# SHORT AND BIASED NOTES ON DARK MATTER

# I. EVIDENCES FOR COLD DARK MATTER
# II. THERMAL FREEZE-OUT & WIMP MODELS
# III. WIMP SEARCHES

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### Introduction

The aim of these lectures is to remind you some fundamental and yet basic aspects of dark matter phenomenology. I will not review all existing dark matter models: there are just too many of them. Furthermore, they are for the essential variations around the idea that dark matter is made of relic particles from the early universe. The most studied possibility is that these particles are WIMPs, or weakly interacting massive particles and this possibility will be the topic of my lectures.
What I will not discuss, but mention for completeness, are the other particle candidates and the prominent astrophysical candidates, primordial black holes. Some other particle candidates are axion-like particles (which are very light, possibly lighter than the neutrinos) and topological defects, like monopoles (which are extremely heavy, $M \sim$ GUT scale — not much considered but that does not mean anything). Primordial black holes are also a possibility, provided they are not too light ($M \gtrsim 10^{16}$ g — about the mass of the Icarus asteroid) so as not too emit too much gamma rays through Hawking evaporation, or too heavy ($M \lesssim 10^{26}$ g — about ten times the mass of the Moon) to be seen through gravitational lensing.

The plan of the lectures is as follows.

**EVIDENCES FOR COLD DARK MATTER** There are evidences for dark matter at different scales in the Universe, from dwark spheroidal galaxies, to galaxies, including ours, cluster of galaxies (important are mergers — like the Bullet Cluster) and the large scale structure of the Universe. The first three cases are briefly discussed. They are easy to grasp. Less obvious is the role of dark matter on large scales. I will discuss the basic picture (Jeans instability in an expanding universe). This is an important, yet basic, lesson. I will do it for massive objects (like WIMPs, or PBH). For light particles (like neutrinos), there is an analogous to the Jeans mass. The latter helps understanding why we think that dark matter is cold and not hot. In between there is the possibility of warm dark matter. I will discuss that only briefly.

**THERMAL FREEZE-OUT & WIMP MODELS** I will discuss the basic picture of freeze-out of particle abundances in the early universe, including hot and cold relics. The latter case is important as it points to the possibility that dark matter is made of weakly interacting massive particles. I will detail the calculations, and illustrate the results by simple models, with some emphasis on the relevance of symmetries. I will also discuss the reason why WIMPs cannot be arbitrarily heavy, because of unitarity constraints.

**WIMP SEARCHES** There are three basic strategies for searches of WIMPs: direct, indirect and production at colliders (ie the LHC). This is the principal reason why WIMPs are so much studied. Production at colliders is not going to be discussed, although I will mention some easy to understand aspects. Instead, I discuss the basics of direct detection and of annihilation into gamma rays. The focus is on easy, yet calculational aspects.

**APPENDIX** There I put some relevant numbers, conversion factors, and equations.

Caveat: These notes are very sketchy and contain essentially only the calculations and some relevant figures. Most words are left for the oral lectures.

## I. EVIDENCES FOR COLD DARK MATTER

### A. Galaxies and dwarf galaxies

This is clearly a huge topic. Perhaps I discuss NFW and isothermal. Well-known problems are the core/cusp, missing satellites issue and too-big-to-fail issues.

The basic observation is the rotation curve (the plot of the rotational velocity as a function of distance to the centre $r$) of spiral galaxies (there are both optical and radio observations). Applying Newton's law to a spherically symmetric distribution of mass gives

$$\frac{GM(\leq r)}{r^2} = \frac{v^2}{r}$$

where $v$ is the rotational velocity. This implies that $v \propto 1/\sqrt{r}$ far from the region where most of the mass exists. From visible matter, this is the core (central region) of the Galaxy. At the position of the Sun, which is about 8 kpc, the velocity should be less than what is observed, $v \approx 220$km/s. Instead a "plateau" is observed, $v(r) \sim$ const, and this in all spiral galaxies. This behaviour is easily if there exists a halo of dark matter on top of the visible matter (visible in optical and through radio observations, that is redshifted 21 cm lines of atomic hydrogen).

$$v \sim \text{const} \quad \rightarrow \quad M(\leq r) \propto r$$

or

$$M(\leq r) = 4\pi \int_0^r \rho(r) r^2 dr \quad \rightarrow \quad \rho(r) \propto r^{-2}$$

for large $r$. This behaviour for $\rho(r)$, which is called the dark matter profile, is characteristic of an isothermal sphere of matter at large $r$ (meaning pressure and matter density are related by the perfect gas law $p = \frac{\rho T}{m}$ with $T$ constant and $m$ the mass of dark matter particles, gas, stars,...). (I set $k_B = 1$). The exact solution is not singular at the origin. Instead it becomes constant. We say that the isothermal distribution is cored. A simple parametrization (ie this is not a solution of any fundamental — numerical or analytical — calculation) is

$$\rho_{\text{ISO}}(r) = \frac{\rho_s}{1 + (r/r_s)^2}$$

where $\rho_s$ is the density at the origin and $r_s$ is the size of the core. The large $r$ behaviour is still $\propto r^{-2}$ meaning that $M$ diverges. This is unphysical for the following reasons. First the thermalization assumption breaks down as for low densities the mean free path becomes too long compared to the age of the system. Second, a galaxy (the same old for cluster of galaxies) are not isolated. Instead there other galaxies (cluster of galaxies) around that can strip the outer matter (stars, gas, dark matter) from each others through tidal interactions.
For the Milky Way, a fit to the local DM density $\rho_\odot = 0.3$ GeV/cm$^3$ and rotation curve corresponds to $r_s = 4.4$ kpc and $\rho_s = 1.4$ GeV/cm$^3$. Notice that for the ISO profile the density of DM at the center is not dramatically different than here.
While this profile is in good agreement with observations (of spiral galaxies, low surface brightness galaxies, dwarf spheroidal galaxies), this is not what comes out of N-body numerical simulations. Instead most simulations see a cuspy (not cored) profile. The most popular example is the Navarro-Frenk-White profile,

$$\rho_{\text{NFW}} = \frac{r_s}{r} \frac{\rho_s}{(1 + r/r_s)^2}$$

with now $r_s \approx 24$ kpc and $\rho_S = 0.2$ GeV/cm$^3$. This profile is singular at the origin (cuspy). This feature is not quite consistent with observations (although the observations are very uncertain for spiral galaxies). A good feature however is the falloff like $r^{-3}$ for large $r$ (this is more consistent with observations and physics). Whether the cusp is physical or becomes more cored if baryon effects are taken into account is at the core of the current research in numerical simulations.
Very briefly, the precise profile of DM is not well-known and understood. Both the NFW (which may predict large flux of particles from dark matter annihilation) and the ISO profile are used as benchmarks profile (clearly the latter is more conservative, and possible more correct). We will get back to this in the last lecture.

### B. Clusters of galaxies and mergers

The first historical evidence for non-luminous matter in the universe is from observations of the dynamics of clusters of galaxies (Coma cluster, called *dunkle Materie* = DM by Fritz Zwicky, 1933). The luminous matter (galaxies) account for 1% of the matter in clusters of galaxies; 9% is in the form of intergalactic gas (ICM = intracluster medium), a hot plasma of ionized hydrogen and helium, representing essentially all the baryonic content (this plasma is visible in X-rays — $T = 10^{7-8}$ K $\equiv 10^{3-4}$eV); 90% is supposed to be made of dark matter, distributed roughly in a spherical halo (hence roughly 10:1 dark matter-to-baryon ratio; this is a factor of 2 larger than the cosmological value. I don't know how serious is that an issue). Whether there is a core or a cusp is unclear.

This sketch of a galaxy cluster is supported by observations of merging clusters of galaxies, the most famous being the Bullet Cluster. More recently many mergers have been observed (in visual, X-rays and through gravitational lensing — the latter measures the total gravitational potential). The observed displacement allows to put constraint on possible interactions between dark matter particles — self-interacting dark matter or SIDM.

### C. Large scale structure and Jeans instability

This is a vast and complex topic. The basic problem is to explain the origin and evolution of inhomogeneities in an otherwise homogeneous and isotropic universe on very large scales.
The density contrast is defined as

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

where $\rho(\vec{x})$ is the actual energy (or mass, we anyway set $c = 1$) distribution, and $\bar{\rho}$ its average value. It is convenient to work in Fourier space

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \delta(\vec{k}) e^{-i\vec{k}\cdot\vec{x}}.$$

In an expanding universe, the physical distances grow proportional to a scale factor, $\vec{x}_{\rm phys} = a(t)\vec{x}$. The Hubble parameter is given by $H = \frac{1}{a}\frac{da}{dt}$, where $t$ is the age of the universe (the same for all observers that see the universe as isotropic and homogeneous on large scales = comoving observers). The distance $\vec{x}$ is a comoving coordinate, in the sense that comoving observers are at rest with respect to these coordinates. In the Fourier transform, it proves useful to use comoving coordinates, so that the conjugate wavenumber $\vec{k}$ is also comoving ($k = 2\pi/\lambda$, with $\lambda$ a time independent characteristic length).
The matter power spectrum $|\delta(k)|^2$ is the basic statistical tool to quantify the distribution of inhomogeneities in the universe. It tells us 1/ on which scales can we say that universe is close to isotropy and homogeneity, 2/ the absolute normalization of the perturbations (how much on a given scale) 3/ the wavenumber dependence. If the inhomogeneities are gaussian distributed, all the statistical information is captured by this 2-point function. Here I follow the notations of Kolb and Turner [1] and define

$$\frac{\delta\rho}{\rho} = \langle \delta(x)\delta(x) \rangle^{1/2}$$

where the average is over all space (as usual, it is assumed that average on space is equivalent to average over the statistical ensemble — that obviously breaks down on the largest scales, where the sampling is poor, i.e. we have only one universe). Using the Fourier transform, one gets

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \frac{1}{V}\int_0^\infty \frac{k^3|\delta(k)|^2}{2\pi^2}\frac{dk}{k}$$

so that the contribution to the density contrast on a scale $\lambda \sim 1/k$ for a log interval $d\ln k$ is given by the density variance. (There are different convention regarding the volume factor. Here it is absorbed in the definition of $\delta(k)$.)

$$\Delta^2(k) = \frac{1}{V}\frac{k^3|\delta(k)|^2}{2\pi^2} \sim \left(\frac{\delta\rho}{\rho}\right)^2_\lambda \tag{1}$$

Figure 1 shows a compilation of data on $\Delta(k)$ on various cosmological scales. The basic info I want to use is that $\frac{\delta\rho}{\rho} \ll 1$ on the largest scales, which of course what we mean by saying that the universe is homogeneous on the large. There is of course much more info in the figure, to which I will briefly come back later.
Since inhomogeneities are small ($\delta\rho/\rho \ll 1$) on large scales, their evolution may be described by linearized equations of motion (first order in a perturbative expansion). This is quite complex in general relativity, so we do it in the newtonian approximation, valid for perturbations within the horizon (at a given time), $\lambda \lesssim 1/H$. We will see that inhomogeneities are unstable and grow (Jeans instability). We do it first for a static distribution of matter. Then for an expanding universe.

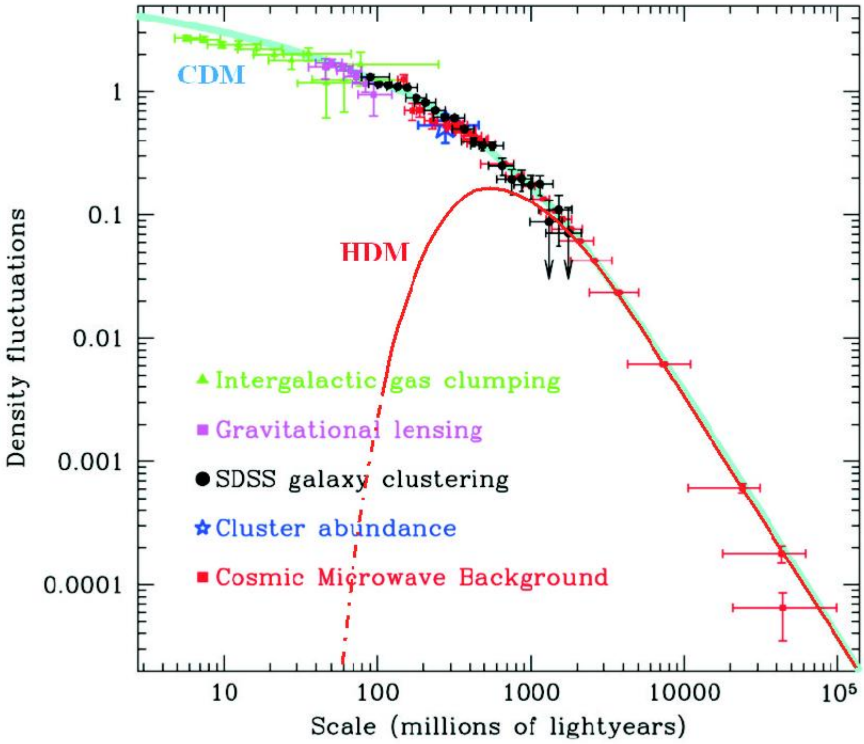


FIG. 1: Plot of variance $\Delta(k)$ ($\equiv$ density fluctuations) from a compilation of observations on different scales, from Lyman-$\alpha$ forest data (intergalactic gas clouds) to the anisotropies in the CMB. The fluctuations averaged on scales larger than $\sim 10$ Mpc (1 pc $\approx 3.3$ ly) are smaller than $O(1)$ and are in a linear regime. The continuous line is for a flat CDM model, with $\Omega_M = 0.28$ and $h = 0.72$. The line labelled HDM corresponds to the hot dark matter prediction. Figure from Max Tegmark.

### 1. *Jeans instability in a non-expanding fluid*

We characterize matter by the density $\rho(x,t)$ and velocity field $\vec{v}(x,t)$. In this fluid approximation, we have 4 basic equations,

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p - \vec{\nabla}\phi \tag{2}$$

$$\nabla^2\phi = 4\pi G\rho$$

$$p = p(\rho) \tag{3}$$

The first equation ensures conservation of matter (continuity), the second is the Euler equation (Newton applied to a fluid) in a Newtonian potential $\phi$, the third one is the Poisson equation and the last one, necessary to close the system, is the equation of state (we assume that the fluid is ideal).
We study small disturbances in the fluid

$$\begin{aligned} \rho &= \rho_0 + \rho_1 \\ p &= p_0 + p_1 \\ \vec{v} &= \vec{v}_0 + \vec{v}_1 \\ \phi &= \phi_0 + \phi_1 \end{aligned} \tag{4}$$

Pressure and density are related by the speed of sound,

$$v_s^2 = \frac{\partial p}{\partial \rho}.$$

To first order we get

$$\begin{aligned}
&\frac{\partial\rho_1}{\partial t}+\rho_0\vec{\nabla}\cdot\vec{v}_1=0\\
&\frac{\partial\vec{v}_1}{\partial t}+\frac{v_s^2}{\rho_0}\vec{\nabla}\rho_1+\vec{\nabla}\phi_1=0\\
&\nabla^2\phi_1=4\pi G\rho_1
\end{aligned} \tag{5}$$

Putting together the above equation, we can derive an equation for $\rho_1$ alone[1]

$$\boxed{\frac{\partial\rho_1}{\partial t^2}-v_s^2\nabla^2\rho_1=4\pi G\rho_0\rho_1} \tag{6}$$

The equation is linear, so we look for plane wave solutions

$$\rho_1\propto\exp\left(-i\vec{k}\cdot\vec{r}+i\omega t\right)$$

and get the mode satisfy the dispersion relation

$$\omega^2=v_s^2k^2-4\pi G\rho_0 \tag{7}$$

There are two possible regimes, depending on the wavenumber $k=|\vec{k}|$. For short wavelengths compared to the so-called Jeans wavenumber $k_J$

$$k>k_J\equiv\sqrt{\frac{4\pi G\rho_0}{v_s^2}}$$

$\omega$ (7) is real and the solutions are oscillatory. For long wavelength on the contrary $\omega$ is imaginary and one solution is exponentially growing. This gravitational collapse is called the Jeans instability.
Physically there is a competition between two opposite effects, pressure and gravity. Pressure, which prevents the growth of density, acts on timescale $\tau_p\sim\lambda/v_s$. Using Newtonian's constant $G$ and themean density $\rho$ we can associate a time scale characteristic of gravitational collapse $\tau_g\sim 1/\sqrt{G\rho}$. If, for a given scale $\lambda$, $\tau_p\ll\tau_g$, pressure wins. Otherwise there is gravitational collapse. The intermediate scale is nothing but

$$\lambda_J\sim\frac{v_s}{\sqrt{G\rho}}\sim\frac{1}{k_J}$$

Notice that $\lambda_J\propto v_s$, so that perturbations in a pressureless gas (eg dark matter) are a priori unstable. The expansion of the universe changes this picture in important ways.

### 2. *Jeans instability in an expanding universe*

The unperturbed solution is now caracterized by (we consider matter, not radiation)

$$\rho_0=\rho_0(t)a^{-3}(t),\qquad \vec{v}_0=\frac{\dot{a}}{a}\vec{r},\qquad \vec{\nabla}\phi_0=\frac{4\pi G\rho_0}{3}\vec{r}$$

Notice that for $r\gtrsim H^{-1}$ with $H=\dot{a}/a$ the velocity of the fluid exceeds the speed of light, so the Newtonian approximation is only valid for scales that are within the horizon $r\lesssim H^{-1}$. [2] For small perturbations, we have

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[1] We assumed that $\phi_0=0$. That this is inconsistent (The Jeans swindle) is another way of seeing that the system has an exponential instability.
[2] It is an interesting exercice to derive the cosmological equations starting from the fluid equations. The equation of continuity gives

$$\frac{d\rho_0}{dt}+\rho_0\vec{\nabla}\cdot\vec{v}_0=0\rightarrow\dot{\rho}_0+3H\rho_0=0.$$

$$\frac{\partial\rho_1}{\partial t}+3H\rho_1+H(\vec{r}\cdot\vec{\nabla})\rho_1+\rho_0\vec{\nabla}\cdot v_1=0$$
$$\frac{\partial\vec{v}_1}{\partial t}+H\vec{v}_1+H(\vec{r}\cdot\vec{\nabla})\vec{v}_1+\frac{v_s^2}{\rho_0}\vec{\nabla}\rho_1+\vec{\nabla}\phi_1=0$$
$$\nabla^2\phi_1=4\pi G\rho_1$$

The density constrat being $\delta=\rho_1/\rho_0$, we expand in Fourier modes as follows

$$\delta(\vec{r},t)=\int\frac{d^3k}{(2\pi)^3}\delta_k(t)e^{-i\vec{k}\cdot\vec{r}/a(t)}$$

In this Fourier decomposition, the wavenumber $\vec{k}$ is as above conjugate to the comoving coordinate $\vec{r}/a=\vec{x}$ (comoving wavenumber). Using this, the equations become

$$\frac{d\delta_k}{d\eta}-i\vec{k}\cdot\vec{v}_k=0$$
$$\frac{d\vec{v}_k}{d\eta}+aH\vec{v}_k-i\vec{k}v_s^2\delta_k-i\vec{k}\phi_k=0$$
$$\phi_k=-\frac{4\pi G\rho_0a^2}{k^2}\delta_k=-\frac{3H^2a^2}{2k^2}\delta_k$$

where we used the conformal time $\eta$ ($dt=ad\eta$) instead of $t$ and, in the last equation, used the Friedmann equation for a flat universe. Notice that the comoving size of the horizon is given by $1/aH$. Notice also that we have done several things. First the Fourier decomposition allows to eliminates the trivial dependence on the expansion. Consider for instance the equation for $\rho_1$. We have

$$\partial_t\rho_1+\rho_1\nabla\cdot v_0+v_0\cdot\nabla\rho_1+\rho_0\nabla\cdot v_1=0$$

Using $\vec{v}_0=H\vec{r}$ we get

$$\partial_t\rho_1+3H\rho_1+H\,r\cdot\nabla\rho_1+\rho_0\nabla\cdot v_1=0$$

Using $\delta(r,t)=\rho_1/\rho_0$ and $\dot{\rho}_0=-3H\rho_0$, we rewrite this as

$$\partial_t\delta+H\,r\cdot\nabla\delta+\nabla\cdot v_1=0$$

Next the Fourier transform gives

$$\partial_t\delta\propto\frac{d}{dt}\delta_k+ik\cdot r\frac{\dot{a}}{a^2}\delta_k$$

while the second term becomes

$$H\,r\cdot\nabla\delta\propto-i\frac{k\cdot r}{a}H\delta_k$$

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For the Euler equation, we must be aware that the equations are given in eulerian coordinates (i.e. we are at fixed point $\vec{x}$ and we look at the fluid passing by). Then

$$\vec{v}_0=H\vec{x}$$

with time dependence only through $H=H(t)$. Then

$$\partial_t\vec{v}_0+\vec{v}_0\cdot\vec{\nabla}\vec{v}_0\equiv\vec{x}\partial_tH+H^2\vec{x}\cdot\vec{\nabla}\vec{x}=\vec{x}\left(\dot{H}+H^2\right)$$

Using $\dot{H}=\ddot{a}/a-H^2$ and the Poisson equation

$$\vec{\nabla}\Phi=\frac{4\pi G}{3}\rho_0\vec{x}$$

gives one of the Einstein equations,

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}\rho_0.$$

so that

$$\frac{d}{dt}\delta_k - \frac{ik\cdot v_k}{a} = 0 \quad \rightarrow \quad \frac{d}{d\eta}\delta_k - ik\cdot v_k = 0.$$

wherewe have dropped the index 1, eg $v_1 \rightarrow v_k$.
At this point, it is useful to consider rotational and irrotational velocity perturbations

$$\vec{v}_1 = \vec{v}_{ir} + \vec{v}_{rot}$$

such that

$$\vec{\nabla}\cdot\vec{v}_{rot} = 0$$

and

$$\vec{\nabla}\times\vec{v}_{ir} = 0$$

To first order these perturbations do not mix. Irrotational perturbation are those that derive from a potential. It is believed that only irrational perturbation are generated in the early universe and that the rotational velocity fields only occur much later in the history of the universe. Notice that rotational perturbations must decay as long as perturbations are dragged by the expansion of the univere, by angular momentum conservation $L \sim (\rho_0 a^3)av_{rot} \sim$ constant implies $v_{rot} \propto 1/a$.
Focusing on irrational velocity perturbations we can write

$$\vec{v}_{ir} = -i\hat{k}v$$

where $\hat{k}$ is a unit vector in the $\vec{k}$ direction. The fluid equations take the particularly simple form:

$$\begin{aligned}
&\dot{\delta}_k - kv = 0\\
&\dot{v} + aHv_k + kv_s^2\delta_k + k\phi_k = 0\\
&\phi_k = -\frac{3H^2a^2}{2k^2}\delta_k
\end{aligned}$$

where the dot stands for derivation with respect to conformal time. We can now write an equation for $\delta_k$ alone:

$$\boxed{\ddot{\delta}_k + aH\dot{\delta}_k + \left(k^2v_s^2 - \frac{3H^2a^2}{2}\right)\delta_k = 0} \tag{8}$$

There are two important differences with respect to Eq.(6). First expansion introduces a friction term, linear in the time derivative of the density field. Second the gravitational source term is decreasing in time, since $H^2 \propto a^{-3}$. Otherwise we recognize the term with the pressure gradient and Newtonian potential as in the static case. We thus expect to have an instability, but that it is going to be of milder form than in a static universe, because of a combination of the friction term and of the dilution of the gravitational term.
To see this explicitely, let us consider a matter dominated, flat universe. Then

$$a \propto t^{2/3} \propto \eta^2$$

and

$$aH = \frac{2}{\eta}$$

The wave equation becomes

$$\ddot{\delta}_k + \frac{2}{\eta}\dot{\delta} + (v_s^2k^2 - \frac{6}{\eta^2})\delta = 0$$

For shortwavelengths (large wavenumbers), $k < k_J$, the solutions are oscillatory. For long wavelengths we expect an instability. To see this we neglect (as an approximation) the pressure term and consider solutions of the form

$$\delta \propto \eta^n$$

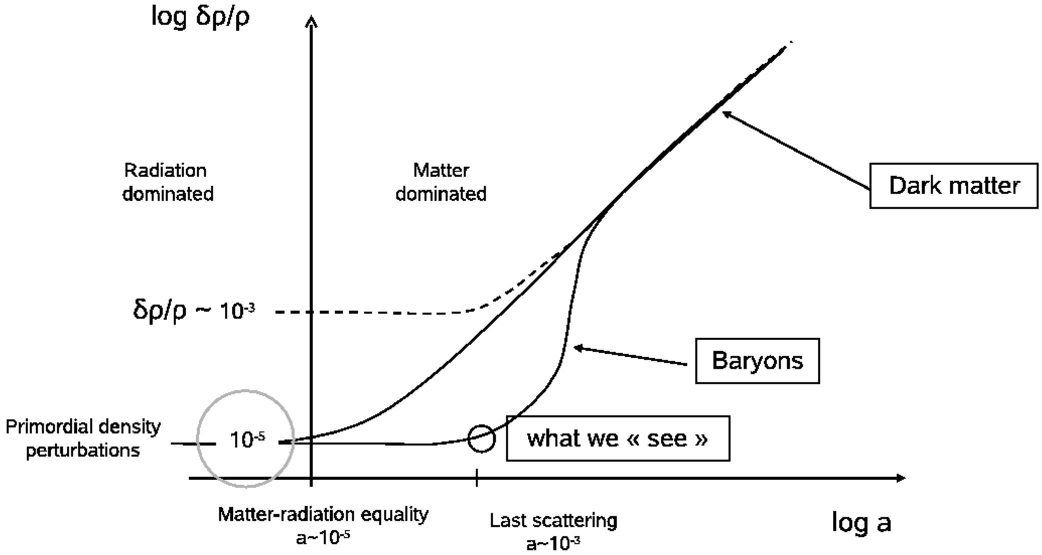


FIG. 2: Sketch of structure formation in the Cold Dark Matter (CDM) scenario for a large scale perturbation. At decoupling, the baryons, which were until then strongly coupled to the photons, fall in the gravitational potential build by the dark matter.

We get the algebraic equation

$$n(n-1)+2n-6=0$$

which has two solutions. One is $n=-3$. The corresponding solution is decaying $\delta_- \propto \eta^{-3}$ and not relevant. The other solution corresponds to $n=2$

$$\delta_+ \propto \eta^2 \sim a(\eta)$$

This is the growing mode. Remarkably this mode growths like the scale factor $a(t)$. It points directly to the existence of dark matter. The argument is quite indirect, but it is robust in the sense that it relies on linearized equations.
It goes as follows. Consider a perturbation that is $\delta = O(1)$ today and assume it is dominantly made of baryons. At the time of decoupling between matter and radiation (surface of last scattering) these perturbations $\delta = O(10^{-3})$, since last scattering occured at $1+z = 1/a \approx 10^3$. Before that time, baryons and photons were strongly coupled, hence the radiation pressure was large and inhomogeneities could not grow.
We cannot see directly the perturbation in baryons, but baryons and photons were in thermal equilibrium, so that $\delta \sim \delta T/T$ (more precisely $\delta = 3\delta T/T$ assuming adiabatic perturbations) where $T$ is the temperature of the CMBR. A given scale is imprinted in a temperature anisotropy of the CMBR. Anisotropies in the temperature of the cosmic microwave background at the level of $10^{-3}$ were searched for and none were found. Instead the COBE satellite found that they are much smaller, $\delta T/T = O(10^{-5})$. Either the Jeans instability is wrong (perhaps gravity has to be modified), or something else is triggering the formation of structures. From the latter has emerged the possibility that there is more matter that does not interact (or little compared to baryons) with light, dark matter.

A few further comments are in order.

1. We used a newtonian approximation, hence we can only described perturbations within the horizon. To study larger scale modes, we need GR. There is an ambiguity in the use of GR because not all the degrees of freedom of the metric are physical. This is like in electromagnetism, and reflect the fact that GR is based on covariance under coordinates transformations. The easiest way out is to fix the gauge.

2. A simple prescription is the following. Consider an unperturbed, flat, matter dominated universe. Then $H^2 = 8\pi G\bar{\rho}/3$. By causality, a perturbation on scale larger than the horizon with an excess of matter compared to the flat case behaves like an independent universe, which may be described by $H^2 = 8\pi/3\rho - k/a^2$. That perturbation has positive curvature ($k > 0$, since $\rho > \bar{\rho}$). Equating the expansion rate (this is called the uniform Hubble flow, analogous to synchronous gauge) we get that

$$\frac{\rho - \bar{\rho}}{\bar{\rho}} = \delta = \frac{3}{8\pi G}\frac{k}{a^2\bar{\rho}} \propto a$$

Hence, in this gauge, we see that matter perturbation grow like $a$ during the matter dominated era both outside and inside of the horizon (for the latter, this is provided there is no pressure — pressure can only act on causal scales).

3. The story is different for sub-horizon perturbations during the radiation dominated era. Assuming (for simplicity) that $\delta_R = 0$. Then the perturbed Poisson equation is still sourced by $\rho_M$ but $\rho_M \ll \rho_R$ in the radiation dominated era. Hence the effect of gravity are smaller and may be neglected to first approximation. For pressureless matter (DM) we then have

$$\ddot{\delta} + \frac{1}{\eta}\dot{\delta} \approx 0$$

(using $a \propto t^{1/2} \sim \eta$ during the RD era) with solution

$$\delta = \delta_0 + \dot{\delta}_0 \log(\eta).$$

Hence matter sub-horizon inhomogeneities grow little during the RD era. This is called the Meszaros effect.

4. In summary, DM and baryonic super-horizon perturbations grow like $a$. As they enter the horizon, the baryonic perturbation do not grow, because they are strongly coupled with the photons. DM perturbations grow little (logarithmically) until matter-radiation equality), then they grow like $a$ again (eventually the cosmological constant will stop that, but this is much later). At decoupling, the pressure drops dramatically and baryons become free. Their perturbations are $O(10^{-5})$, much smaller than that build by the dark matter and they fall in the potential well. This scenario, called Cold Dark Matter, is depicted in Fig.2

5. What is the connection between this picture and the power spectrum shown in Fig. 1? The whole picture of Jeans instability in an expanding universe is that there must be primordial inhomogeneities (initial conditions). A standard assumption is that the have a simple power law spectrum

$$|\delta(k)| = AVk^n.$$

From Eq.(1), we have

$$\left(\frac{\delta\rho}{\rho}\right)_{\rm HOR} \sim k^{(3+n)/2}$$

where we consider a perturbation at the horizon crossing for a perturbation on a given scale $\lambda \sim 1/H$ (this is a good timing, because it avoid subtleties related to gauge choices inherent to perturbations beyond the horizon and because causal evolution has not yet modified the primordial perturbation). Observations (for instance $n \gg -3$ would lead to formation of primordial black holes, which are essentially excluded by non-observation of gamma rays from Hawking evaporation), and also a theory, called inflation, imply that $n \approx -3$. This scale invariant spectrum of inhomogeneities is associated to the names of Harrison and Zeldovich. This may be translated to a prediction for the power spectrum at a given fixed time, for instance sometime after matter radiation equality. Modes that are smaller than the horizon at equality (corresponding to a comoving scale of about 600 Mpc), have entered the horizon during the RD era, and have suffered little change (see item 3). Those correspond to the "plateau" in Fig.1. Those beyond the horizon are evolving like $a$ and thus are smaller by a factor $\sim a_{EQ}/a_{\rm HOR}$. Since $k_{\rm HOR} \approx a_{\rm HOR}H \propto a_{\rm HOR}^{-1/2}$, we conclude that $\delta\rho/\rho \propto k^2$ for $k \lesssim k_{EQ}$. This corresponds to the RHS part $\propto 1/\lambda^2$ of Fig.1, $\lambda \gtrsim 600$ Mpc.

6. Why the acronym Cold Dark Matter? It basically stands for cold dark matter, that is matter that has little kinetic energy as they enter the horizon. The opposite is Hot Dark Matter. For instance light neutrinos. Hot Dark Matter are basically relativistic particles, so they may not be described with our simplified equations. Still, there is something analogous to the Jeans scale for Hot Dark Matter in the sense that only perturbations larger than that scale can collapse. The difference with that CDM is that smaller scale are erased, through a phenomenon called free streaming. The argument goes as follows, here presented for an hypothetical neutrino like particle. Neutrinos of mass $m_\nu$ are relativistic until a temperature $T \sim m_\nu$.

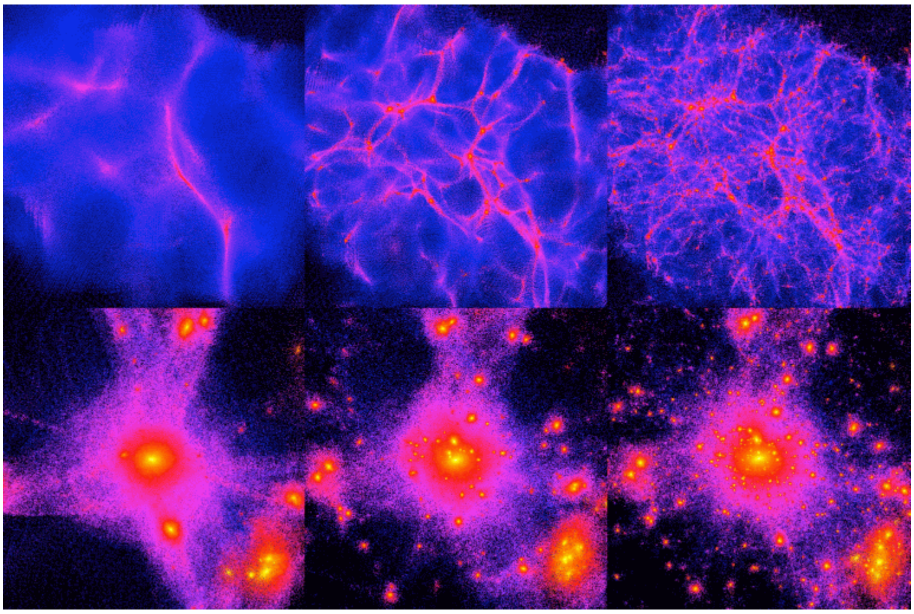

FIG. 3: Numerical simulation of structures with hot, warm and cold dark matter (Credit - university of Zurich)

Before this, they are relativistic and cannot gravitationally collapse. The corresponding minimum mass that can collapse (call it the Jeans mass) is of the order of $M_{\rm min} \sim d_\nu^3 m_\nu n_\nu(T\sim m_\nu) \sim d_\nu^3 m_\nu^4$ where $d_\nu$ is the size of the horizon when $T\sim m_\nu$, or $d_\nu \sim 1/H \sim m_{\rm pl}/m_\nu^2$. All together, we have $M_{\rm min}\sim m_{\rm pl}^3 m_\nu^{-2}$. A more detailled calculation gives [2]

$$M_{\rm min} \approx 3.2\,10^{15}\left(\frac{30\mathrm{eV}}{m_\nu}\right)^2 M_\odot$$

on a comoving scale

$$d_\nu \approx 40\left(\frac{30\mathrm{eV}}{m_\nu}\right)^2 \mathrm{Mpc}$$

In this picture, no structure lighter than about a supercluster can form. Smaller structures form later, by fragmentation (top-down formation). This is opposite to CDM, in which case small scale structures form first (bottom-up formation). HDM is incompatible with observations, see Fig.1 and Fig.3 for numerical simulations. (There is an intermediate possibility, not hot, not cold, just warm enough to erase small structures — this warm dark matter — like DM satellites in the Milky Way.)

### D. Summary of cosmological constraints on DM

There is much more in the cosmological observations than we could address. In particular precise measurements of the CMBR anisotropies (WMAP and now Planck, see [3] for the underlying physics), together with large scale structure observations and measurements of the recession of distant galaxies (the Hubble parameter $H_0$, as well as the accelerated expansion through dark energy) determine that

$$\Omega_{DM}h^2 = 0.119 \qquad \Omega_B h^2 = 0.022 \quad \text{with} \quad h = 0.68 \quad (h^2\approx 0.5)$$

The measurement of $\Omega_B$ is consistent with the determination of that from primordial nucleosynthesis (cf particle data group)

$$0.021 \leq \Omega_B h^2 \leq 0.025 \quad (95\%\mathrm{CL}).$$

Notice that SM neutrinos would be HDM, hence cannot be the dominant form of DM. From freeze-out (see next lecture), we know that there are about 100 neutrinos per cm$^3$ per family. We also know that $\Delta m_{23}^2 \approx 10^{-3}$ eV$^2$ (from neutrino oscillations) and that $m_\nu \lesssim 2$ eV (from precise measurement of tritium decay). This implies that

$$0.001 \lesssim \Omega_\nu \lesssim 0.12$$

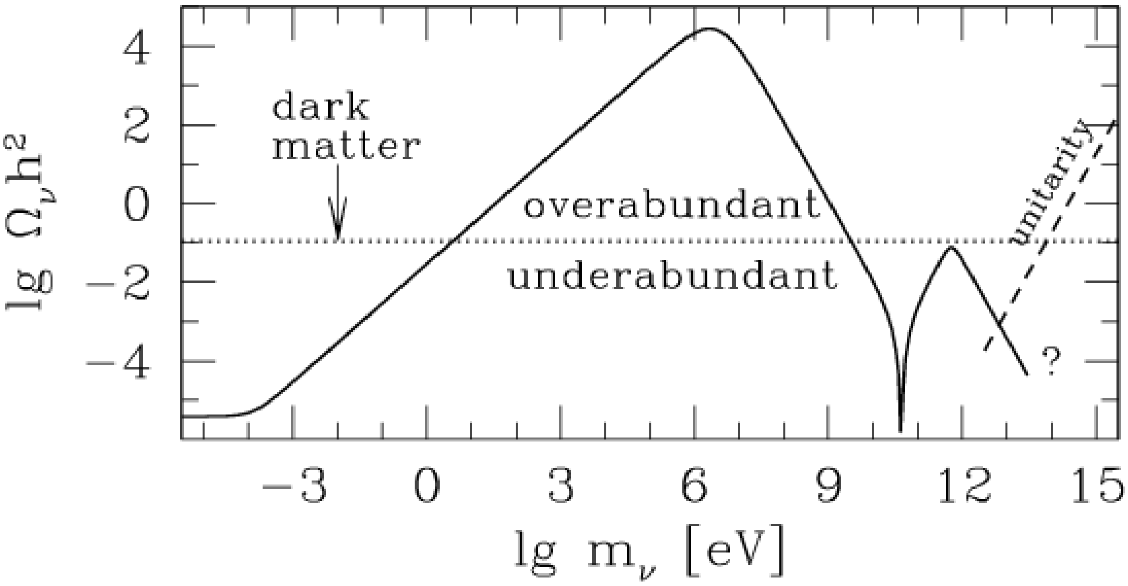


FIG. 4: Relic density of a thermal dirac neutrino with SM interactions. There are essentially three solutions consistent with cosmological observations (the horizontal dotted line): a few eV candidate (HDM), a few GeV one (a WIMP = CDM) and a very heavy, about 100 TeV candidate (a WIMP too, but with strong interactions).

## II. THERMAL FREEZE-OUT AND WIMP MODELS

### A. The case for particle dark matter

Cosmological observations imply that dark matter is not made of ordinary baryons. The most studied possibility is that it is made of new particles, beyond the standard model (SM) of particle physics. To first approximation, a dark matter candidate is a neutral (so as to be essentially invisible), massive (for gravitational collapse) and stable. None of this is actually truly required (there are many exceptions) but is a good starting point, in part because it is not that difficult to implement these properties from a particle physics perpesctive.

This being said, the DM may be approached from two rather distinct perspectives. In a top-down approach dark matter is a by-product of a more fundamental theory. Two standard example is 1/ supersymmetric extensions of the SM (SUSY), which predict that the lightest supersymmetric particle is stable (better be neutral, hence the name neutralino) and 2/ the strong CP problem, which predict the existence of an axion (the axion can in principle decay into 2 photons, but a) it is very light and b) very weakly coupled to photons, so its lifetime is huge). Both the neutralino and the axion still stand as good DM candidates. Yet, most of the activity of the last few years has been in building alternative models. This is motivated by different factors, the most important one being the recurent appearance of observational (gamma rays or positron excesses, problem with structure formation,...) or experimental (DAMA, etc.) anomalies, which don't (or did not) quite fit in the standard picture. The other important motivation is that it is important to explore alternatives, if anything as a way of comparison. In this part of the lectures, I will be agnostic and will merely discuss the standard mechanism of dark matter production as a relic from the early universe.

### B. Freeze-out as the origin of DM

To be specific, I will focus on Fig.4, taken from [4] (but based on an original drawing by Kimmo Kainulainen). The model candidate whose behaviour is depicted in the figure is a massive Dirac neutrino interacting through SM electroweak interactions. To make it fancy, wwe could call it Higgsino for reasons that will become clearer later (see section II C); we keep the name neutrino and keep the notation $m_\nu$ for its mass. There are three possible solutions, which are associated to quite distinct dynamics, so it is interesting to consider it for pedagogical reasons, even though the two light solutions are excluded (the few eV one because it is HDM, the few GeV one by direct detection — see last lecture— and also by constraints from the width of the Z boson). We will analyzed the behaviour of the abundance curve in details (and eventually will correct it).

*1. Freeze-out while relativistic: hot relics*

We start from the low mass range. If $m_\nu \lesssim T_0 = 2.75\text{K} = 2.4\cdot 10^{-4}$ eV, the particle is still relativistic today, and behaves like radiation (i.e. definitely not DM and $\Omega_\nu$ is independent of the mass, hence the plateau for $m_\nu \lesssim 10^{-4}$ eV). The relic abundance (noted $Y$) of the neutrino is given by

$$Y_\nu(T_f) = \frac{n_\nu(T_f)}{s(T_f)} = \frac{\frac{3}{4}\frac{\zeta(3)}{\pi^2} g_\nu}{\frac{2\pi^2}{45} h_{\text{eff}}(T_f)} = \frac{135\zeta(3)}{8\pi^4}\frac{g_\nu}{h_{\text{eff}}(T_f)} \approx 0.21 \frac{g_\nu}{h_{\text{eff}}(T_f)}$$

This is a standard result, but let me explain it in details. Here $n_\nu$ is the density of DM particle and $s$ the entropy density. I use units in which $\hbar = kc = k_B = 1$ and $g_\nu$ is the degeneracy factor of the neutrino (to be discussed later) and $h_{\text{eff}}$ the effective number of degrees of freedom that contribute to the entropy density (see Fig.5). This ratio stays constant if the number of neutrino (per comoving volume) does not change (the entropy per comoving volume is constant in a uniform and isotropic expanding universe — the expansion is adiabatic). So both are taken here at the so-called freeze-out temperature $T_f$ (the factor of $3/4\zeta(3) = 0.90$ is there because our candidate is a fermion — a boson would have no factor of $3/4$— and because quantum statistic effects are non-negligible for relativistic particles, well it's a 10-20% effect), $T_f$ being the temperature at which the neutrino interactions decouple. The rule of thumb for this (see later) is that the interaction rate $\Gamma$ must be less than the expansion rate $H$. The latter is given by

$$H = 1.66 g_{\text{eff}} \frac{T^2}{M_{\text{pl}}}$$

during the radiation dominated era (RD) with $M_{\text{pl}} \approx 10^{19}$ GeV and $g_{\text{eff}}$ counts the effective number of degrees of freedom that contribute to the expansion rate. For the former, we assume that the neutrino interact through Z gauge boson exchange (the details are not important at this level), so that

$$\Gamma \sim G_F^2 T^5$$

Equating the two quantities gives the freeze-out temperature, $T_f \sim 1$ MeV (check this using $m_{\text{pl}} \approx 10^{19}$ GeV and $G_F \approx 10^{-5}$ GeV$^{-2}$). Hence $m_\nu \lesssim 1$ MeV neutrinos decoupled from the thermal bath when relativistic. This mass corresponds to the break in Fig.4 around $m_\nu \sim 1$ MeV. To understand the behaviour of the curve between $m_\nu \sim 10^{-4}$ eV and 1 MeV, we need to relate $Y_\nu(T_f)$ to $\Omega_\nu$. The latter is defined by

$$\Omega_\nu = \frac{\rho_\nu(T_0)}{\rho_{c0}} = \frac{m_\nu s(T_0) Y_\nu(T_f)}{\rho_{c0}}$$

where $\rho_{c0} = 3H_0^2/8\pi G = h^2 10^4$ eV/cm$^3$ (ie equivalent to about 5 nucleons per cubic meter) is the critical energy density of the universe today. The factor of $s(T_0)$ is there because $n_\nu(T_0) = s(T_0)/s(T_f) n_\nu(T_f) \equiv g_s(T_0)T_0^3/g_s(T_f)T_f^3 n_\nu(T_f)$. An important effect is that the effective number of degrees of freedom that contribute to the entropy of the universe changes, $g_s(T_0) \ll g_s(T_f)$. Specifically for the SM neutrinos (and to first approximation for our candidate) $g_S(T_f) = 10.75$ while $g_s(T_0) = 3.9$. This dilution may be expressed through a distinct temperature for decoupled neutrinos, $T_\nu/T_\gamma = (4/11)^{1/3}$, see Kolb & Turner.[3] Putting everything together ($s_0 \approx 10^3\text{cm}^{-3}$), we get

$$\Omega_\nu h^2 \approx 0.02 \left(\frac{g_\nu}{4}\right)\left(\frac{m_\nu}{\text{eV}}\right)$$

which explains the linear dependence in $m_\nu$ of the abundance. For $g = 4$, then $m_\nu \approx 5$ eV fits the observed relic abundance, consistent with Fig.4.

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[3] Here the effect of the SM neutrinos mass is neglected. The counting is then as follows:

$$h_{\text{eff}}(1\text{MeV}) = 2 + 7/8\cdot(4 + 2N_\nu) = 10.75$$

and

$$h_{\text{eff}}(T_0) = 2 + 7/82N_\nu(4/11)^3 = 3.9.$$

for $N_\nu = 3$ for three light species. In principle we should also count the number of degrees of freedom of the DM neutrino. If it is Dirac and both chirality couple to gauge bosons (a vector-like particle), then $\Delta N_\nu = 2$.

A few words. First we took $g = 4$ because we expect both chiralities of the neutrino to be coupled to the $Z$ with same strength. This is unlike the SM neutrinos, which are either Dirac, but with a RH component that is sterile and probably decoupled much earlier, so effectively $g = 2$, or Majorana, for which $g = 2$. Second, the value we got (which anyway is bad DM, since it is hot), may be turned into a bound, which is a version of the bound derived by Cowsik and McLelland [5] (the bound was about $m_\nu \lesssim 90$ eV, which comes about taking $\Omega_\nu h^2 \lesssim 1$ — old days — and $g_\nu = 2$).[4] Last, if the particle decouples much earlier (but was still relativistic), then the mass may be larger, but not much than a few hundreds eV. Indeed all the SM degrees of freedom of the SM amount to $h_{\rm eff} = 106.75$, and are relativistic for $T \gtrsim 200$ GeV. The extra factor of about 10 from entropy dilution gives $m_\nu \sim 50$ eV. This is too hot DM, current bounds requiring that $m_{DM} \gtrsim$ keV (this is from Lyman-$\alpha$ forest observations [6]).

### 2. *Freeze-out while non-relativistic*

For $m_\nu \gtrsim 1$ MeV, the neutrino is non-relativistic at freeze-out. Determining the relic abundance is a bit more complicated and requires solving a Boltzmann equation that describes departure from chemical equilibrium. Focusing on our Dirac neutrino, the equation for $n_\nu$ takes a very simple form,

$$\frac{dn_\nu}{dt} + 3Hn_\nu = \langle \sigma v \rangle \left( n_\nu^{\rm eq} n_{\bar\nu}^{\rm eq} - n_\nu n_{\bar\nu} \right)$$

The derivation of this equation is standard, and may be found in many textbook. The LHS encapsulate the fact that the density tends to dilute like $a^{-3}$. Setting the RHS to zero(no interactions) gives

$$\frac{1}{n}\frac{dn}{dt} = -3\frac{1}{a}\frac{da}{dt} \longrightarrow n \propto a^{-3}$$

The effect of interactions (here annihilations of neutrino and antineutrino pairs in SM Particles) is captured by the RHS. The factor between angle brackets is the thermal average of the cross section of annihilation times the relative velocity ($v = 2p/m$ for NR particles in the CM frame). The second term represents the annihilation of neutrino-antineutrino pairs, the first term is the reversed process, production of neutrino-antineutrino pairs by SM particles in the thermal bath. The basic assumption is that the SM particles are in thermal equilibrium, so that for a $2 \leftrightarrow 2$ process

$$\nu + \bar\nu \leftrightarrow f + \bar f$$

(we have here in mind annihilation through the Z-boson into SM fermion-antifermion particles — for NR $\nu$, this is possible provided $m_f \lesssim \nu$ give or take the velocity distribution tail of the neutrinos), while in principle the first term on the RHS is proportioal to $n_f n_{\bar f}$, setting $n_{f,\bar f} = n_{f,\bar f}^{\rm eq}$ and using detailed balance gives $n_f n_{\bar f} \to n_f^{\rm eq} n_{\bar f}^{\rm eq} = n_\nu^{\rm eq} n_{\bar\nu}^{\rm eq}$, which is clearly a useful simplification. I wrote the equation for the neutrino abundance, but it is clearly the same for antineutrinos. The total abundance is given by $n = n_\nu + n_{\bar\nu}$ and in absence of asymmetry, we may rewrite the Boltzmann equation as

$$\frac{dn}{dt} + 3Hn = \frac{\langle \sigma v \rangle}{2} \left( n_{\rm eq}^2 - n^2 \right)$$

Notice the factor of $1/2$ for non-identical DM particles; it's absent for self-conjugate (eg Majorana DM) particles. In the sequel, we absorbe this factor in the definition of the cross section (so the BE looks the same as for self-conjugate particles). Just remember that there is a factor of 2 if the DM particles are non-identical.

The Boltzmann equation is valid in any regime, including for decoupling while the particles are relativistic, as in the previous section. Let's specialize to the case of decoupling while the particles are non-relativistic. Then the equilibrium density of neutrinos is given by a Maxwell-Boltzmann distribution

$$n_\nu^{\rm eq} = n_{\bar\nu}^{\rm eq} = g_\nu \left( \frac{m_\nu T}{2\pi^2} \right)^{3/2} e^{-m_\nu/T} \qquad \text{for} \qquad m_\nu \gtrsim T$$

---

[4] There is another simple bound, due to Tremaine and Gunn, that excludes such light particles. It rests on the fact that neutrinos are fermions, and have limited occupation number. For a degenerate sphere of mass $M_\nu$ and radius $R$ made of neutrinos of mass $m_\nu$, we must have at most $M_\nu \lesssim m_\nu R^3 \int d^3p \sim m_\nu R^3 (m_\nu v)^3$ where $v$ is the mean velocity. For a gravitationally bound system, $GM_\nu/R \approx v^2$ by the virial theorem. This gives a lower bound on the mass $m_\nu \gtrsim (G^3 M R^3)^{-1/8}$ called the Tremaine-Gunn bound. The constraint is stronger for small system (which are also denser). For dwarf spheroidal galaxies, this crude bound implies that $m_\nu >$ few eV.

while for $m_\nu \lesssim T$, we may write (neglecting the quantum effects — this is not important at the present level)

$$n_\nu^{\rm eq} = g_\nu \frac{1}{\pi^2} T^3 \qquad \text{for} \qquad m_\nu \lesssim T$$

To understand the behaviour of the solution of the BE, it is convenient first to rewrite it for the abundance $Y = n/s$. Also instead of time, we may use $x = m_\nu/T$ which is also monotically increasing in time (since $T \propto 1/a$ et $a \propto t^{1/2}$ in the RD era)[5]. Using $dx/dt = xH$, we get

$$xH\frac{dY}{dx} = s\langle\sigma v\rangle \left(Y_{\rm eq}^2 - Y^2\right) \tag{9}$$

or

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = \frac{\langle\sigma v\rangle n_{\rm eq}}{H}\left(1 - \frac{Y^2}{Y_{\rm eq}^2}\right)$$

The latter form puts forward the fact that the relevant parameter to describe decoupling is the ratio of the interaction rate to the expansion rate,

$$\kappa = \frac{\Gamma_{\rm int}}{H} = \frac{\langle\sigma v\rangle n_{\rm eq}}{H}$$

When this ratio is $\kappa \gg 1$, interactions are fast, and equilibrium is maintained, as $Y \approx Y_{\rm eq}$ to keep the RHS term $O(1)$ ($dY/dx \sim Y_{\rm eq}/x$). On the contrary, for $\kappa \ll 1$, interactions cannot keep up with expansion, and $Y$ departs from $Y_{\rm eq}$. Decoupling occurs for the intermediate value, $\kappa \approx 1$. Lets us assumes that

$$\frac{\langle\sigma v\rangle n_{\rm eq}}{H} \approx 1 \qquad \text{at} \qquad x = x_f \gg 1 \tag{10}$$

whose precise value is to determined by this condition (see later). Then rapidly $Y \gg Y_{\rm eq}$ since the latter is decreasing exponentially $Y_{\rm eq} \propto \exp(-x)$, and we may neglect the first term on the RHS. Then

$$\frac{dY}{dx} \approx -\frac{s\langle\sigma v\rangle}{xH} Y^2$$

In general the thermally average annihilation cross section is temperature dependent (see later). Let us parameterize this as $\langle\sigma\rangle = \sigma_0 x^{-n}$, assuming a simple power law dependence. Then, using (we neglect for simplicity the temperature dependence of the number of degrees of freedom over the range of integration)

$$s \propto T^3 \sim x^{-3} \qquad H \propto T^2 \sim x^{-2}$$

we may rewrite the BE at decoupling as

$$\frac{dY}{dx} = -\frac{\lambda}{x^{n+2}} Y^2 \tag{11}$$

with

$$\lambda = \left(\frac{s\sigma_0}{H}\right)_{x=1} = 8.1\,10^9\, g_*^{1/2} \left(\frac{\sigma_0}{\rm pbarn}\right)\left(\frac{m_\nu}{\rm GeV}\right)$$

---

[5] The evolution of $T \propto 1/a$ is affected is there is the number of degrees of freedom change. The classical example is again the annihilation of electron-positron pairs into $\gamma$, which transfer entropy to the photons, and implies that $T$ evolves more slowly than $1/a$. Instead, one may use entropy conservation, $s \propto a^{-3}$ together with $s = 2\pi^2/45 h_{\rm eff}(T) T^3$ to trace the evolution of $T$. Taking this effect introduces a correction to the BE equation (9), which amounts at replacing the number of effective degrees of freedom $h_{\rm eff}$ by

$$h_*(T) = h_{\rm eff}\left(1 + \frac{1}{3}\frac{d\ln h_{\rm eff}}{d\ln T}\right)$$

in the expression of the entropy density $s$ (we use the same notation for $s$ to avoid cluttering of symbols.

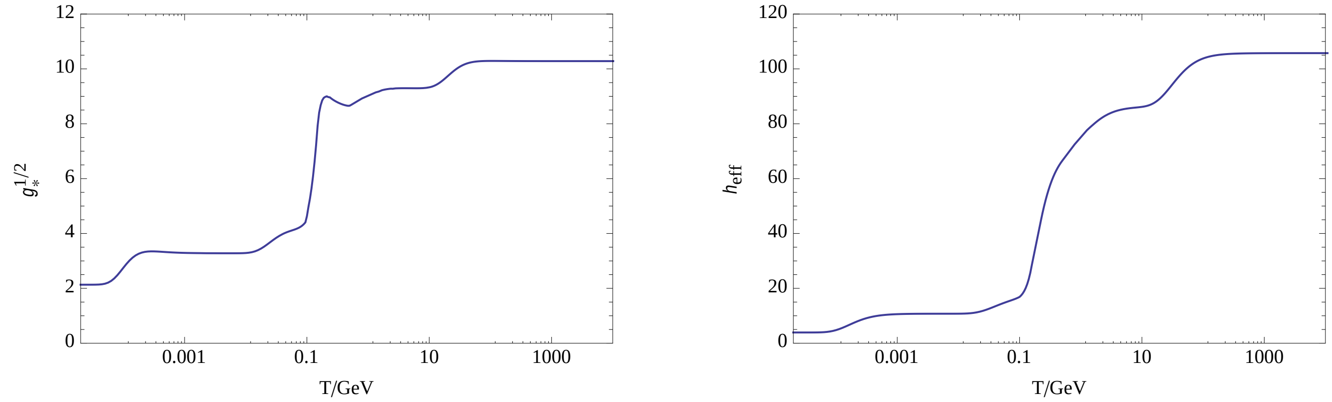


FIG. 5: Plot of the degrees of freedom factors $h_{\rm eff}(T)$ and $g_*^{1/2}$ in function of the temperature. The sharp drop around $T \sim 200$ MeV is due to QCD confinement of quarks and gluons into hadronic degrees of freedom (see for instance [7])

(were here $g_*^{1/2} = h_*(T_f)/g_{\rm eff}^{1/2}(T_f)$, see footnote 5).[6] The Boltzmann equation (11) is now simple to solve. Integrating from $x_f$ to some large value $x_\infty \gg x_f$, we get

$$\frac{1}{Y(x_\infty)} - \frac{1}{Y(x_f)} = -\frac{\lambda}{n+1}\left(\frac{1}{x_f^{n+1}} - \frac{1}{x_\infty^{n+1}}\right)$$

For $x_\infty \gg x_f$ and assuming $Y(x_f) \gg Y(x_\infty)$ (and $n \geq 0$), the relic abundance is approximately given by

$$Y(x_\infty) \approx (n+1)\frac{x_f^{n+1}}{\lambda} \propto \frac{1}{m_\nu \langle \sigma v \rangle}$$

which is the main result: the relic abundance is inversely proportional to the annihilation cross section. Since $\Omega_{\rm dm} \propto m_{\rm dm} Y_\infty \propto 1/\langle \sigma v \rangle$, matching to cosmological observations point to a specific value of the annihilation cross section.
Using $s_0 = 3.010^3$ cm$^{-3}$ and $\rho_c = 1.05 h^2 10^{-5}$ GeV cm$^{-3}$, we get finally

$$\Omega_{\rm dm} = \frac{m_{\rm dm} Y_\infty s_0}{\rho_c} \longrightarrow \boxed{\Omega_{\rm dm} h^2 = \frac{0.04(n+1)x_f^{n1+}}{g_*^{1/2}}\left(\frac{\rm pbarn}{\sigma_0}\right)}$$

The last two quantities we need are the number of degrees of freedom, ore more precisely $g_*^{1/2}$, and $x_f$. The former is easy to calculate for an ideal gas. A complication the QCD phase transition, during which there is a dramatic change in the nature of hadronic degrees of freedom (see for instance [7]) and for which the equation of state $p = p(\rho)$ is to be determined by lattice theory. In Fig.5 we show both $g_*^2$ and the related quantity $h_{\rm eff}$. For this we used the points tabulated in the micromegas code [8]. Beside a drop in the number of degrees of freedom around the QCD phase transition ($T_{\rm QCD} \sim 200$ MeV), the factor $g_*^{1/2} = O(10)$ or $O(3)$ for respectively $T \gtrsim T_{\rm QCD}$ or $T \lesssim T_{\rm QCD}$. Whether DM annihilate after or before the QCD transition depends on its mass through the parameter $x_f$, which is given implicitely by Eq.(10). Using the expression of $H$, we may rewrite this condition as

$$x_f^{-1/2+n} e^{x_f} = 0.038 M_{\rm pl}\, m_\nu\, \sigma_0 \frac{g_\nu}{g_{\rm eff}^{1/2}}$$

which may be solved by iteration for $x_f$,

$$x_f \approx 20.9 + \left(\frac{1}{2} - n\right)\log(x_f) + \log\left(\frac{m_\nu}{\rm GeV}\right) + \log\left(\frac{\sigma_0}{\rm pbarn}\right) + \log\left(\frac{g_\nu}{g_{\rm eff}^{1/2}}\right)$$

---

[6] While freeze-out takes place earlier if the number of degrees of freedom increases, this effects is overcomed by the larger entropy of the universe; this is why $\Omega_{\rm dm}$ is a decreasing function of $g_*$.

It seems that the calculation is circular, since we need $\sigma_0$ to calculate $x_f$ but we may eliminate $\sigma_0$ in terms of $\Omega_{\rm dm}$, so that at the end $x_f$ is a function of $m_{\rm dm}$ only. A first approximate solution of this transcendental relation may be found by simply substituting $x_f \to 18.3 - \log\Omega_{\rm dm}$ on the RHS, which gives $x_F \approx 25$. This value is rather stable, as the dependence on $m_{\rm dm}$ is only logarithmic. Putting all this together, we get for, for $n = 0$, $g_*^{1/2} \approx 10$ and $x_f \approx 25$,

$$\Omega_{\rm dm} h^2 \approx 0.1 \left(\frac{\rm pbarn}{\sigma_0}\right)$$

This shows that an annihilation cross section of $\sigma_0 \approx 1$ pbarn $\equiv 3\cdot 10^{-26}$ cm$^3$s$^{-1}$ would fit the observations. More precise calculations may be found in [9] (see also [10]) but our estimate is not too bad. It is sometime said that a pbarn annihilation cross section point to an electroweak scale particle. This is because

$$\sigma_0 \sim \frac{\alpha^2}{\pi}\frac{1}{M^2} \approx \left(\frac{100\text{GeV}}{M}\right)^2 \text{pbarn}$$

Well, that may be, but there is much freedom, some of which I will discuss.
Focusing on the case of the neutrino, we may now understand the behaviour of $\Omega_\nu$ for $m_\nu \gtrsim 1$ MeV. The annihilation cross section is through the $Z$ gauge boson into SM light fermions. A direct calculation shows that its annihilation is an s-wave state, so that $\langle \sigma v\rangle = \sigma_0$ (ie $n = 0$), with

$$\sigma_0 \sim G_F^2 m_\nu^2$$

Assuming $m_\nu \ll m_Z$, the amplitude is $\propto G_F$, then the rate is $\propto m_\nu^2$, which explains why the relic abundance is $\propto m_\nu^{-2}$ in Fig.4. Taking into account the annihilation into SM particles ligther than $m_{\rm nu}$ I got, using Calchep (yes, I got lazy), $\sigma_0 \approx 2$ pbarn for $m_\nu = 6$ GeV (annihilation is slightly before the QCD phase transition). For annihilation into non-identical particles, the effective cross section has to be divided by a factor of 2. This is consistent with Fig.4. Historically, this candidate is associated to the names of Lee and Weinberg, who derived a lower bound on a massive Dirac neutrino by requiring that $\Omega_{\rm dm} \lesssim 1$, which corresponds to a mass of about 1 GeV [11]. I am not totally sure, but I think this is the first work on WIMPs.
For larger $m_\nu$, the cross section keeps increasing, up to $m_\nu \sim m_Z/2$ which has a very large cross section. This is reason for the dip in Fig.4. For even larger masses, things become more complicated, and the details of the figure depends on the assumptions made. Clearly a Dirac neutrino interacting only through the $Z$ is not consistent, since $Q = T_3 + Y/2$ requires that the neutrino has at least one charged partner. The simplest implementation of this is a Dirac doublet,

$$L = \begin{pmatrix} N \\ E \end{pmatrix}$$

with $Y = 1$. This candidate, which has the same quantum numbers as the Brout-Englert-Higgs doublet (a higgsino) may be vector-like (both chiralities coupled to $SU(2)$), or other representations are required if it is chiral. I the latter case, it looks like a fourth generation lepton. In both cases, stability requires to implement a discrete symmetry ($Z_2$ in this case) so as to avoid mixing with SM leptons.
For $m_\nu \geq M_W$, gauge boson pairs production becomes possible and, on general grounds, we expect that the cross section decreases with $m_\nu$, with eventually $\sigma_0 \propto \alpha^2/m_\nu^2$. This is not precisely what is shown in the Figure. Instead there is a dashed line, which obviously $\propto m_\nu^2$, and with name tag "unitarity". Let me discuss this, because it is a general result, before going to some further comments on WIMP models and their relic abundance.

### 3. *Griest-Kamionkowski unitarity limit*

The basic question we want to address is whether a WIMP can be arbitrarily heavy? Since the relic abudance is fixed by the annihilation cross section of the WIMP particle, the question is whether there is a limit on this cross section. Matching to the relic abundance will in turn give a bound on the possible mass. This has been studied by Griest and Kamionkowski, and I will refer to this as the Griest-Kamionkowski bound [12].
The consider the limit set by unitarity on a process $a + b \to c + d$ in a state of total angular momentum $J$. The basic output is that the total reaction cross section (that is total cross section minus the elastic one) is bounded from above by

$$\sigma_J(a + b \to c + d) \leq \frac{\pi(2J+1)}{p_i^2}$$

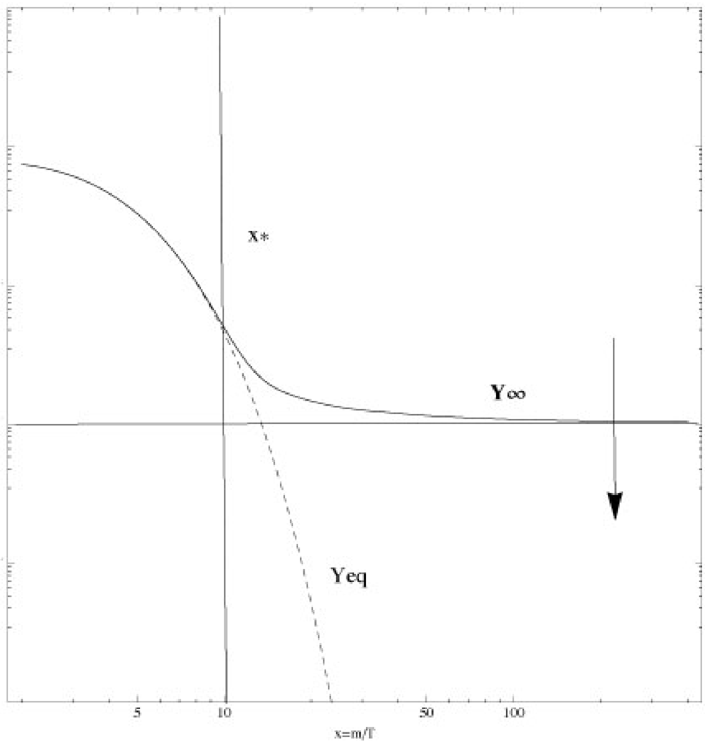


FIG. 6: Typical evolution of the dark matter abundance, here called comoving number density.

For non-relativistic annihilation, $p_i^2 = E^2 - m_{\rm dm}^2 \approx m_{\rm dm}^2 v_{\rm rel}^2/4$. The most stringent bound is on $J = 0$, so that

$$\sigma_0 v_{\rm rel} \lesssim \frac{4\pi}{m_{\rm dm}^2 v_{\rm rel}^2}$$

Using $\langle v_{\rm rel}^2\rangle = 6/x_f$ and requiring $\langle\sigma v_{\rm rel}\rangle/2 \approx 1$ pbarn, gives an upper bound on a Dirac neutrino mass (notice the factor of 2),

$$m_{\rm dm} \lesssim 110\,\mathrm{TeV}$$

consistent with value from the dashed line in Figure 4 (for a Majorana particle the bound is weaker by a factor of $\sqrt{2}$. For heavier particles, the annihilation cross section is too small, and they would be over abundant (or they must be unstable). In practice, for a given model, the real cross section are smaller than that imposed by unitarity, so WIMPs to be substantially lighter. We see a few example of this in the next section.

### C. Minimal dark matter models

As said in the introduction, the neutralino remains the best motivated WIMP. Yet, a recent trend is to study simple models. I discussed some of these in this section.
Our Dirac neutrino is actually best seen as a simple implementation of this idea, as partner of an $SU(2)$ doublet. Its partner is like a heavy electron. This particle is better be heavier than the neutrino. This may be achieved in two ways. Through Yukawa coupling with other, for instance $SU(2)$ singlets states, which introduces isospin breaking when the BEH develops a vacuum expectation value. Alternatively, in absence of such states, one may isospin breaking through loop corrections, in the vein of Minimal Dark Matter [13]. For our vector-like Dirac doublet the Lagrangian takes the simple form

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar\chi(i\not{D} + M)\chi$$

Exchange of $Z$ and $W$ bosons at one-loop gives [13]

$$M_E - M_N \approx (1 + \cos\theta_W^{-1})\alpha_2 M_W \sin^2\frac{\theta_W}{2} \approx 350\,\mathrm{MeV}$$

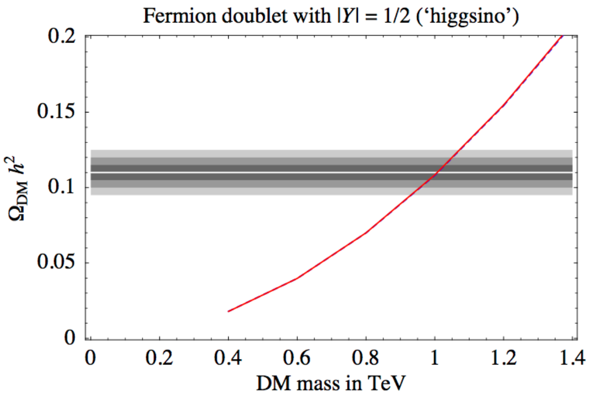


FIG. 7: Relic abundance of a heavy fermion doublet candidate.

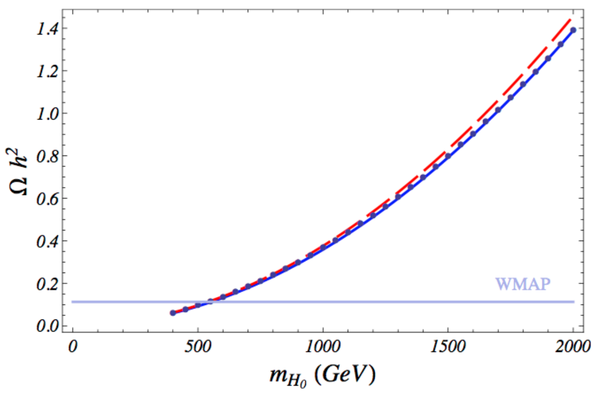


FIG. 8: Relic abundance of heavy inert dark matter candidate.

with $\sin^2\theta_W \approx 0.24$. Interestingly, this is a generic result. If splitting occurs through radiative corrections, the neutral particle is always the lightest component of an $SU(2)$ multiplet. The relation between the relic abundance and the DM mass is shown in Fig.7. So why does Fig.4 shows a different behaviour with a cross section that is increasing with mass? The reason has to do with the source of isospin breaking. A similar behaviour occurs in models with coupling to electroweak gauge bosons. To be concrete, I illustrate this with the inert doublet model, a scalar analogous of our neutrino doublet. In this case the DM is one of the neutral components of scalar doublet

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H_0 + iA_0) \end{pmatrix}$$

In this model the spectrum is fixed by a potential, which assuming a discrete $Z_2$ symmetry, reads

$$V = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2)^2 + h.c.\right]. \tag{12}$$

The $SU(2)\times U(1)$ symmetry is broken by the vacuum expectation value of $H_1$,

$$\langle H_1 \rangle = \frac{v}{\sqrt{2}}$$

with $v = -\mu_1^2/\lambda_1 = 246$ GeV while, assuming $\mu_2^2 > 0$,

$$\langle H_2 \rangle = 0.$$

The mass of the Brout-Englert-Higgs particle ($h$ or Higgs for short) is

$$M_h^2 = -2\mu_1^2 \equiv 2\lambda_1 v^2 \tag{13}$$

while the mass of the charged, $H^+$, and two neutral, $H_0$ and $A_0$, components of the field $H_2$ are given by

$$\begin{aligned} M_{H^+}^2 &= \mu_2^2 + \lambda_3 v^2/2 \\ M_{H_0}^2 &= \mu_2^2 + (\lambda_3+\lambda_4+\lambda_5)v^2/2 \\ M_{A_0}^2 &= \mu_2^2 + (\lambda_3+\lambda_4-\lambda_5)v^2/2. \end{aligned} \tag{14}$$

For appropriate quartic couplings, $H_0$ or $A_0$ is the lightest component of the $H_2$ doublet.
Consider now the amplitude for $H_0H_0 \to ZZ$. The $Z$ may be transversaly polarized, $Z = Z_T$, or, through the BEH mechanism, have a longitudinal component $Z = Z_L$. The latter is essentially a Goldstone scalar in disguise. At large masses, $M_{H_0} \gg M_Z$, the two amplitudes have a quite distinct behaviour. For transversaly polarized Z bosons, the amplitude is

$$\mathcal{M}_T = \mathcal{M}(H_0H_0 \to Z_TZ_T) \approx \frac{g^2}{2c_w^2}$$

Squaring and integrating over phase space gives $\sigma_T \propto 1/M_{H_0}^2$ as expected for a good theory. The longitudinal component is given by (for $M_{H_0} \gg M_Z$),

$$\mathcal{M}_L \approx \frac{g^2}{2c_w^2}\frac{M_{A_0}^2 - M_{H_0}^2}{M_Z^2} \propto \lambda$$

where $\lambda$ is the quartic coupling responsible for the splitting between the $A_0$ and $H_0$. This is a realization of the equivalence principle (not the one of GR), which state the large energy behaviour of longitudinal gauge bosons is equivalent to the emission of pseudo-Goldstone particles. Now if $\Delta M^2 \propto M_{H_0}^2$, clearly $\sigma_L \propto M_{H_0}^2/M_Z^4$ and increases for larger masses. This is akin to the behaviour shown in Fig.4. This is however pathological, since $\Delta M^2 \propto \lambda v^2$. While $M_{H_0}$ can be arbitrarily heavy, the mass splitting is limited to be $O(4\pi)$ (and this is probably optimistic), or $\Delta M \lesssim 0.9$ TeV. In practice one may use this effect to move up the mass of the $H_0$ to large values. A viable $H_0$ candidate has a mass between $M_{H_0} \approx 530$ GeV (zero splitting) and $\approx 58$ TeV ($\lambda \approx 4\pi$) [14, 15]. The former solution is shown in Fig.8. Hidden in both the cases I have discussed (the Higgsino and Inert Doublets) are two important, albeit quite technical, complications.

First, the dark matter candidates have partners that are almost degenerate in mass, $\Delta M/M \ll 1$. At freeze-out, the partners may annihilate with each others, or can co-annihilate with the DM candidate itself. These processes are often important, and may in some occasion actually be dominant. A simple example is the following. Consider the following piece of Lagrangian

$$\Delta\mathcal{L} = yS\bar{Q}q_R \tag{15}$$

with $S$ a real scalar (the DM), $q_R$ a SM quark and $Q$ a new heavy (vector-like) quark. Annihilation may occur through $SS \to \bar{q}q$ with the heavy quark in the t-channel and this is all if $m_Q \gg m_S$. But if $m_Q \gtrsim m_S$, then $\bar{Q}Q \to gg$ or $\bar{Q}Q \to \bar{q}q$ are important, since they are strongly coupled particles. This is even more true if the Yukawa coupling $y$ is small. Another possibility is $SQ \to qg$, which is an instance of co-annihilation. If $y$ is relatively small, an efficient way toward freeze-out is to go through the heavy quark annihilation and, possibly through the co-annihilation channel. It's like having two reservoirs, one with $S$ and one with $Q/\bar{Q}$ particles. As the temperature lowers, the $Q/\bar{Q}$ reservoir empties easely. If the $S$ can transform into $Q$ particles efficiently, then the $S$ reservoir can refil the $Q$ reservoir, and empty itself. The key is whether $S \leftrightarrow Q$ processes are fast enough. This is usually the case, since $S+q \to Q+g$ has generically a large rate compared to $H$ at the time of freeze-out, simply because the target particles are light SM degrees of freedom. Assuming that this and related processes are fast enough, then it is possible to show that the BE equation driving the relic abundance takes the simple form of Eq.(9) provided that $n$ represent the abundance of the DM and all its partners (all the odd particles),

$$n \to \sum_i n_i$$

and the annihilation cross section is replaced by (see [16])

$$\langle\sigma v_{\rm rel}\rangle = \sum_{i,j}\langle\sigma_{ij} v_{\rm rel}\rangle \frac{n_i^{\rm eq} n_j^{\rm eq}}{n_{\rm eq}^2}$$

This simple reformulation take into account the that, if the mass gap is $\Delta M$, then the relative abundance at temperature $T$ of the partner compared to the DM candidate is $\exp -\Delta M/T$, and so is more important for nearly degenerate particles (the rule of thumb is that co-annihilation is important if $\Delta M/M \lesssim 0.1$). Also, it takes into account the fact that eventually the $i$ partners all decay in the DM particle so that any $n_i \to n_{\rm dm}$. Taking all the possible processes into account by hand is cumbersome. In practice one uses numerical tools, the most flexible one being micromegas [8] (which also solves the BE numerically).

A further complication is due to the fact that the particles in the above mentionned processes annihilate nearly at rest. Non-perturbative effects, in particular possible attractive or repulsive interactions, are important. These are most important if the two co-annihilating particles may exchange a vector or scalar particle (it must be clearly a boson) that is substantially lighter $m \ll m_{\rm dm}$. This is a fortiori the case for the photon or the gluons (if the co-annihiating particle are charged or colored) and more generally for particles heavier than the $W,Z$ gauge bosons or the Higgs. Since this happens in the early universe it is wise to take into account that the exchanged particle may get a thermal mass (actually a Debye mass, since the effect is essentially that of a Coulomb interaction). This so-called Sommerfeld effect considerably complicates the calculation of the relic abundance, in particular for heavy particles and each model has to be treated case by case. I will give a talk about, not this, but the relevance of the Sommerfeld effect for annihilation in the galaxy (this is in some way simpler, since in that case co-annhilation is absent, but in some other aspects it is more complicated, because the signal is very sensitive to details of the parameters).

### D. Discrete symmetries, stability and selection rules

Discrete symmetries are useful in dark matter physics. At the most elementary level, the stability of DM may be imposed by the introduction of a discrete, eg $Z_2$ symmetry, say

$$\chi \stackrel{Z_2}{\longleftrightarrow} -\chi$$

while

$$\Phi \stackrel{Z_2}{\longleftrightarrow} \Phi$$

where $\Phi$ generically represent SM degrees of freedom (bosons or fermions). If the symmetry is conserved, the $\chi$ particle is absolutely stable,

$$\chi \not\rightarrow \Phi + \Phi'$$

while allowing for annihilation processes, say

$$\chi + \chi \rightarrow \Phi + \Phi'$$

(possible if $\chi$ is a real particle — Majorana fermion, real scalar, or photon-like).
There is some arbitrariness in imposing by hand a discrete $Z_2$ symmetry. By Lorentz covariance fermions come in pairs in a Lagrangian. Hence the SM fermions may be even or odd under a $Z_2$ (in GUT framework, they are naturally odd, see for instance [17]). Stability requires that a fermionic DM must have opposite parity to that of SM fermions. The SM doublet must however be even, because of the Yukawa couplings, hence any scalar DM must be odd (and the $Z_2$ symmetry must remain unbroken in vacuum). The simplest realization of this is a singlet scalar [18, 19] (Zee called that a "phantom"),

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\partial_\mu S \partial^\mu S - V(S) + \lambda S^2 H^\dagger H \tag{16}$$

with $S \rightarrow -S$ under a $Z_2$ parity, and of course $V(-S) = V(S)$. The last term becomes $\lambda v h S^2$, where $v = 246$ GeV, so that the $S$ interact with the visible sector through the Higgs particle $h$. This is called the Higgs portal. This is very interesting conceptually, because of its relation to the mass term in the Higgs potential, which sets the scale for electroweak symmetry breaking [20]. Stability of the $S$ particle requires at the very least that the $S$ does not develop a vacuum expectation value, which is far from being guaranteed, but can be imposed by tuning the parameters of the potential. Whether this is enough is far from being clear...
Although common practice (to be begin with the MSSM) and easy to implement, imposing a discrete symmetry is essentially an act of despair. To compare, the stability of matter in the SM is rooted in much deeper principles: it is explained either by gauge invariance (the electron is absolutely stable because it the lightest electrically charged particle) or (we believe) is accidental. There is a global $U(1)_B$ symmetry in the SM, but is broken by anomalies and a fortiori in grand unified frameworks, like $SU(5)$. At some level, we think that for instance

$$p \rightarrow e^+ + \pi^0$$

should happen. However the current constraint on that process is $\tau_{1/2} \gtrsim 8.10^{33}$ years [21], much longer than the lifetime of the universe. There is a good reason for this, which has to do with the fact that the proton and the pion are composite particles, made of quarks. The effective coupling is not between two fundamental fermions (the proton and the positron) and a boson (the pion), which could be written as a renormalizable Yukawa coupling,

$$\mathcal{L} \supset g\,(pe)\,\pi_0 + h.c.$$

(in which case the decay rate would be $\Gamma \sim g^2 m_p$, or $\tau \sim g^{-2} 10^{-24}$ sec – the coupling would need to be extraordinarily small, $g \sim 10^{-32}$). Instead the effective coupling is between at the very least 4 fermionic fields (3 quarks and 1 lepton),

$$\mathcal{L}_{eff} \supset \frac{1}{\Lambda^2}\,(qq)(ql)$$

which is dimension 6 operator. For $\Lambda \sim 10^{16-17}$ GeV, characteristic of GUT scales, the lifetime of the proton is consistent with current constraints (see [22]). Bottom line is that the proton is naturally long-lived because it is composite (and, admittedly, because the supposed scale of GUT is so large).
A portal is a generic name. Here is another instance. A fermionic singlet is like a sterile neutrino, $N$, which may have a coupling

$$\Delta\mathcal{L} = y\bar{L}HN + \frac{1}{2}M_N NN$$

This is bad, a priori, but not so if the Yukawa coupling is small enough [23]. Decay occurs through

$$N \to \nu + Z \to \nu + (\bar{\nu} + \nu)$$

(if decay into $e^+e^-$ is kinematically impossible), with a life time that scales like

$$\tau \sim \frac{M_Z^4}{M_N^5}\theta^{-2} \approx 10^{26}\,\mathrm{sec}\left(\frac{M_N}{1\mathrm{keV}}\right)^{-5}\left(\frac{\theta^2}{10^{-8}}\right)^{-1}$$

(the lifetime of the universe is about $\tau_U \approx 14\cdot 10^9$ years $\approx 5\cdot 10^{17}$ sec). In this model, the scale $M_N \sim$ keV is correlated to that of the mixing angle $\theta \sim yv/M_N$ through the requirement that the SM neutrinos have a mass compatible with observations $\lesssim 1$ eV. A keV scale dark matter is warm, not cold, but that could be useful to explain small scale issues with cold dark matter [24]. This is clearly a neat model.

Symmetries also impose selection rules on possible dark matter processes, in particular annihilation. In the freeze-out mechanism, annihilation takes place at a temperature that is substantially smaller than the mass of the dark matter, ie when the DM is non-relativistic $x_f = m_{DM}/T_{\mathrm{f}} = O(25)$. In a thermal bath of non-relativistic particles $\langle E\rangle = m + \frac{3}{2}T \equiv m + \frac{1}{2}m\langle v^2\rangle$. Taking into account that $v_{\mathrm{rel}} \equiv 2v$ (in the CM frame), the average relative velocity at freeze-out is $\langle v_{\mathrm{rel}}^2\rangle = 6/x_{\mathrm{fo}} = O(0.3) \lesssim 1$. Accordingly the average cross section of WIMPs is generally presented in an expansion in the relative velocity (see eg [9]),

$$\langle\sigma v_{\mathrm{rel}}\rangle = a + b\langle v_{\mathrm{rel}}^2\rangle + c\langle v_{\mathrm{rel}}^4\rangle + \ldots$$

The first term is velocity independent and corresponds to annihilation into a s-wave orbital angular momentum state. The second term is a p-wave, the third one d-wave, etc.
While it is often assumed that DM annihilation in a s-wave state, this is not always the case. A standard example is the annihilation of a Majorana DM particle in a pair of light fermions through a chiral coupling (This is the case for instance for a Wino particle). Schematically the process is

$$\chi\chi \to f_L\bar{f}_L$$

by which I mean that $\bar{f}_L$ is the CP conjugate state of the $f_L$. Of course they have opposite helicity. Let's consider that the $\chi$ pair are in a s-wave. They are self-conjugate particles, so the state must be antisymmetric under the exchange of the $\chi$, so they are in a spin zero state. Using the spectroscopic notation, the pair of $\chi$ are in a state $^{2S+1}L_J(J^{PC}) = {}^1S_0(0^{-+})$ [7]
In the CM frame, the outgoing fermion-antifermion pair have opposite momenta, say $\vec{p}$, and opposite helicity, so that $|-1/2 - 1/2| = 1$. The total helicity being the projection of the total angular momentum of the pair on the momentum $\vec{p}$, we must have $|h_1 + h_2| \leq J$. Since $J = 0$, this process is forbidden by angular momentum conservation. The way around is to allow for helicity flip, through insertion of a mass term, $m_f$, or $f_L \to f_R$. We conclude that

$$\sigma v|_{^1S_0} \propto m_f^2$$

If $m_f$ is small, $m_f \ll m_\chi$, annihilation in a p-wave state may be dominant over that in the s-wave. In that case the Majorana pair must be in a S=1 triplet state, corresponding to a CP even initial state, and the annihilation cross section is p-wave,

$$\sigma v \propto v_{\mathrm{rel}}^2$$

It is instructive to consider this basic phenomenology from an effective Lagrangian perspective. For the $S = 0$ state, the possible coupling between a pair of Majorana and fermions is

$$O_1 = m_f\,\bar{\chi}\chi\,\bar{\Psi}_f\Psi_f$$

---

[7] The parity and charge conjugation quantum numbers are also given for reference. Notice that this state is CP-even. This is often as a synonym for s-wave annihilation, because $P|\phi\rangle = -(-1)^L|\phi\rangle$ for a fermion pair. The $(-1)^L$ is just the usual parity of $L$ angular momentum waves (spherical harmonics), the minus sign is from the intrinsic parity of fermions (for bosons there is no minus sign). The transformation under $C$, which is well defined for a self-conjugate state (a fermion-antifermion pair, and a fortiori for a pair of identical Majorana particles), may be obtained by considering the exchange of the two particles, which must be odd according to the Pauli principle. The interchange of spatial coordinates introduces a factor of $(-)^L$, that of the spin coordinates $(-)^{S+1}$ and the charge variable by $C$. Combining these transformations should give a factor of $-1$ for fermion exchange, $(-)^{L+S+1}C = -1$ so that $C = (-)^{L+S}$ for fermions (minus this for bosons). Since $P = -(-)^L$, $CP = (-)^{S+1}$. As $P$ and $C$ are not defined for processes involving chiral states and/or weak interactions, it's better to use $CP$ to classify the possible states.

which involves both chiralites of the $f$. For the $S=1$ state, instead,

$$O_2 = \bar{\chi}\gamma_\mu\gamma_5\chi\,\bar{\psi}_f\gamma^\mu\gamma_5\psi_f$$

which is axial-vector because self-conjugate particle has vanishing vector current. How about d-wave? I know of only one example, based on the Lagrangian (15). Annihilation of a pair of real scalar state $S$ in an s-wave state into a pair of SM fermion requires a mass insertion, because the initial state is $J=0$. The effective coupling is then

$$O_3 = m_f S^2\bar{\psi}_f\psi_f$$

Hence in the massless limit, s-wave is suppressed. Interestingly, a p-wave annihilation is also suppressed. The simple way to see this is to consider the possible effective operator. A p-wave corresponds to a current, but there is no such quantity for a real scalar. A complex scalar $\phi^\star \neq \phi$ is fine, in which case

$$O_4 = (\phi^\star\partial_\mu\phi - \partial_\mu\phi^\star\phi)\bar{\psi}_f\gamma^\mu\psi_f$$

but for a real scalar, the next possibility is

$$O_5 = \partial_\mu S\partial_\nu S\,\Theta_f^{\mu\nu}$$

where $\Theta_f^{\mu\nu}$ is the stress-energy tensor of the SM fermion. This rank-2 operator transforms as a spin 2 object, and so the corresponding annihilation cross section is d-wave, $\sigma \propto v_{\rm rel}^4$ in the massless limit. This may seem academic, but has in general dramatic consequences for indirect searches.

## III. WIMP SEARCHES

### A. Direct detection

Let me go back to our Dirac neutrino candidate. The few GeV candidate is of course excluded by constraints on the invisible width of the $Z$, which is consistent with 3 light neutrinos. It is also excluded by direct detection, and this since many years. It is instructive to work this out.

Direct detection aims at observing the dark matter that is in the vicinity of the solar system. The flux is quite large. Using $\rho_0 \approx 0.3\text{GeV/cm}^{-3}$ and $v_c \approx 220$km/s (the orbit velocity of the Sun around the Galactic centre), the flux is naively $O(10^5\text{cm}^{-2}\cdot\text{s}^{-1})$ for a 100 GeV particle. As a way of comparison, this is much more than muon (about 1 per cm$^2$ per second), and about a tenth of that of solar neutrinos. The problem is that, like neutrinos, DM, or more precisely WIMPs, interact very weakly with matter. The strategy is to look for recoil of nucleus $N$ (of mass $N$) inside of low-noise detectors. The event rate (hits per unit time per interval of recoil energy and per mass of target) is given by [25]

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_N m_\chi}\int_{v_{\rm min}}^\infty v f(v)\frac{d\sigma}{dE_R}d^3v$$

where $E_R$ is the recoil energy of the nucleus (what is measured), and $f(v)$ is the velocity distribution of the DM particles at the position of the Sun. This is generally assumed to be Maxwell distributed,

$$f(v) = \frac{1}{\sqrt{2\pi\sigma}}e^{-v^2/2\sigma^2}$$

with variance $\sigma \approx 270$ km/s. It can be shown that exponential falloff for large velocities leads to a similar falloff of the event rate for large recoil energy,

$$\frac{dR}{dE_R} \sim \left(\frac{dR}{dE_R}\right)_{E_R=0} F^2(E_R)\,e^{-E_R/E_c}$$

where $E_c \sim \mu^2 v_c^2/m_N \sim 10^{-6}\mu^2/m_N$, with $\mu = m_N m_\chi/(m_\chi+m_N)$ the reduced mass of the nucleus and the dark matter particle and $F(E_R)$ is a nuclear form factor. For $m_\chi \lesssim m_N$, $E_c \sim m_\chi^2 v_c^2/m_N$; otherwise $E_c \sim m_N v_c^2$. Hence, in the former case the event rate at large $E_R$ falls more rapidly for a heavy target than for a light one. At the same time, total rate is larger for nucleus with larger mass number, $\sim A^2$ (so the experimentalists have to compromise). This is at least provided elastic collisions are coherent (and that the contribution of the nucleons add up constructively, see below). Coherent scattering requires that the energy transferred

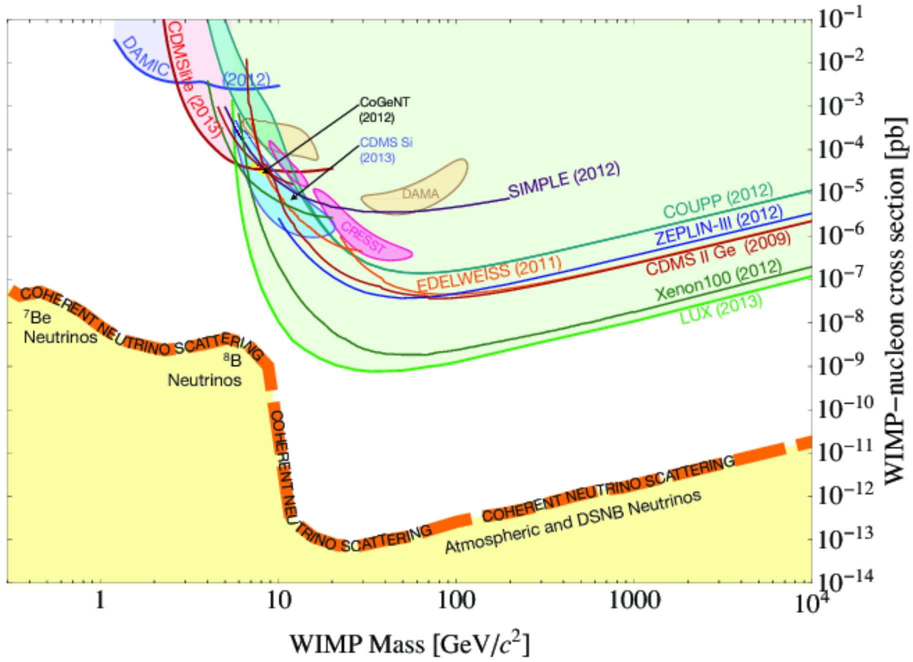


FIG. 9: A compilation of current exclusion limits. Also shown is the ultimate background due to various neutrino sources [26].

$E_R$ is smaller than the binding energy of the nucleus. Simple kinematics reveals that in the lab frame, the recoil energy is related to the velocity of the incoming DM particle by

$$v\cos\theta = \sqrt{\frac{m_N E_R}{2\mu_N^2}}.$$

For fixed $E_R$, there is a minimum velocity $v_{\min}$ (the lower limit in the integral over velocities given above) corresponding to a collision with $\theta = 0$; $E_R \lesssim \mu_N^2 v^2/m_N$ is typically in the 0-100 keV range, which is indeed below the binding energy of target nuclei.
The nuclear form factor $F(E_R)$ (ie the Fourier transform of the nucleon density in the nucleus) introduces a further suppression when the momentum transferred $q = \sqrt{2m_N E_R}$ is larger than $1/r_N \sim A^{-1/3}$ fm$^{-1}$ $\sim 200/A^{1/3}$ MeV. Typical targets have $A = O(10^2)$ (Ge has $A \approx 70$ and Xe $A \approx 130$). For $E_R \lesssim 100$ keV, $q \lesssim 100$ MeV, so that $qr_R$ may be larger than 1, leading to further suppression of the event rates, as nuclear form factor decreases rapidly with $qr_N$.
As for the cross section itself, it depends on the DM candidate. The cross sections fall in two categories: spin-independent (SI) or spin-dependent (SD), depending on whether DM couples to the spin of the nucleus. This is the case for Majorana particles, through their axial-vector coupling, while SI interactions are due to scalar or vector couplings. Here I will only consider for simplicity SI cross sections, which are relevant for a large class of candidates, including our Dirac neutrino. SD cross sections are proportional to $J(J+1)$ with $J$ the spin of the nucleus, while SI is typically $A^2$; nucleus spins may be large, but yet are much smaller than the mass number, so SD searches, when relevant, tend to be more sensitive.
The SI elastic scattering DM-N differential cross section is in general parameterized as follows the

$$\frac{d\sigma}{dE_R} = \frac{1}{v^2}\frac{m_N\sigma_n^0}{2\mu_N^2}\frac{[Zf_p + (A-Z)f_n]^2}{f_n^2}F^2(E_R)$$

where the parameters $f_p$ and $f_n$ encapsulate the coupling of DM to the nucleons and $\sigma_n^0$ is the effective DM-neutron cross section at zero momentum transfer; the limit are usually quoted in terms of the latter quantity vs the mass of DM. (Warning: there are quite a few assumptions behind this expression.)
In practice detectors has a minimum threshold recoil energy, of the order of a few keV typically, so there are (essentially) no constraints for light dark matter, say less than a few GeV. For larger masses, the abundance of dark matter $n = \rho/m_{\rm dm}$ so the limits on $\sigma_n$ decreases like $1/m_{\rm dm}$, as shown in the familiar exclusion plots, see Fig.9. Maximal sensitivity is around $m_\chi \sim m_N$, which is a compromise between maximal energy transfer (the DM particle comes to rest for collisions with $\theta = 0$) and the exponential falloff of the event rate for large $E_R$.

How about our Dirac neutrino? The neutron elastic cross section is actually quite large [27]

$$\sigma_n^0 = \frac{G_F^2}{2\pi}\mu_N^2 \approx 7.4\cdot 10^{-39}\,\mathrm{cm}^2$$

or about $10^{-2}$ pbarn, assuming $m_\nu \gtrsim m_n$.[8] This is has been searched for as far as 1987 [28], and a Dirac neutrino with mass between 20 GeV and 1 TeV has been excluded. Light Dirac neutrinos are excluded by the invisible width of the Z. How about heavy candidates? The current upper limit is of course much stronger, excluding Dirac neutrinos beyond 1000 TeV (this is a crude limit, obtained by eye inspection). This is beyond the mass scale set by unitarity limit, so it cannot be a WIMP. Actually any (not just our Dirac neutrino) WIMP with vector coupling to the $Z$ gauge boson is excluded by direct searches. SD limits are substantially weaker, so a heavy Majorana coupled to the Z, and there are many, to start with the neutralino, is still a viable option. The SD constraints are weaker by about 5 orders of magnitude.

For the case of the Dirac neutrino, the coupling was through the $Z$ gauge boson. Vector couplings to nucleons are relatively easy to handle, because a vector boson couples only to the constituent quarks, and not the sea quarks or gluons. The latter is obvious. The reason why sea quarks don't participate is because they come in pairs quark-antiquark, which average to zero (at three level). For the sake of comparison, let us consider another important channel for SI direct detection: the coupling of DM through the BEH particle $h$. This is not academic. In many models, this is the dominant channel. I mentionned already the case of a scalar singlet $S$ or the lightest neutral component of the inert doublet. Then comes the question of determining the coupling of this particle to the nucleons. Clearly the coupling to light quarks must very inefficient, since the up and down quarks are very light ($m_u \sim 2$ MeV and $m_d \sim 6$ MeV) an thus weakly coupled to the $h$ (ie $y_u \sim 10^{-5}$). This is not quite the case for the heavy quarks, and a fortiori for the top quark, since its Yukawa is $y_t \approx 1$. Of course these quarks are only virtual, but since both quarks and antiquarks contribute, their contribution sum up. Let's see how this can be taken into account. The argument is sophisticated, so take what you can out of it.

The key reference is an article by Shifman, Vainshtein and Zakharov [29]. It is one of my favorite paper. The story start with the stress-energy tensor of QCD, $\Theta_{\mu\nu}$. For a scale invariant theory we expect that the trace of the stress-energy tensor is vanishing,

$$\Theta^\mu_\mu = 0.$$

Quark masses explicitely break scale invariance and contribute to the RHS,

$$\Theta^\mu_\mu = m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + \sum_Q m_Q\bar{Q}Q$$

where the sum is over the heavy quarks ($c$, $b$, $t$). But even if the quarks were massless, at the quantum level there is an extra contribution, called the trace anomaly

$$\Theta^\mu_\mu = -\left(11 - \frac{2}{3}n_q - \frac{2}{3}n_Q\right)\frac{\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu} m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + \sum_Q m_Q\bar{Q}Q$$

This quantum is somewhat analogous to the chiral anomaly. Here the extra contribution proportional to the gluon field may be related to the fact that quantum correction break scale invariance (because of the renomalization requires the introduction of an explicit scale – the renormalization scale — incidentally, the coefficient of the first term is proportional to the QCD $\beta$ function at 1-loop, with $n_q = 3$ light quarks and $n_Q = 3$ heavy quarks). This being accepted, we may take the matrix element of $\Theta^\mu_\mu$ between a nucleon state,

$$\langle N|\Theta^\mu_\mu|N\rangle \equiv m_N\langle N|N\rangle$$

If this sounds strange, just think that what we are doing is identifying the stress-energy tensor expressed in terms of quarks and gluons into one expressed in terms of hadronic states. Then the matrix element just pick the contribution to the nucleon $N$, with a coefficient which is its mass. The next step is to take into account the fact that the heavy quarks are heavy, so we integrate them out (ie we put them in loops and expand in power of $1/m_Q$). Then by consistency, we should get

$$\Theta^\mu_\mu = -\left(11 - \frac{2}{3}n_q\right)\frac{\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu} m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + O\left(\frac{1}{m_Q^2}\right)$$

---

[8] This is for a 4th generation Dirac neutrino with $Y = 1$ interacting vectorially through the $Z$. In this case $f_n \equiv f_u + 2f_d = -1/4$ and $f_p = 2f_u + f_d = 1/4 - \sin^2\theta_W$. If the Dirac neutrino is vector-like, then both the L and R components have the same coupling, and the $1/4 \to 1/2$. In that case, the cross section is 4 times larger.

since there are no more heavy quarks as $m_Q \to \infty$. In other words, the contribution to the heavy quarks is equivalent to the substitution

$$\sum_Q m_Q \bar{Q}Q \to -\frac{2}{3} n_Q \frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{a\mu\nu} + O\left(\frac{1}{m_Q^2}\right)$$

Weird, but this may checked diagrammatically. The heavy quarks are in triangular loop, with two gluons attached.
What have we learned? Two things. First in the chiral limit (or just accepting that the light quarks masses contribute little to the nucleon mass, we get

$$m_N = \langle N| -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{a\mu\nu} |N\rangle$$

so that the mass of the nucleon comes from the gluons (this does not contradicts the intuitive idea that the light quarks gets a consistuent mass — ie are dressed with gluons). Next we may extract the effective coupling of the BEH. At the quark level,

$$\Delta \mathcal{L} = -h\left(\sum_q \frac{m_q}{v}\bar{q}q + \sum_Q \frac{m_Q}{v}\bar{Q}Q\right)$$

Taking the matrix element of these Yukawa couplings gives

$$-h\langle N|\left(\sum_q \frac{m_q}{v}\bar{q}q + \sum_Q \frac{m_Q}{v}\bar{Q}Q\right)|N\rangle$$

Neglecting the contribution of the light quarks and using the substitution from above, we get

$$-\frac{h}{v}\langle N|\left(-\frac{2}{3}n_Q\frac{\alpha_s}{8\pi}G^a_{\mu\nu}G^{a\mu\nu}\right)|N\rangle \equiv -h\frac{2}{3\cdot 9}n_Q\frac{m_N}{v}\langle N|N\rangle$$

which leads to the identification

$$-\frac{y_N}{\sqrt{2}}h\bar{N}N \equiv -\frac{2n_Q}{27}\frac{m_N}{v}h\bar{N}N$$

At the end of the day, the BEH couples to the nucleons through the heavy quarks. The effective Yukawa coupling is given by (for $n_Q = 3$)

$$y_N = \frac{2\sqrt{2}}{9}\frac{m_N}{v} \approx 10^{-3}$$

Notice that $y_N v/\sqrt{2} \approx 0.2$ GeV; this does not mean that the Higgs field contributes to $\sim 1/5$ of the mass of the nucleons.[9] Here we have neglected the contribution of the light quarks. Consequently the coupling to the proton or the neutron is the same. Now for the strange quark, it is not clear at this stage whether this is a good approximation; the corrections can of course be worke d out [30].
The simplest DM model with interaction through the Higgs portal is a scalar singlet (16). Since there is only are only two (relevant) free parameters, $m_S$ and the coupling $\lambda$, one can univocally relate the relic abundance and the direct detection SI cross section. In Fig.10 we show a plot of the WIMP candidates in the plane $\lambda - m_S$ (the plot is from [31], with $\lambda \to \lambda_{eff}$). The dip is due to the Higgs resonance. Most of the parameter space is excluded by LUX exclusion limits and constraints on the invisible BR of the BEH. There are only two viable regions: around the $h$ dip and for masses larger than $m_S \gtrsim 100$ GeV. There are other constraints, in particular around the $h$ resonance, due to annihilation of the $S$ into gamma rays, see e.g. [32]: those are most relevant for light candidates [33]. This brings to constraints from indirect searches for DM.

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[9] Instead, and merely, it is a measure of the effective coupling of the Higgs to nucleons. The contribution of heavy quarks (which are virtual particles in nucleons) to the mass of nucleons is instead estimated to be $Om_\pi^2/m_Q^2$, which for instance for the charm is at most $10^{-3}$ GeV [29].

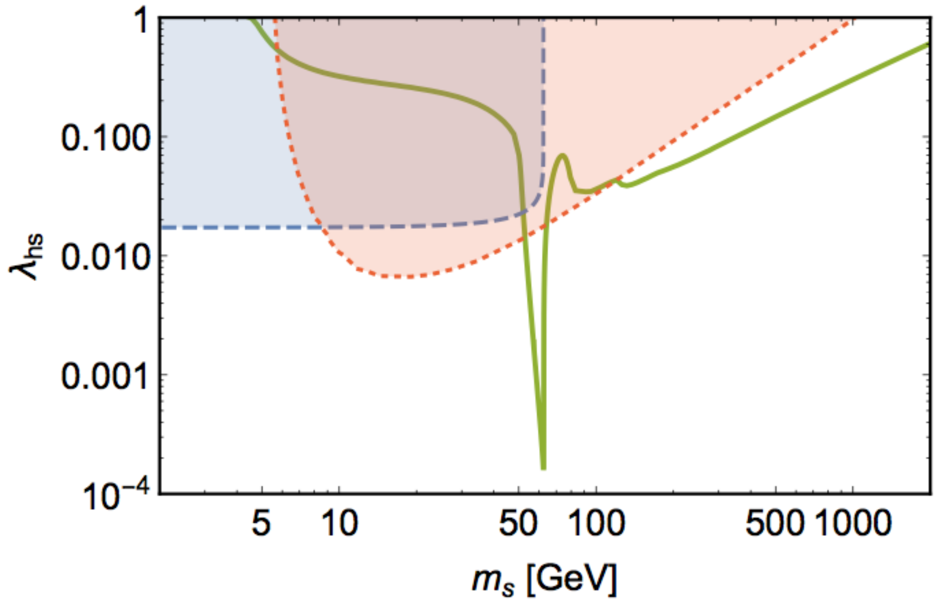


FIG. 10: Singlet scalar candidates constraints. The green line corresponds to the relic abundance, the pink region is excluded by LUX and the blue one by the invisible branching of the Higgs.

### B. Indirect detection: a case study

This is (again) a huge field. DM annihilation may be probed through the production of their SM products in cosmic rays, ie 1) gamma rays from $\pi^0$ decay (the latter are produced through hadronization of quarks, which may be produced directly or from say, the decay of a $\tau$; most interesting are mono-energetic gamma rays — I'll come back to this rightaway-), 2) neutrinos from the GC, or from the Sun or the Earth, and 3) antimatter in cosmic rays [34]. There are also a whole bunch of more or less indirect probes, like excess of synchroton radiation (produced by electrons and positrons), the deposition of heat in the CMBR, spallation of products from big bang nucleosynthesis, etc. Explain all of this would be hopeless. Instead I will focus on a so-called smoking gun in the form of gamma ray spectral features.

Gamma-rays emission directly from DM annihilation is simple to handle (synchroton or other repocessed emission is more complicated). Assuming annihilation cross section to velocity independent, the spectral gamma-ray flux (the number count of gamma per energy interval, per area per unit of time – we will use GeV$^{-1}\cdot$ cm$^{-2}\cdot$s$^{-1}$) observed at distance $r$ from a source and integrated over the volume of the annihilation region is given by

$$\frac{d\phi_\gamma}{dE} = \frac{1}{4\pi}\frac{dN_\gamma}{dE}\frac{\sigma v}{m_\chi^2}\int_{V_{\rm ann}}\frac{1}{r^2}\frac{\rho^2(r)}{2}\,d^3x \quad \text{for } \chi=\overline{\chi} \tag{17}$$

where $dN_\gamma/dE$ is the spectrum of gamma rays produced per annihilation. The factor of $1/2$ comes from the counting of the number of possible pairs $\frac{1}{2}\frac{N}{V}\frac{N-1}{V} \equiv \rho^2/2m_\chi^2$.[10] This is traditionnally rewritten as

$$\frac{d\phi_\gamma}{dEd\Omega} = r_s\rho_s^2\,\frac{dN_i}{dE}\,\frac{\sigma v}{8\pi m_\chi^2}\,J(\theta)$$

where $r_s$ is the distance to the source (eg $r_s \approx 8$ kpc for emission from the GC) and the factor $J$ contains all the astrophysics and is defined as the integral of $\rho^2(r)$ along the line of sight (los) (see Fig.11)

$$J(\theta) = \int_{\rm los}\frac{ds}{r_0}\left(\frac{\rho(r(s,\theta))}{\rho_s}\right)^2. \tag{18}$$

[10] For non-identical particles $\bar\chi \neq \chi$ there is an extra factor of $1/2$ for $\frac{N_{\bar\chi}}{V}\times\frac{N_\chi}{V} \equiv \rho^2/4/m_\chi^2$.

The behaviour of $J(\theta)$ for the Milky Way for various profiles is shown in Fig.12. Observations are more than often made over a

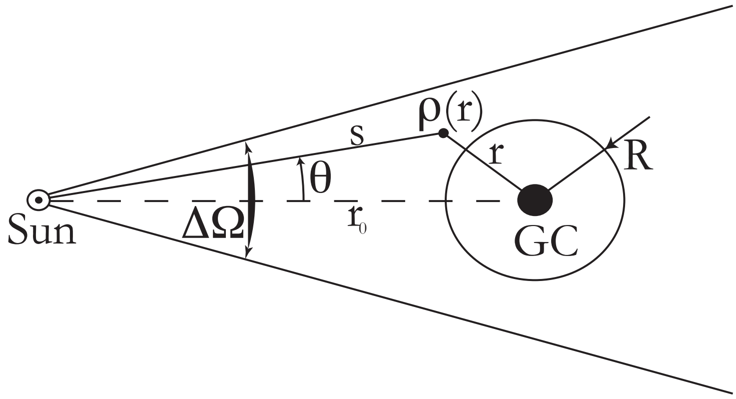


FIG. 11: Sketch of the relevant parameters for the integration of $J$ (courtesy S. Andreas).

region of size $\Delta\Omega$, so that $J$ is replaced by its average $\bar{J} = 1/\Delta\Omega \int d\Omega J$. Different profiles lead to very different predictions for

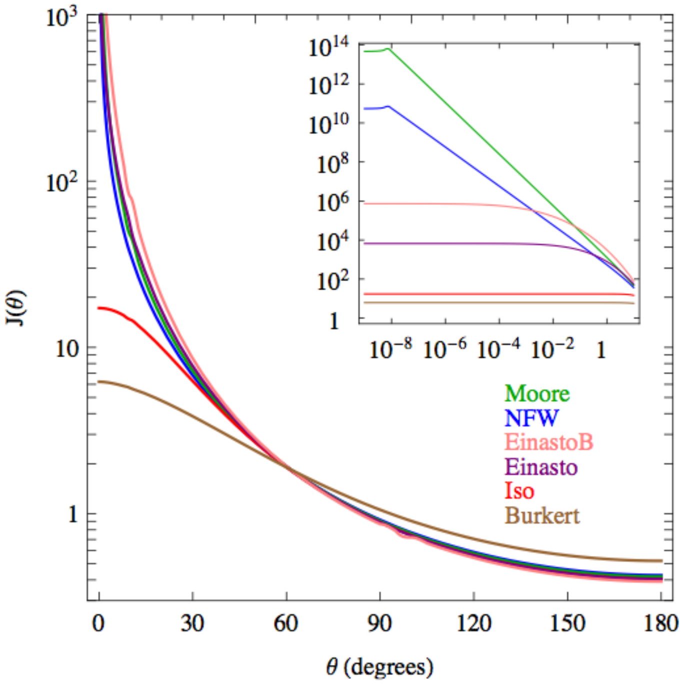


FIG. 12: $J(\theta)$ factor for various profiles as function of $\theta$ (in degrees, including the inset)[34]

the flux, specially if the integration region is small compared to the natural scale of the halo, cuspy profiles (like NFW) leading to much large $\bar{J}$ factors than cored profile (like isothermal); some values are given in Table I. More precise expressions may be found in [34].
The simplest instance is annihilation of DM into digammas, in which case $dN/dE_\gamma = 2\delta(E_\gamma - m_\chi)$. In the mass range relevant for WIMPs, there is no astrophysical source of digammas, so background is less an issue than for other searches. Furthermore observation would give directly the mass of DM, since annihilation in astrophysical site is essentially at rest, $E_\gamma = m_{\rm dm}$. The drawback is that, generally, annihilation into digammas is a loop suppressed process, and so subdominant. This effect is somewhat compensated by the diminution of the background (provided there is no significant continuum contribution, which is NOT generically the case). Another kind of possible feature is so-called virtual internal bremsstrahlung. This kind of emission is quite

| $\Delta\Omega$ | $\theta$ range | NFW | ISO |
|---|---|---|---|
| $10^{-5}$ | $< 0.1^\circ$ | $\sim 10^5$ | $\sim 20$ |
| $10^{-3}$ | $< 1^\circ$ | $\sim 10^4$ | $\sim 20$ |
| 0.1 | $10^\circ \times 10^\circ$ | $\sim 10^2$ | $\sim 15$ |

TABLE I: $\bar{J}$ for different regions centered on the GC for the NFW and Isothermal profiles.

distinct from standard Bremsstrahlung, so let me explain a bit what may be going on. By standard Bremsstrahlung I mean the emission of soft and collinear photons by charged particles on mass-shell. This typically leads to infrared divergences, which properly taken into account, sum up to multiplying the leading cross section by so-called splitting functions, for instance

$$\frac{d\sigma(\chi\chi \to l\bar{l}\gamma)}{dx} \approx \frac{\alpha}{\pi} F(x) \log\left(\frac{s(1-x)}{m_l^2}\right) \sigma(\chi\chi \to l\bar{l})$$

where $x$ is the fraction of the center of mass energy (squared) taken away by the emitted photon. The factorization of these so-called Sudakov factors is central to many soft processes in QED or QCD. The important point is that the Bremsstrahlung cross section is proportional to the leading order cross section. Closer examination also reveals that the splitting function $F$ typically decreases with $E_\gamma$. Hence this kind of Bremsstrahlung is essentially soft. Virtual Bremsstrahlung is an extra contribution that may arise if there is an intermediate electrically charged particle $L$ in the $t$ channel of, say, mass $M_L$ (see Fig.13). In that case

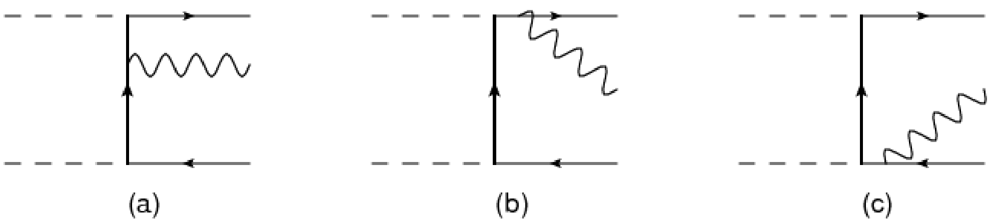

FIG. 13: Diagrams contributing to the amplitudes for $\chi\chi \to \bar{l}l\gamma$ process.

the amplitude is proportional to the

$$\mathcal{M} \propto \left((p_\chi - p_\gamma)^2 - M_L^2\right)^{-1} \sim \left(M_\chi^2 - 2M_\chi E_l - M_L^2\right)^{-1}$$

where I have assumed that $\chi$ annihilation is at rest and that $M_l \ll M_\chi$. Clearly, if $M_L^2 \sim M_\chi^2$, the amplitude increases for $E_l \to 0$, hence when $E_\gamma \to M_\chi$. We thus expect the signal to be peaked around the DM mass, and this the more the DM and the particle in the t-channel are close in mass. Interestingly different models predict the same signal, independent of the nature of the DM particle; it's a purely kinematic effect. Such a spectrum $dN/dE_\gamma$ is shown in Fig.14
Most interested is the situation in which $\sigma(\chi\chi \to \bar{l}l)$ is suppressed in the non-relativistic limit. This is the case for Majorana particles (which is p-wave suppressed for annihilation into light fermion pairs) and more generally for identical particles (a real scalar is even more suppressed). In that case the dominant contribution to emission from, say the GC, is gamma-rays through virtual internal Bremsstrahlung [35]. Since the signal is generically peaked around $E_\gamma \sim M_\chi$, it cannot in general be descriminated from a true gamma-ray line; hence, to first approximation we may use the Fermi limits on gamma-ray line searches. We show that in Fig.15 and 16 for respectively a Majorana and a real scalar dark matter candidate [36]. The latter is very constrained; the former less.

This is just the beginning. Further constraints should be considered because there are new, charged particles. Those could be produced at colliders. If we extend this framework to colored particles, the relevant collider is the LHC, where heavy colored particles are being searched for. This is somewhat distinct from the standard strategy for DM searches at colliders, that are more focused on missing $E_T$ signals (with jets, or gammas, etc.). In the present context, the strongest constraints are related to the mediator particle, here in a t-channel. Somewhat similar statements can be made for mediator in an s-channel. But this is another story. So I stop here.

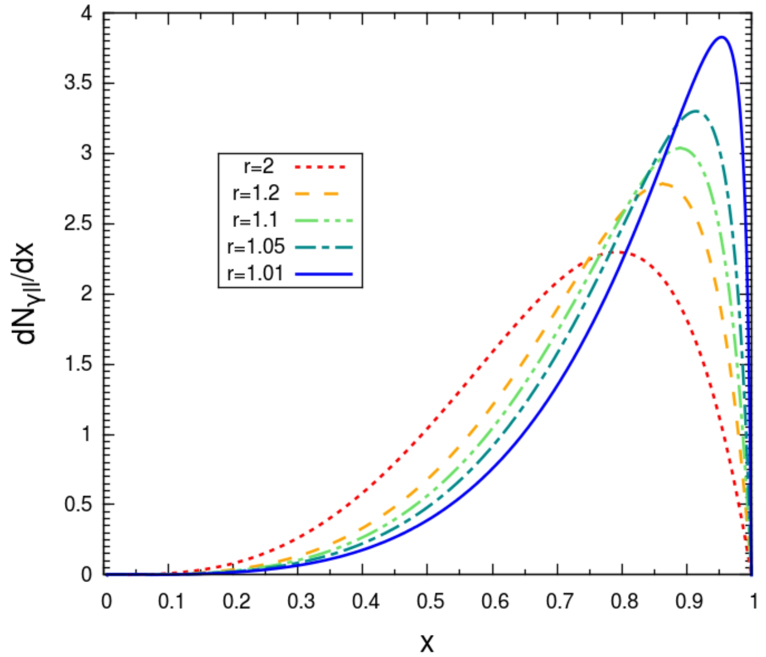


FIG. 14: Spectra $dN_{\bar{l}l\gamma}/dx$ as a function of $x = E_\gamma/M_\chi$ for several values of $r = M/M_\chi$ with $M$ the mass of the particle in the t-channel.

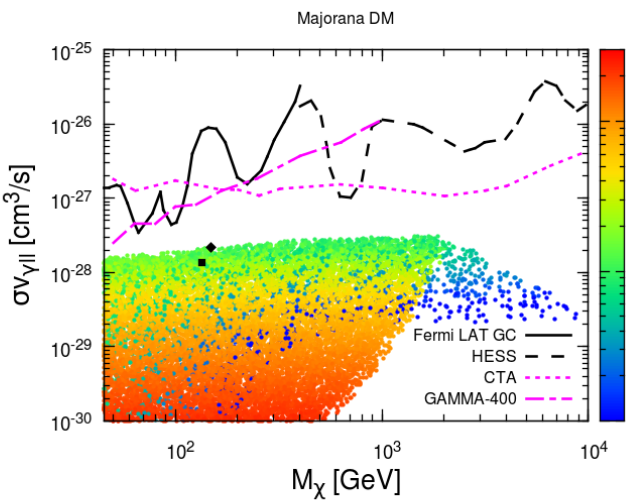


FIG. 15: Expected VIB cross section for Majorana WIMP candidates, together with constraints from Fermi-LAT, HESS and the future CTA.

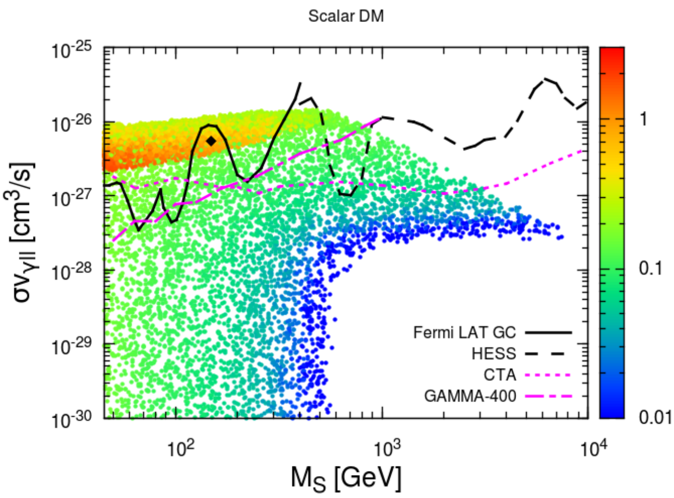


FIG. 16: The same for a real scalar candidate.

## IV. APPENDICES

### A. Some definitions

Friedmann equation:

$$H = \frac{\dot{a}}{a} \qquad H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

During the radiation dominated era (RD), curvature may be neglected and

$$\rho = \rho_R = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$$

where $g_{\text{eff}}$ counts the number of relativistic degrees of freedom at a given temperature $T$ (all the particles for which $m \lesssim T$), taking into account that some degrees of freedom may have a distinct temperature (the temperature of reference is that of the photons),

$$g_{\text{eff}} = \sum_B g_B \left(\frac{T_B}{T}\right)^4 + \frac{7}{8}\sum_F g_F \left(\frac{T_F}{T}\right)^4$$

For instance for SM neutrinos, $T_F/T = (4/11)^{1/3} \approx 0.71$ for $T \lesssim m_e$. Above this temperature, the neutrinos are in thermodynamic equilibrium with the photons and they share the same temperature.
During the RD era,

$$H \approx 1.66\, g_{\text{eff}}^{1/2} \frac{T^2}{M_{\text{pl}}}.$$

### B. Some numbers

Radiation-matter equality:

$$\begin{aligned} a_{EQ} &= 4.15 \times 10^{-5}(\Omega_m h^2)^{-1} \\ k_{EQ} &= 0.073\Omega_m h^2 \text{Mpc}^{-1} \approx 0.015\, h\text{Mpc}^{-1} \\ \lambda_{EQ} &\approx 600\text{Mpc} \end{aligned}$$

Cosmological and astrophysical parameters:

$$\begin{aligned} M_{\text{pl}} &\equiv G^{-1/2} = 1.2 \cdot 10^{19}\,\text{GeV} \\ \rho_c &\equiv \frac{3H^2}{8\pi G} \approx 5\,\text{GeV/m}^3 \end{aligned}$$

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