{align*}
Scenarios to constrain CKM
Scenarios to constrain CKM: the strategy

### Closure test
- Fix CKM parameters to current values
- Assign ad-hoc “true” values to Had. amplitudes
- Deduce corresponding values of physical observables
- Explore constraints on CKM parameters assuming very small uncertainties on observables
- Had. amplitudes constrained to follow naïve hierarchy pattern
  - $T^+ > T^{00} > N^{0+}$ and $P^+ > P_{EW} > P_{EW}^C$
- Furthermore, $P_{EW}$ constrained to match CPS/GPSZ assumption
  - $|P_{EW}/(T^+ + T^{00})| = 0.0135$ and $\arg(P_{EW}) = \arg(T^+ + T^{00})$
- This ad-hoc choice of “true” values roughly reproduces current BF and $A_{CP}$ (c.f. table)

<table>
<thead>
<tr>
<th>Hadron par.</th>
<th>magnitude</th>
<th>phase (deg)</th>
<th>Physical observable</th>
<th>Measurement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^+ -$</td>
<td>2.540</td>
<td>0.00</td>
<td>$B(B^0 \rightarrow K^{*+}\pi^-)$</td>
<td>8.2 ± 0.9</td>
<td>7.1</td>
</tr>
<tr>
<td>$T^{00}$</td>
<td>0.762</td>
<td>75.74</td>
<td>$B(B^0 \rightarrow K^{*0}\pi^0)$</td>
<td>3.3 ± 0.6</td>
<td>1.6</td>
</tr>
<tr>
<td>$N^{0+}$</td>
<td>0.143</td>
<td>108.37</td>
<td>$B(B^+ \rightarrow K^{*+}\pi^0)$</td>
<td>9.2 ± 1.5</td>
<td>8.5</td>
</tr>
<tr>
<td>$P^+$</td>
<td>0.091</td>
<td>-6.48</td>
<td>$B(B^+ \rightarrow K^{*0}\pi^+)$</td>
<td>11.6 ± 1.2</td>
<td>10.9</td>
</tr>
<tr>
<td>$P_{EW}$</td>
<td>0.038</td>
<td>15.15</td>
<td>$A_{CP}(B^0 \rightarrow K^{*+}\pi^-)$</td>
<td>-24.0 ± 7.0</td>
<td>-12.9</td>
</tr>
<tr>
<td>$P_{EW}^C$</td>
<td>0.029</td>
<td>101.90</td>
<td>$A_{CP}(B^0 \rightarrow K^{*0}\pi^0)$</td>
<td>-15.0 ± 13.0</td>
<td>-46.5</td>
</tr>
<tr>
<td>$V_{us}V_{ub}^*P^+$</td>
<td>1.801</td>
<td></td>
<td>$A_{CP}(B^+ \rightarrow K^{*+}\pi^-)$</td>
<td>-0.52 ± 15.0</td>
<td>-35.4</td>
</tr>
<tr>
<td>$V_{us}V_{ub}^*T^+$</td>
<td>0.300</td>
<td></td>
<td>$A_{CP}(B^+ \rightarrow K^{*0}\pi^-)$</td>
<td>+5.0 ± 5.0</td>
<td>+3.9</td>
</tr>
<tr>
<td>$N^{0+}/T^{00}$</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{EW}/P^+$</td>
<td>0.420</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{EW}/(T^+ + T^{00})$</td>
<td>1.00</td>
<td>/R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_{EW}/P_{EW}^C$</td>
<td>0.762</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Explored hypothesis**
- CPS/GPSZ-like assumption
- Hypothesis on the annihilation
Scenarios to constrain CKM: CPS/GPSZ-like (I)

- The CKM $\alpha$ is extracted from $B \to \pi\pi, \rho\pi$ and $\rho\rho$ isospin analysis by neglecting the $P_{EW}$ contributions to the decay amplitudes.
- A similar approach is tested here [CPS PRD74:051301, GPSZ PRD75:014002]

Only for $P_{EW} = 0$

- Yields constraint on $\rho - \eta$ following $\alpha$ contours.
- But fails (by large amounts!) to reproduce true $\alpha$.

If $P_{EW} \neq 0$ (fixed to its true value)

- Yields unbiased constraint.
- Which does not follow $\alpha$ contour.
Scenarios to constrain CKM: CPS/GPSZ-like (II)

- **P\_EW set to zero**: The excluded area with CL > 0.05 shows the toy scenario. The p-value is 0.0 in the generation value.

- **P\_EW in (0, 0.5xgen value)**: The excluded area with CL > 0.05 shows the toy scenario. The p-value is 0.0 in the generation value.

- **P\_EW in (0, 1.0xgen value)**: The excluded area with CL > 0.05 shows the toy scenario. The p-value is 0.0 in the generation value.

- **P\_EW in (0, 2.0xgen value)**: The excluded area with CL > 0.05 shows the test scenario. The p-value is 0.0 in the generation value.
Scenarios to constrain CKM: CPS/GPSZ-like (II)

The method is overly sensitive to the assumed $P_{EW}$ values

- A strong hypothesis on $P_{EW}$ provides a strong, but biased constraint
- Relaxing the hypothesis spoils the predictability of the method
  - Remember the “true” $P_{EW}$ is 1.35% smaller than the tree amplitudes!
  - But its impact is strongly enhanced by the CKM factors on penguin terms
Scenarios to constrain CKM: hypothesis on $N^{0+}$ (I)

- CKM enhancement does not affect tree terms
- Furthermore, the annihilation $N^{0+}$ is naïvely expected to be small
- May be constrained from theory and/or from annihilation-dominated modes

Hypotheses in the $|N^{0+}/T^+|$ provides a “β-like” constraint in $\rho-\eta$
Scenarios to constrain CKM: hypothesis on $N^{0+}$ (II)

$|N^{0+}/T^+| < 1.5 \times \text{(gen val)}$

$|N^{0+}/T^+| < 5 \times \text{(gen val)}$

$|N^{0+}/T^+| < 10 \times \text{(gen val)}$

$|N^{0+}/T^+| < 15 \times \text{(gen val)}$
Scenarios to constrain CKM: hypothesis on $N^{0+}$ (II)

Relaxing the hypothesis on $N^{0+}$ yields only a mild deterioration on the constraints.

$|N^{0+}/T^+| < 1.5 \times \text{(gen val)}$

$|N^{0+}/T^+| < 5 \times \text{(gen val)}$
Scenarios to constrain CKM: hypothesis on $N^{0+}$ (III)

$\beta$ coverage vs Upper bound on $|N^{0+}/T^{+}|$ (in units of the generation value)

Even assuming a 500% uncertainty on the $N^{0+}/T^{+}$ bound, the theoretical error is less than 9 degrees.
Scenarios to constrain CKM: Summary

- **CPS/GPSZ-like hypothesis:**
  - Conservative values on the uncertainty of the $P_{EW}$ prediction gives uncontrollable effects of the $\rho-\eta$ constraints
  - ⇒ *The method is dominated by the theoretical uncertainties*
  - This is expected due to the CKM enhancement ($|V_{ts} V^*_{tb} / V_{us} V^*_{ub}| \sim 55$) of “penguin” w.r.t “tree” terms

- **Hypothesis on the annihilation ($N^{0+}$)**
  - It is possible to set a constraint in $\rho-\eta$ by just setting a upper bound on the $|N^{0+}/T^+|$?
  - Constraint on CKM less sensitive to theoretical uncertainties as there is no CKM enhancement

  *Uncertainty of 500% on $|N^{0+}/T^+|$ gives a theory error of less than 9 degrees*

  - Possibility to get bounds on the annihilation from data by measuring the annihilation-dominated mode $B^+_s \rightarrow K^0 K^+$ which is U-spin related to $B^0 \rightarrow K^0 \pi^+$

  ⇒ Accessible to LHCb
Current constraints on Hadronic amplitudes
Experimental inputs: **BABAR** (I)

**BABAR** $B^0 \rightarrow K^+\pi^-\pi^0$ analysis: [PRD83:112010 (2011)]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>A(K^+\pi^+)/A(K^+\pi^-)</td>
</tr>
<tr>
<td>Re$(K^0\pi^0/K^+\pi^-)$</td>
<td>$0.80 \pm 0.20$</td>
</tr>
<tr>
<td>Im$(K^0\pi^0/K^+\pi^-)$</td>
<td>$-0.32 \pm 0.42$</td>
</tr>
<tr>
<td>Re$(\bar{K}^0\pi^0/K^+\pi^-)$</td>
<td>$1.00 \pm 0.15$</td>
</tr>
<tr>
<td>Im$(\bar{K}^0\pi^0/K^+\pi^-)$</td>
<td>$-0.07 \pm 0.53$</td>
</tr>
<tr>
<td>$B(K^0\pi^0)$</td>
<td>$(3.30 \pm 0.64) \times 10^{-6}$</td>
</tr>
</tbody>
</table>

**BABAR** $B^0 \rightarrow K^0_s\pi^+\pi^-$ analysis: [PRD80:112001 (2009)]

Two minima differing by 0.16 2NLL units

**Global minimum**

- Re$(K^-\pi^+/K^+\pi^-)$ = $0.43 \pm 0.41$;
- Im$(K^-\pi^+/K^+\pi^-)$ = $-0.69 \pm 0.26$;
- $B(K^+\pi^-)$ = $(8.3 \pm 1.2) \times 10^{-6}$;

**Full Correlation matrix**

$$
\begin{pmatrix}
1.0 & 0.93 & 0.02 \\
1.0 & -0.08 & \\
1.0 & & 
\end{pmatrix}
$$

**Local Minimum**

- Re$(K^-\pi^+/K^+\pi^-)$ = $-0.82 \pm 0.09$;
- Im$(K^-\pi^+/K^+\pi^-)$ = $-0.05 \pm 0.43$;
- $B(K^+\pi^-)$ = $(8.3 \pm 1.2) \times 10^{-6}$;

**Full Correlation matrix**

$$
\begin{pmatrix}
1.0 & -0.20 & 0.22 \\
1.0 & -0.01 & \\
1.0 & & 
\end{pmatrix}
$$
Experimental inputs: \textbf{BABAR (II)}

\textbf{BABAR} \( B^+ \to K^+ \pi^- \pi^+ \) analysis: \textcolor{blue}{PRD78:012004 (2008)}

\[
\frac{|A(K^*\pi^-)/A(K^*\pi^+)|}{1.033 \pm 0.047;}
\]
\[
B(K^*\pi^+) = (10.8 \pm 1.4) \times 10^{-6};
\]
\[
\begin{pmatrix}
1.0 & 0.02 \\
0.02 & 1.0
\end{pmatrix}
\]

\textbf{BABAR} \( B^+ \to K^0_s \pi^+ \pi^0 \) analysis: \textcolor{blue}{ArXiv : 1501.00705 [hep-ex] (2015)} \textcolor{red}{New Result!}

- Currently in communication with authors to get full set of observables and correlation matrices
- The results shown in next slides just use
  - \( B(K^{*+}\pi^0) = (9.2 \pm 1.5) \times 10^{-6}; \)
  - \( C(K^{*+}\pi^0) = -0.52 \pm 0.15; \Rightarrow \sim 3.5\sigma \) significance
**Experimental inputs: Belle**

**Belle** $B^0 \to K^0_S \pi^+ \pi^-$ analysis: PRD75:012006 (2007) and PRD79:072004 (2009)

two minima differing by 7.5 2NLL units

- **Global minimum**
  
  \[
  \text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.79 \pm 0.14;
  \]
  
  \[
  \text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = -0.21 \pm 0.40;
  \]
  
  \[
  B(K^{*+}\pi^-) = (8.4 \pm 1.5) \times 10^{-6};
  \]

  **Full Correlation matrix**

  \[
  \begin{pmatrix}
  1.0 & 0.62 & 0.0 \\
  0.62 & 1.0 & 0.0 \\
  0.0 & 0.0 & 1.0
  \end{pmatrix}
  \]

- **Local Minimum**
  
  \[
  \text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.81 \pm 0.11;
  \]
  
  \[
  \text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.01 \pm 0.44;
  \]
  
  \[
  B(K^{*+}\pi^-) = (8.4 \pm 1.5) \times 10^{-6};
  \]

  **Full Correlation matrix**

  \[
  \begin{pmatrix}
  1.0 & 0.01 & 0.0 \\
  0.01 & 1.0 & 0.0 \\
  0.0 & 0.0 & 1.0
  \end{pmatrix}
  \]

**Belle** $B^+ \to K^+ \pi^- \pi^+$ analysis: PRL96:251803 (2006)

- $|A(K^0\pi^-)/A(K^0\pi^+)| = 0.86 \pm 0.09$;

- $B(K^0\pi^+) = (9.7 \pm 1.1) \times 10^{-6}$;

  **Full Correlation matrix**

  \[
  \begin{pmatrix}
  1.0 & 0.0 \\
  0.0 & 1.0
  \end{pmatrix}
  \]

**No Belle results on:**

- $B^0 \to K^+ \pi^+ \pi^0$ and $B^+ \to K^0_S \pi^+ \pi^0$
Combining *BABAR* + *Belle*: $B^0 \rightarrow K^0_s \pi^+ \pi^-$

- Two solutions for both *BABAR* and *Belle* analyses
  - Combine all possible combinations of *BABAR* and *Belle* solutions taking into account the difference in 2NLL
  - Results: 4 solutions differing in $\chi^2$: 0, 7.7, 8.4 and 97.2. Consider only the global minimum
Combining $BABAR \ + Belle: B^+ \rightarrow K^+\pi^−\pi^+$  

- Single solution for both $BABAR$ and Belle
Decay amplitudes ($\delta_i$ and $\phi_i$ are weak/strong phases)

\[ A = M_1 \exp(i\delta_1)\exp(i\phi_1) + M_2 \exp(i\delta_2)\exp(i\phi_2) \]
\[ A = M_1 \exp(i\delta_1)\exp(-i\phi_1) + M_2 \exp(i\delta_2)\exp(-i\phi_2) \]

\[ A_{CP} = 2\frac{\sin(\Delta \delta)\sin(\Delta \phi)}{(M_1/M_2) + (M_2/M_1) + 2\cos(\Delta \delta)\cos(\Delta \phi)} \]

In our case $\Delta \phi = \arg(V_{ts}V^*_{tb}/V_{us}V^*_{ub}) = 2\gamma \neq 0$

If $A_{CP}$ is significantly different from zero then

- $|CKM^*(P/T)| \sim 1$
- $\arg(P/T) \neq 0$

3\sigma significance for $C(B^0 \rightarrow K^{*+}\pi^-)$

\[ A(B^0 \rightarrow K^{*+}\pi^-) = V_{us}V^*_{ub}T^+ + V_{ts}V^*_{tb}P^+ \]

- Two solutions with same $\chi^2$ (Sol A and B)
- Both inconsistent with $\arg(P/T) = 0/\pi$
- Only solution A has $|CKM^*(P/T)| \sim 1$
Results on Had. Amplitudes: CP violation (II)

- Decay amplitudes ($\delta_i$ and $\phi_i$ are weak/strong phases)
  \[ A = M_1 \exp(i\delta_1)\exp(i\phi_1) + M_2 \exp(i\delta_2)\exp(i\phi_2) \]
  \[ A = M_1 \exp(i\delta_1)\exp(-i\phi_1) + M_2 \exp(i\delta_2)\exp(-i\phi_2) \]
  \[ A_{CP} = \frac{\sin(\Delta \delta)\sin(\Delta \phi)}{(M_1/M_2) + (M_2/M_1) + 2\cos(\Delta \delta)\cos(\Delta \phi)} \]

- In our case $\Delta \phi = \arg(V_{ts}V^{*}_{tb}/V_{us}V^{*}_{ub}) = 2\gamma \neq 0$

- If $A_{CP}$ is significantly different from zero then
  - $|\text{CKM}^*(P/T)| \sim 1$
  - $\arg(P/T) \neq 0$

- 3.4$\sigma$ significance for $C(B^+\rightarrow K^{*+}\pi^0)$
  \[ \sqrt{2}A(B^+\rightarrow K^{*+}\pi^0) = V_{us}V^{*}_{ub}(P^{+}-P_{\text{EW}}^{C}+P_{\text{EW}})+V_{ts}V^{*}_{tb}(T^{+}+T_{00}^{0}-N_{0}^{0}) \]
  - Both solutions inconsistent with $\arg(P/T) = 0/\pi$ and with $|\text{CKM}^*(P/T)| \sim 1$
  - Appearance of other local minima
Results on Had. Amplitudes: CP violation (III)

\[ A(B^+ \rightarrow K^0\pi^+) = V^{us}\bar{V}^{ub}N^{0+} + V^{ts}\bar{V}^{tb}(-P^+ + P^C_{EW}) \]

\[ A(B^0 \rightarrow K^0\pi^0) = V^{us}\bar{V}^{ub}T^{00} + V^{ts}\bar{V}^{tb}(-P^+ + P^C_{EW}) \]

\[ A_{CP}(K^0\pi^0) \text{ and } A_{CP}(K^0\pi^+) \]
consistent with zero @ 1\(\sigma\)

P/T constraints are consistent either with
- \(|\text{CKM}*(P/T)| \gg 1 \text{ or } \ll 1\)
- \(\text{arg}(P/T) = 0 \text{ or } \pm\pi\)

\[ \log_{10}((-P^+ + P^C_{EW})/N^{0+}) \]

\[ \log_{10}((-P^+ + P^C_{EW})/N^{0+}) \]
Two solutions for the $P^{+}/T^{+}$ with same $\chi^2$ (sol: A and B)

- Those generate multiple solution on the other had. parameters

Essentially no constrain on $N^{0+}$ and $T^{00}$

Marginal agreement with CPS/GPSZ prediction
CPS/GPSZ prediction

\[ P_{EW}/(T^+ + T^{00}) = R(1-r_{VP})/(1+r_{VP}) \]
with \( R = 1.35\% \) and \( |r_{VP}| < 5\% \)

The current experimental constraints in poor agreement with the CPS/GPSZ prediction

Marginal agreement only reached by inflating the uncertainty on \( |r_{VP}| \) up to 30\%
Results on Had. Amplitudes: Hierarchies (I)

- Current data favours a relatively high $P_{EW}$
- This result is mainly driven by the $K^{*+}\pi^-/K^{*0}\pi^0$ phase differences measured in $B^0\to K^+\pi^-\pi^0$
- Without these phases there is good agreement among the experimental observables ($\chi^2 = 1.29$, p-Value $\sim 1.1\sigma$)
- Adding the phases brings slight tension ($\Delta\chi^2 = 7.7$, $2.6\sigma$)
- Only one experiment has performed the $B^0\to K^+\pi^-\pi^0$ analysis
- An independent confirmation is needed to claim non-zero (and large!) value of $P_{EW}$

Constraints on $|P_{EW}/P^{+*}|$

Excluding $K^{*+}\pi^-/K^{*0}\pi^0$ phases

- $\chi^2 = 1.29$
- pValue $\sim 26\%$ ($1.1\sigma$)
- Upper bound

Including $K^{*+}\pi^-/K^{*0}\pi^0$ phases

- $\chi^2 = 9.10$
- PValue $\sim 1.1\%$ ($2.6\sigma$)
- Precise measurement
Results on Had. Amplitudes: Hierarchies (II)

- Essentially no constraint is possible on $N^{0+}/T^{00}$ with current data
- Strong constrain on $P_{EW}^C/P_{EW}^E$
  - 2 solutions at ~0.8 and ~1.0
  - Result on $P_{EW}^C/P_{EW}^E$ is also consequence of the large $P_{EW}^E$
  - Needs also confirmation for the $B^0 \rightarrow K^+\pi^-\pi^0$ analysis

Constraint on $N^{0+}/T^{00}$

Constraint on $|P_{EW}^C/P_{EW}^E|$

Constraint on $P_{EW}^C/P_{EW}$
Prospects for future LHCb and Belle-II data
Prospects for LHCb and Belle-II (I)

Assume future experiments will measure central values used in the closure test study

LHCb will have high statistic measurements in the fully charged modes:

\[ B^0 \rightarrow K^0_s(\rightarrow \pi^+\pi^-)\pi^+\pi^- \text{ and } B^+ \rightarrow K^+\pi^-\pi^+ \]

- Expect a significant improvement of signal/background ratio w.r.t BABAR/Belle
- Error on \( \Delta\phi(K^*-\pi+/K^*+\pi-) \) scale as \( 1/\sqrt{Q} \) (effective tagging efficiency)
  \( \Rightarrow \) degrade the error by a factor \( \sqrt{30.5/2.38} \sim 3.6 \)
- Resolution in Dalitz plot \( \Rightarrow \) negligible effect according to LHCb experts
- Scale the errors by the expected statistics
- LHCb will have signal for \( B^0 \rightarrow K^+\pi^-\pi^0/B^+ \rightarrow K^0_s\pi^+\pi^0 \), but difficult to anticipate performances due to \( \pi^0 \) reconstruction efficiency and resolution

Belle II will measure all modes: \( B^0 \rightarrow K^0_s\pi^+\pi^- \), \( B^0 \rightarrow K^+\pi^-\pi^0 \), \( B^+ \rightarrow K^+\pi^-\pi^+ \) and \( B^+ \rightarrow K^0_s\pi^+\pi^0 \)

- Experimental environment similar to BABAR/Belle. Will scale uncertainties by luminosity
  \( \Rightarrow \) errors should get reduced by a factor of \( \sqrt{(50\text{ab}^{-1}/1.0\text{ab}^{-1})} \sim 7 \)

Both LHCb and Belle II will be able to measure \( B^+ \rightarrow K^+\pi^-\pi^+ \) mode with high precision

- Will be able to well define the signal model and probe line-shapes of the main components
- Model systematics will be significantly reduced \( \Rightarrow \) assume negligible model uncertainty
### Prospects for LHCb and Belle-II (II)

#### Expected evolution of the uncertainties on the observables

<table>
<thead>
<tr>
<th>Observable</th>
<th>Analysis</th>
<th>Current</th>
<th>LHCb (run1+run2)</th>
<th>Belle-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re(A(K^<em>\pi^-)/A(K^</em>\pi^-))</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.11</td>
<td>0.04</td>
<td>0.014</td>
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<tr>
<td>Im(A(K^<em>\pi^-)/A(K^</em>\pi^-))</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.16</td>
<td>0.11</td>
<td>0.023</td>
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<tr>
<td>B(K^*\pi^+)x10^{-6}</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.69</td>
<td>0.32</td>
<td>0.094</td>
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<tr>
<td>B(K^*\pi^0)\times10^{-6}</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.06</td>
<td>0.06</td>
<td>0.008</td>
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<tr>
<td>Re(A(K^0\pi^+)/A(K^*\pi^-))</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.11</td>
<td>0.11</td>
<td>0.016</td>
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<tr>
<td>Im(A(K^0\pi^+)/A(K^*\pi^-))</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.23</td>
<td>0.23</td>
<td>0.033</td>
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<tr>
<td>Re(A(K^0\pi^+)/A(K^*\pi^-))</td>
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<td>0.10</td>
<td>0.10</td>
<td>0.014</td>
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<tr>
<td>Im(A(K^0\pi^+)/A(K^*\pi^-))</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.30</td>
<td>0.30</td>
<td>0.042</td>
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<tr>
<td>B(K^0\pi^+)\times10^{-6}</td>
<td>B^0\rightarrow K^0\pi^+\pi^-</td>
<td>0.35</td>
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<tr>
<td>B(K^0\pi^+)</td>
<td>A(K^0\pi^+)</td>
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<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.04</td>
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<tr>
<td>B(K^0\pi^+)\times10^{-6}</td>
<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.81</td>
<td>0.50</td>
<td>0.113</td>
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<tr>
<td>B(K^0\pi^+)</td>
<td>A(K^0\pi^+)</td>
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<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.15</td>
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<td>0.16</td>
<td>0.16</td>
<td>0.023</td>
</tr>
<tr>
<td>Im(A(K^*\pi^+)/A(K^0\pi^+))</td>
<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.30</td>
<td>0.30</td>
<td>0.042</td>
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<tr>
<td>Re(A(K^*\pi^+)/A(K^0\pi^+))</td>
<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.21</td>
<td>0.21</td>
<td>0.030</td>
</tr>
<tr>
<td>Im(A(K^*\pi^+)/A(K^0\pi^+))</td>
<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.13</td>
<td>0.13</td>
<td>0.018</td>
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<tr>
<td>B(K^0\pi^+)\times10^{-6}</td>
<td>B^+\rightarrow K^0\pi^+\pi^+</td>
<td>0.92</td>
<td>0.92</td>
<td>0.130</td>
</tr>
</tbody>
</table>

- LHCb cannot perform B-counting like in B-factories
- BF are normalized w.r.t modes measured somewhere else (mainly @ B-factories)
- Error contribution from norm. modes not scaling with stat.
- B(B^0\rightarrow K^*\pi^-) norm. mode: B(B^0\rightarrow K^0\pi^+\pi^-) (σ_{rel} ~4%) 
- B(B^+\rightarrow K^0\pi^-) norm. mode: B(B^+\rightarrow K^*\pi^-) (σ_{rel} ~5%)
Main impact of LHCb data on $N^{0+}$, $P^{+-}$ and $P^\text{EW}$

Only upper limits on $T^{00}$ and $P^\text{EW}$
With LHCb + Belle-II data will be able to make precision measurements on the hadronic parameters

Precision test of QCD predictions
Summary and Outlook
**Summary and Outlook (I)**

- **B → K*π system has a large amount of physical observables among charmless decays**
  - Charmless B decay system with as many observables as unknowns
  - Large potential for phenomenology of charmless B decays
    - Model-independent extraction of hadronic parameters (assuming CKM and SU(2) as only hypotheses)
    - Extraction of CKM parameters limited by hadronic uncertainties

- **Extraction of CKM parameters**
  - α-like constraints spoiled by sensitivity to electroweak penguins
  - β-like constraints in the vanishing annihilation approximation
    - Future constraints from annihilation-dominated B → PV modes could be used
    - LHCb measurement of B_{(s)} → K*K will play an important contribution to this program
Study of hadronic amplitudes with available experimental data
- For the first time, at least one complete amplitude analysis of each $B \to K\pi\pi$ mode available
- Evidence of CP-violation provides strong constraints on the relevant tree-to-penguin ratios
- Loose bounds on colour-suppressed tree and annihilation amplitudes
- Current data favours relatively large EWPs
  - Mainly driven by $BABAR \ B^0 \to K^+\pi^−\pi^0$ analysis
  - If confirmed, would set evidence for EWPs in charmless $B$ decays
  - Until now, EWPs only established in $\epsilon'/\epsilon \neq 0$ (radiative $B$ decays are different operators)

Expect significant improvements with LHCb and Belle II data
- Model-independent measurement of all hadronic parameters
  - Both amplitudes and phases can be measured with outstanding accuracy
- Results on hadronic $B \to K^*\pi$ parameters can be used as “standard candles” to study other $B_{(s)} \to PV$ modes
  - $B_s \to K^*\pi$, $B_{(s)} \to K^*K$, $B_{(s)} \to \rho K$
Back up Slides
Parameterization

Parameterizing Decay amplitude using Isobar Model:

\[ A(DP) = \sum a_j F_j(DP) \]
\[ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \]

Shapes of intermediate states over DP

\[ F^L_j(DP) = R_j(m) \times X_L(|\vec{p}^*| r) \times X_L(|\vec{q}| r) \times T_j(L, \vec{p}, \vec{q}) \]

Effective Range Term

\[
R_j(m_{K\pi}) = \frac{m_{K\pi}}{q \cot \delta_B - iq} + e^{2i\delta_B} \frac{m_0 \Gamma_0 \frac{m_0}{q_0}}{\left( m_0^2 - m_{K\pi}^2 \right) - im_0 \Gamma_0 \frac{q}{m_{K\pi}} \frac{m_0}{q_0}}
\]

LASS lineshape.

Gounaris-Sacurai:

S-wave \(K\pi\): Nucl. Phys., B296:493, 1988

Relativistic Breit-Wigner:

Dalitz Plot

Isobar Model
## B → ρK System: Physical Observables

### Observables:

- 4 BFs and 4 $A_{CP}$ from DP and Q2B analyses.
- 1 phase differences:

\[
2\beta_{\text{eff}} = \arg((q/p)\overline{A}(B^0 \rightarrow \rho^0 K^0)A^*(B^0 \rightarrow \rho^0 K^0)) \text{ from } B^0 \rightarrow K^0_s \pi^+\pi^-
\]

### Equations:

\[
A(B^0 \rightarrow \rho^+ K^-) = V_{us} V^*_{ub} t^+ + V_{ts} V^*_{tb} p^+
\]

\[
A(B^+ \rightarrow \rho^0 K^+) = V_{us} V^*_{ub} n^{0+} + V_{ts} V^*_{tb} (-p^+ + p^C_{EW})
\]

\[
\sqrt{2}A(B^+ \rightarrow \rho^+ K^0) = V_{us} V^*_{ub} (t^+ + t^0 c - n^{0+}) + V_{ts} V^*_{tb} (p^+ - p^C_{EW} + p_{EW})
\]

\[
\sqrt{2}A(B^0 \rightarrow \rho^0 K^0) = V_{us} V^*_{ub} t^0 c + V_{ts} V^*_{tb} (-p^+ + p_{EW})
\]

11 QCD and 2 CKM = 13 unknowns

---

**Under constraint system. Still some constrains possible**

**A total of 9 observables**
Global phase between $K^*\pi$ and $\rho K$ now accessible:
- $K^*\pi$: 11 hadronic parameters (1 global phase fixed)
- $\rho K$: 12 parameters
- CKM: 2 parameter

A total of $= 25$ unknowns

Observables:
- $K^*\pi$ only: 13 observables
- $\rho K$ only: 9 observables
- 7 phase differences from: interference between $K^*\pi$ and $\rho K$ resonances contributing to the same DP
  - $\phi = \arg(A(B^0 \to \rho^0 K^0)A^*(B^0 \to K^*\pi^-))$ from $B^0 \to K^0_s \pi^+\pi^-$
  - $\phi = \arg(A(B^0 \to \rho^- K^+)A^*(B^0 \to K^*\pi^-))$ and CP conjugated from $B^0 \to K^+\pi^-\pi^0$
  - $\phi = \arg(A(B^0 \to \rho^0 K^+)A^*(B^0 \to K^0\pi^+))$ and CP conjugated from $B^+ \to K^+\pi^-\pi^+$
  - $\phi = \arg(A(B^0 \to \rho^+ K^0)A^*(B^0 \to K^0\pi^+))$ and CP conjugated from $B^+ \to K^0\pi^+\pi^0$

A total of 29 experimentally independent observables
Neglecting $P_{EW}$, the amplitude combinations:

\[ 3A_{3/2} = A(B^0 \rightarrow K^*\pi^-) + \sqrt{2}.A(B^0 \rightarrow K^0\pi^0) = V_{us} V^*_{ub} (T^+ + T^{00}) \]

\[ 3\bar{A}_{3/2} = \bar{A}(\bar{B}^0 \rightarrow K^*\pi^+) + \sqrt{2}.\bar{A}(\bar{B}^0 \rightarrow K^{0}\pi^0) = V^*_{us} V_{ub} (T^+ + T^{00}) \]

which gives: $R_{3/2} = (q/p)(3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$
Neglecting $P_{EW}$, the amplitude combinations:

$$3A_{3/2} = A(B^0 \rightarrow K^{*+}\pi^-) + \sqrt{2}.A(B^0 \rightarrow K^0\pi^0) = V_{us}V^*_{ub}(T^+ + T^{00})$$

$$3\Delta A_{3/2} = \Delta A(B^0 \rightarrow K^+\pi^-) + \sqrt{2}.\Delta A(B^0 \rightarrow K^0\pi^0) = V^*_{us}V_{ub}(T^+ + T^{00})$$

which gives: $R_{3/2} = (q/p)(3A_{3/2})/(3\Delta A_{3/2}) = e^{-2i\beta}e^{-2i\gamma} = e^{-2i\alpha}$

From experiment:

$$\phi = \arg(A(B^0 \rightarrow K^{*+}\pi^-)A^*(B^0 \rightarrow K^0\pi^0))$$

$$\bar{\phi} = \arg(\Delta A(B^0 \rightarrow K^+\pi^-)A^*(B^0 \rightarrow K^0\pi^0))$$

Measured from an amplitude analysis of $B^0 \rightarrow K\pi\pi^0$ decays
Neglecting $P_{EW}$, the amplitude combinations:

\[
3A_{3/2} = A(B^0 \to K^{*-} \pi^-) + \sqrt{2} A(B^0 \to K^* \pi^-) = V_{us} V^*_{ub} (T^+ + T^{00})
\]

\[
3\bar{A}_{3/2} = \bar{A}(\bar{B}^0 \to K^{*-} \pi^+) + \sqrt{2} \bar{A}(\bar{B}^0 \to K^* \pi^+) = V^*_{us} V_{ub} (T^+ + T^{00})
\]

which gives: $R_{3/2} = (q/p)(3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$

From experiment:

\[
\phi = \arg(A(B^0 \to K^{*-} \pi^-)A^*(B^0 \to K^* \pi^-))
\]

\[
\bar{\phi} = \arg(\bar{A}(\bar{B}^0 \to K^{*-} \pi^+)\bar{A}^*(\bar{B}^0 \to K^* \pi^+))
\]

Measured from an amplitude analysis of $B^0 \to K^+ \pi^- \pi^0$ decays

\[
\Delta\phi = \arg((q/p)\bar{A}(\bar{B}^0 \to K^{*-} \pi^+)\bar{A}^*(B^0 \to K^* \pi^-))
\]

Measured from a time-dependent amplitude analysis of $B^0 \to K^0 \pi^+ \pi^-$ decays
Scenarios to constrain CKM: hypothesis on $N^{0+}$ (III)

$|N^{0+}/T^+| < 1.5 \times \text{gen val}$

$|N^{0+}/T^+| < 5 \times \text{gen val}$

$|N^{0+}/T^+| < 10 \times \text{gen val}$

$|N^{0+}/T^+| < 15 \times \text{gen val}$
CPS/GPSZ theoretical prediction

Effective Hamiltonian of $B \rightarrow K^*\pi$

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1,2} c_i (\Omega_u Q_i^u + \Omega_c Q_i^c) - \Omega_t \sum_{i=3}^{10} c_i Q_i \right\} + \text{h.c.}, \text{ with } \Omega_q = V_{qs} V_{qb}$$

Hierarchy of Wilson coefficients for electro-weak operators $|c_{9,10}| \gg |c_{7,8}|$

$$\mathcal{H}_{EW}^{[\Delta I=1]} = \frac{3}{2} \frac{c_9 + c_{10}}{2} [Q_1^u + Q_2^u]_{\Delta I=1} + \frac{3}{2} \frac{c_9 - c_{10}}{2} [Q_1^u - Q_2^u]_{\Delta I=1}$$

Electro-weak Hamiltonian

$$\mathcal{H}_{CC}^{[\Delta I=1]} = \frac{c_1 + c_2}{2} [Q_1^u + Q_2^u]_{\Delta I=1} + \frac{c_1 - c_2}{2} [Q_1^u - Q_2^u]_{\Delta I=1}$$

Current-current Hamiltonian

Using

$$\left( \frac{c_9 + c_{10}}{c_1 + c_2} \approx -0.0084 \right) \simeq \left( \frac{c_9 - c_{10}}{c_1 - c_2} \approx +0.0084 \right)$$

$$\mathcal{H}_{EW}^{[\Delta I=1]} = R \frac{c_1 + c_2}{2} [Q_1^u + Q_2^u]_{\Delta I=1} - R \frac{c_1 - c_2}{2} [Q_1^u - Q_2^u]_{\Delta I=1}$$

$$R = (3/2) (c_9 + c_{10}) / (c_1 + c_2)$$

$$R = (1.35 \pm 0.12)\%$$

Obtain the relation $P_{EW}^{\Delta} = R_{\text{eff}} (T^+ + T^0)$, with $R_{\text{eff}} = R (1 + r_{VP}) / (1 - r_{VP})$.

$$r_{VP} = \frac{\langle K^*\pi (I = 3/2) | Q_- | B \rangle}{\langle K^*\pi (I = 3/2) | Q_+ | B \rangle}, \quad Q_\pm = (Q_1 \pm Q_2) / 2.$$ 

$$r_{VP} = \left| \frac{f_{K^*} F_0^{B \rightarrow \pi} - f_\pi A_0^{B \rightarrow K^*}}{f_{K^*} F_0^{B \rightarrow \pi} + f_\pi A_0^{B \rightarrow K^*}} \right| \lesssim 0.05$$

Feb. 25th 2015
Outlook

CKM constraints

- $B^0 \rightarrow K^{*0}\pi^+$ and $B^0_s \rightarrow K^{*0}K^+$ modes related by U-spin
  - $\Rightarrow$ expects the same annihilation amplitude ($N^{0+}$) up to U-spin breaking effects
  - $A(B^0_s \rightarrow K^{*0}K^+) = V_{ud}^{*} V_{ub}^{*} N^{0+} + V_{ts}^{*} V_{tb}^{*} P^{0+}$
- LHCb will measure this modes in the near future
- Can include this mode in our phenomenological framework to set a bound on $N^{0+}$ and be able to set constraints on CKM

Extending the $B \rightarrow K^{*}\pi$ system: include $B \rightarrow \rho K$ modes

- $B \rightarrow \rho K$ resonances also contribute to the $B \rightarrow K\pi\pi$ final states and have same isospin relations as $B \rightarrow K^{*}\pi \Rightarrow$ same number of hadronic parameters
- Smaller number of observables (9) than $B \rightarrow K^{*}\pi$ (13), but can measure interference phases (7) between $B \rightarrow K^{*}\pi$ and $B \rightarrow \rho K$ modes
- Combined system $B \rightarrow K^{*}\pi + B \rightarrow \rho K$
  - Unknowns: 11 + 12 hadronic from $B \rightarrow K^{*}\pi$ and $B \rightarrow \rho K + 2$ CKM = 25
  - Observables: 13 + 9 from $B \rightarrow K^{*}\pi$ and $B \rightarrow \rho K + 7$ phase differences = 28
  - Still need hypothesis on hadronic or CKM to raise reparametrization invariance