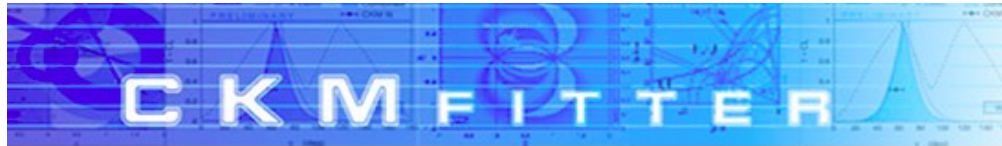


Phenomenology of $B \rightarrow K\pi\pi$ modes and Prospects with LHCb and Belle-II data

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IPHC – CNRS Strasbourg

On behalf of the CKMfitter Group



Outline

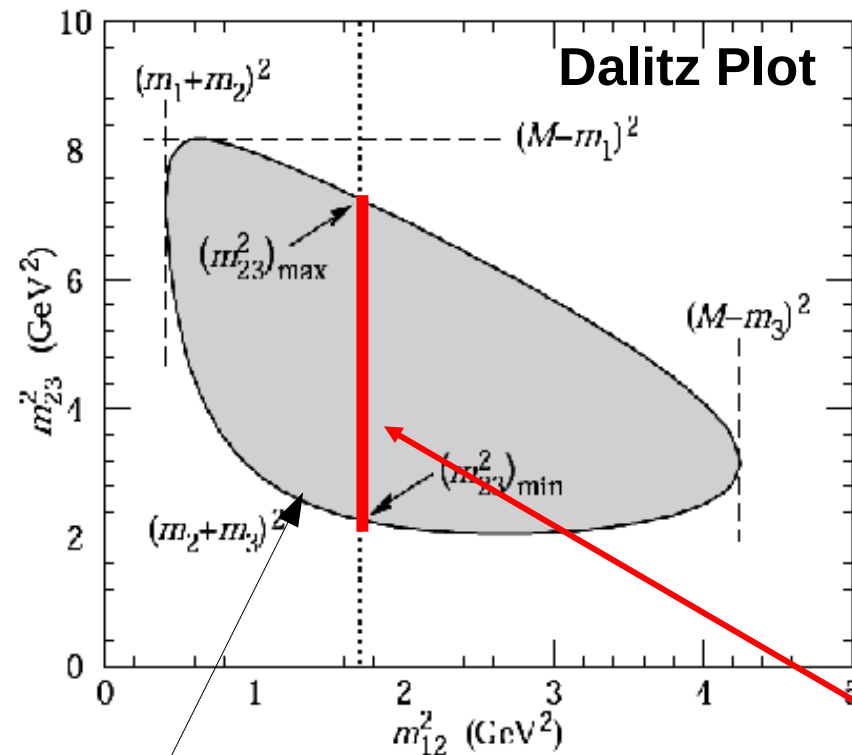
- **Amplitude analyses**
- **The phenomenological framework**
- **Some theoretical scenarios for constraining CKM**
- **Constraints on hadronic amplitudes using the latest $B \rightarrow K^* \pi$ measurements**
- **Prospects for future LHCb and Belle-II data**
- **Summary and outlook**

Amplitude Analyses

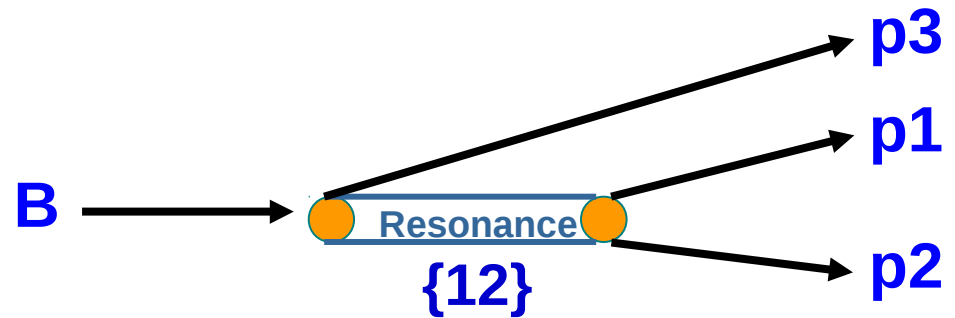
Dalitz Plot (DP)

Three body decays described by two parameters

Mandelstam variables $m_{ij}^2 = (p_i + p_j)^2$



Kinematic boundary



$$P \rightarrow P_{res} + p_3$$

$$P_{res} \rightarrow p_1 + p_2$$

(Distribution around resonance mass)

Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot
Isobar Model

$$\left\{ \begin{array}{l} A(DP) = \sum a_j F_j(DP) \\ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \end{array} \right.$$

Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot
Isobar Model

$$\left\{ \begin{array}{l} A(DP) = \sum_j a_j F_j(DP) \\ \bar{A}(DP) = \sum_j \bar{a}_j \bar{F}_j(DP) \end{array} \right.$$

Shapes of intermediate
states over DP

Isobar amplitudes:
Weak phases information

Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot
Isobar Model

$$\begin{cases} A(DP) = \sum a_j F_j(DP) \\ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \end{cases}$$

Shapes of intermediate
states over DP

$$F_j^L(DP) = R_j(m) \times X_L(|\vec{p}^*| r) \times X_L(|\vec{q}| r) \times T_j(L, \vec{p}, \vec{q})$$

Line-shape

Kinematic part

For $B \rightarrow K\pi\pi$

Relativistic Breit-Wigner: $K^*(892)\pi$

Flatté: $f_0(980)K$

Gounaris-Sakurai: $\rho(770)K$

S-wave $K\pi$: LASS

Non-resonant: Different parameterizations

Other contributions:

Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot

Isobar Model

$$\left\{ \begin{array}{l} A(DP) = \sum a_j F_j(DP) \\ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \end{array} \right.$$

Time-dependent DP PDF ($|q/p| = 1$)

$$f(\Delta t, DP, q_{\text{tag}}) \propto (|A|^2 + |\bar{A}|^2) \frac{e^{-|\Delta t|/\tau}}{4\tau} \left(1 + q_{\text{tag}} \frac{2\text{Im}[(q/p)\bar{A}A^*]}{|A|^2 + |\bar{A}|^2} \sin(\Delta m_d \Delta t) - q_{\text{tag}} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos(\Delta m_d \Delta t) \right)$$

mixing and decay CPV

Direct CPV



Only different from zero for final states
accessible to both B^0 and \bar{B}^0

(e.g. $B^0 \rightarrow K_s^0 \pi^+ \pi^-$)

Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot

Isobar Model

$$\left\{ \begin{array}{l} A(DP) = \sum a_j F_j(DP) \\ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \end{array} \right.$$

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mixing and decay CPV

Direct CPV

Sensitivity to phase difference between amplitudes in the same DP plane (B or \bar{B})

Sensitivity to phase differences between a_j and \bar{a}_j amplitudes
Includes q/p mixing phase

Amplitude Analyses: Parametrization

Parametrizing Decay amplitude using Isobar Model:

Dalitz Plot

Isobar Model

$$\left\{ \begin{array}{l} A(DP) = \sum a_j F_j(DP) \\ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \end{array} \right.$$

Time-dependent DP PDF ($|q/p| = 1$)

$$f(\Delta t, DP, q_{\text{tag}}) \propto (|A|^2 + |\bar{A}|^2) \frac{e^{-|\Delta t|/\tau}}{4\tau} \left(1 + q_{\text{tag}} \frac{2\text{Im}[(q/p)\bar{A}A^*]}{|A|^2 + |\bar{A}|^2} \sin(\Delta m_d \Delta t) - q_{\text{tag}} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \cos(\Delta m_d \Delta t) \right)$$

mixing and decay CPV

Direct CPV

Complex amplitudes a_j and \bar{a}_j determine DP interference pattern. Modules and phases can be directly fitted on data

Amplitude Analyses: What can be measured?

- Any function of the isobar parameters which does not depend on conventions is a physical observable

Examples

- Direct CP-asymmetries:

$$A_{CP}^j = \frac{|\bar{a}_j|^2 - |a_j|^2}{|\bar{a}_j|^2 + |a_j|^2}$$

- Branching Fractions:

$$B_j \propto \iint (|a_j|^2 + |\bar{a}_j|^2) F_j(DP) dDP$$

- Phase differences in the same B or \bar{B} DP: $\varphi_{ij} = \arg(a_i/a_j)$ $\bar{\varphi}_{ij} = \arg(\bar{a}_i/\bar{a}_j)$

- Phase differences between B and \bar{B} DP: $\Delta\varphi_j = \arg(\bar{a}_j/a_j)$

- All amplitude analyses should provide the complete set of isobar parameters together with the full statistical and systematic covariance matrices
- This allows to properly use all the available experimental information and to correctly interpret the results

Amplitude Analyses: the signal model

- Isobar model needs predefined list of components with their lineshapes: **signal model**
- No straightforward way of determining the signal model from theory
- The signal model is mainly determined from data
 - Use previous experimental results to come out with a smart guess of this predefined list
⇒ Raw Signal Model (RSM)
 - Use the data to test for additional contributions which could eventually be added to RSM
⇒ building of “Nominal Signal Model”
 - Minor contributions treated as systematics ⇒ **Model uncertainties**
 - Additional model errors: uncertainties on line-shapes (e.g. non-resonant and $K\pi$ S-wave)
- SU(3) prediction: same components should contribute to SU(3) related final states
 - Final states with high efficiency and low background can be used to build the signal model
 - This model can then be used coherently among SU(3) related final states
 - This implies correlations of the model uncertainties of the SU(3) related final states which need to be evaluated ⇒ **currently it is assumed no correlation**
- **We strongly recommend to analyst of all $B \rightarrow hhh$ ($h = \pi, K$) modes to work in coordination, ideally the same set of conventions should be used by all experiments**

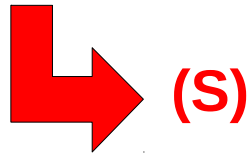
Phenomenological Framework

B → K* π System: Isospin relations

SU(2) Isospin relations:

$$A^{0+} + \sqrt{2}A^{+0} = \sqrt{2}A^{00} + A^{+-}$$

$$\overline{A}^{0+} + \sqrt{2}\overline{A}^{+0} = \sqrt{2}\overline{A}^{00} + \overline{A}^{+-}$$



(S)

$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-}$$

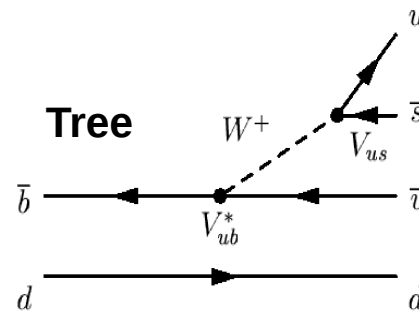
$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

$$\sqrt{2}A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T_C^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

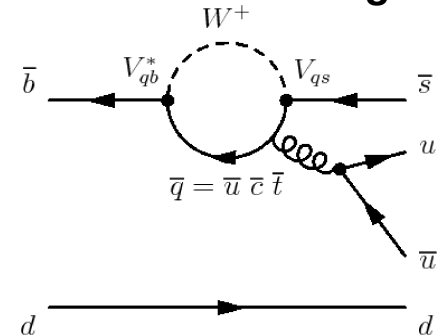
$$\sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T_C^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})$$

$B^0 \rightarrow K^{*+} \pi^-$

Tree



Penguin



- Due to CKM unitarity the hadronic amplitudes receive contributions of different topologies. In the above convention they are referred by the main contributions
 - T^{+-} and P^{+-} : colour allowed three and penguin
 - N^{0+} : annihilation contributions
 - T_C^{00} : colour suppressed tree
 - P_{EW} and P_{EW}^C : colour allowed and colour suppressed electroweak penguins

B→K*π System: extraction of α (CPS/GPSZ)

$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-}$$

$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

$$\sqrt{2}A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T_C^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

$$\sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T_C^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})$$

(S)

Neglecting P_{EW} , the amplitude combinations:

$$3A_{3/2} = A(B^0 \rightarrow K^{*+} \pi^-) + \sqrt{2}A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00})$$

$$3\bar{A}_{3/2} = \overline{A(B^0 \rightarrow K^{*-} \pi^+)} + \sqrt{2}\overline{A(B^0 \rightarrow \bar{K}^{*0} \pi^0)} = V_{us}^* V_{ub} (T^{+-} + T^{00})$$

which gives: $R'_{3/2} = (3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\gamma}$

CPS PRD74:051301
GPSZ PRD75:014002

The actually physical observable is
(invariant under phase redefinitions)

$$R_{3/2} = (q/p)(3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$$

B→K*π System: unknowns and observables count

$$\begin{aligned}
 A(B^0 \rightarrow K^{*+} \pi^-) &= V_{us} V_{ub}^* T^{+-} + V_{ts} V_{tb}^* P^{+-} \\
 A(B^+ \rightarrow K^{*0} \pi^+) &= V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C) \\
 \sqrt{2} A(B^+ \rightarrow K^{*+} \pi^0) &= V_{us} V_{ub}^* (T^{+-} + T_C^{00} - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW}) \\
 \sqrt{2} A(B^0 \rightarrow K^{*0} \pi^0) &= V_{us} V_{ub}^* T_C^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW})
 \end{aligned} \tag{S}$$

11 QCD and 2 CKM = 13 unknowns

Observables:

- 4 BFs and 4 A_{CP} from DP and Q2B analyses.
- 5 phase differences:
 - $\Delta\phi = \arg((q/p) \overline{A}(\overline{B^0} \rightarrow \overline{K^{*+}} \pi^+) A^*(B^0 \rightarrow K^{*+} \pi^-))$: $B^0 \rightarrow K_s^0 \pi^+ \pi^-$
 - $\phi = \arg(A(B^0 \rightarrow K^{*0} \pi^0) A^*(B^0 \rightarrow K^{*+} \pi^-))$ and $\overline{\phi} = \arg(\overline{A}(\overline{B^0} \rightarrow \overline{K^{*0}} \pi^0) \overline{A}^*(\overline{B^0} \rightarrow \overline{K^{*+}} \pi^+))$ from $B^0 \rightarrow K^+ \pi^- \pi^0$
 - $\phi = \arg(A(B^+ \rightarrow K^{*0} \pi^+) A^*(B^+ \rightarrow K^{*+} \pi^0))$ and $\overline{\phi} = \arg(\overline{A}(B^- \rightarrow \overline{K^{*0}} \pi^-) \overline{A}^*(B^- \rightarrow \overline{K^{*+}} \pi^0))$ from $B^+ \rightarrow K^0 \pi^+ \pi^0$

A total of 13 observables

Event if $N(\text{unknowns}) = N(\text{obs})$, reparametrization invariance prevents the simultaneous extraction of all CKM and hadronic parameters without additional information

PRD71:094008 (2005)

$B \rightarrow K^* \pi$ System: two strategies

Scenario 1: set some constraints on hadronic parameters:

- If $\text{Had} \rightarrow \text{Had} + \delta\text{Had}$ gives $\text{CKM} \rightarrow \text{CKM} + \delta\text{CKM}$ 😊

Ex.: α from $B \rightarrow \pi\pi$

- If $\text{Had} \rightarrow \text{Had} + \delta\text{Had}$ gives $\text{CKM} \rightarrow \text{CKM} + \Delta\text{CKM}$ ☹️

Goal: test CPS/GPSZ method

Scenario 2: CKM from external input (global fit) and fit hadronic parameters:

- **Uncontroversial: only assumes CKM unitarity**

- inputs:

- * Fix CKM parameters from global fit
- * $B \rightarrow K\pi\pi$ experimental measurements

- output:

- * **Prediction of unavailable observables**
- * **Exploration of hadronic amplitudes \Rightarrow test of QCD predictions**

$B \rightarrow K^* \pi$ System: CPS/GPSZ theoretical prediction

CPS PRD74:051301
GPSZ PRD75:014002

GPS/CPSZ: relation between the P_{EW} and $T_{3/2} = T^{+-} + T^{00}_C$

- $B \rightarrow \pi\pi$: $P_{EW} = R T_{3/2}$, $R = 1.35\%$ and real. (**SU(2) and Wilson coeff. $|c_{8,9}|$ small**).
P and T CKM of same order $\rightarrow P_{EW}$ negligible
- $B \rightarrow K\pi$: $P_{EW} = R T_{3/2}$ (**same as $\pi\pi$ and SU(3)**)
P amplified CKM wrt. T ($|V_{ts} V_{tb}^* / V_{us} V_{ub}^*| \sim 55$) $\rightarrow P_{EW}$ non-negligible
- $B \rightarrow K^* \pi$: $P_{EW} = R_{eff} T_{3/2}$
 - $R_{eff} = R(1 - r_{VP}) / (1 + r_{VP})$
 - r_{VP} **complex** \rightarrow **vector-pseudoscalar phase space**
 - **GPSZ estimation $|r_{VP}| < 5\%$**

$B \rightarrow K^* \pi$ System: proposed parametrization of observables

$$B^0 \rightarrow K_S^0 \pi^+ \pi^-$$

$$\begin{aligned} &B(B^0 \rightarrow K^{*+} \pi^-) \\ &A_{CP}(B^0 \rightarrow K^{*+} \pi^-) \\ &\Delta\phi(B^0 \rightarrow K^{*+} \pi^-) \end{aligned}$$



$$\begin{aligned} &\text{Re}(A(K^{*-} \pi^+)/A(K^{*+} \pi^-)) \\ &\text{Im}(A(K^{*-} \pi^+)/A(K^{*+} \pi^-)) \\ &B(B^0 \rightarrow K^{*+} \pi^-) \end{aligned}$$

$$B^0 \rightarrow K^+ \pi^- \pi^0$$

$$\begin{aligned} &B(B^0 \rightarrow K^{*+} \pi^-) \\ &A_{CP}(B^0 \rightarrow K^{*+} \pi^-) \\ &B(B^0 \rightarrow K^{*0} \pi^0) \\ &A_{CP}(B^0 \rightarrow K^{*0} \pi^0) \\ &\phi(K^{*0} \pi^0 / K^{*+} \pi^-) \\ &\phi(\bar{K}^{*0} \pi^0 / K^{*-} \pi^+) \end{aligned}$$



$$\begin{aligned} &|\bar{A}(K^{*-} \pi^+)/A(K^{*+} \pi^-)| \\ &\text{Re}(A(K^{*0} \pi^0)/A(K^{*+} \pi^-)) \\ &\text{Im}(A(K^{*0} \pi^0)/A(K^{*+} \pi^-)) \\ &\text{Re}(A(\bar{K}^{*0} \pi^0)/A(K^{*-} \pi^+)) \\ &\text{Im}(A(\bar{K}^{*0} \pi^0)/A(K^{*-} \pi^+)) \\ &B(B^0 \rightarrow K^{*0} \pi^0) \end{aligned}$$

$$B^+ \rightarrow K^+ \pi^- \pi^+$$

$$\begin{aligned} &B(B^+ \rightarrow K^{*0} \pi^+) \\ &A_{CP}(B^+ \rightarrow K^{*0} \pi^+) \end{aligned}$$



$$\begin{aligned} &|A(\bar{K}^{*0} \pi^-)/A(K^{*0} \pi^-)| \\ &B(B^+ \rightarrow K^{*0} \pi^+) \end{aligned}$$

$$B^+ \rightarrow K_S^0 \pi^+ \pi^0$$

$$\begin{aligned} &B(B^+ \rightarrow K^{*0} \pi^+) \\ &A_{CP}(B^+ \rightarrow K^{*0} \pi^+) \\ &B(B^+ \rightarrow K^{*+} \pi^0) \\ &A_{CP}(B^+ \rightarrow K^{*+} \pi^0) \\ &\phi(K^{*+} \pi^0 / K^{*0} \pi^+) \\ &\phi(K^{*-} \pi^0 / \bar{K}^{*0} \pi^-) \end{aligned}$$



$$\begin{aligned} &|A(K^{*-} \pi^0)/A(K^{*+} \pi^0)| \\ &\text{Re}(A(K^{*+} \pi^0)/A(K^{*0} \pi^+)) \\ &\text{Im}(A(K^{*+} \pi^0)/A(K^{*0} \pi^+)) \\ &\text{Re}(A(K^{*-} \pi^0)/A(\bar{K}^{*0} \pi^-)) \\ &\text{Im}(A(K^{*-} \pi^0)/A(\bar{K}^{*0} \pi^-)) \\ &B(B^+ \rightarrow K^{*+} \pi^0) \end{aligned}$$

Scenarios to constrain CKM

Scenarios to constrain CKM: the strategy

Closure test

- Fix CKM parameters to current values
- Assigning ad-hoc “true” values to Had. amplitudes
- Deduce corresponding values of physical observables
- Explore constraints on CKM parameters assuming very small uncertainties on observables
- Had. amplitudes constrained to follow naïve hierarchy pattern
 - $T^{+-} > T^{00} > N^{0+}$ and $P^{+-} > P_{EW} > P_{EW}^C$
- Furthermore, P_{EW} constrained to match CPS/GPSZ assumption
 - $|P_{EW}/(T^{+-} + T^{00})| = 0.0135$ and $\arg(P_{EW}) = \arg(T^{+-} + T^{00})$
- This ad-hoc choice of “true” values roughly reproduces current BF and A_{CP} (c.f. table)

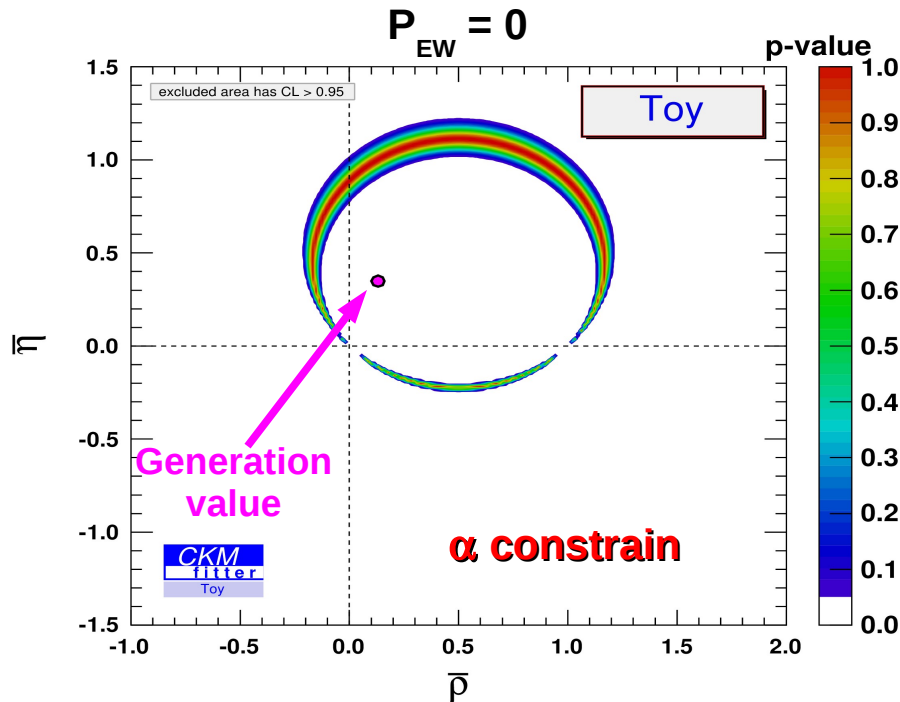
Hadronic par.	magnitude	phase (deg)	Physical observable	Measurement	Value
T^{+-}	2.540	0.00	$\mathcal{B}(B^0 \rightarrow K^{*+}\pi^-)$	8.2 ± 0.9	7.1
T^{00}	0.762	75.74	$\mathcal{B}(B^0 \rightarrow K^{*0}\pi^0)$	3.3 ± 0.6	1.6
N^{0+}	0.143	108.37	$\mathcal{B}(B^+ \rightarrow K^{*+}\pi^0)$	9.2 ± 1.5	8.5
P^{+-}	0.091	-6.48	$\mathcal{B}(B^+ \rightarrow K^{*0}\pi^+)$	11.6 ± 1.2	10.9
P_{EW}	0.038	15.15	$A_{CP}(B^0 \rightarrow K^{*+}\pi^-)$	-24.0 ± 7.0	-12.9
P_{EW}^C	0.029	101.90	$A_{CP}(B^0 \rightarrow K^{*0}\pi^0)$	-15.0 ± 13.0	-46.5
$\left \frac{V_{ts} V_{tb}^*}{V_{us} V_{ub}^*} \frac{P^{+-}}{T^{+-}} \right $	1.801		$A_{CP}(B^+ \rightarrow K^{*+}\pi^0)$	-0.52 ± 15.0	-35.4
$ T^{00}/T^{+-} $	0.300		$A_{CP}(B^+ \rightarrow K^{*0}\pi^+)$	$+5.0 \pm 5.0$	+3.9
$ N^{0+}/T^{00} $	0.187				
$ P_{EW}/P^{+-} $	0.420				
$ P_{EW}/(T^{+-} + T^{00}) /R$	1.000				
$ P_{EW}/P_{EW}^C $	0.762				

Explored hypothesis

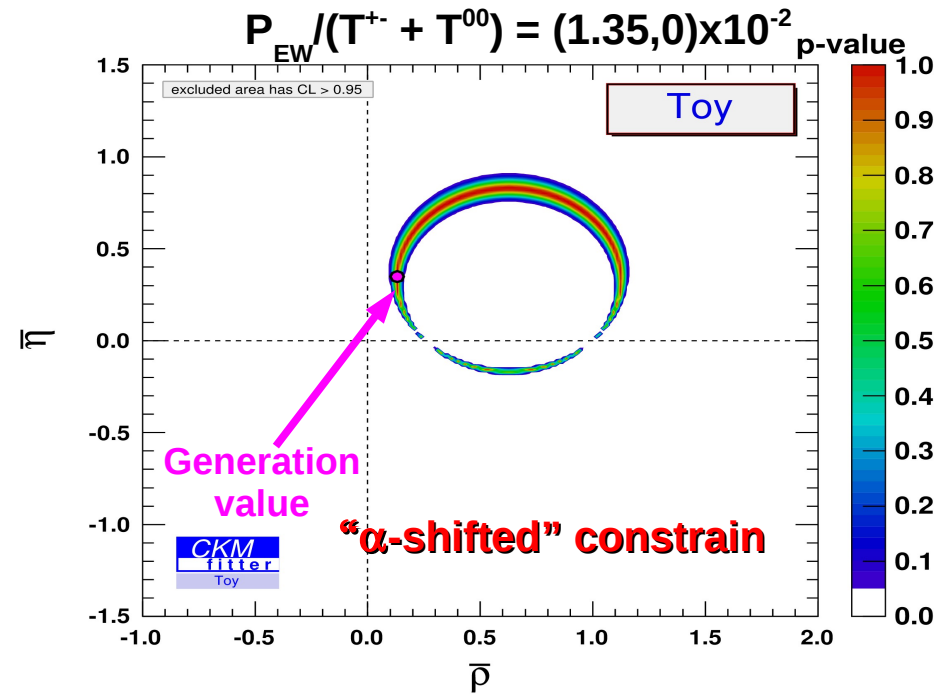
- CPS/GPSZ-like assumption
- Hypothesis on the annihilation

Scenarios to constrain CKM: CPS/GPSZ-like (I)

- The CKM α is extracted from $B \rightarrow \pi\pi$, $\rho\pi$ and $\rho\rho$ isospin analysis by neglecting the P_{EW} contributions to the decay amplitudes
- A similar approach is tested here [CPS PRD74:051301](#), [GPSZ PRD75:014002](#)

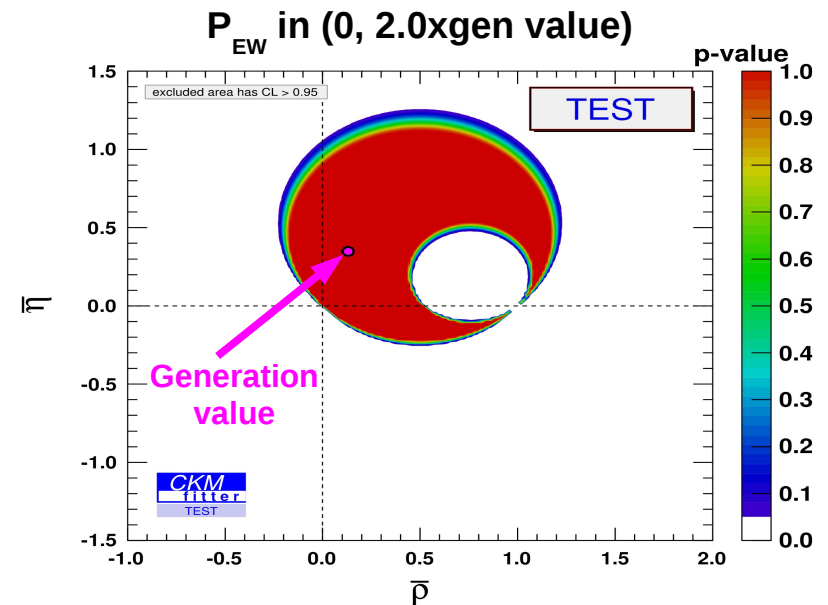
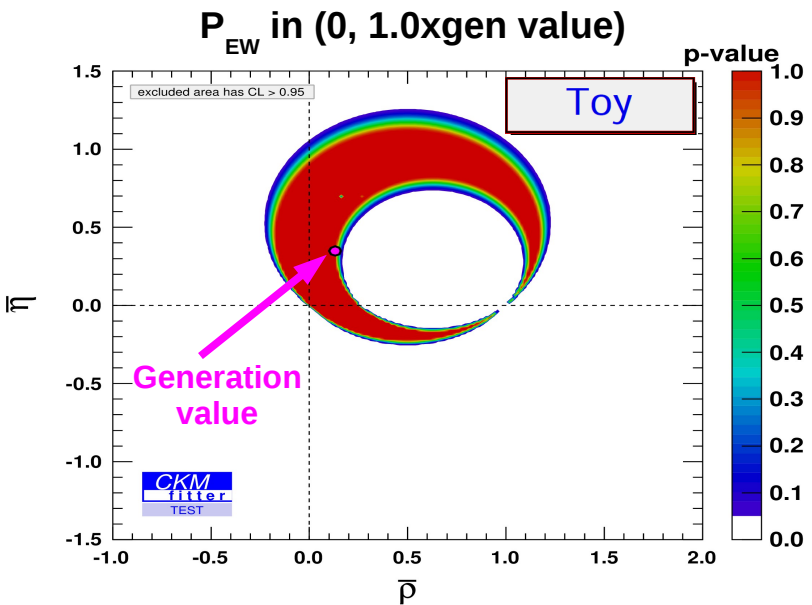
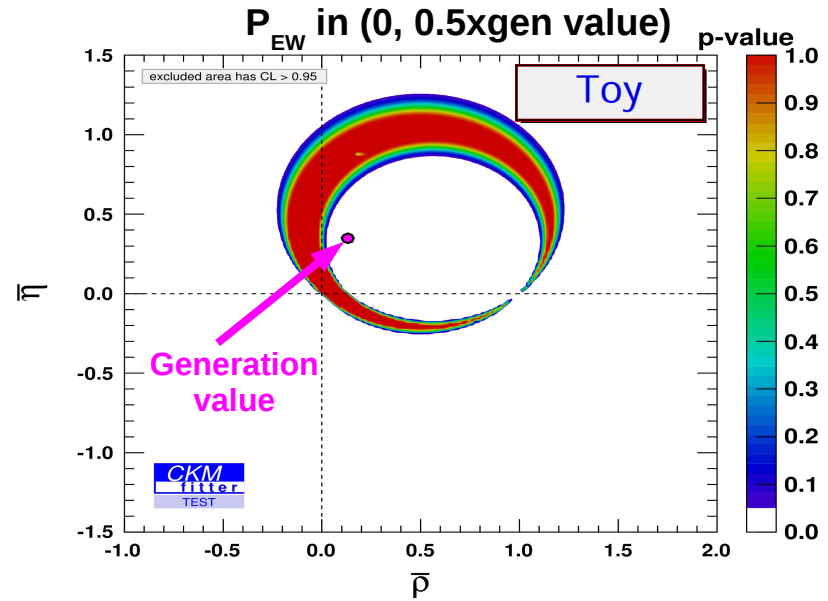
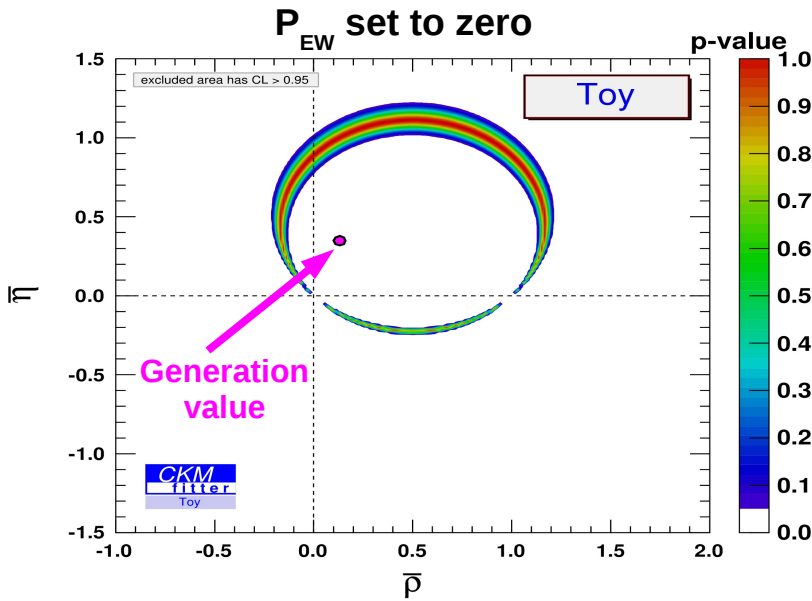


- Yields constraint on ρ - η following α contours
- But fails (by large amounts!) to reproduce true α

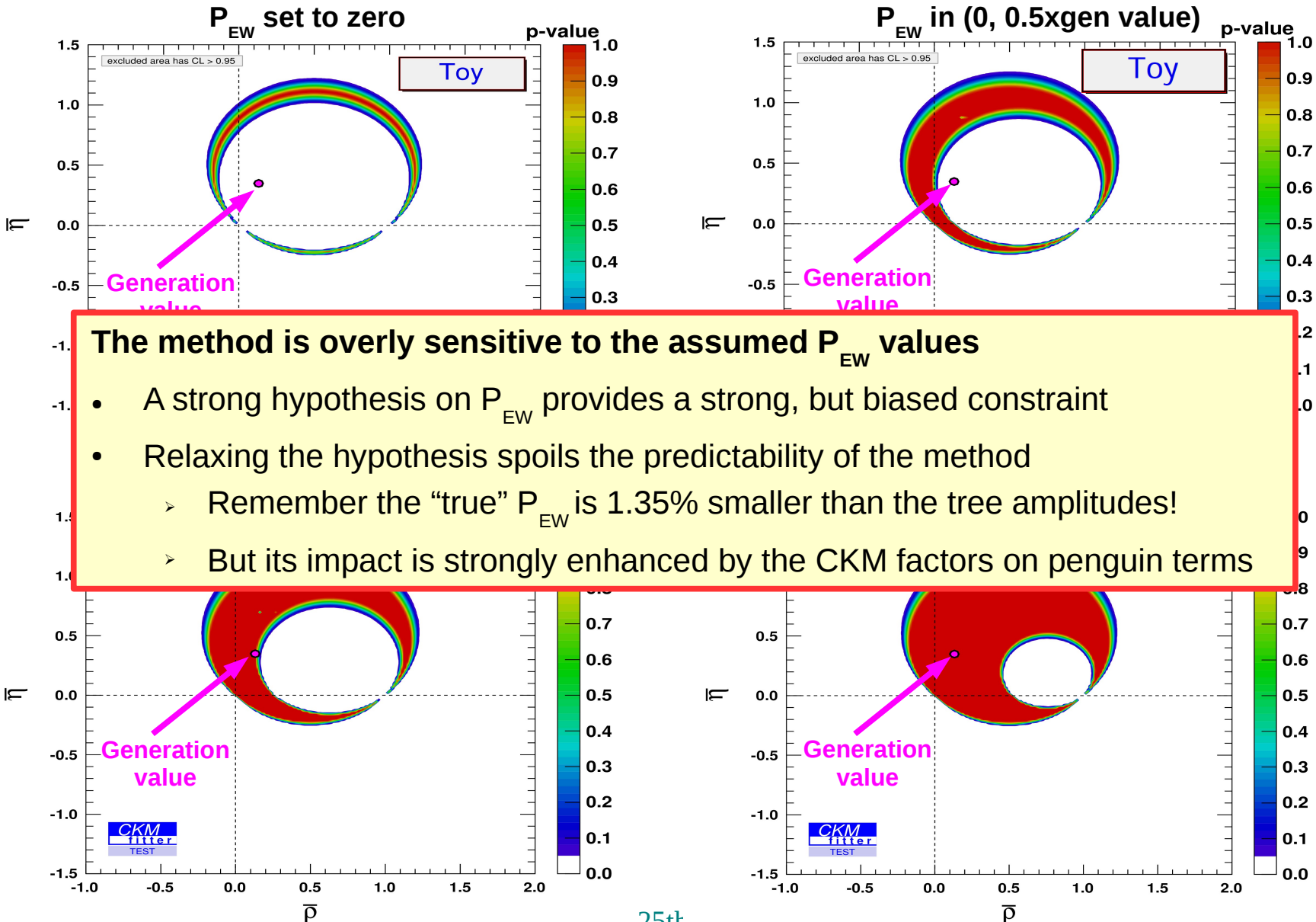


- Yields unbiased constraint
- Which does not follow α contour

Scenarios to constrain CKM: CPS/GPSZ-like (II)



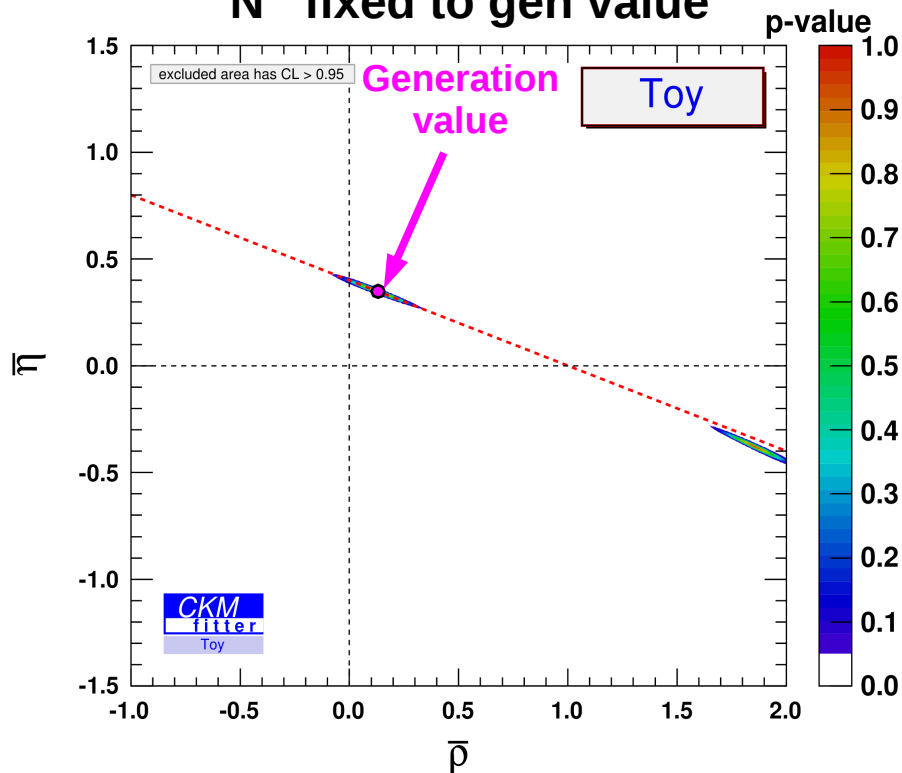
Scenarios to constrain CKM: CPS/GPSZ-like (II)



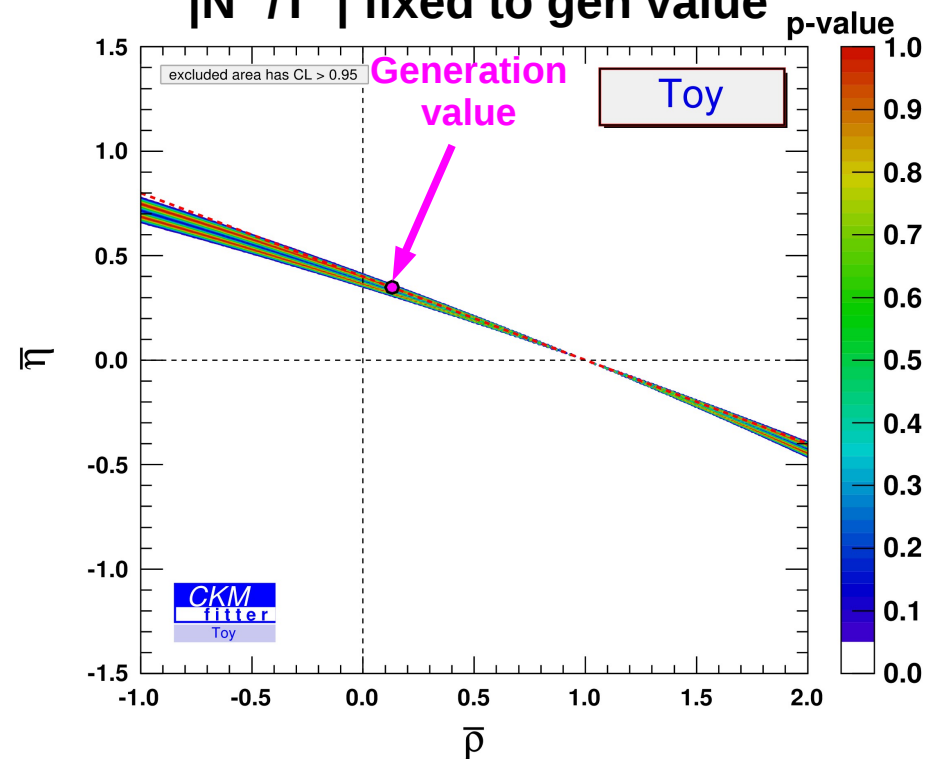
Scenarios to constrain CKM: hypothesis on N^{0+} (I)

- CKM enhancement does not affect tree terms
- Furthermore, the annihilation N^{0+} is naïvely expected to be small
- May be constrained from theory and/or from annihilation-dominated modes

N^{0+} fixed to gen value

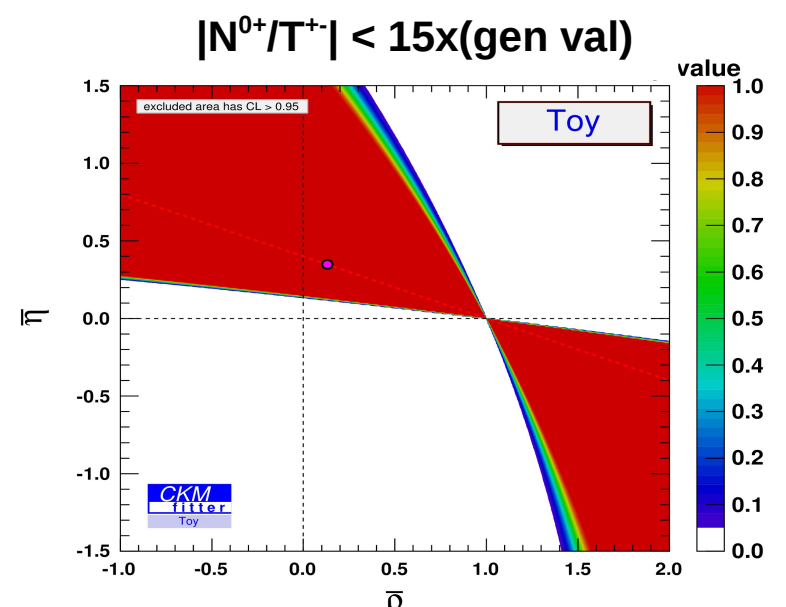
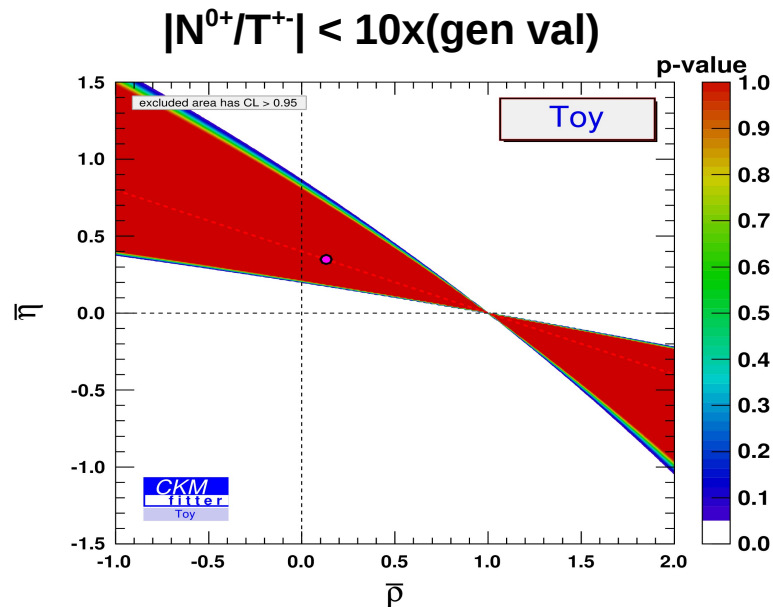
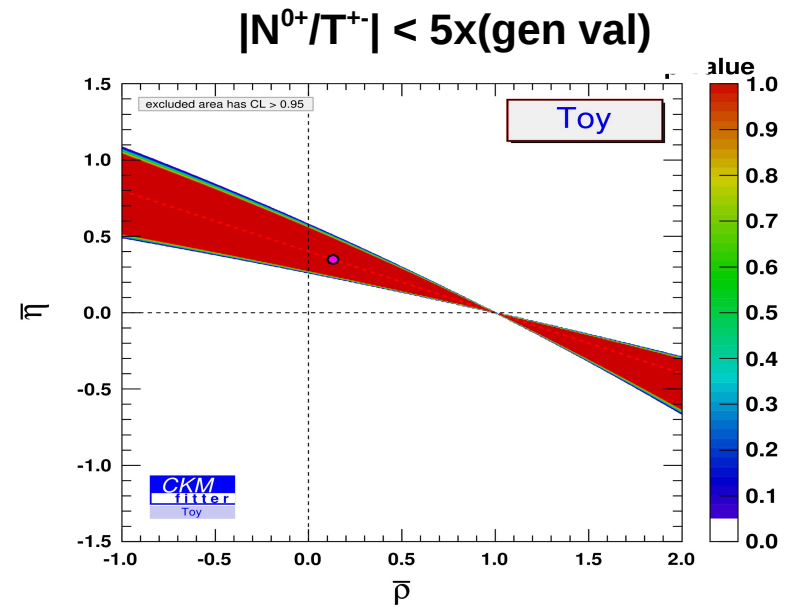
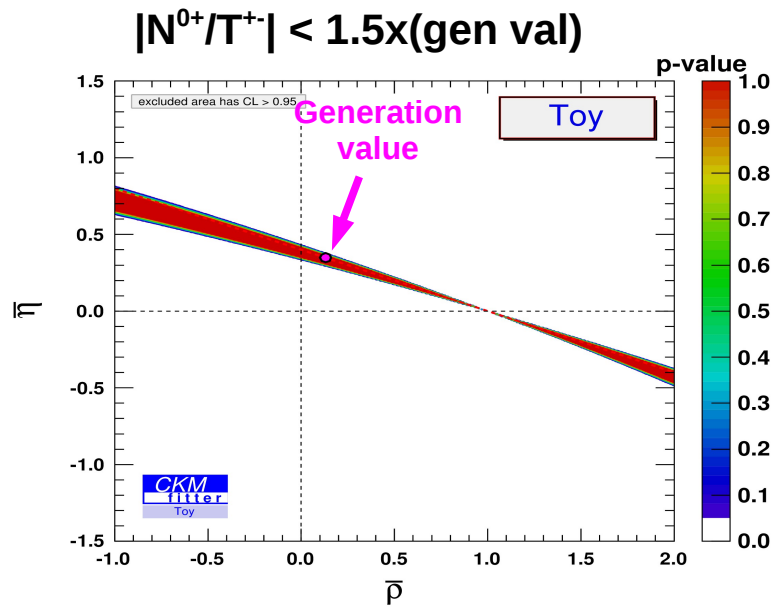


$|N^{0+}/T^{+-}|$ fixed to gen value

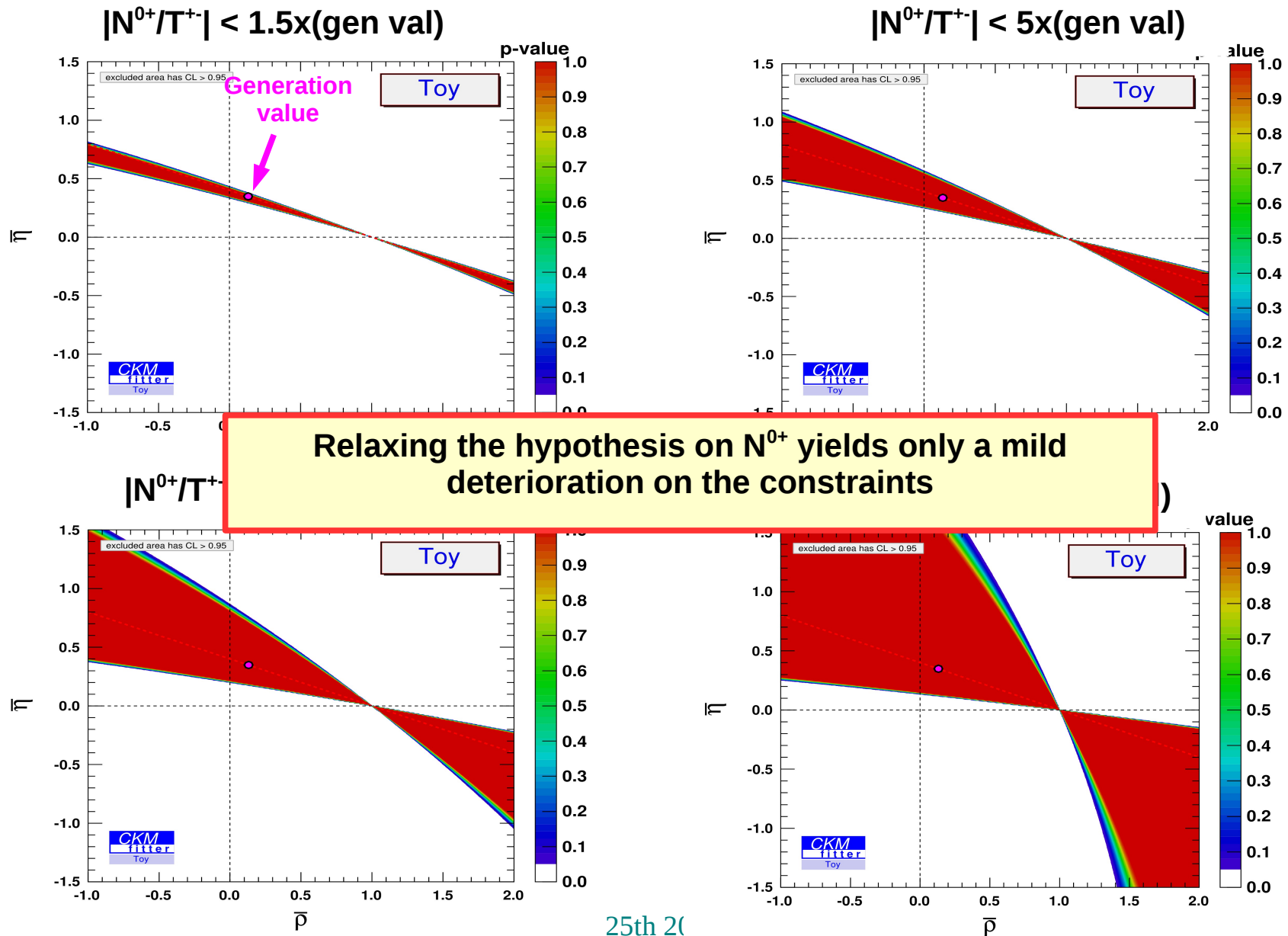


Hypotheses in the $|N^{0+}/T^{+-}|$ provides a “ β -like” constraint in ρ - η

Scenarios to constrain CKM: hypothesis on N^{0+} (II)

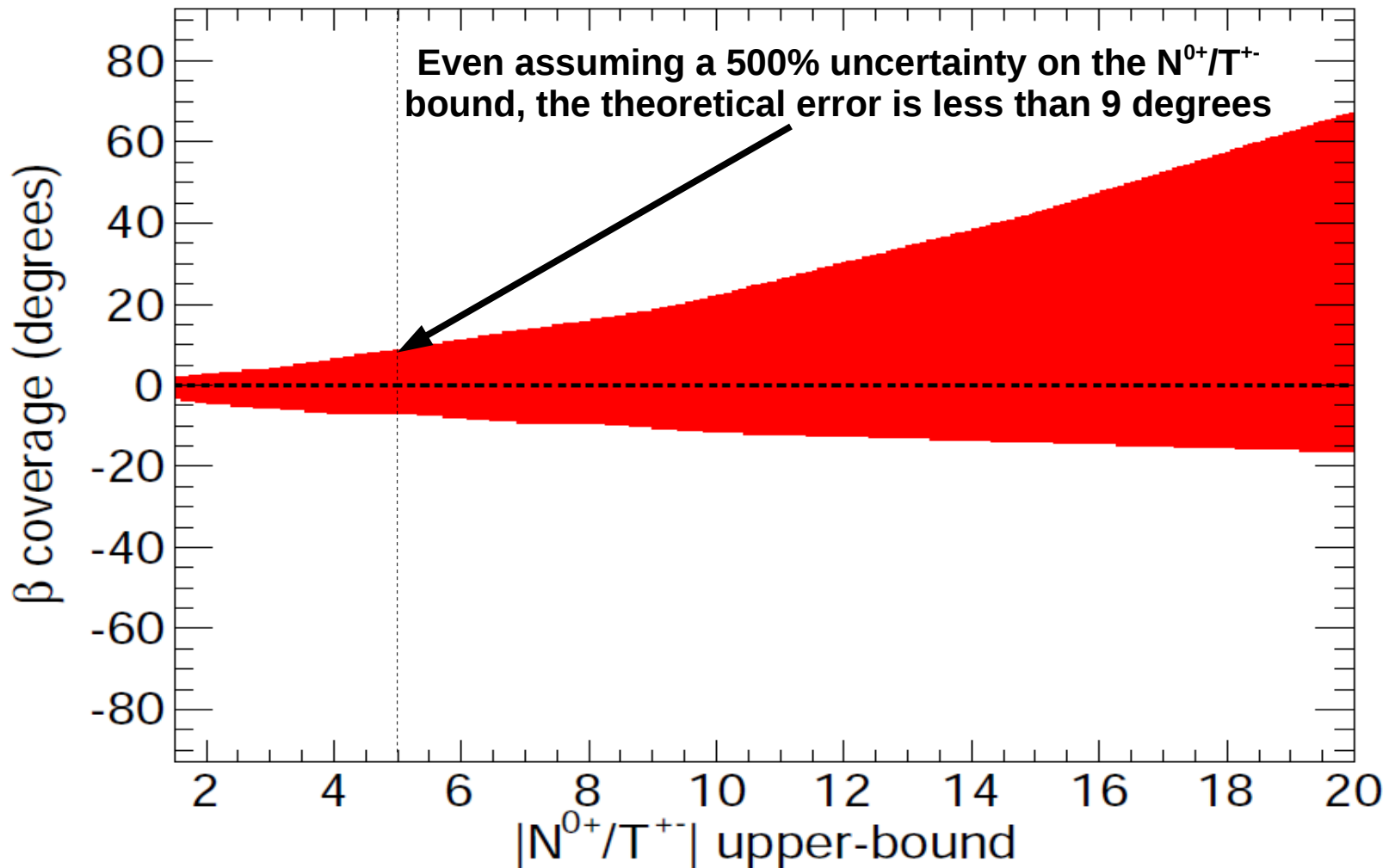


Scenarios to constrain CKM: hypothesis on N^{0+} (II)



Scenarios to constrain CKM: hypothesis on N^{0+} (III)

β coverage vs Upper bound on $|N^{0+}/T^{+-}|$ (in units of the generation value)



Scenarios to constrain CKM: Summary

■ CPS/GPSZ-like hypothesis:

- Conservative values on the uncertainty of the P_{EW} prediction gives uncontrollable effects of the ρ - η constraints
 \Rightarrow The method is dominated by the theoretical uncertainties
- This is expected due to the CKM enhancement ($|V_{ts} V_{tb}^* / V_{us} V_{ub}^*| \sim 55$) of “penguin” w.r.t “tree” terms

■ Hypothesis on the annihilation (N^{0+})

- It is possible to set a constraint in ρ - η by just setting an upper bound on the $|N^{0+}/T^{+}|$
- Constraint on CKM less sensitive to theoretical uncertainties as there is no CKM enhancement
Uncertainty of 500% on $|N^{0+}/T^{+}|$ gives a theory error of less than 9 degrees
- Possibility to get bounds on the annihilation from data by measuring the annihilation-dominated mode $B_s^+ \rightarrow K^{*0} K^+$ which is U-spin related to $B^0 \rightarrow K^{*0} \pi^+$
 \Rightarrow Accessible to LHCb

Current constraints on Hadronic amplitudes

Experimental inputs: *BABAR* (I)

■ *BABAR* $B^0 \rightarrow K^+ \pi^- \pi^0$ analysis: PRD83:112010 (2011)

$$|A(K^{*-} \pi^+)/A(K^{*+} \pi^-)| = 0.74 \pm 0.09$$

$$\text{Re}(K^{*0} \pi^0/K^{*+} \pi^-) = 0.80 \pm 0.20;$$

$$\text{Im}(K^{*0} \pi^0/K^{*+} \pi^-) = -0.32 \pm 0.42;$$

$$\text{Re}(\overline{K}^{*0} \pi^0/K^{*-} \pi^+) = 1.00 \pm 0.15$$

$$\text{Im}(\overline{K}^{*0} \pi^0/K^{*-} \pi^+) = -0.07 \pm 0.53;$$

$$B(K^{*0} \pi^0) = (3.30 \pm 0.64) \times 10^{-6}$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.06 & 0.02 & -0.35 & -0.11 & -0.06 \\ & 1.0 & 0.78 & 0.30 & -0.01 & 0.29 \\ & & 1.0 & -0.06 & 0.00 & -0.10 \\ & & & 1.0 & 0.30 & 0.42 \\ & & & & 1.0 & -0.02 \\ & & & & & 1.00 \end{pmatrix}$$

■ *BABAR* $B^0 \rightarrow K_s^0 \pi^+ \pi^-$ analysis: PRD80:112001 (2009)

two minima differing by 0.16 2NLL units

Global minimum

$$\text{Re}(K^{*-} \pi^+/K^{*+} \pi^-) = 0.43 \pm 0.41;$$

$$\text{Im}(K^{*-} \pi^+/K^{*+} \pi^-) = -0.69 \pm 0.26;$$

$$B(K^{*+} \pi^-) = (8.3 \pm 1.2) \times 10^{-6};$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.93 & 0.02 \\ & 1.0 & -0.08 \\ & & 1.0 \end{pmatrix}$$

Local Minimum

$$\text{Re}(K^{*-} \pi^+/K^{*+} \pi^-) = -0.82 \pm 0.09;$$

$$\text{Im}(K^{*-} \pi^+/K^{*+} \pi^-) = -0.05 \pm 0.43;$$

$$B(K^{*+} \pi^-) = (8.3 \pm 1.2) \times 10^{-6};$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & -0.20 & 0.22 \\ & 1.0 & -0.01 \\ & & 1.0 \end{pmatrix}$$

Experimental inputs: *BABAR* (II)

■ *BABAR* $B^+ \rightarrow K^+ \pi^- \pi^+$ analysis: PRD78:012004 (2008)

$$|A(K^{*0} \pi^-)/A(K^{*0} \pi^+)| = 1.033 \pm 0.047; \quad \text{Full Correlation matrix}$$
$$B(K^{*0} \pi^+) = (10.8 \pm 1.4) \times 10^{-6}; \quad \begin{pmatrix} 1.0 & 0.02 \\ & 1.0 \end{pmatrix}$$

■ *BABAR* $B^+ \rightarrow K_s^0 \pi^+ \pi^0$ analysis: ArXiv : 1501.00705 [hep-ex] (2015)

New Result!

- Currently in communication with authors to get full set of observables and correlation matrices
- The results shown in next slides just use
 - $B(K^{*+} \pi^0) = (9.2 \pm 1.5) \times 10^{-6}$;
 - $C(K^{*+} \pi^0) = -0.52 \pm 0.15; \Rightarrow \sim 3.5\sigma$ significance

Experimental inputs: Belle

- **Belle $B^0 \rightarrow K_S^0 \pi^+ \pi^-$ analysis:** PRD75:012006 (2007) and PRD79:072004 (2009)

two minima differing by 7.5 2NLL units

Global minimum

$$\text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.79 \pm 0.14;$$

$$\text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = -0.21 \pm 0.40;$$

$$B(K^{*+}\pi^-) = (8.4 \pm 1.5) \times 10^{-6};$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.62 & 0.0 \\ & 1.0 & 0.0 \\ & & 1.0 \end{pmatrix}$$

Local Minimum

$$\text{Re}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.81 \pm 0.11;$$

$$\text{Im}(K^{*-}\pi^+/K^{*+}\pi^-) = 0.01 \pm 0.44;$$

$$B(K^{*+}\pi^-) = (8.4 \pm 1.5) \times 10^{-6};$$

Full Correlation matrix

$$\begin{pmatrix} 1.0 & 0.01 & 0.0 \\ & 1.0 & 0.0 \\ & & 1.0 \end{pmatrix}$$

- **Belle $B^+ \rightarrow K^+ \pi^- \pi^+$ analysis:** PRL96:251803 (2006)

$$|A(K^{*0}\pi^-)/A(K^{*0}\pi^+)| = 0.86 \pm 0.09;$$

Full Correlation matrix

$$B(K^{*0}\pi^+) = (9.7 \pm 1.1) \times 10^{-6};$$

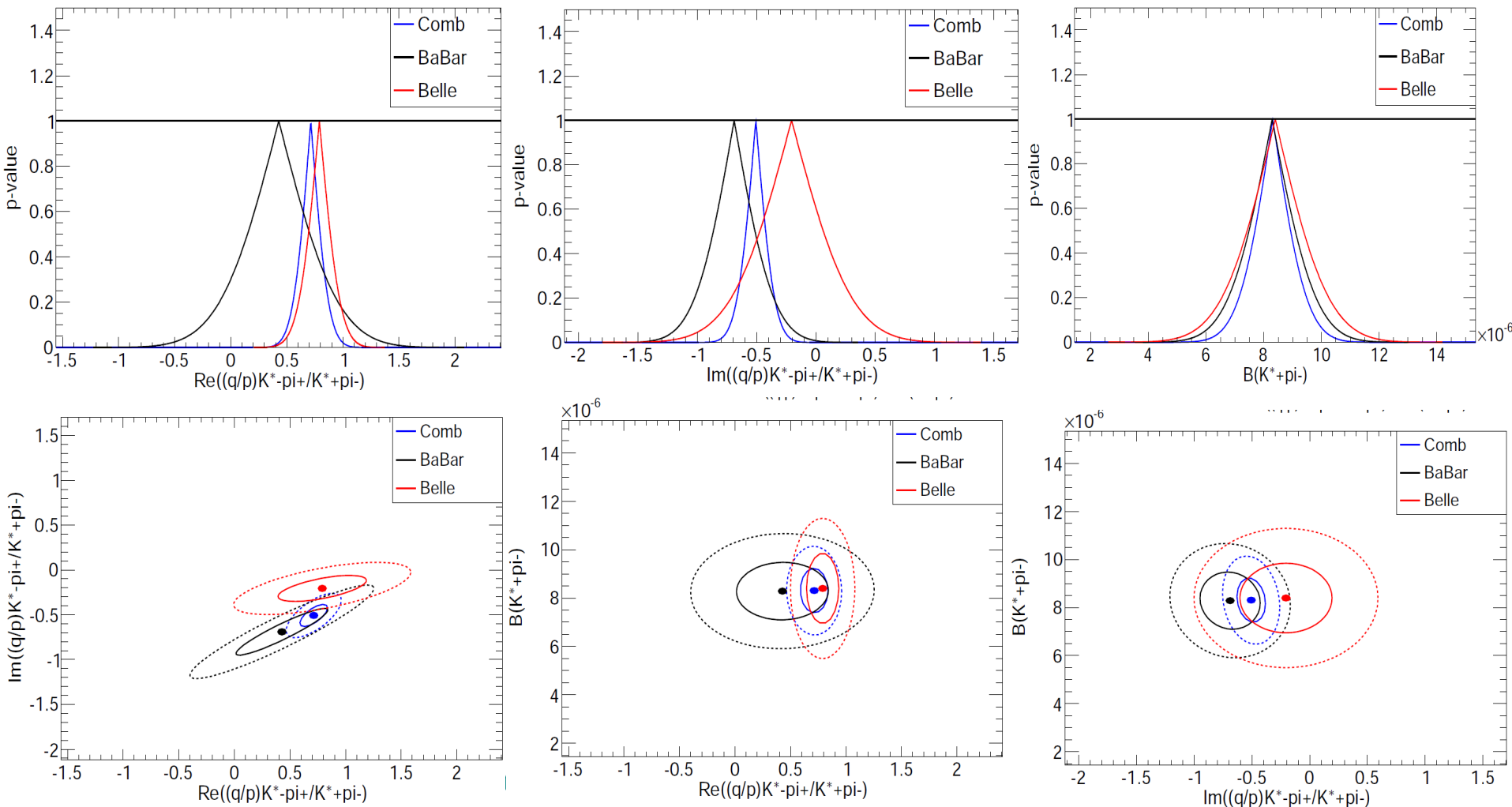
$$\begin{pmatrix} 1.0 & 0.0 \\ & 1.0 \end{pmatrix}$$

- **No Belle results on:**

$$B^0 \rightarrow K^+ \pi^+ \pi^0 \text{ and } B^+ \rightarrow K_S^0 \pi^+ \pi^0$$

Combining *BABAR* + Belle: $B^0 \rightarrow K_S^0 \pi^+ \pi^-$

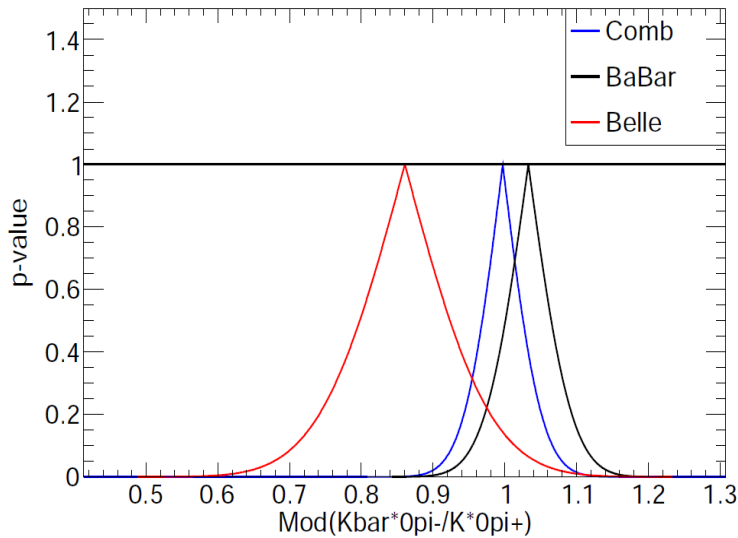
- Two solutions for both *BABAR* and Belle analyses
 - Combine all possible combinations of *BABAR* and Belle solutions taking into account the difference in 2NLL
 - Results: 4 solutions differing in χ^2 : 0, 7.7, 8.4 and 97.2. Consider only the global minimum



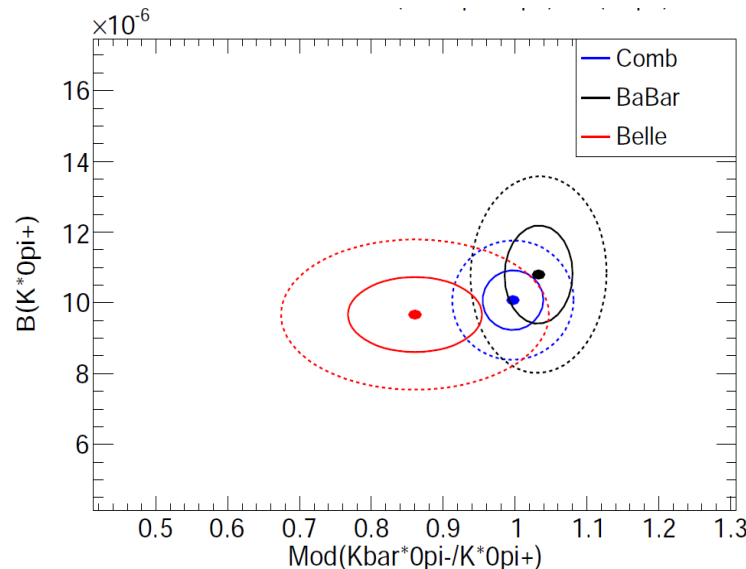
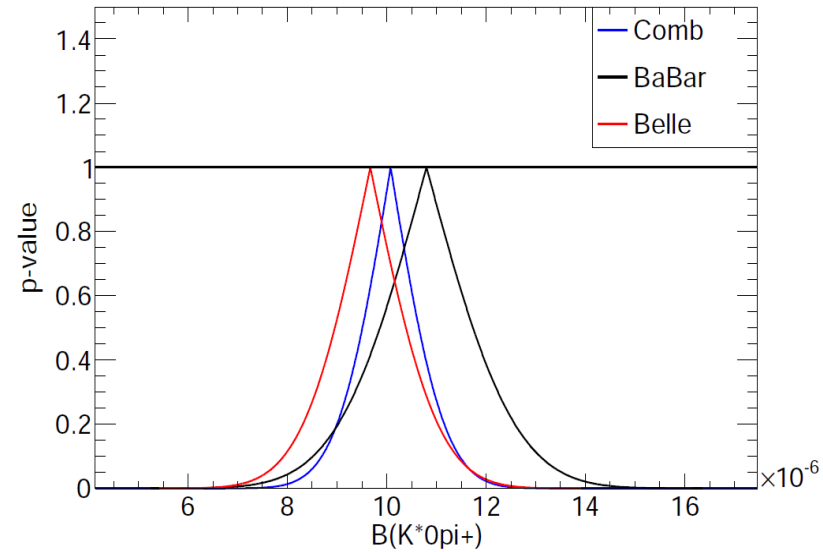
Combining *BABAR* + Belle: $B^+ \rightarrow K^+ \pi^- \pi^+$

- Single solution for both *BABAR* and Belle

Likelihood vs $\text{Mod}(K\bar{K}^0\pi^-/K^+\pi^0\pi^+)$



Likelihood vs $B(K^0\pi^+)$



Results on Had. Amplitudes: CP violation (I)

- Decay amplitudes (δ_i and φ_i are weak/strong phases)

$$A = M_1 \exp(i\delta_1) \exp(i\varphi_1) + M_2 \exp(i\delta_2) \exp(i\varphi_2)$$

$$A = M_1 \exp(i\delta_1) \exp(-i\varphi_1) + M_2 \exp(i\delta_2) \exp(-i\varphi_2)$$

$$A_{CP} = 2 \frac{\sin(\Delta\delta) \sin(\Delta\varphi)}{(M_1/M_2) + (M_2/M_1) + 2 \cos(\Delta\delta) \cos(\Delta\varphi)}$$

- In our case $\Delta\varphi = \arg(V_{ts} V_{tb}^* / V_{us} V_{ub}^*) = 2\gamma \neq 0$

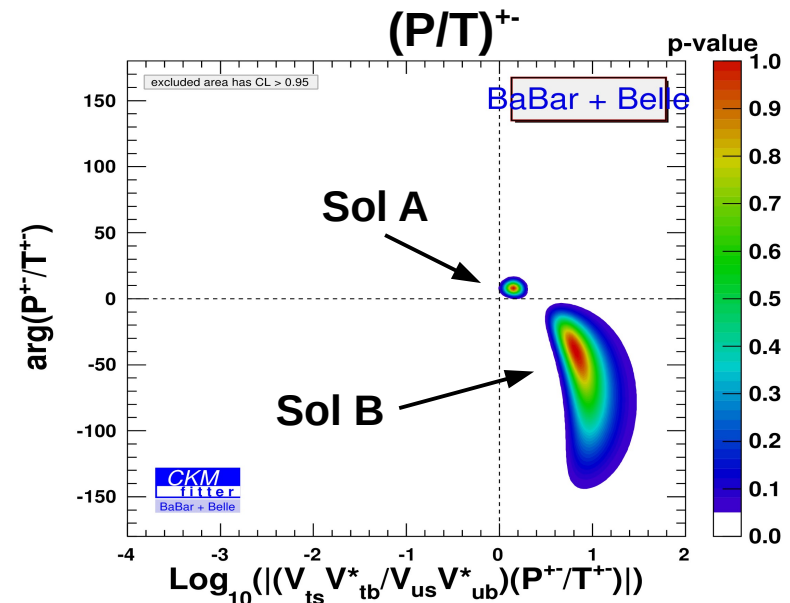
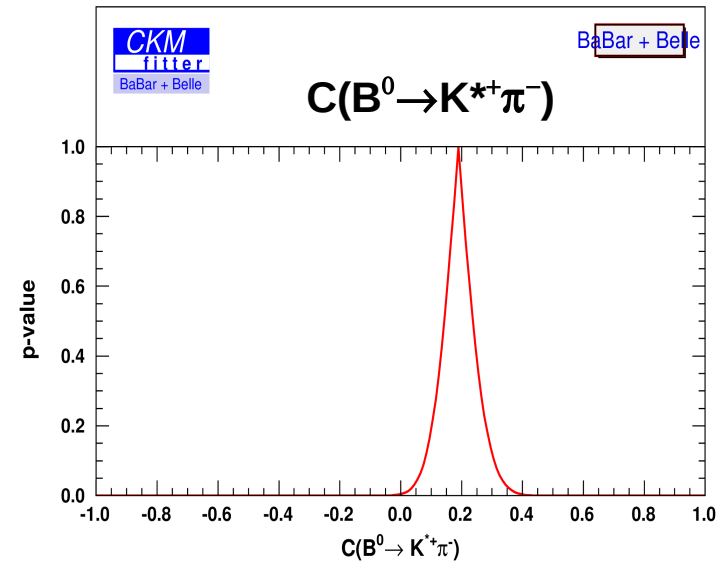
- If A_{CP} is significantly different from zero then

- $|\text{CKM}^*(P/T)| \sim 1$
- $\arg(P/T) \neq 0$

- 3σ significance for $C(B^0 \rightarrow K^{*+} \pi^-)$

$$A(B^0 \rightarrow K^{*+} \pi^-) = V_{us} V_{ub}^* T^{*+} + V_{ts} V_{tb}^* P^{*+}$$

- Two solutions with same χ^2 (Sol A and B)
- Both inconsistent with $\arg(P/T) = 0/\pi$
- Only solution A has $|\text{CKM}^*(P/T)| \sim 1$



Results on Had. Amplitudes: CP violation (II)

- Decay amplitudes (δ_i and ϕ_i are weak/strong phases)

$$A = M_1 \exp(i\delta_1) \exp(i\phi_1) + M_2 \exp(i\delta_2) \exp(i\phi_2)$$

$$A = M_1 \exp(i\delta_1) \exp(-i\phi_1) + M_2 \exp(i\delta_2) \exp(-i\phi_2)$$

$$A_{CP} = 2 \frac{\sin(\Delta\delta) \sin(\Delta\phi)}{(M_1/M_2) + (M_2/M_1) + 2 \cos(\Delta\delta) \cos(\Delta\phi)}$$

- In our case $\Delta\phi = \arg(V_{ts} V_{tb}^* / V_{us} V_{ub}^*) = 2\gamma \neq 0$

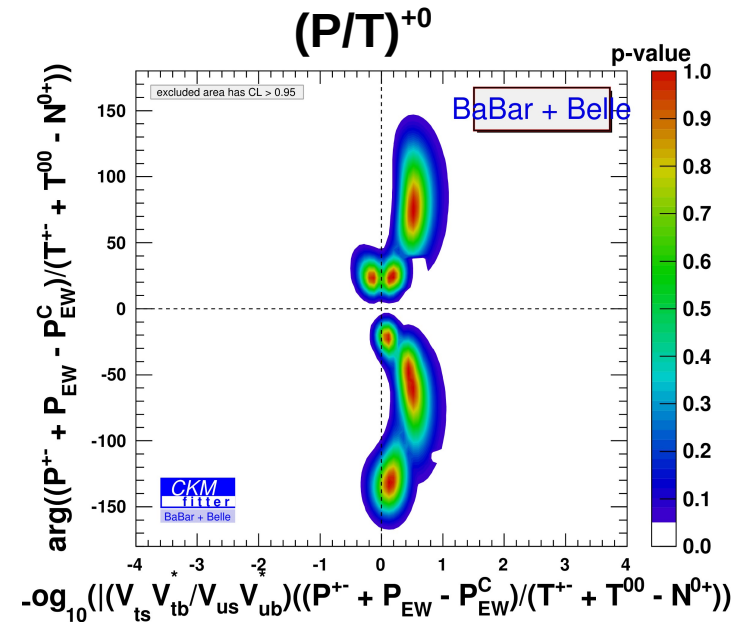
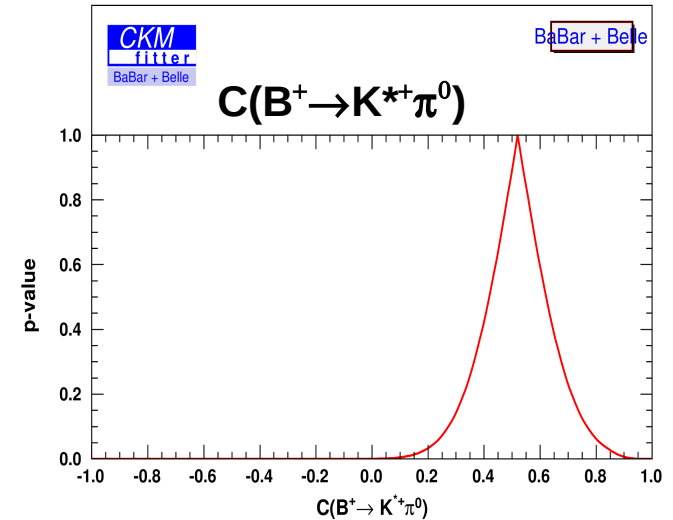
- If A_{CP} is significantly different from zero then

- $|\text{CKM}^*(P/T)| \sim 1$
- $\arg(P/T) \neq 0$

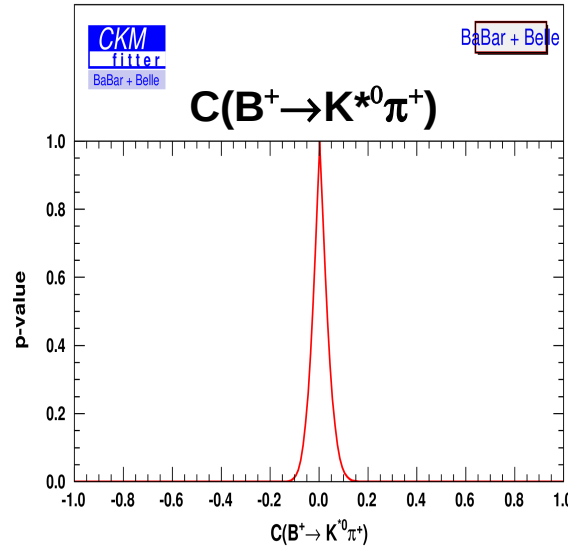
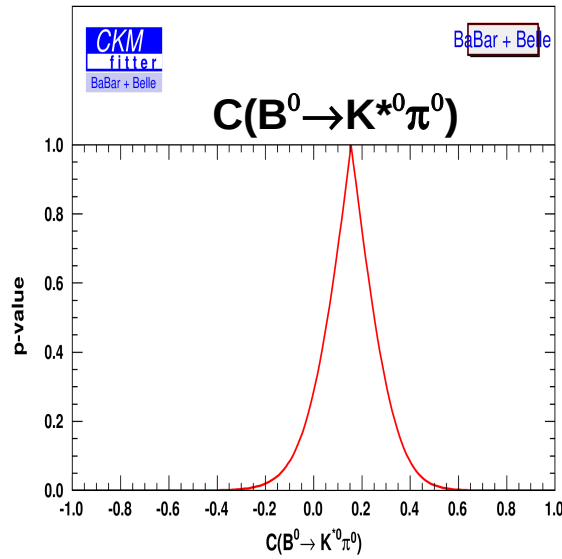
- 3.4 σ significance for $C(B^+ \rightarrow K^{*+} \pi^0)$

$$\sqrt{2} A(B^+ \rightarrow K^{*+} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00}_C - N^{0+}) + V_{ts} V_{tb}^* (P^{+-} - P_{EW}^C + P_{EW})$$

- Both solutions inconsistent with $\arg(P/T) = 0/\pi$ and with $|\text{CKM}^*(P/T)| \sim 1$
- Appearance of other local minima



Results on Had. Amplitudes: CP violation (III)

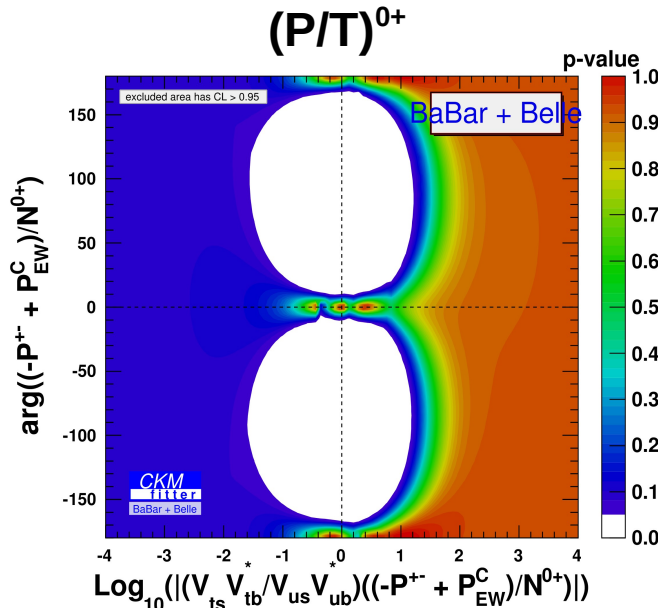
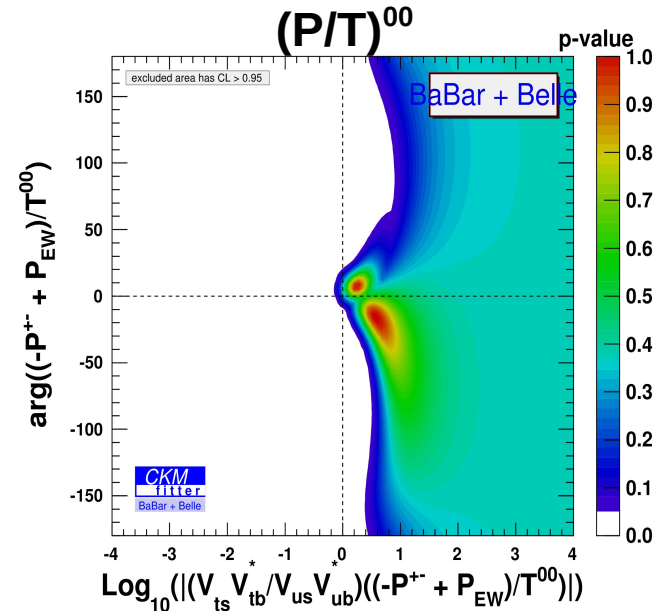


$$A(B^+ \rightarrow K^{*0} \pi^+) = V_{us} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

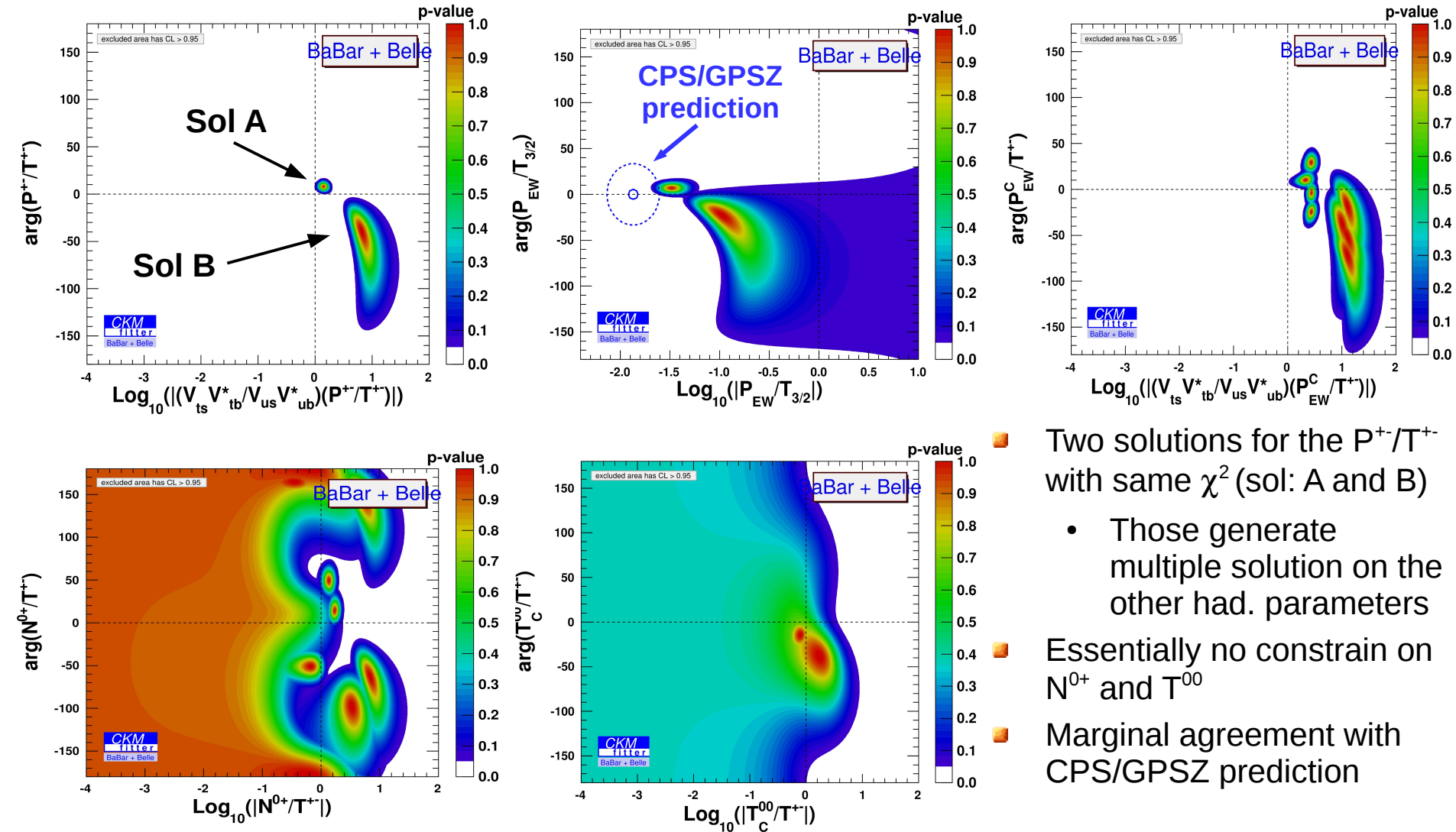
$$A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* T^{00} + V_{ts} V_{tb}^* (-P^{+-} + P_{EW}^C)$$

- $A_{CP}(K^{*0} \pi^0)$ and $A_{CP}(K^{*0} \pi^+)$ consistent with zero @ 1σ
- P/T constraints are consistent either with

- $|CKM^*(P/T)| \gg 1$ or $\ll 1$
- $\arg(P/T) = 0$ or $\pm\pi$



Results on Had. Amplitudes: all together

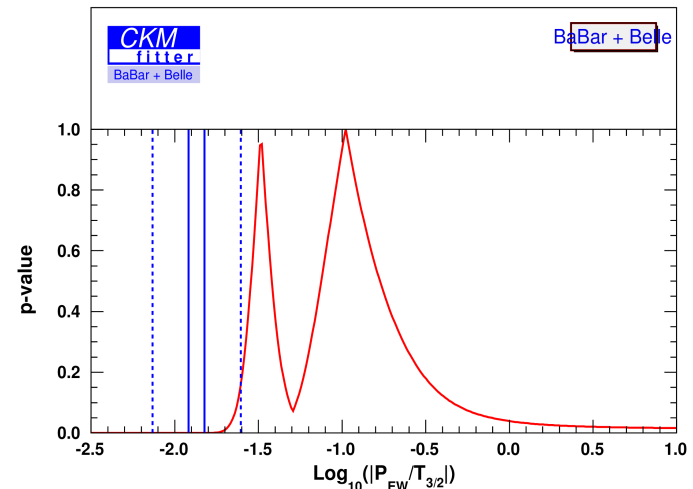
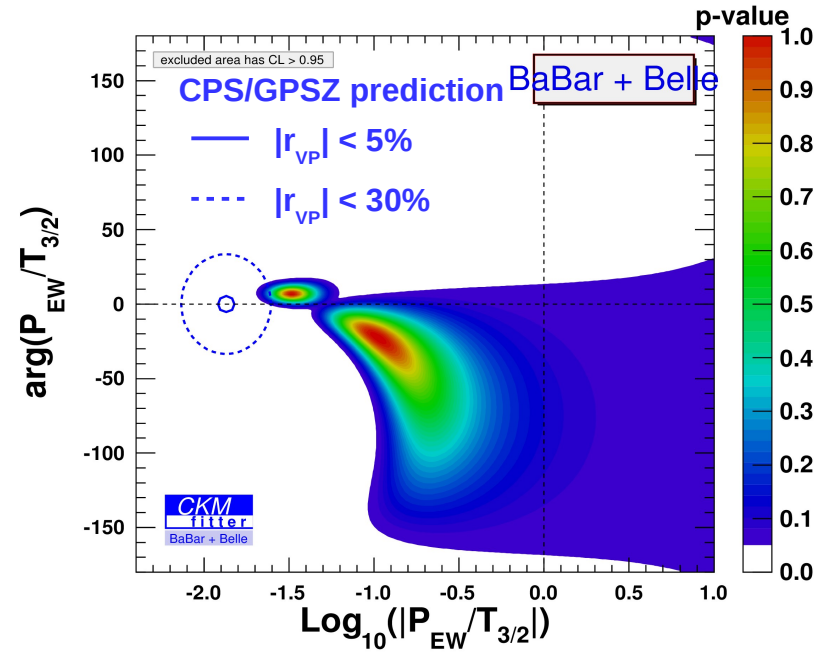


- Two solutions for the P^+/T^+ with same χ^2 (sol: A and B)
 - Those generate multiple solution on the other had. parameters
- Essentially no constrain on N^{0+} and T^{00}
- Marginal agreement with CPS/GPSZ prediction

Results on Had. Amplitudes: agreement with CPS/GPSZ

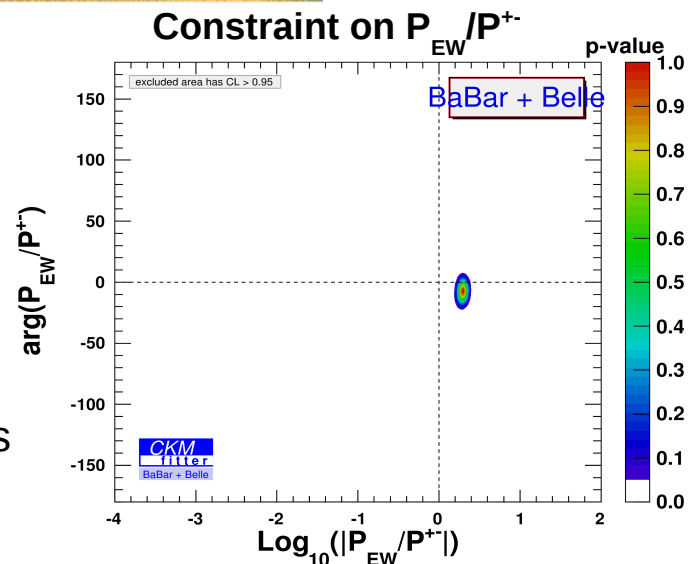
- CPS/GPSZ prediction

$$P_{EW}/(T^{+-} + T^{00}) = R(1-r_{VP})/(1+r_{VP})$$
 with $R = 1.35\%$ and $|r_{VP}| < 5\%$
- The current experimental constraints in poor agreement with the CPS/GPSZ prediction
- Marginal agreement only reached by inflating the uncertainty on $|r_{VP}|$ up to 30%

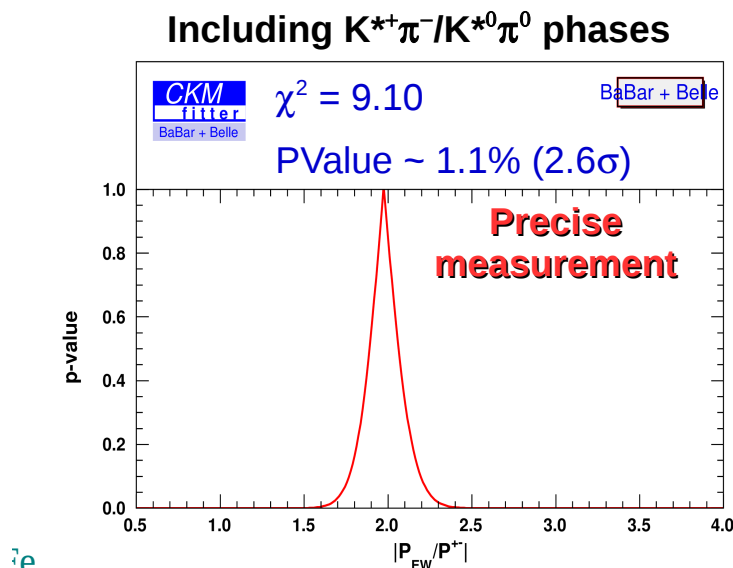
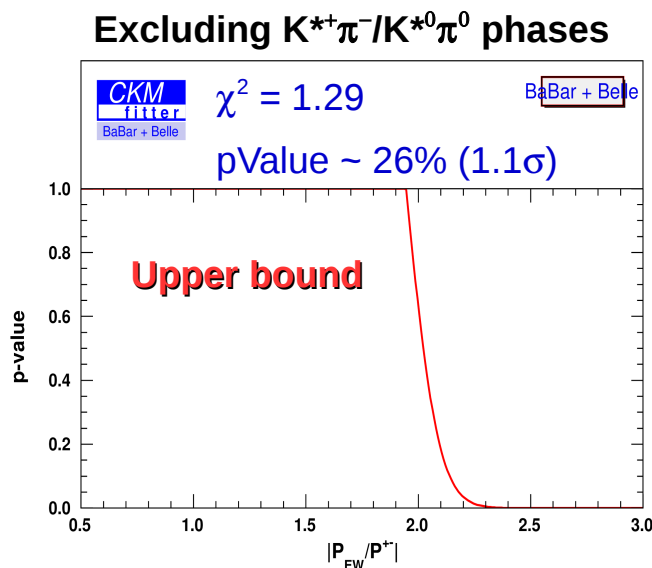


Results on Had. Amplitudes: Hierarchies (I)

- Current data favours a relatively high P_{EW}
- This results is mainly driven by the $K^{*+}\pi^-/K^{*0}\pi^0$ phase differences measured in $B^0 \rightarrow K^+\pi^-\pi^0$
- Without these phases there is good agreement among the experimental observables ($\chi^2 = 1.29$, p-Value $\sim 1.1\sigma$)
- Adding the phases brings slight tension ($\Delta\chi^2 = 7.7$, 2.6σ)
- Only one experiment has performed the $B^0 \rightarrow K^+\pi^-\pi^0$ analysis
- An independent confirmation is needed to claim non-zero (and large!) value of P_{EW}

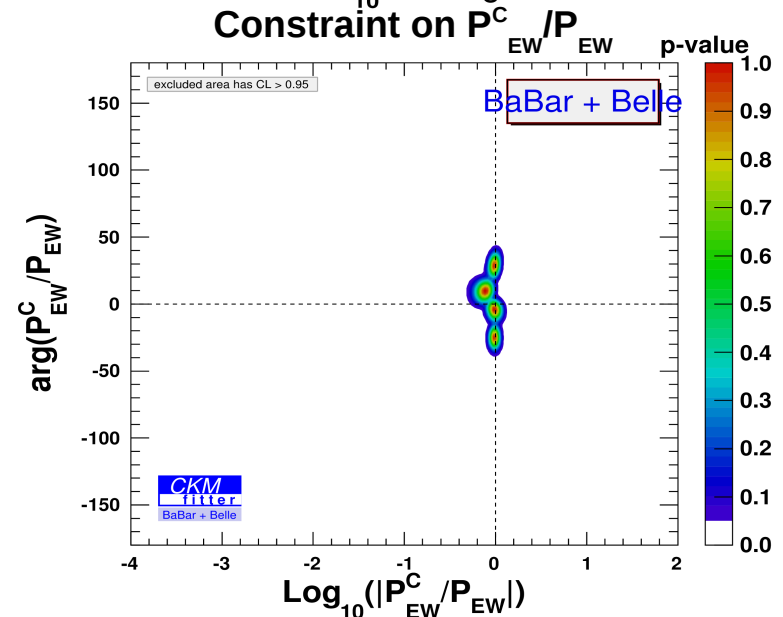
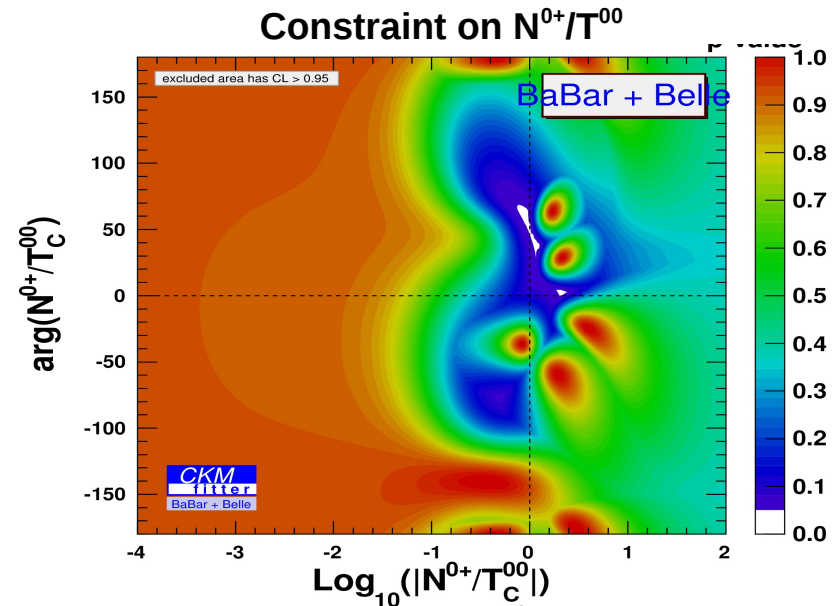
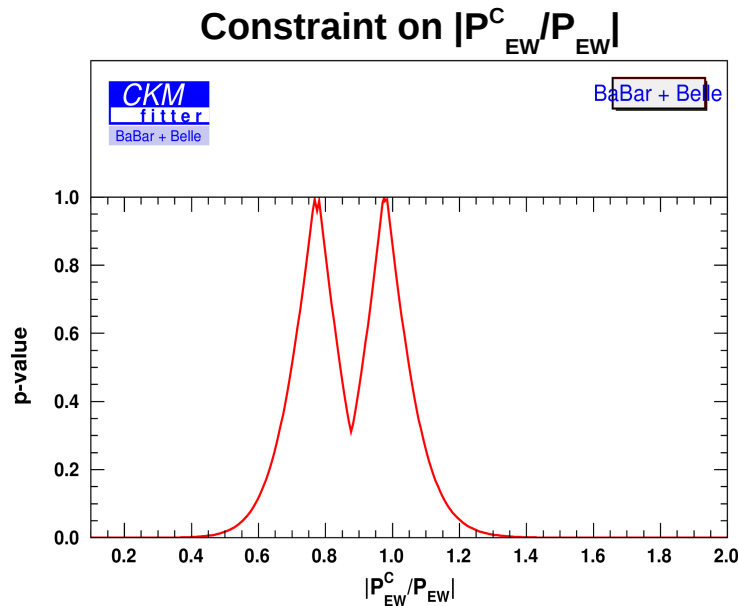


Constraints on $|P_{EW}/P^{+-}|$



Results on Had. Amplitudes: Hierarchies (II)

- Essentially no constraint is possible on N^{0+}/T^{00} with current data
- Strong constrain on P_{EW}^C/P_{EW}
 - 2 solutions at ~ 0.8 and ~ 1.0
 - Result on P_{EW}^C/P_{EW} is also consequence of the large P_{EW}
 - Needs also confirmation for the $B^0 \rightarrow K^+ \pi^- \pi^0$ analysis



Prospects for future LHCb and Belle-II data

Prospects for LHCb and Belle-II (I)

- Assume future experiments will measure central values used in the closure test study
- LHCb will have high statistic measurements in the fully charged modes:**
 $B^0 \rightarrow K_S^0 (\rightarrow \pi^+ \pi^-) \pi^+ \pi^-$ and $B^+ \rightarrow K^+ \pi^- \pi^+$
 - Expect a significant improvement of signal/background ratio w.r.t *BABAR/Belle*
 - Error on $\Delta\phi(K^*-\pi^+/K^{*-}\pi^-)$ scale as $1/\sqrt{Q}$ (effective tagging efficiency)
 \Rightarrow degrade the error by a factor $\sqrt{30.5/2.38} \sim 3.6$
 - Resolution in Dalitz plot \Rightarrow negligible effect according to LHCb experts
 - Scale the errors by the expected statistics
 - LHCb will have signal for $B^0 \rightarrow K^+ \pi^- \pi^0 / B^+ \rightarrow K_S^0 \pi^+ \pi^0$, but difficult to anticipate performances due to π^0 reconstruction efficiency and resolution
- Belle II will measure all modes: $B^0 \rightarrow K_S^0 \pi^+ \pi^-$, $B^0 \rightarrow K^+ \pi^- \pi^0$, $B^+ \rightarrow K^+ \pi^- \pi^+$ and $B^+ \rightarrow K_S^0 \pi^+ \pi^0$**
 - Experimental environment similar to *BABAR/Belle*. Will scale uncertainties by luminosity
 \Rightarrow errors should get reduced by a factor of $\sqrt{(50\text{ab}^{-1}/1.0\text{ab}^{-1})} \sim 7$
- Both LHCb and Belle II will be able to measure $B^+ \rightarrow K^+ \pi^- \pi^+$ mode with high precision**
 - Will be able to well define the signal model and probe line-shapes of the main components
 - Model systematics will be significantly reduced \Rightarrow assume negligible model uncertainty

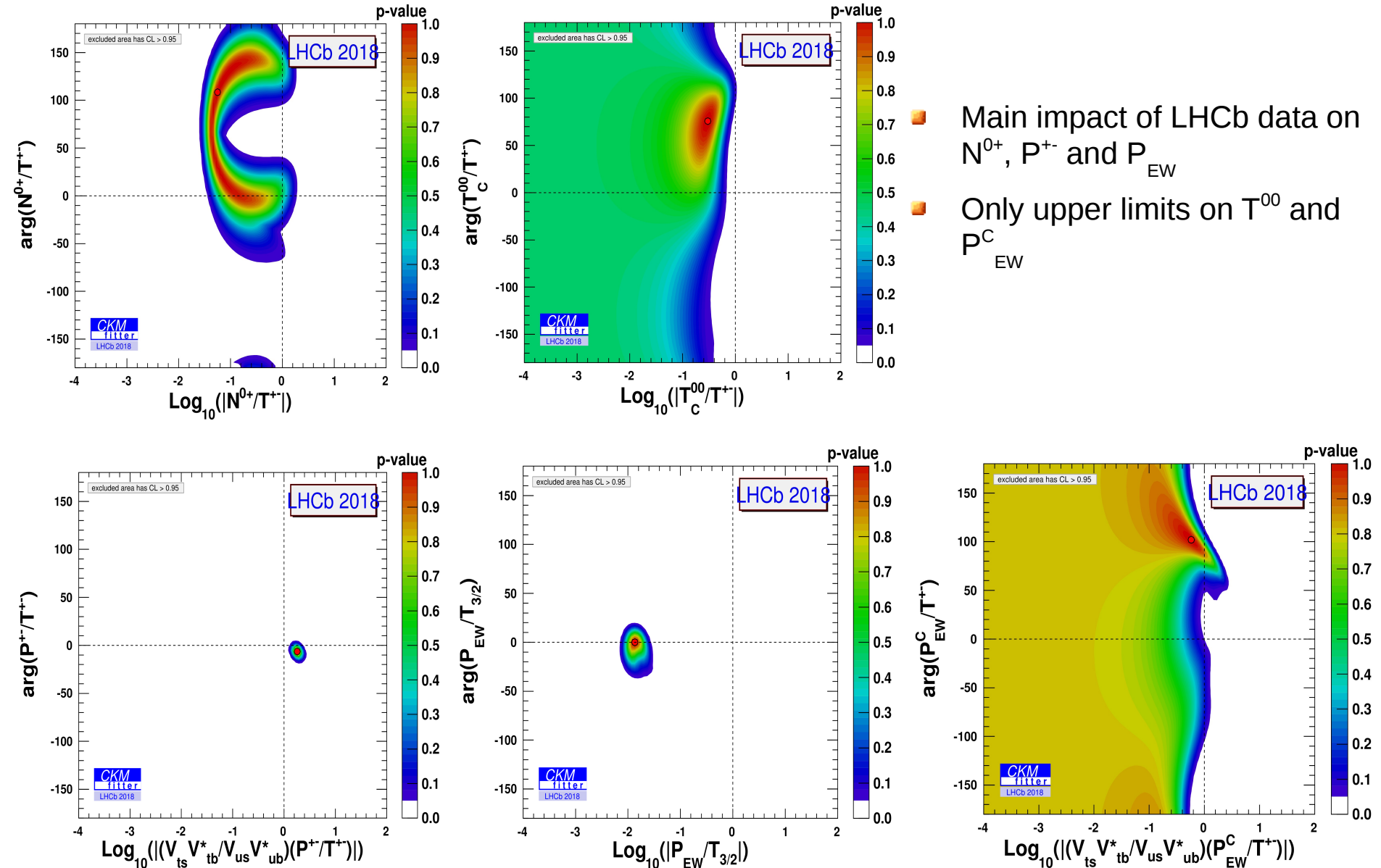
Prospects for LHCb and Belle-II (II)

Expected evolution of the uncertainties on the observables

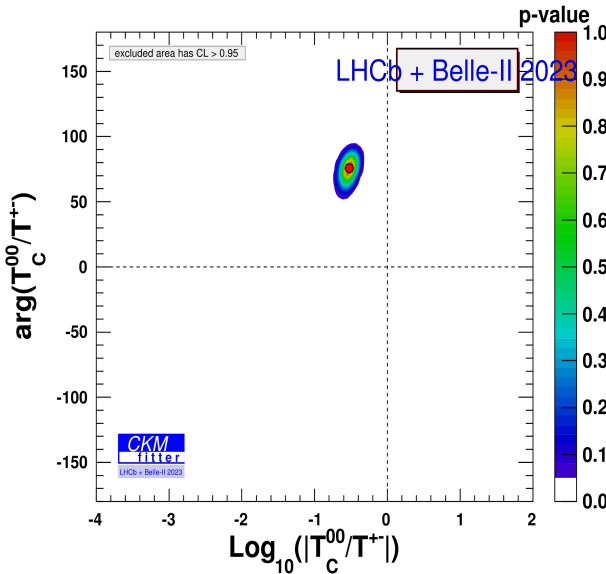
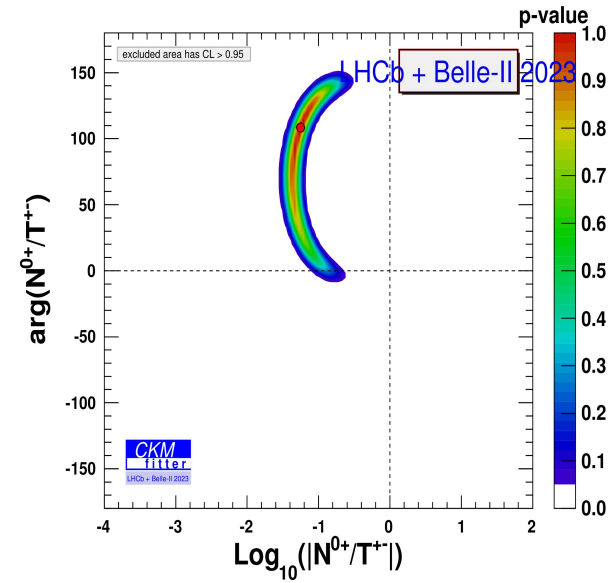
Observable	Analysis	Current	LHCb (run1+run2)	Belle-II
$\text{Re}(A(K^{*-}\pi^+)/A(K^{*+}\pi^-))$	$B^0 \rightarrow K^0 \pi^+ \pi^-$	0.11	0.04	0.014
$\text{Im}(A(K^{*-}\pi^+)/A(K^{*+}\pi^-))$	$B^0 \rightarrow K^0 \pi^+ \pi^-$	0.16	0.11	0.023
$B(K^{*-}\pi^+) \times 10^{-6}$	$B^0 \rightarrow K^0 \pi^+ \pi^-$	0.69	0.32	0.094
$ A(K^{*-}\pi^+)/A(K^{*+}\pi^-) $	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.06	0.06	0.008
$\text{Re}(A(K^{*0}\pi^0)/A(K^{*+}\pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.11	0.11	0.016
$\text{Im}(A(K^{*0}\pi^0)/A(K^{*+}\pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.23	0.23	0.033
$\text{Re}(A(\overline{K}^{*0}\pi^0)/A(K^{*+}\pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.10	0.10	0.014
$\text{Im}(A(\overline{K}^{*0}\pi^0)/A(K^{*+}\pi^-))$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.30	0.30	0.042
$B(K^{*0}\pi^0) \times 10^{-6}$	$B^0 \rightarrow K^+ \pi^- \pi^0$	0.35	0.35	0.05
$ A(\overline{K}^{*0}\pi^-)/A(K^{*0}\pi^+) $	$B^+ \rightarrow K^+ \pi^- \pi^+$	0.04	0.005	0.004
$B(K^{*0}\pi^+) \times 10^{-6}$	$B^+ \rightarrow K^+ \pi^- \pi^+$	0.81	0.50	0.113
$ A(K^{*-}\pi^0)/A(K^{*+}\pi^0) $	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.15	0.15	0.021
$\text{Re}(A(K^{*+}\pi^0)/A(K^{*0}\pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.16	0.16	0.023
$\text{Im}(A(K^{*+}\pi^0)/A(K^{*0}\pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.30	0.30	0.042
$\text{Re}(A(K^{*+}\pi^0)/A(\overline{K}^{*0}\pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.21	0.21	0.030
$\text{Im}(A(K^{*+}\pi^0)/A(\overline{K}^{*0}\pi^+))$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.13	0.13	0.018
$B(K^{*+}\pi^0) \times 10^{-6}$	$B^+ \rightarrow K^0 \pi^+ \pi^0$	0.92	0.92	0.130

- LHCb cannot perform B-counting like in B-factories
- BF are normalized w.r.t modes measured somewhere else (mainly @ B-factories)
- Error contribution from norm. modes not scaling with stat.
- $B(B^0 \rightarrow K^{*+}\pi^-)$ norm. mode: $B(B^0 \rightarrow K^0 \pi^+ \pi^-)$ ($\sigma_{\text{rel}} \sim 4\%$)
- $B(B^+ \rightarrow K^{*0}\pi^+)$ norm. mode: $B(B^+ \rightarrow K^+ \pi^- \pi^+)$ ($\sigma_{\text{rel}} \sim 5\%$)

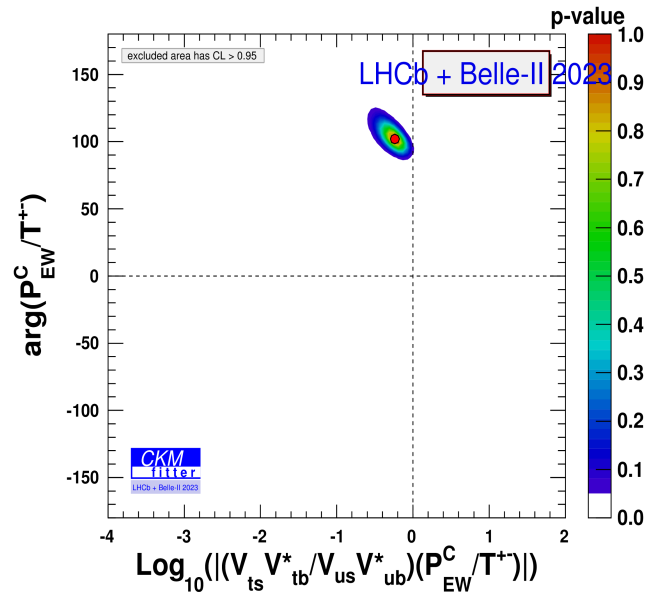
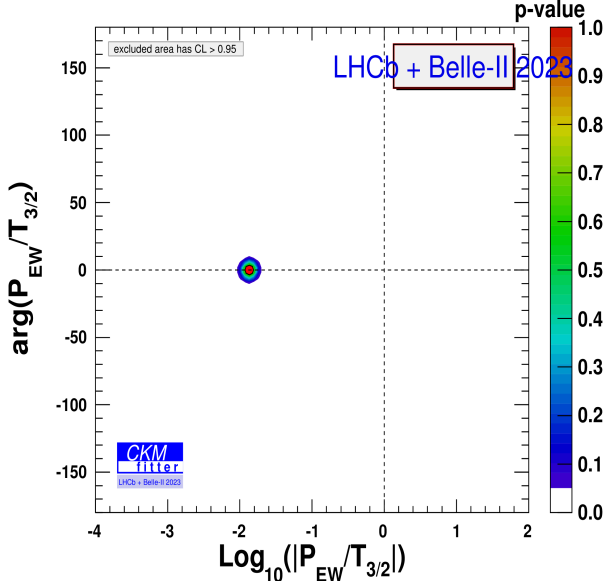
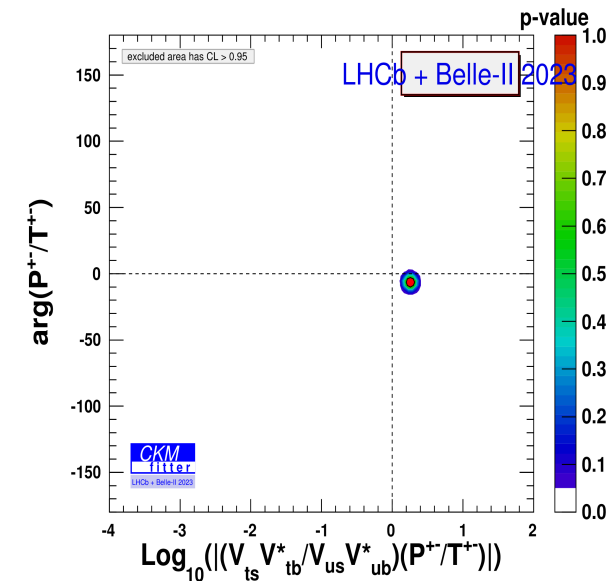
Had. Pars. : LHCb (run1+run2) 2018



Had. Pars. : LHCb + Belle-II 2023



- With LHCb + Belle-II data will be able to make precision measurements on the hadronic parameters
- Precision test of QCD predictions



Summary and Outlook

Summary and Outlook (I)

■ **$B \rightarrow K^* \pi$ system has a large amount of physical observables among charmless decays**

- Charmless B decay system with as many observables as unknowns
- Large potential for phenomenology of charmless B decays
 - Model-independent extraction of hadronic parameters (assuming CKM and SU(2) as only hypotheses)
 - Extraction of CKM parameters limited by hadronic uncertainties

■ **Extraction of CKM parameters**

- α -like constraints spoiled by sensitivity to electroweak penguins
- β -like constraints in the vanishing annihilation approximation
 - Future constraints from annihilation-dominated $B \rightarrow PV$ modes could be used
 - LHCb measurement of $B_{(s)} \rightarrow K^* K$ will play an important contribution to this program

Summary and Outlook (II)

■ Study of hadronic amplitudes with available experimental data

- For the first time, at least one complete amplitude analysis of each $B \rightarrow K\pi\pi$ mode available
- Evidence of CP-violation provides strong constraints on the relevant tree-to-penguin ratios
- Loose bounds on colour-suppressed tree and annihilation amplitudes
- Current data favours relatively large EWPs
 - Mainly driven by $BABAR$ $B^0 \rightarrow K^+\pi^-\pi^0$ analysis
 - If confirmed, would set evidence for EWPs in charmless B decays
 - Until now, EWPs only established in $\varepsilon'/\varepsilon \neq 0$ (radiative B decays are different operators)

■ Expect significant improvements with LHCb and Belle II data

- Model-independent measurement of all hadronic parameters
 - Both amplitudes and phases can be measured with outstanding accuracy
- Results on hadronic $B \rightarrow K^*\pi$ parameters can be used as “standard candles” to study other $B_{(s)} \rightarrow PV$ modes
 - $B_s \rightarrow K^*\pi$, $B_{(s)} \rightarrow K^*K$, $B_{(s)} \rightarrow \rho K$

Back up Slides

Parameterization

Parameterizing Decay amplitude using Isobar Model:

Dalitz Plot
Isobar Model

$$\begin{cases} A(DP) = \sum a_j F_j(DP) \\ \bar{A}(DP) = \sum \bar{a}_j \bar{F}_j(DP) \end{cases}$$

Shapes of intermediate states over DP

$$F_j^L(DP) = R_j(m) \times X_L(|\vec{p}^*| r) \times X_L(|\vec{q}| r) \times T_j(L, \vec{p}, \vec{q})$$

Rela

Gou

S-wave $K\pi$:

$$R_j(m_{K\pi}) = \underbrace{\frac{m_{K\pi}}{q \cot \delta_B - iq}}_{\text{Effective Range Term}} + e^{2i\delta_B} \frac{m_0 \Gamma_0 \frac{m_0}{q_0}}{(m_0^2 - m_{K\pi}^2) - im_0 \Gamma_0 \frac{q}{m_{K\pi}} \frac{m_0}{q_0}}$$

LASS lineshape.

Nucl. Phys., B296:493, 1988

B→pK System: Physical Observables

$$A(B^0 \rightarrow \rho^+ K^-) = V_{us} V_{ub}^* t^{+-} + V_{ts} V_{tb}^* p^{+-}$$

$$A(B^+ \rightarrow \rho^0 K^+) = V_{us} V_{ub}^* n^{0+} + V_{ts} V_{tb}^* (-p^{+-} + p_{EW}^C)$$

$$\sqrt{2}A(B^+ \rightarrow \rho^+ K^0) = V_{us} V_{ub}^* (t^{+-} + t_C^{00} - n^{0+}) + V_{ts} V_{tb}^* (p^{+-} - p_{EW}^C + p_{EW})$$

$$\sqrt{2}A(B^0 \rightarrow \rho^0 K^0) = V_{us} V_{ub}^* t_C^{00} + V_{ts} V_{tb}^* (-p^{+-} + p_{EW})$$

11 QCD and 2 CKM = 13 unknowns

Same Isospin relations as $K^* \pi$

Observables:

- 4 BFs and 4 A_{CP} from DP and Q2B analyses.

- 1 phase differences:

$$* 2\beta_{eff} = \arg((q/p) \overline{A}(\overline{B}^0 \rightarrow \rho^0 \overline{K}^0) A^*(B^0 \rightarrow \rho^0 K^0)) \text{ from } B^0 \rightarrow K_S^0 \pi^+ \pi^-$$

Under constraint system. Still some constraints possible

A total of 9 observables

$\rho K + K^* \pi$ system: Physical Observables

Global phase between $K^* \pi$ and ρK now accessible:

- $K^* \pi$: 11 hadronic parameters (1 global phase fixed)
- ρK : 12 parameters
- CKM: 2 parameter

A total of = 25 unknowns

Observables:

- $K^* \pi$ only: 13 observables
- ρK only: 9 observables
- 7 phase differences from: interference between $K^* \pi$ and ρK resonances contributing to the same DP
 - $\phi = \arg(A(B^0 \rightarrow \rho^0 K^0) A^*(B^0 \rightarrow K^{*+} \pi^-))$ from $B^0 \rightarrow K_S^0 \pi^+ \pi^-$
 - $\phi = \arg(A(B^0 \rightarrow \rho^- K^+) A^*(B^0 \rightarrow K^{*+} \pi^-))$ and CP conjugated from $B^0 \rightarrow K^+ \pi^- \pi^0$
 - $\phi = \arg(A(B^0 \rightarrow \rho^0 K^+) A^*(B^0 \rightarrow K^{*0} \pi^+))$ and CP conjugated from $B^+ \rightarrow K^+ \pi^- \pi^+$
 - $\phi = \arg(A(B^0 \rightarrow \rho^+ K^0) A^*(B^0 \rightarrow K^{*+} \pi^0))$ and CP conjugated from $B^+ \rightarrow K^0 \pi^+ \pi^0$

A total of 29 experimentally independent observables

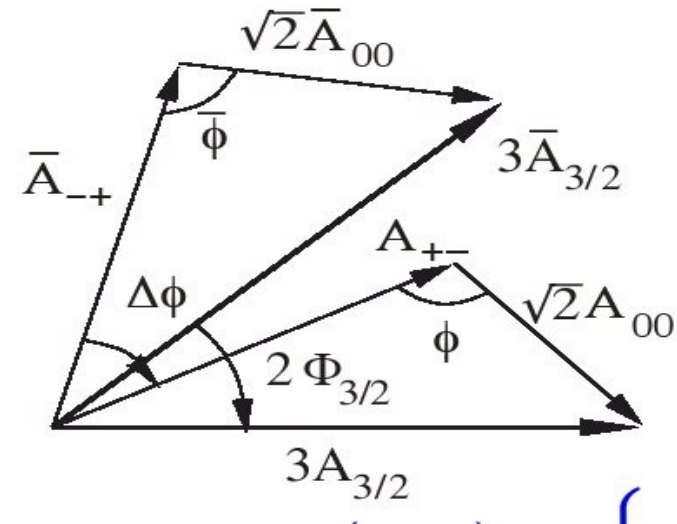
B→K*π System: extraction of α (CPS/GPSZ)

Neglecting P_{EW} , the amplitude combinations:

$$3A_{3/2} = A(B^0 \rightarrow K^{*+} \pi^-) + \sqrt{2} A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00})$$

$$3\bar{A}_{3/2} = \bar{A}(B^0 \rightarrow K^{*-} \pi^+) + \sqrt{2} \bar{A}(B^0 \rightarrow \bar{K}^{*0} \pi^0) = V_{us}^* V_{ub} (T^{+-} + T^{00})$$

which gives: $R_{3/2} = (q/p)(3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$



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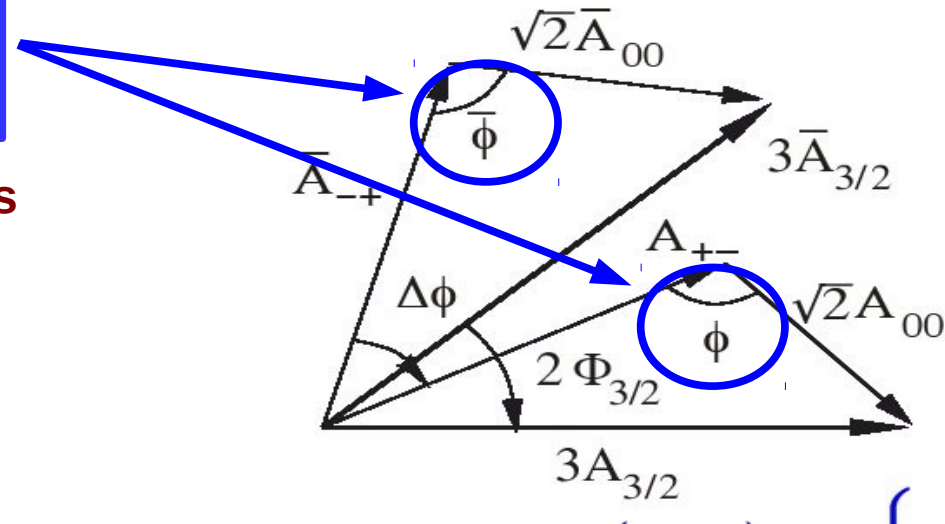
which gives: $R_{3/2} = (q/p)(3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$

From experiment:

$$\phi = \arg(A(B^0 \rightarrow K^{*+} \pi^-) A^*(B^0 \rightarrow K^{*0} \pi^0))$$

$$\bar{\phi} = \arg(\bar{A}(B^0 \rightarrow K^{*-} \pi^+) \bar{A}^*(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0))$$

Measured from an amplitude analysis
of $B^0 \rightarrow K^+ \pi^- \pi^0$ decays



B→K*π System: extraction of α (CPS/GPSZ)

Neglecting P_{EW} , the amplitude combinations:

$$3A_{3/2} = A(B^0 \rightarrow K^{*+} \pi^-) + \sqrt{2} A(B^0 \rightarrow K^{*0} \pi^0) = V_{us} V_{ub}^* (T^{+-} + T^{00})$$

$$3\bar{A}_{3/2} = \bar{A}(\bar{B}^0 \rightarrow K^{*-} \pi^+) + \sqrt{2} \bar{A}(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0) = V_{us}^* V_{ub} (T^{+-} + T^{00})$$

which gives: $R_{3/2} = (q/p)(3A_{3/2})/(3\bar{A}_{3/2}) = e^{-2i\beta} e^{-2i\gamma} = e^{-2i\alpha}$

From experiment:

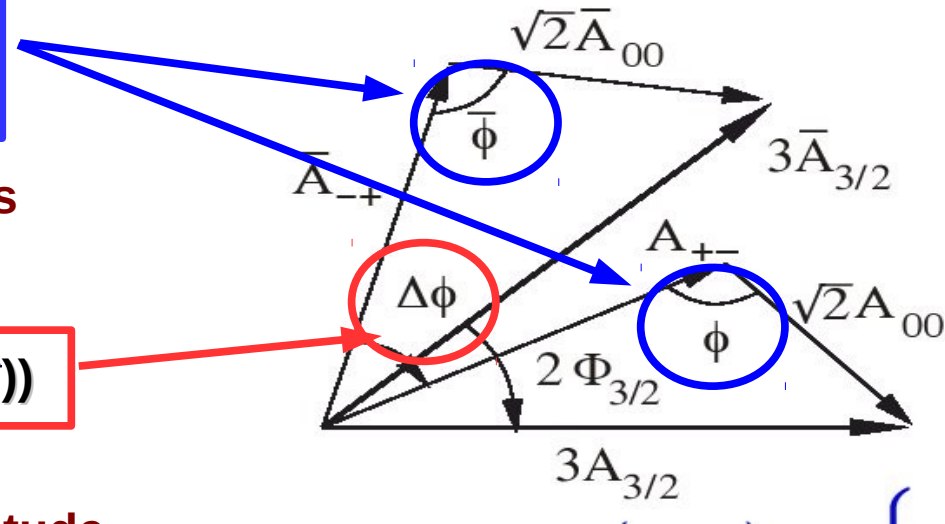
$$\phi = \arg(A(B^0 \rightarrow K^{*+} \pi^-) A^*(B^0 \rightarrow K^{*0} \pi^0))$$

$$\bar{\phi} = \arg(\bar{A}(\bar{B}^0 \rightarrow K^{*-} \pi^+) \bar{A}^*(\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0))$$

Measured from an amplitude analysis
of $B^0 \rightarrow K^+ \pi^- \pi^0$ decays

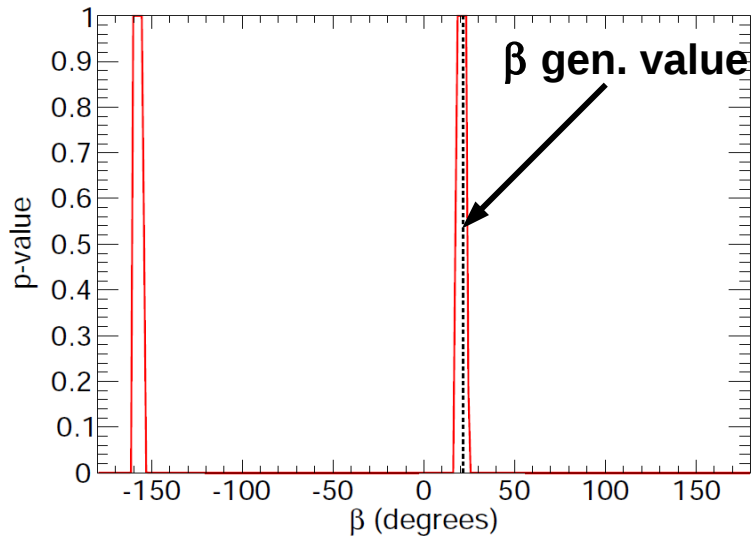
$$\Delta\phi = \arg((q/p) \bar{A}(\bar{B}^0 \rightarrow K^{*-} \pi^+) A^*(B^0 \rightarrow K^{*+} \pi^-))$$

Measured from a time-dependent amplitude
analysis of $B^0 \rightarrow K_s^0 \pi^+ \pi^-$ decays

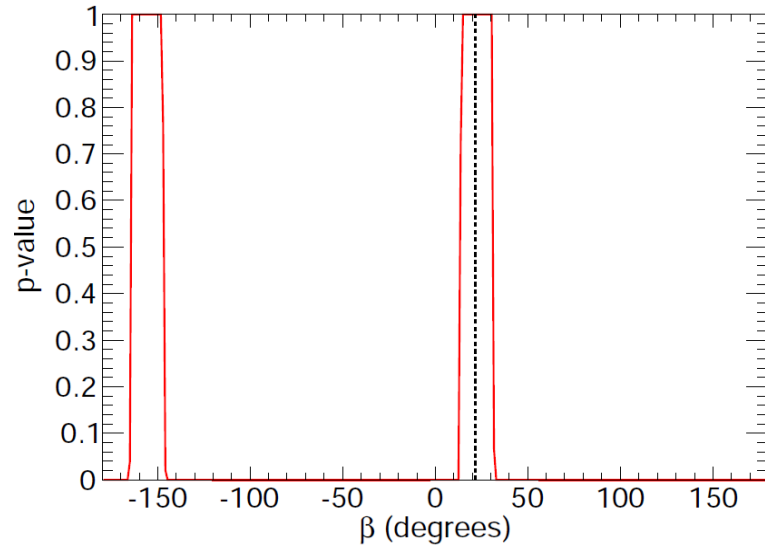


Scenarios to constrain CKM: hypothesis on N^{0+} (III)

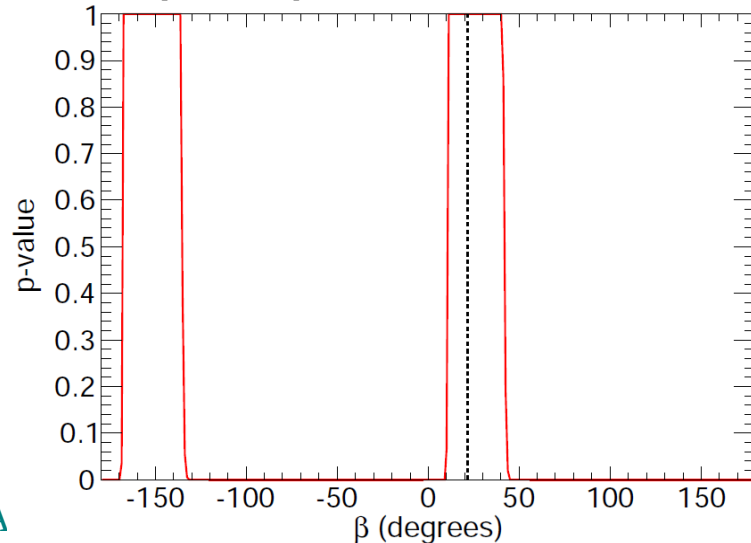
$|N^{0+}/T^{+-}| < 1.5 \times (\text{gen val})$



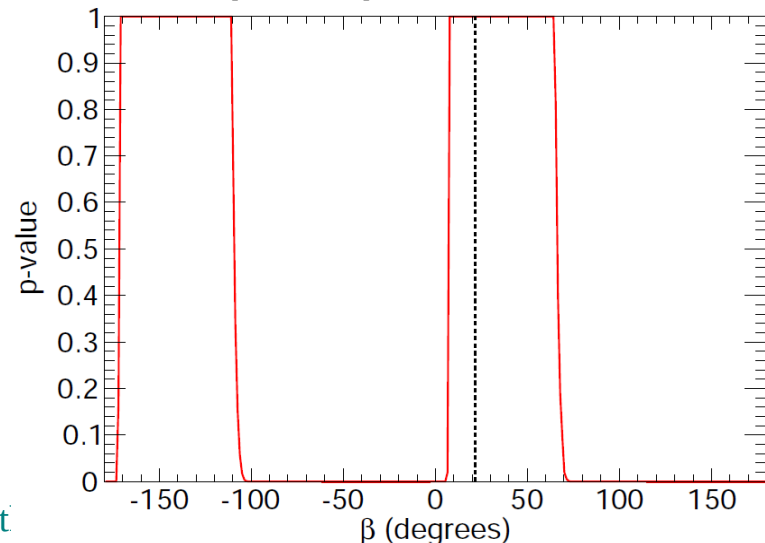
$|N^{0+}/T^{+-}| < 5 \times (\text{gen val})$



$|N^{0+}/T^{+-}| < 10 \times (\text{gen val})$

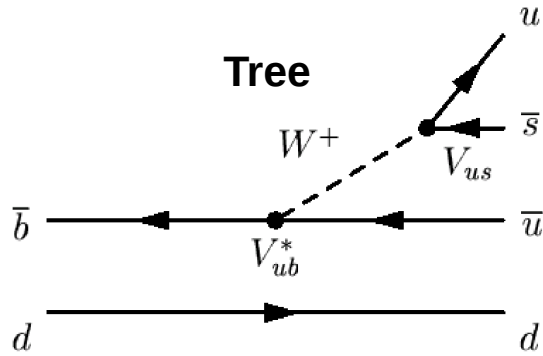


$|N^{0+}/T^{+-}| < 15 \times (\text{gen val})$

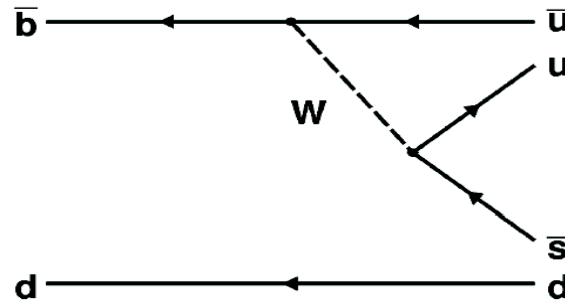


Feynman Diagrams

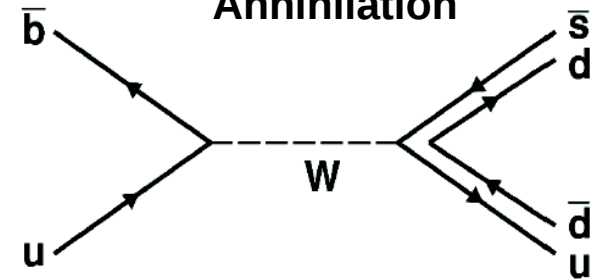
Tree



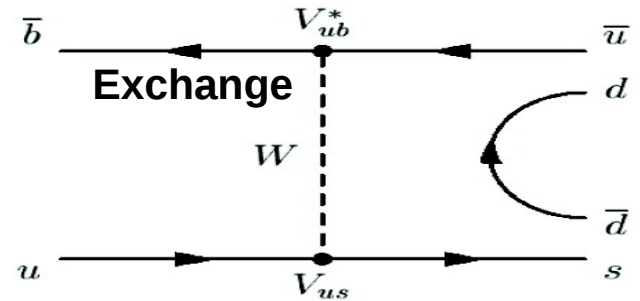
Colour suppressed three



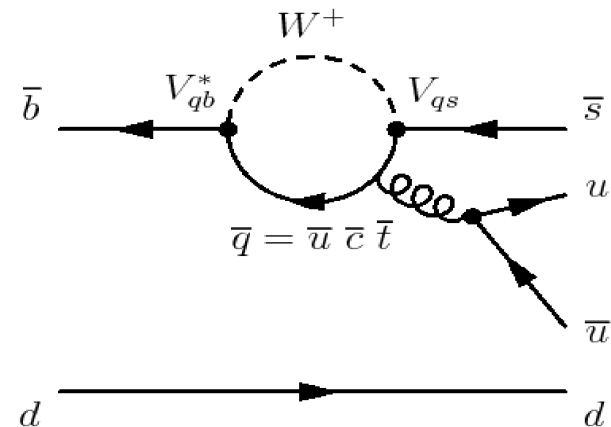
Annihilation



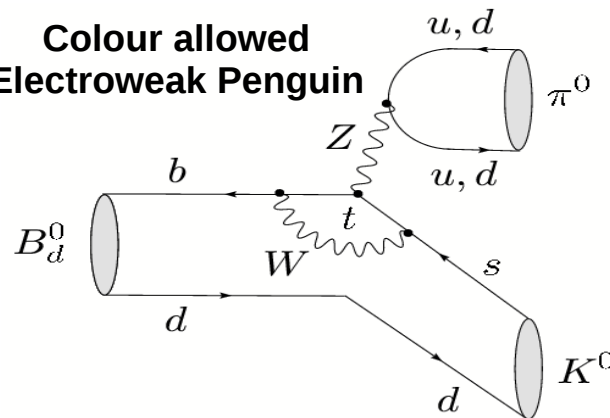
Exchange



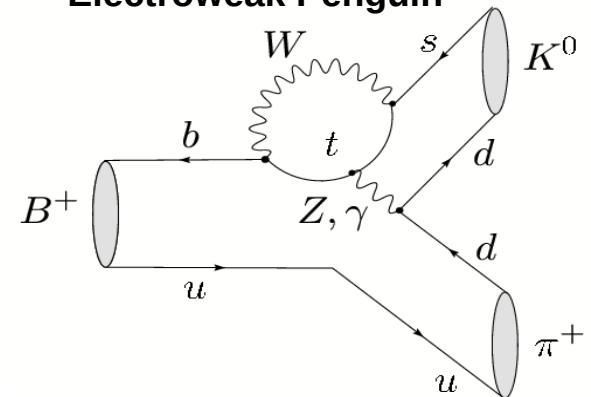
Gluonic Penguin



Colour allowed Electroweak Penguin



Colour suppressed Electroweak Penguin



CPS/GPSZ theoretical prediction

- Effective Hamiltonian of $B \rightarrow K^* \pi$

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1,2} c_i (\Omega_u Q_i^u + \Omega_c Q_i^c) - \Omega_t \sum_{i=3}^{10} c_i Q_i \right\} + \text{h.c.}, \text{ with } \Omega_q = V_{qs} V_{qb}^*$$

- Hierarchy of Wilson coefficients for electro-weak operators $|c_{9,10}| \gg |c_{7,8}|$

$$[\mathcal{H}_{EWP}]_{\Delta I=1} = \frac{3}{2} \frac{c_9 + c_{10}}{2} [Q_1^u + Q_2^u]_{\Delta I=1} + \frac{3}{2} \frac{c_9 - c_{10}}{2} [Q_1^u - Q_2^u]_{\Delta I=1} \quad \text{Electro-weak Hamiltonian}$$

$$[\mathcal{H}_{CC}]_{\Delta I=1} = \frac{c_1 + c_2}{2} [Q_1^u + Q_2^u]_{\Delta I=1} + \frac{c_1 - c_2}{2} [Q_1^u - Q_2^u]_{\Delta I=1} \quad \text{Current-current Hamiltonian}$$

- Using $\left(\frac{c_9 + c_{10}}{c_1 + c_2} \simeq -0.0084 \right) \simeq \left(\frac{c_9 - c_{10}}{c_1 - c_2} \simeq +0.0084 \right)$

$$[\mathcal{H}_{EWP}]_{\Delta I=1} = R \frac{c_1 + c_2}{2} [Q_1^u + Q_2^u]_{\Delta I=1} - R \frac{c_1 - c_2}{2} [Q_1^u - Q_2^u]_{\Delta I=1}$$

$$R = (3/2)(c_9 + c_{10})/(c_1 + c_2)$$

$$R = (1.35 \pm 0.12)\%$$

- Obtain the relation $P_{EW} = R_{\text{eff}} (T^{+-} + T^{00})$,

$$\text{with } R_{\text{eff}} = R(1 + r_{VP})/(1 - r_{VP}) \quad r_{VP} = \frac{\langle K^* \pi(I=3/2) | Q_- | B \rangle}{\langle K^* \pi(I=3/2) | Q_+ | B \rangle}, \quad Q_{\pm} = (Q_1 \pm Q_2)/2.$$

$$r_{VP} = \left| \frac{f_{K^*} F_0^{B \rightarrow \pi} - f_{\pi} A_0^{B \rightarrow K^*}}{f_{K^*} F_0^{B \rightarrow \pi} + f_{\pi} A_0^{B \rightarrow K^*}} \right| \lesssim 0.05$$

Outlook

■ CKM constraints

- $B^0 \rightarrow K^{*0} \pi^+$ and $B_s^0 \rightarrow K^{*0} K^+$ modes related by U-spin
 \Rightarrow expects the same annihilation amplitude (N^{0+}) up to U-spin breaking effects
$$A(B_s^0 \rightarrow K^{*0} K^+) = V_{ud} V_{ub}^* N^{0+} + V_{ts} V_{tb}^* P_s^{0+}$$
- LHCb will measure this modes in the near future
- Can include this mode in our phenomenological framework to set a bound on N^{0+} and be able to set constraints on CKM

■ Extending the $B \rightarrow K^* \pi$ system: include $B \rightarrow \rho K$ modes

- $B \rightarrow \rho K$ resonances also contribute to the $B \rightarrow K \pi \pi$ final states and have same isospin relations as $B \rightarrow K^* \pi \Rightarrow$ same number of hadronic parameters
- Smaller number of observables (9) than $B \rightarrow K^* \pi$ (13), but can measure interference phases (7) between $B \rightarrow K^* \pi$ and $B \rightarrow \rho K$ modes
- Combined system $B \rightarrow K^* \pi + B \rightarrow \rho K$
 - Unknowns: 11 + 12 hadronic from $B \rightarrow K^* \pi$ and $B \rightarrow \rho K$ + 2 CKM = 25
 - Observables: 13 + 9 from $B \rightarrow K^* \pi$ and $B \rightarrow \rho K$ + 7 phase differences = 28
 - Still need hypothesis on hadronic or CKM to raise reparametrization invariance