



# Global Collective Flow in Heavy Ion Reactions

Laszlo P. Csernai, University of Bergen  
Based on: Csernai, Stöcker, J Phys G 41 (2014) 124001

CERN - Heavy Ion Forum, Jan. 23, 2015

## CERN Heavy Ion Forum

## Global Collective Flow in Heavy Ion Reactions

by Laszlo Csernai (Department of Physics and Technology)

Friday, 23 January 2015 from **10:00** to **11:00** (Europe/Zurich)  
at CERN ( 4-3-006 - TH Conference Room )

**Description** Heavy ion reactions recently are analyzed for fluctuations, which are clearly the dominant sources of azimuthal flow asymmetries in central collisions. In peripheral collisions on the other hand there are azimuthal asymmetries arising from the finite impact parameter and the consequently large angular momentum of the participant system. Experimental results clearly indicate the existence of these two mechanisms in the flow patterns. The talk will discuss how to separate and analyze these two types of flow patterns in peripheral collisions.

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Extension	10350426
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Description	Heavy ion reactions recently are analyzed for fluctuations, which are clearly the dominant sources of azimuthal flow asymmetries in central collisions. In peripheral collisions on the other hand there are azimuthal asymmetries arising from the finite impact parameter and the consequently large angular momentum of the participant system. Experimental results clearly indicate the existence of these two mechanisms in the flow patterns. The talk will discuss how to separate and analyze these two types of flow patterns in peripheral collisions.
Auto-join URL	<a href="http://vidyoportal.cern.ch/flex.html?roomdirect.htm&amp;key=LhyJ1W4sC2KWe0ULzK7ks1mU">http://vidyoportal.cern.ch/flex.html?roomdirect.htm&amp;key=LhyJ1W4sC2KWe0ULzK7ks1mU</a>

<https://indico.cern.ch/event/358426/>



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L.P. Csernai 2

# Fluctuations and polarization

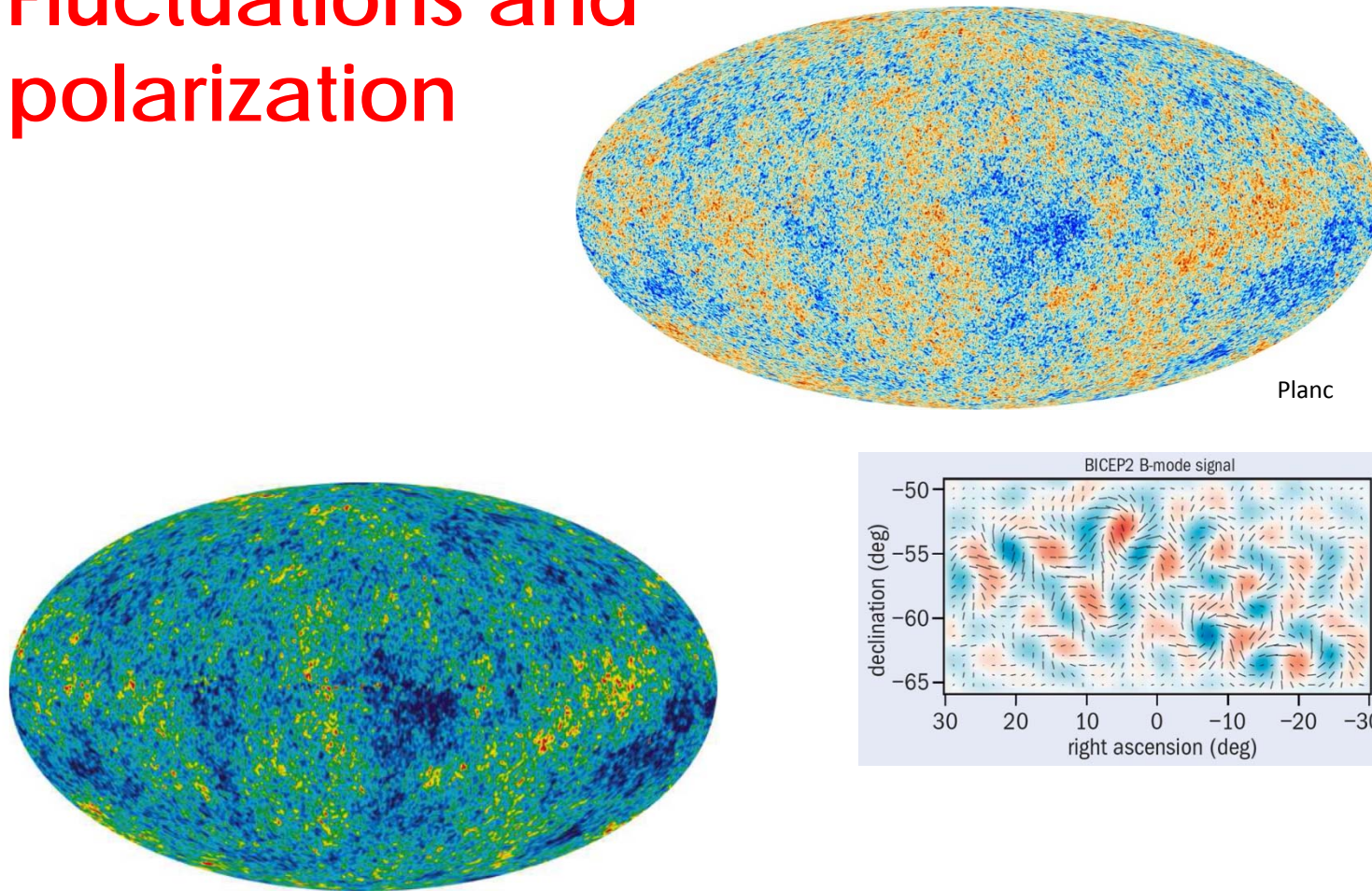


Figure 32: The CMB radiation temperature fluctuations from the 5-year WMAP data seen over the full sky. The average temperature is 2.725K, and the colors represents small temperature fluctuations. Red regions are warmer, and blue colder by about 0.0002 K.

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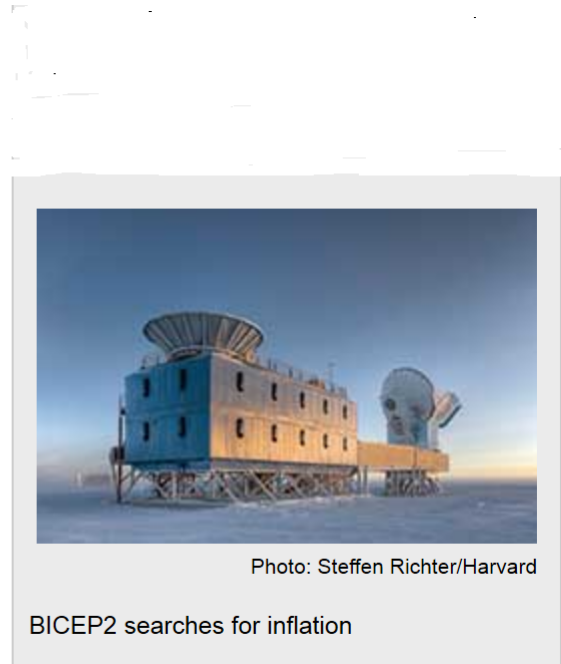
### Stories in 2014

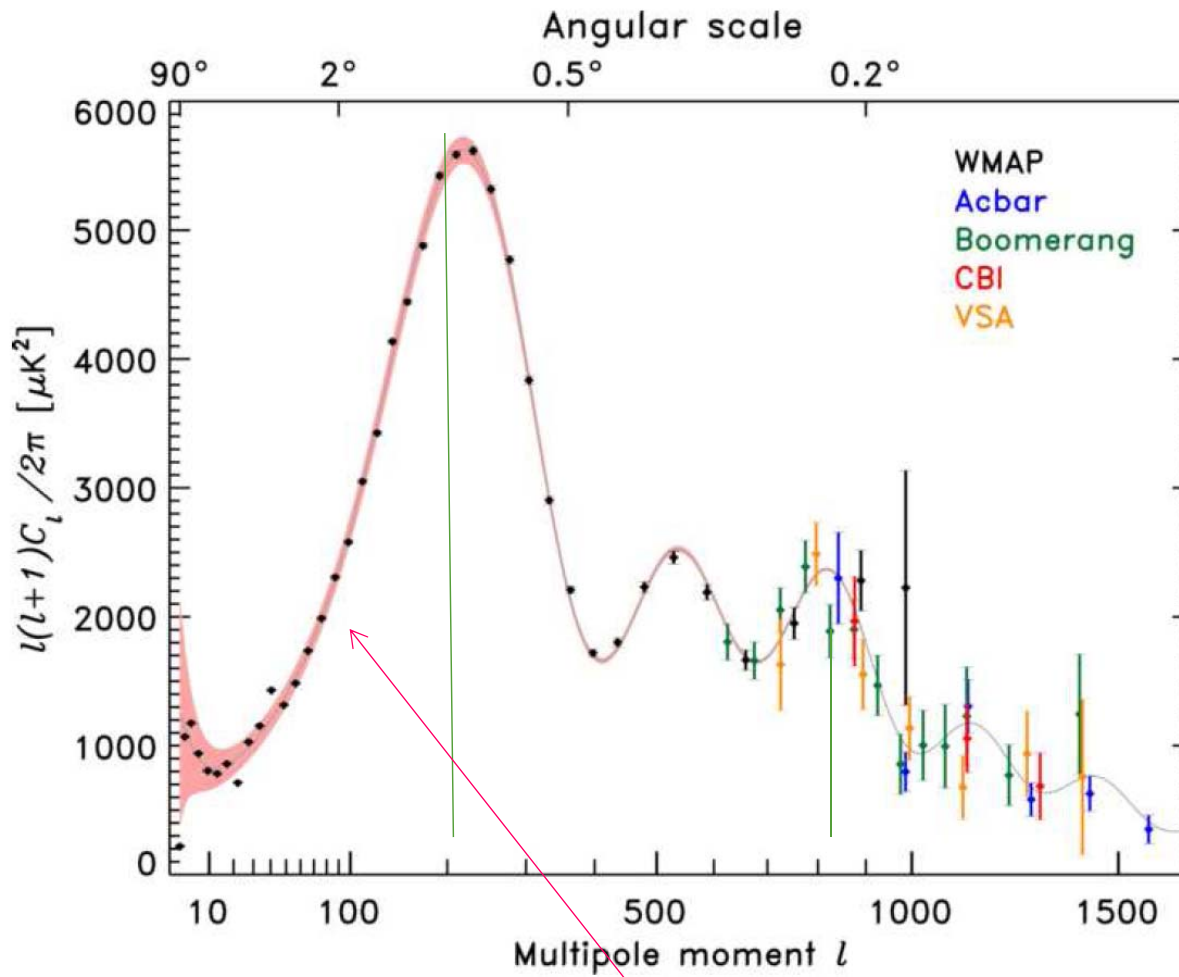
## Top Ten Physics News Stories in 2014

Every year, APS News looks back to see which physics news stories grabbed the attention of the public. This list is not necessarily a compilation of the most important advances or discoveries of the year, but rather the ones that seemed to garner the most headlines and column-inches. In (roughly) chronological order, the top ten physics stories of 2014 were:

### BICEP2

In **March** the scientific team behind the BICEP2 telescope at the South Pole made the sensational announcement that they had seen the first evidence of “B-mode” polarization in the cosmic microwave background (CMB) radiation. At the time it was held up as “the smoking gun” for evidence of gravitational waves left over from a period of rapid inflation in the early universe. However, soon after the announcement, doubts about the data started to emerge, and it was unclear if the team could definitively rule out the effect of cosmic dust. In the resulting scientific paper, published in **June**, the team acknowledged that dust may have affected the observations, but nevertheless they still felt the gravitational wave signal was real. In **September** a new report from the ESA’s Planck satellite reinforced concerns about the initial results, but the two teams are continuing to work together to resolve the discrepancies. Also in **December**, independent of the BICEP2 research, Planck’s team announced that they had finished processing the data from the satellite’s four-year run and had created the most detailed map of the CMB.

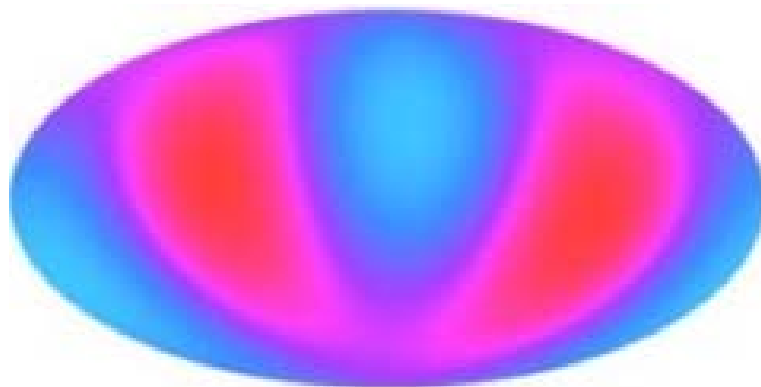




Longer tail on the negative ( low  $\ell$  ) side ! (see discussion of “Skewness” later)

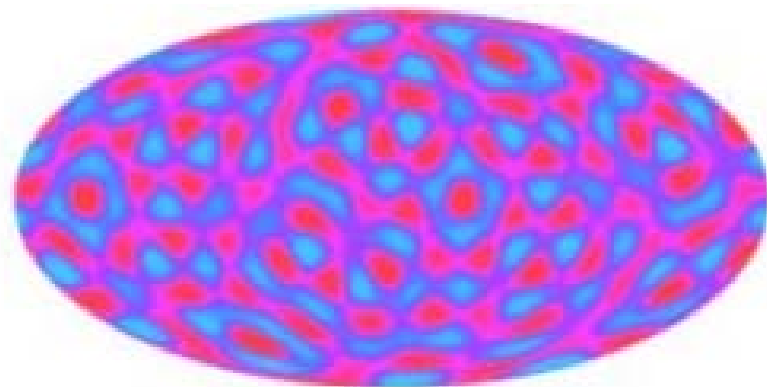
# In Central Heavy Ion Collisions

~ like Elliptic flow,  $v_2$



$l=2$

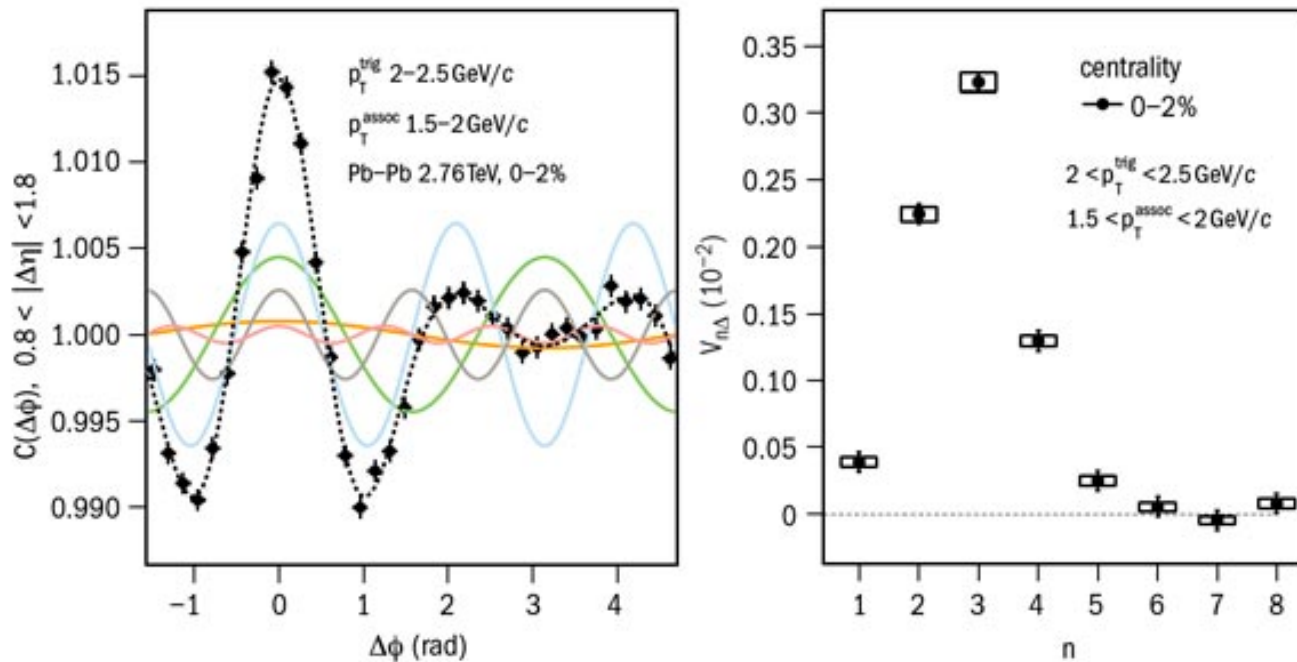
~ spherical with many (16) nearly equal perturbations



$l=16$

Sep 23, 2011

## ALICE measures the shape of head-on lead-lead collisions



**Flow originating from initial state fluctuations is significant and dominant in central and semi-central collisions (where from global symmetry no azimuthal asymmetry could occur, all Collective  $v_n = 0$ ) !**

# Critical Fluctuations



[L.P. Csernai, Z. Neda / Physics Letters B 337 (1994) 25-29]

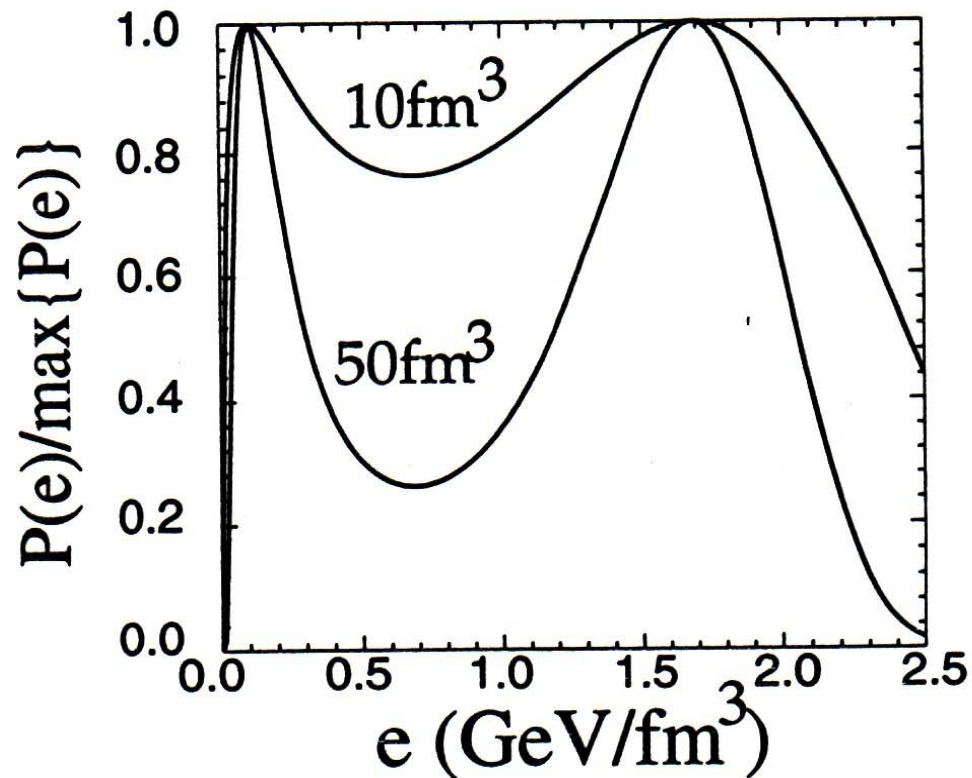


Fig. 2. *The relative probability of finding a state of a given energy density,  $e$ , in a system of given volume,  $\Omega = 10, 50 \text{ fm}^3$ , at a constant temperature,  $T = T_c$ .*



# Fluctuations in Hadronizing QGP

L.P. Csernai<sup>1,2</sup>, G. Mocanu<sup>3</sup> and Z. Néda<sup>3</sup> *PHYSICAL REVIEW C* **85**, 068201 (2012)

Higher order moments can be obtained from fluctuations around the critical point. →  
 Skewness & Kurtosis are calculated for the QGP → HM phase transition

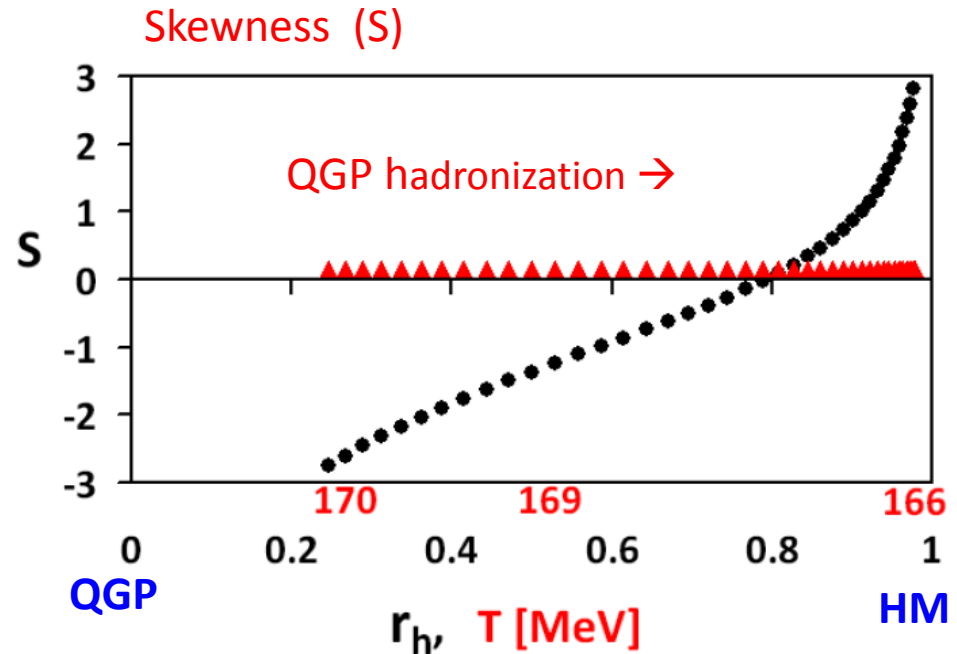
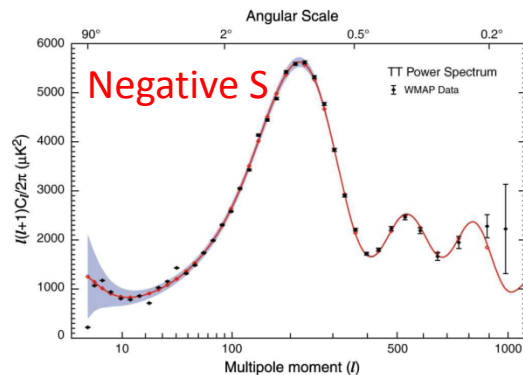
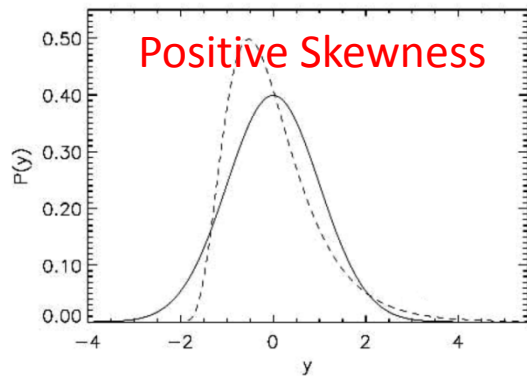


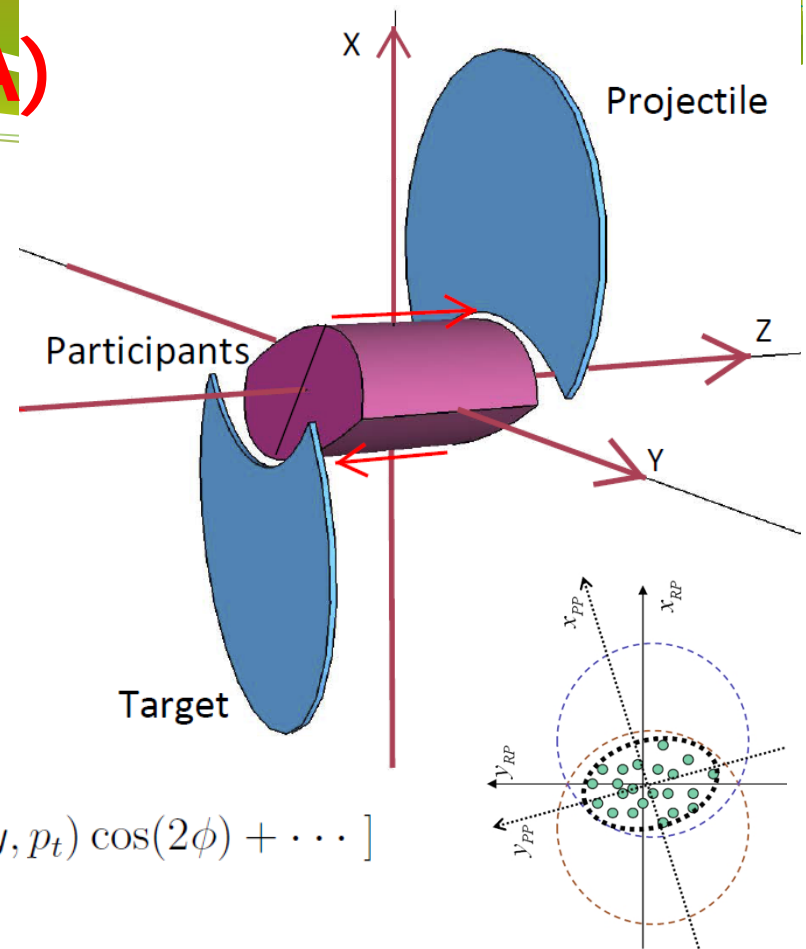
FIG. 4: (color online) Skewness as a function of the volume abundance of the hadronic matter (denoted as  $r_h$ , where 1 represents complete hadronization). The temperature scale is also indicated for clarity, the identifiers represent increments of 0.1 MeV in  $T$ . Results for  $\Omega = 500 fm^3$ .

Negative Skewness indicates Freeze-out mainly still on the QGP side.

# Peripheral Collisions (A+A)

Csernai & Stöcker, arXiv: 1406.1153v2 [nucl-th]

- ❑ Global Symmetries
- ❑ Symmetry axes in the global CM-frame:
  - ❑ ( $y \leftrightarrow -y$ )
  - ❑ ( $x, z \leftrightarrow -x, -z$ )
  - ❑ Azimuthal symmetry:  $\phi$ -even ( $\cos n\phi$ )
  - ❑ Longitudinal  $z$ -odd, (rap.-odd) for  $v_{\text{odd}}$
  - ❑ Spherical or ellipsoidal flow, expansion



Theory:

$$\frac{d^3 N}{dy dp_t d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dy dp_t} [1 + 2v_1(y, p_t) \cos(\phi) + 2v_2(y, p_t) \cos(2\phi) + \dots]$$

Experiment:

$$\frac{d^3 N}{dy dp_t d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dy dp_t} [1 + 2v_1(y - \underline{y_{CM}}, p_t) \cos(\phi - \underline{\Psi_{RP}}) + 2v_2(y - \underline{y_{CM}}, p_t) \cos(2(\phi - \underline{\Psi_{RP}})) + \dots]$$

- ❑ Fluctuations
- ❑ Global flow and Fluctuations are simultaneously present  $\rightarrow \exists$  interference
  - ❑ Azimuth - Global: even harmonics - Fluctuations : odd & even harmonics
  - ❑ Longitudinal - Global:  $v_1, v_3$  y-odd - Fluctuations : odd & even harmonics
  - ❑ The separation of Global & Fluctuating flow is a must !! (not done yet)

# Anisotropic Flow

[R.Snellings, J Phys G 41 (2014)]

Used by most experimental groups today.

$$\frac{dN}{d\varphi} = \frac{\bar{N}}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \bar{v}_n \cos(n(\varphi - \bar{\Psi}_n)) \right) \quad (1)$$

where  $\bar{N} \equiv \langle N \rangle$  is the mean number of selected particles per event,  $\varphi$  the azimuthal angle, and  $\bar{\Psi}_n$  the mean angle of the  $n$ -th harmonic flow plane.

This is a complete ortho-normal series only if all  $\Psi_n$ -s are given in the same reference frame with respect to some physical axis frame of the reaction, e.g. the

$$\bar{v}_n(p_T, y) = \langle \langle \cos[n(\varphi - \bar{\Psi}_n)] \rangle \rangle \quad \text{or equivalently}$$

$$\bar{v}_n(p_T, y) = \langle \langle e^{in\varphi} e^{-in\bar{\Psi}_n} \rangle \rangle,$$

$$\bar{v}_n(p_T, y) = \text{Re} \langle \langle e^{in\varphi} e^{-in\bar{\Psi}_n} \rangle \rangle$$

where  $\langle \langle \dots \rangle \rangle$  denotes an average in the  $(p_T, y)$  bin

# Anisotropic Flow

[R.Snellings, J Phys G 41 (2014)]

## 3.1. *Experimental Methods* (for evaluating $v_n$ )

$$\begin{aligned}\text{Re} \langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle &= \langle\langle e^{in(\varphi_1 - \bar{\Psi}_n - (\varphi_2 - \bar{\Psi}_n))} \rangle\rangle, \\ &= \langle\langle e^{in(\varphi_1 - \bar{\Psi}_n)} \rangle\rangle \langle e^{-in(\varphi_2 - \bar{\Psi}_n)} \rangle + \delta_{2,n}, \\ &= \langle \underline{v_n^2} + \delta_{2,n} \rangle,\end{aligned}$$

e.g. with 4 particle cumulant method:

$$c_n\{4\} \equiv \langle\langle\langle e^{in(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle\rangle\rangle - 2 \langle\langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle\rangle^2 = \langle -v_n^4 + \delta_{4,n} \rangle.$$

Reaction plane (RP) is lost, P/T side of RP is also lost.

# We need an EbE reference angle (e.g. the RP). Can we find it?

flow vector  $Q_n \equiv \sum_{i=1}^M e^{in\varphi_i} \longrightarrow Q_1 \equiv \sum_{i=1}^M e^{i\varphi_i} \longrightarrow Q_1^p \equiv \sum_{i=1}^M |\vec{p}_i| e^{i\varphi_i} = 0$

By Danielewicz and Odyniec (DO)  $\rightarrow$  Separate forward & backward pt.  $\rightarrow$  **c.m.**

$${}^{DO}Q_1^p \equiv \sum_{i=1}^M |\vec{p}_i| y_i e^{i\varphi_i} \neq 0. \quad {}^wQ_1^p \equiv |\vec{p}_t| \sum_{i=1}^M y_i e^{i\varphi_i} \neq 0$$

$$\tan(\bar{\Psi}_{RP}) = \frac{\text{Im } {}^{DO}Q_1^p}{\text{Re } {}^{DO}Q_1^p}.$$

Or one can approximate this as:

$$\tan(\bar{\Psi}_{RP}) \approx \frac{\text{Im } Q_1^y}{\text{Re } Q_1^y}$$

$$\text{where } Q_1^y \equiv \sum_{i=1}^M y_i e^{i\varphi_i} \neq 0$$

Weighting with  $y \rightarrow$  dominates large rapidities  $\rightarrow$  Use a segmented ZDC to find the **RP!**

In addition we should find the participant **c.m.** Separate out longitudinal fluctuations.

## Removing self-correlations ( ← DO )

$$|Q_n|^2 = \sum_{i,j=1}^M e^{in(\varphi_i - \varphi_j)} = M + \sum_{i \neq j} e^{in(\varphi_i - \varphi_j)}$$

$$\langle e^{in(\varphi_1 - \varphi_2)} \rangle \equiv \frac{1}{M(M-1)} \sum_{i \neq j} e^{in(\varphi_i - \varphi_j)} = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$V_n^2 \approx c_n\{2\} \equiv \langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle = \frac{\sum_{i=1}^N |Q_{in}|^2 - M_i}{\sum_{i=1}^N M_i(M_i - 1)}$$

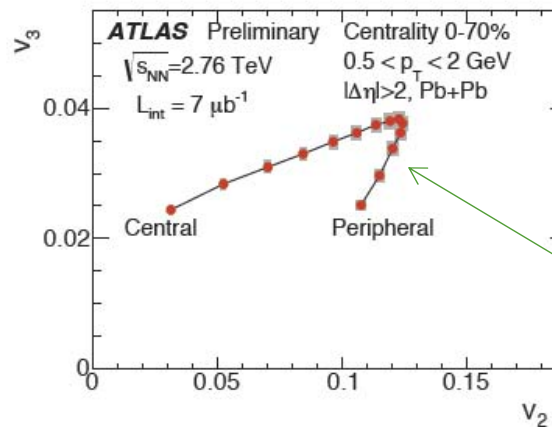
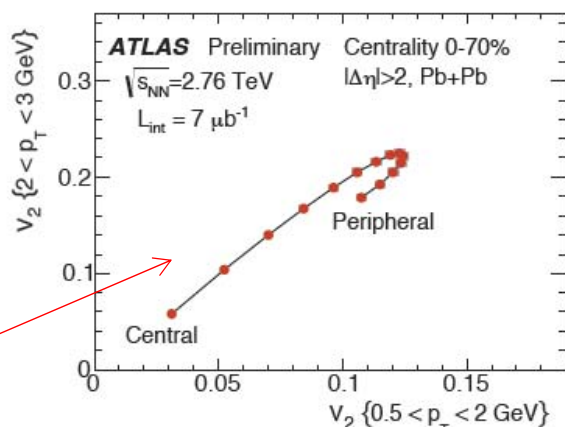
The sign of odd harmonics is lost

$$v_n\{2\} = \sqrt{\langle v_n^2 \rangle},$$

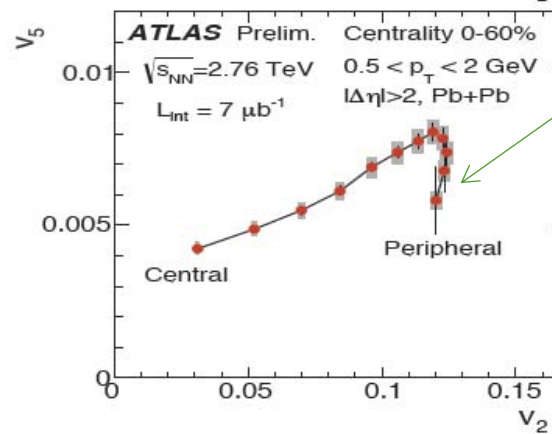
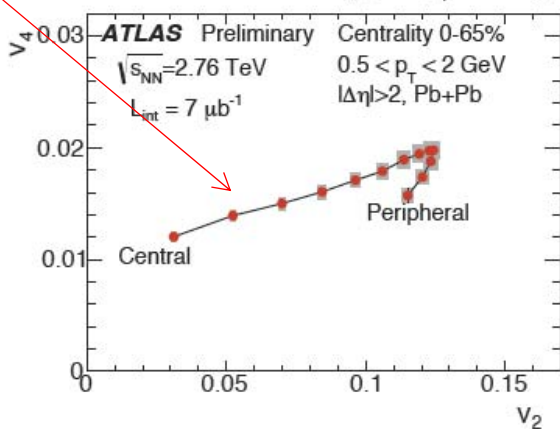
$$v_n\{4\} = \sqrt[4]{2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle},$$

# Event-shape engineering

Fluctuations



Global flow

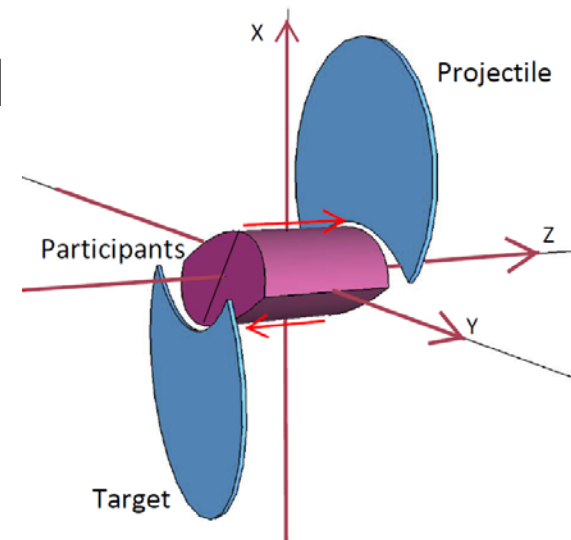


Correlation between flow coefficients:

- Non monotonic variation

# Two types of flow processes from: Fluctuations and/or Global Collective Flow

- How to split these two:
- In theoretical models
  - Mode-by-mode hydrodynamics,  
[S. Floerchinger, U.A. Wiedemann, Phys. Rev. C 88,  
044906 (2013), Phys. Rev. C 89, 034914 (2014), Phys.  
Lett. B 728, 407 (2014)]
- In experiments it is more involved
  - Average many events
  - But keeping the symmetries





# Method to compensate for C.M. rapidity fluctuations

1. Determining experimentally  $E_B$  the C.M. rapidity
2. Shifting each event to its own C.M. and evaluate flow-harmonics there

L.P. Csernai<sup>1,2</sup>, G. Eyyubova<sup>3</sup> and V.K. Magas<sup>4</sup>

PHYSICAL REVIEW C 86, 024912 (2012)

## Determining the C.M. rapidity:

The rapidity acceptance of a central TPC is usually constrained (e.g for ALICE  $|\eta| < \eta_{lim} = 0.8$ , and so:  $|\eta_{C.M.}| \ll \eta_{lim}$ , so it is not adequate for determining the C.M. rapidity of participants.

Participant rapidity from spectators

$$E_B = A_B m_{B\perp} \cosh(y^B) = E_{tot} - E_A - E_C,$$

$$M_B = A_B m_{B\perp} \sinh(y^B) = -(M_A + M_C)$$

$$E_A = A_P m_N \cosh(y_0),$$

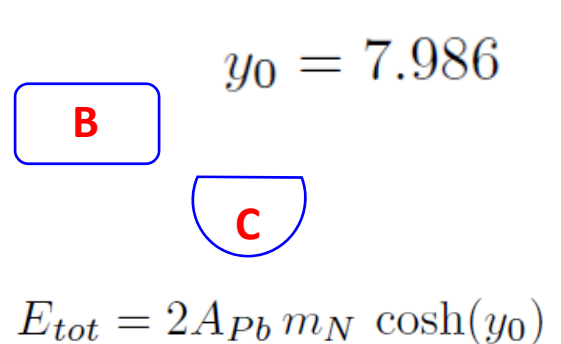
$$E_C = A_T m_N \cosh(-y_0),$$

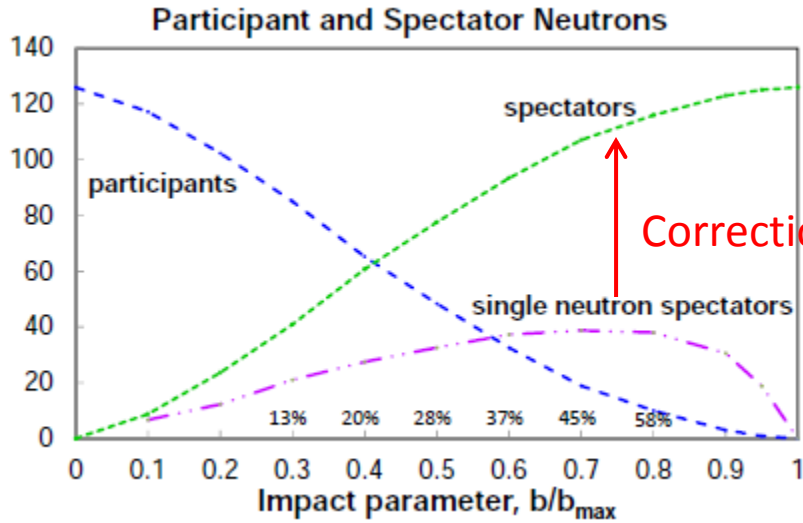
give the spectator numbers,  $A_P$  and  $A_T$ , and 

$$M_A = A_P m_N \sinh(y_0),$$

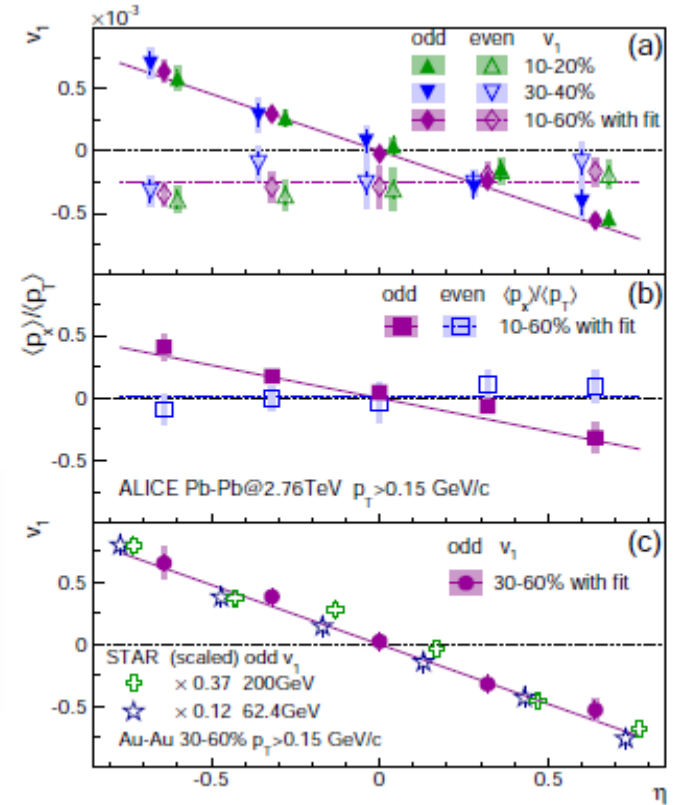
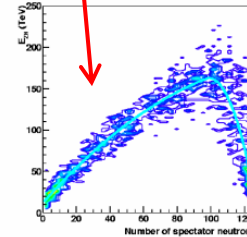
$$M_C = A_T m_N \sinh(-y_0),$$

$$y_E^{CM} \approx y^B = \text{artanh} \left( \frac{-(M_A + M_C)}{E_{tot} - E_A - E_C} \right)$$





Single neutron spectators are based on nuclear multi fragmentation studies → in experiment should be taken from data [ALICE estimate from 1984 →]



Results from preliminary ALICE data]

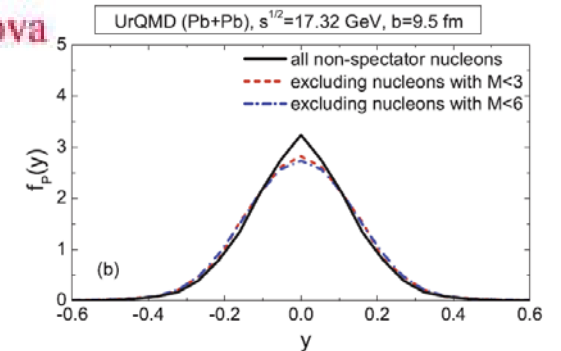
<https://twiki.cern.ch/twiki/bin/viewauth/ALICE/FlowGyulnaraEyyubova>

Results from preliminary ALICE data show the average and EbE fluctuations →

$$v_1^{\text{odd}} = \sim -0.0025 \quad v_1^{\text{even}} = \sim 0$$

ALICE PRL 2013:

$$v_1^{\text{odd}} = \sim -0.0005 \quad v_1^{\text{even}} = \sim -0.00025$$



V. Vovchenko, et al. FIG. 8. (Color online) Participant center-of-mass rapidity distribution. PHYSICAL REVIEW C 88, 014901 (2013)

## Azimuthal Flow analysis with Fluctuations today

In contrast to the above formulation

$$\frac{d^3 N}{dy dp_t d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dy dp_t} \left[ 1 + 2v_1(y, p_t) \cos(\phi - \Psi_1^{EP}) + 2v_2(y, p_t) \cos(2(\phi - \Psi_2^{EP})) + \dots \right],$$

Here  $\Psi_n^{EP}$  maximizes  $v_n(y, p_t)$  in a rapidity range

Is this a complete ortho-normal series? Yes, if the  $\Psi_n^{EP}$  values are defined .....

We can see this by using:  $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$ ,  $\rightarrow$

# Azimuthal FLOW analysis with Fluctuations today

Is this a complete ortho-norml series? Yes, if the  $\Psi_n^{EP}$  values are defined .....

We can see this by using:  $\cos(\alpha-\beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$ ,  $\rightarrow$

terms of the harmonic expansion

$$v_n \cos[n(\phi - \Psi_n^{EP})] = \underbrace{v_n \cos(n\Psi_n^{EP})}_{\text{In Collider}} \cos(n\phi) + \underbrace{v_n \sin(n\Psi_n^{EP})}_{\text{In EbE: CM,RP}} \sin(n\phi)$$

$$\begin{aligned} \Phi_n^{EP} &\equiv \Psi_n^{EP} - \Psi_{RP} \\ \phi' &\equiv \phi - \Psi_{RP} \end{aligned} \quad \begin{array}{l} \swarrow \\ \nwarrow \end{array} \text{Reaction Plane (EbE)}$$

And the two coefficients:

$${}^c v'_n \equiv v_n \cos(n(\Psi_n^{EP})) \quad {}^c v'_n = {}^c v'_n(y - y_{CM}, p_t)$$

$${}^s v'_n \equiv v_n \sin(n(\Psi_n^{EP})) \quad {}^s v'_n = {}^s v'_n(y - y_{CM}, p_t)$$

$\rightarrow$  terms of the harmonic expansion

$$v_n \cos[n(\phi - \Psi_n^{EP})] = v_n \cos[n(\phi' - \Phi_n^{EP})] = {}^c v'_n \cos(n\phi') + {}^s v'_n \sin(n\phi').$$

In Collider

In EbE: CM,RP

In EbE: CM,RP

## Now: Separating Global Collective Flow & Fluctuations

the Global Collective flow in the configuration space has to be  $\pm y$  symmetric  
 the coefficients of the  $\sin(n\phi')$  terms should vanish:  $s'_n = 0$

$c'_n$  for odd harmonics have to be odd functions of  $(y - y_{CM})$   
 for even harmonics have to be even functions of  $(y - y_{CM})$   
 $s'_n$  can be due to fluctuations only

Let us now introduce the rapidity variable  $\mathbf{y} \equiv y - y_{CM}$

and let us construct even and odd combinations from the data:

$$v_n^{\frac{Coll.}{odd}} \cos[n(\phi - \Psi_n^{EP})] = \frac{1}{2} [c'_n(\mathbf{y}, p_t) \pm c'_n(-\mathbf{y}, p_t)] \cos(n\phi')$$

$$v_n^{\frac{Fluct.}{odd}} \cos[n(\phi - \Psi_n^{EP})] = \frac{1}{2} [c'_n(\mathbf{y}, p_t) \mp c'_n(-\mathbf{y}, p_t)] \cos(n\phi') + \underbrace{s'_n(\mathbf{y}, p_t)} \sin(n\phi')$$

fluctuations must have the same magnitude for sine and cosine components  
 & for odd and even rapidity components.

[Csernai L P, Eyyubova G and Magas V K, Phys. Rev. C **86** (2012) 024912.]

[Csernai L P and Stoecker H, (2014) arXiv: 1406.1153v2 .]

# Development of $v_1(y)$ at increasing beam energies

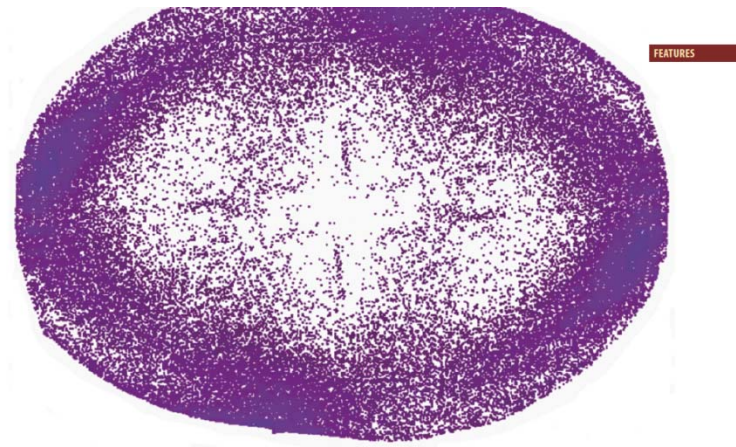
$v_1(y)$  observations show a central antiflow slope,  $\partial v_1(y)/\partial y$ , which is gradually decreasing with increasing beam energy [23]:

$$\frac{\partial v_1(y)_{odd}}{\partial y} = \begin{cases} -1.25\% & \text{for } 62.4 \text{ GeV (STAR)} \\ -0.41\% & \text{for } 200.0 \text{ GeV (STAR)} \\ -0.15\% & \text{for } 2760.0 \text{ GeV (ALICE)} \end{cases}$$

This can be attributed to smaller increase of  $p_T$  and the pressure, and the shorter interaction time, and **also to increasing rotation**.

In [Cs., Magas, Stöcker, Strottman, PRC84 (2011)] we predicted this rotation, but the turnover depends on the balance between rotation, expansion and freeze out. Apparently expansion is still faster and freeze out is earlier, so the turn over to the Positive side is not reached yet.

Interesting collective flow phenomena in low viscosity QGP →



**The Quark-Gluon Plasma, a nearly perfect fluid**

• L. Cifarelli<sup>1</sup>, L.P. Csernai<sup>2</sup> and H. Stöcker<sup>2</sup> - DOI: 10.1051/epj/2012206

# Detection of Global Collective Flow

We are will now discuss rotation (eventually enhanced by KHI). For these, the separation of Global flow and Fluctuating flow is important. (See ALICE v1 PRL (2013) Dec.)

- One method is polarization of emitted particles
  - This is based equilibrium between local thermal vorticity (orbital motion) and particle polarization (spin).
  - Turned out to be more sensitive at RHIC than at LHC (although  $L$  is larger at LHC)  
[Becattini F, Csernai L P and Wang D J, *Phys. Rev. C* **88** (2013) 034905.]
  - At FAIR and NICA the *thermal* vorticity is still significant (!) so it might be measurable.
- The other method is the Differential HBT method to analyze rotation:
  - [LP. Csernai, S. Velle, DJ. Wang, *Phys. Rev. C* **89** (2014) 034916]
  - We are going to present this method now

## Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

Laszlo P. Csernai,<sup>1,2</sup> Joseph I. Kapusta,<sup>3</sup> and Larry D. McLerran<sup>4</sup>

<sup>1</sup>*Section for Theoretical Physics, Department of Physics, University of Bergen, Allegaten 55, 5007 Bergen, Norway*

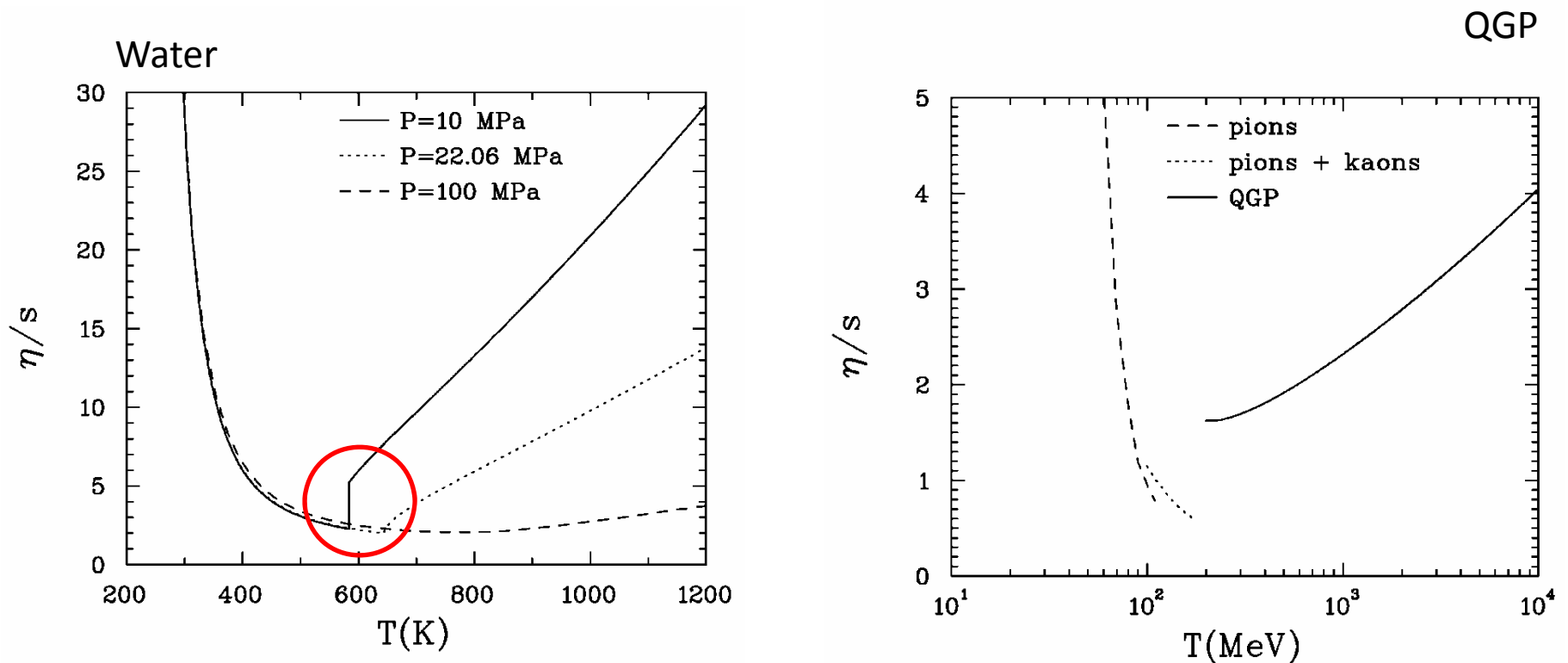
<sup>2</sup>*MTA-KFKI, Research Institute of Particle and Nuclear Physics, 1525 Budapest 114, P. O. Box 49, Hungary*

<sup>3</sup>*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*

<sup>4</sup>*Nuclear Theory Group and Riken Brookhaven Center, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA*

(Received 12 April 2006; published 12 October 2006)

Viscosity vs.  $T$  has a minimum at the 1<sup>st</sup> order phase transition. This might signal the phase transition if viscosity is measured. At lower energies this was done.





# Kelvin-Helmholtz instability in high-energy heavy-ion collisions

L.P. Csernai<sup>1,2,3</sup>, D.D. Strottman<sup>2,3</sup>, and Cs. Anderlik<sup>4</sup>  
 PHYSICAL REVIEW C **85**, 054901 (2012)

ROTATION

KHI →

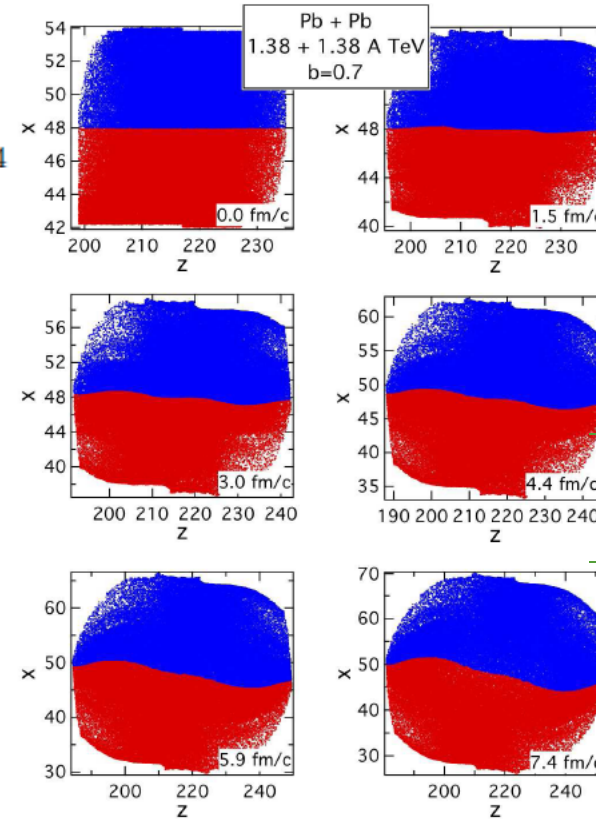
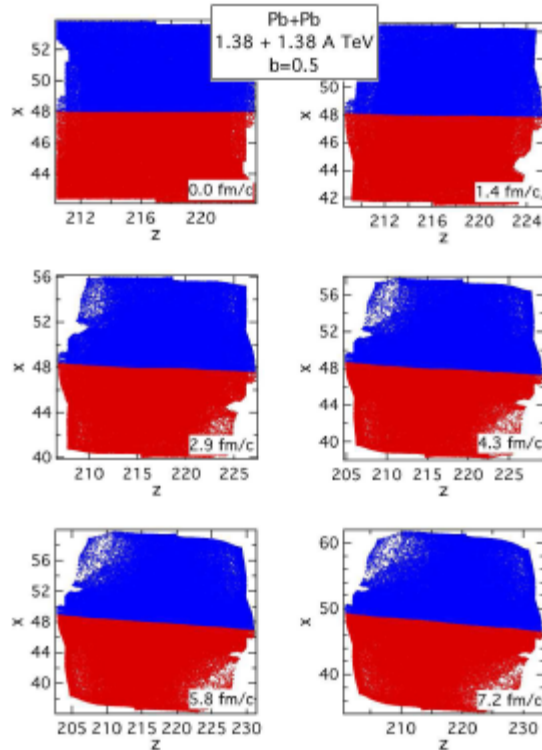


FIG. 1: (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a 1.38A + 1.38A TeV peripheral,  $b = 0.7b_{\max}$ , Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are  $dx = dy = dz = 0.4375\text{fm}$  and the time-step is  $0.04233\text{ fm/c}$ . The number of randomly placed marker particles in each fluid cell is  $8^3$ . The axis-labels indicate the cell numbers in the  $x$  and  $z$  (beam) direction. The initial development of a KH type instability is visible from  $t = 1.5$  up to  $t = 7.41\text{ fm/c}$  corresponding from 35 to 175 calculation time steps).



# Weighted Vorticity

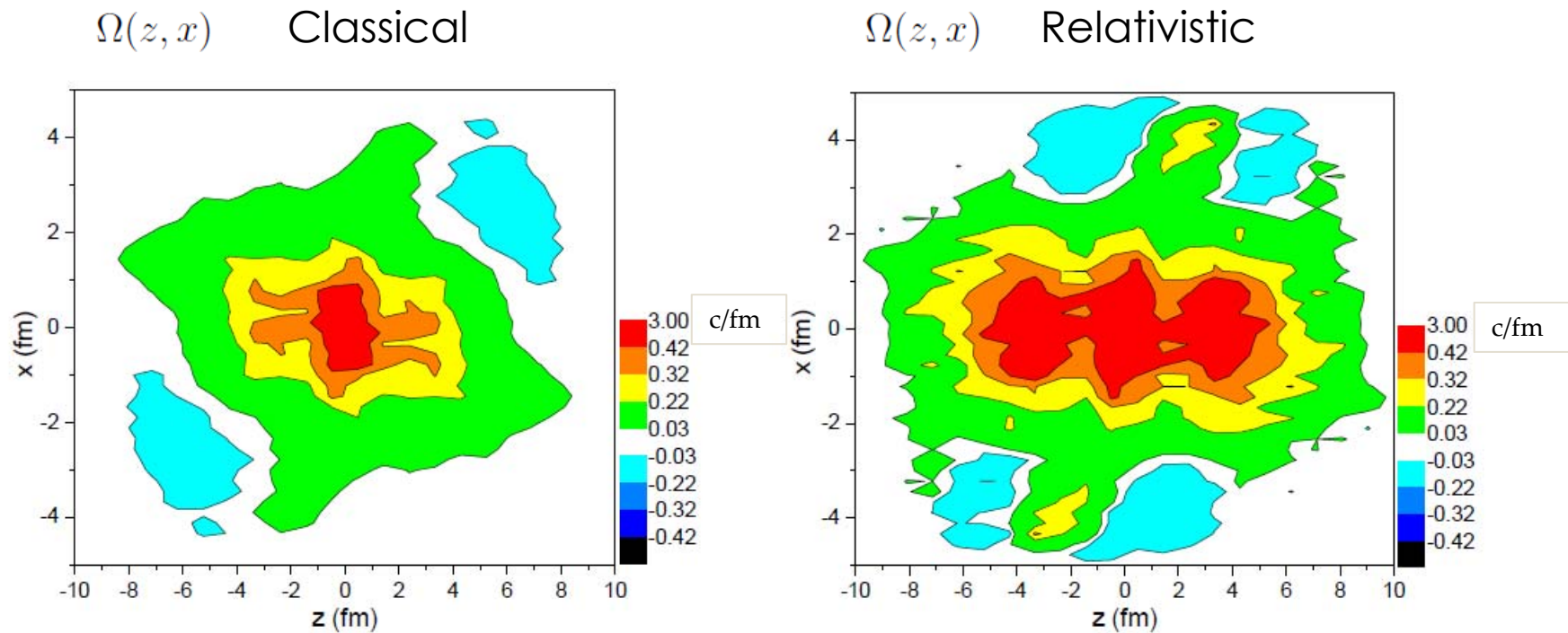
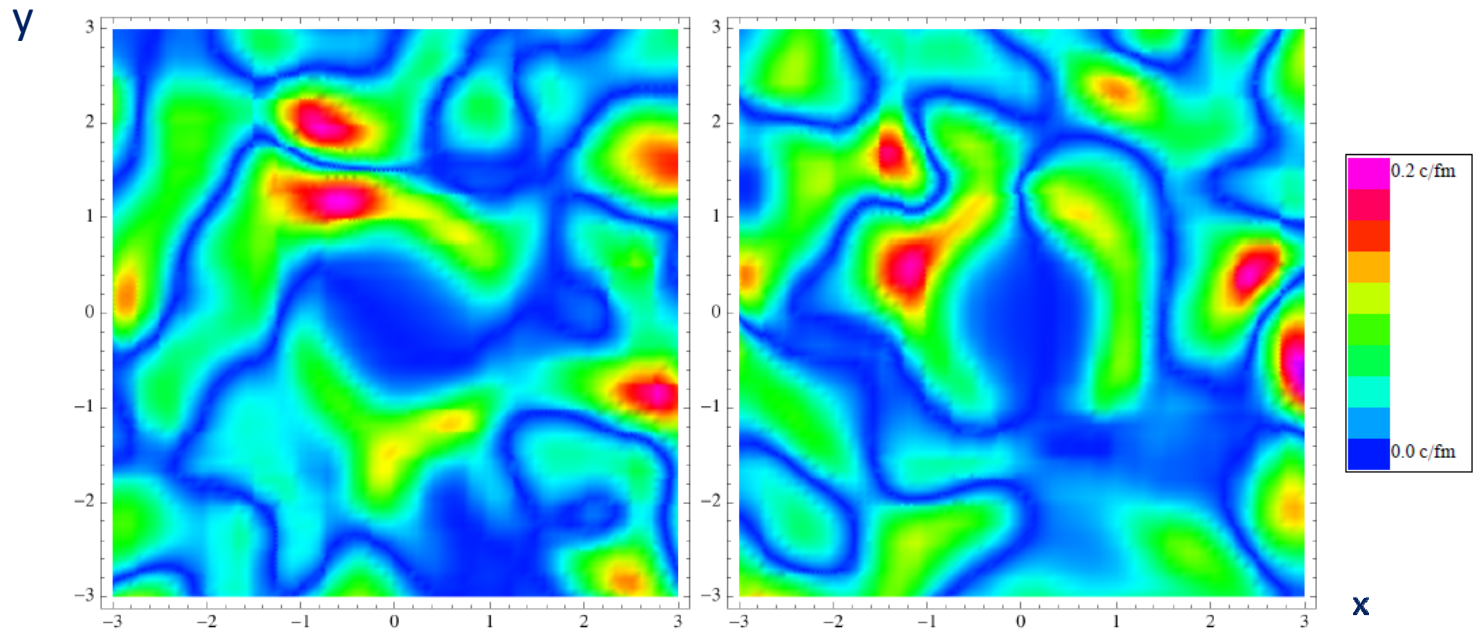


FIG. 5: The classical (left) and relativistic (right) weighted vorticity calculated for all  $[x-z]$  layers at  $t=3.56$  fm/c. The collision energy is  $\sqrt{s_{NN}} = 2.76$  TeV and  $b = 0.7b_{max}$ , the cell size is  $dx = dy = dz = 0.4375$  fm. The average vorticity in the reaction plane is 0.0538 / 0.10685 for the classical / relativistic weighted vorticity respectively.

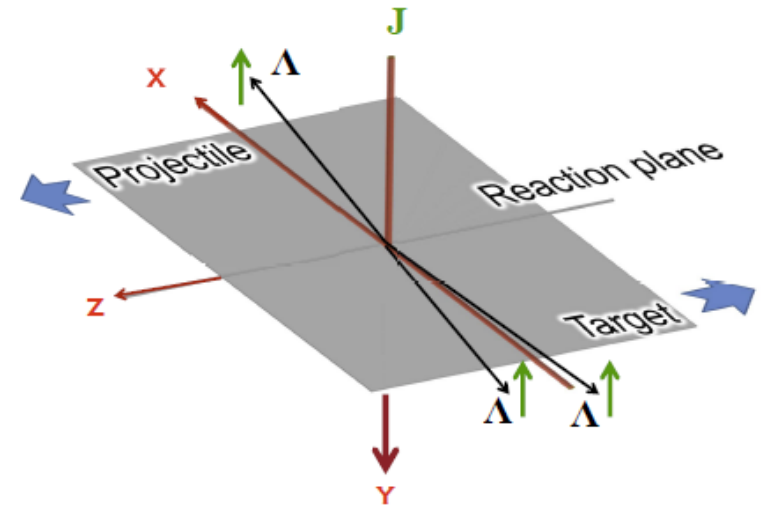
# Onset of turbulence around the Bjorken flow

S. Floerchinger, U.A. Wiedemann, JHEP 100,1111 (2011); J. Phys. G 38, 124171 (2011)



- Initial state Event by Event vorticity and divergence fluctuations.
- Amplitude of random vorticity and divergence fluctuations are the same
- In dynamical development viscous corrections are negligible ( $\rightarrow$  no damping)
- Initial transverse expansion in the middle ( $\pm 3\text{fm}$ ) is neglected ( $\rightarrow$  no damping)
- High frequency, high wave number fluctuations **may feed** lower wave numbers

# Detecting rotation: Lambda polarization

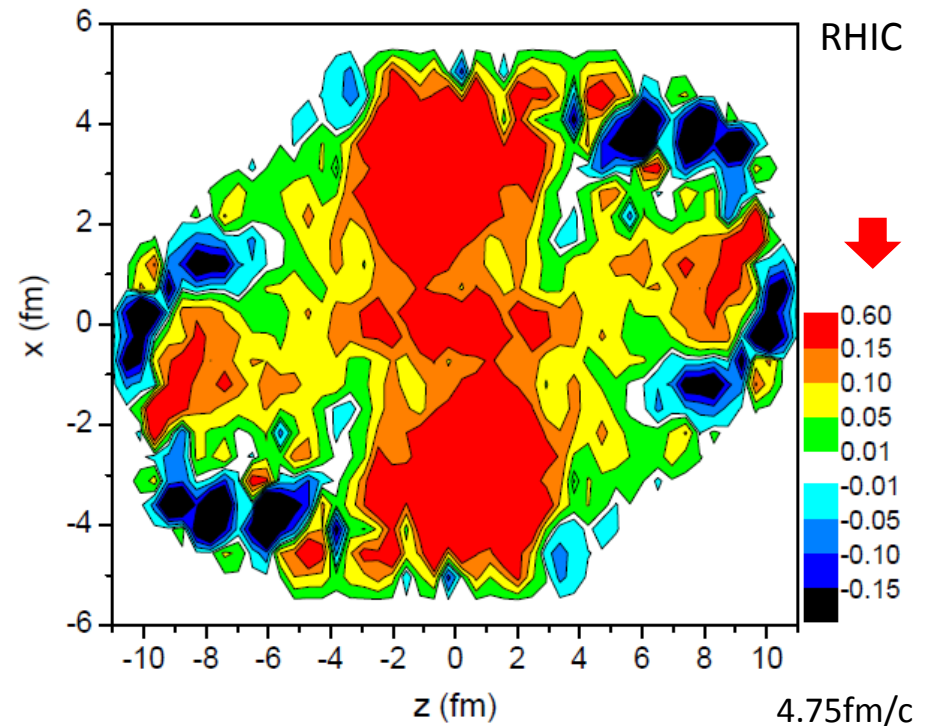
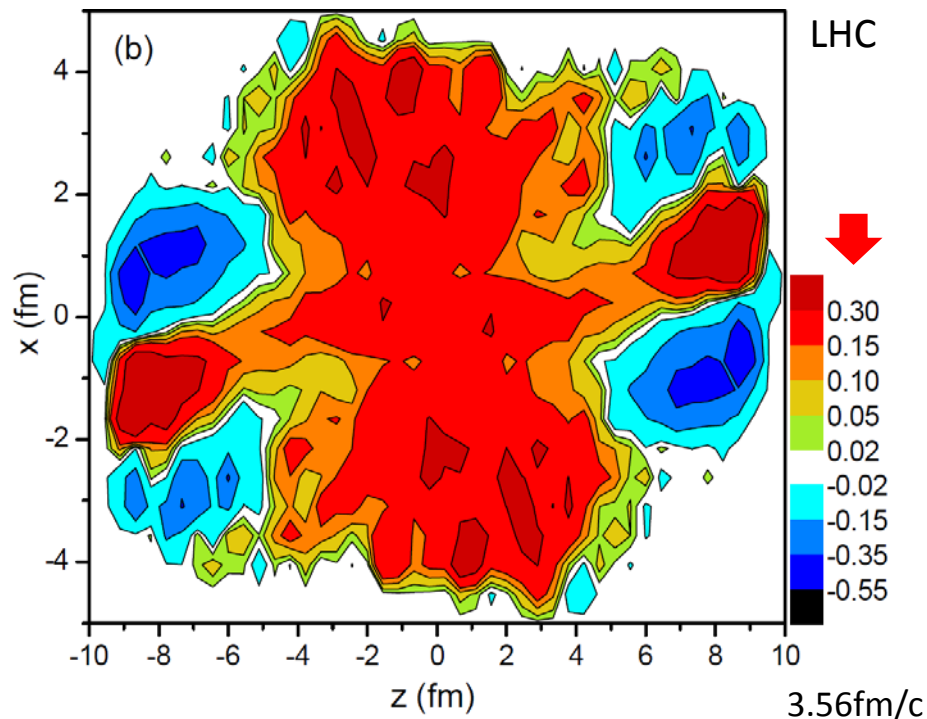


$$\Pi(p) = \frac{\hbar \varepsilon \int dV n_F (\nabla \times \beta)}{8m \int dV n_F}$$

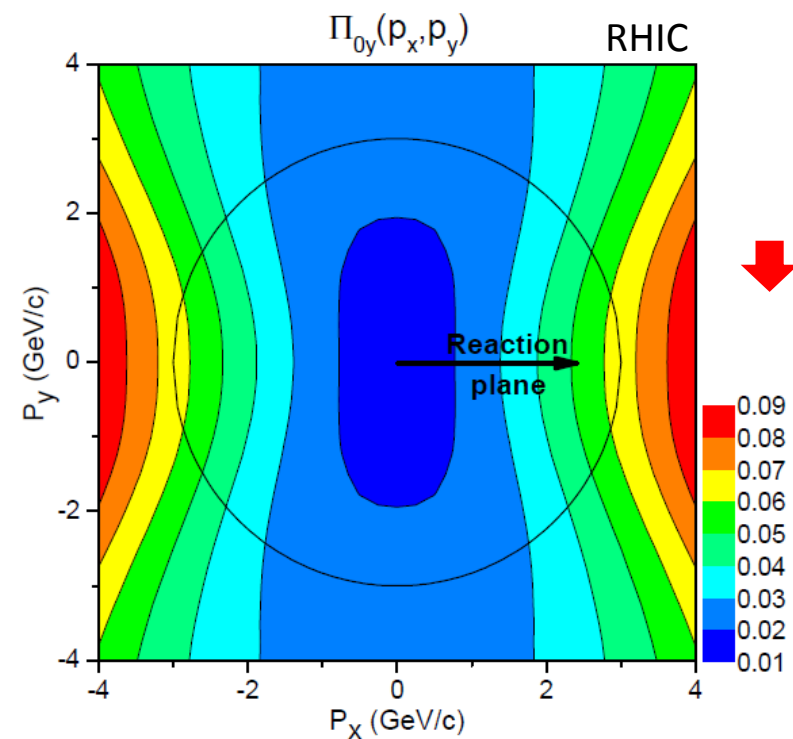
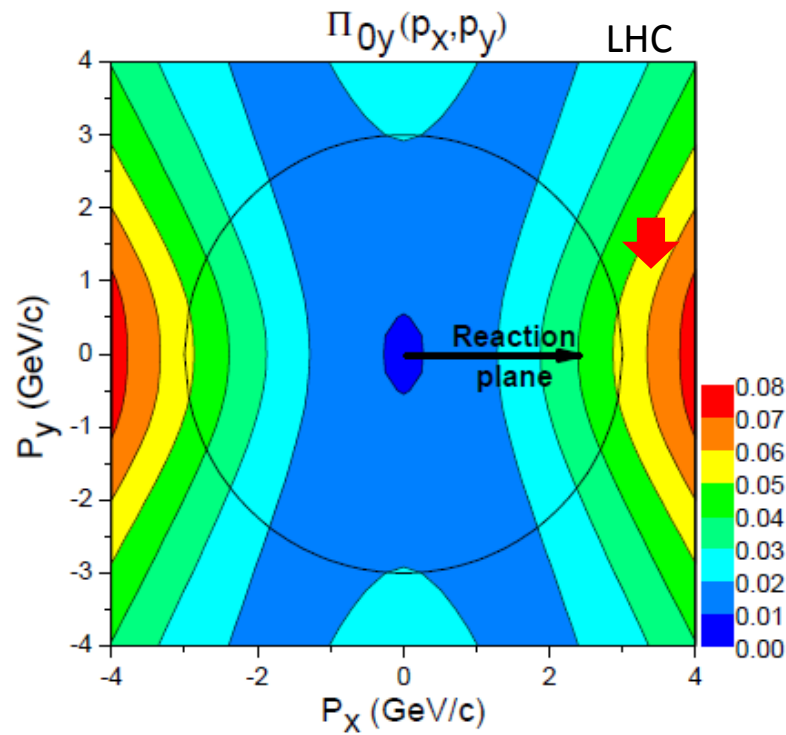
$$\beta^\mu(x) = (1/T(x))u^\mu(x) \quad \leftarrow \text{From hydro}$$

$$\Pi_0(p) = \Pi(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon + m)} \Pi(p) \cdot \mathbf{p}$$

[ F. Becattini, L.P. Csernai, D.J. Wang,  
Phys. Rev. C **88**, 034905 (2013)]



# Lambda polarization



- The **POLARIZATION** of  $\Lambda$  and  $\bar{\Lambda}$  due to thermal equipartition with local vorticity is slightly stronger at RHIC than at LHC due to the much higher temperatures at LHC.
- Although early measurements at RHIC were negative, these were averaged over azimuth! We propose selective measurement in the reaction plane (in the +/- x direction) in the EbE c.m. frame. Statistical error is much reduced now, so significant effect is expected at  $p_x \geq 3$  GeV/c.

# Measurements: Inconclusive (2007)

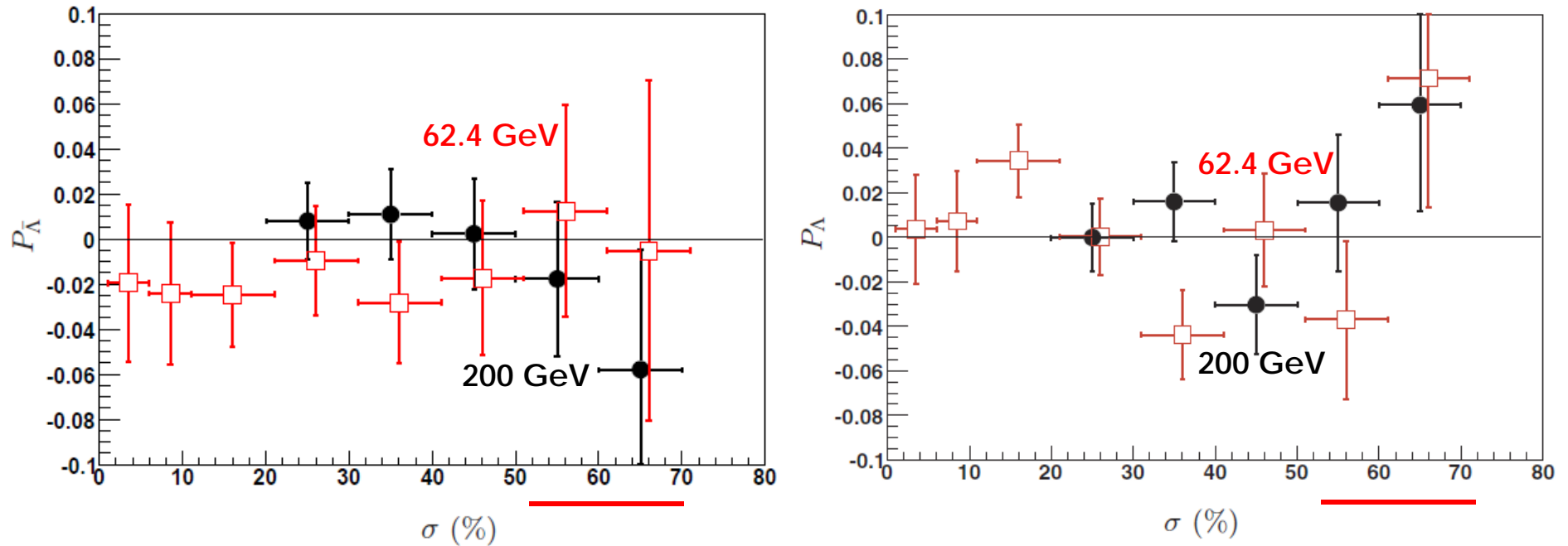


FIG. 8: (Color online) Global polarization of  $\bar{\Lambda}$ -hyperons as a function of centrality. Filled circles show the results for Au+Au collisions at  $\sqrt{s_{NN}}=200$  GeV (centrality region 20-70%) and open squares indicate the results for Au+Au collisions at  $\sqrt{s_{NN}}=62.4$  GeV (centrality region 0-80%). Only statistical uncertainties are shown.

**C.M. &  $RP_{(p/T)}$  should be precisely determined, & only at large  $p_x$  !**

B.I. Abelev et al., (STAR) PHYSICAL REVIEW C 76, 024915 (2007)

# Differential HBT method

PHYSICAL REVIEW C **89**, 034916 (2014)

FIG. 2. (Color online) Differential correlation function,  $\Delta C(k, q)$ , at the final time with and without rotation.

We can rotate the frame of reference:

$$k'(\alpha) = \begin{Bmatrix} k_{x'} \\ k_{z'} \end{Bmatrix} = \begin{Bmatrix} k_x \cos \alpha - k_z \sin \alpha \\ k_z \cos \alpha + k_x \sin \alpha \end{Bmatrix}.$$

$$\rightarrow \Delta C_\alpha(k', q'),$$

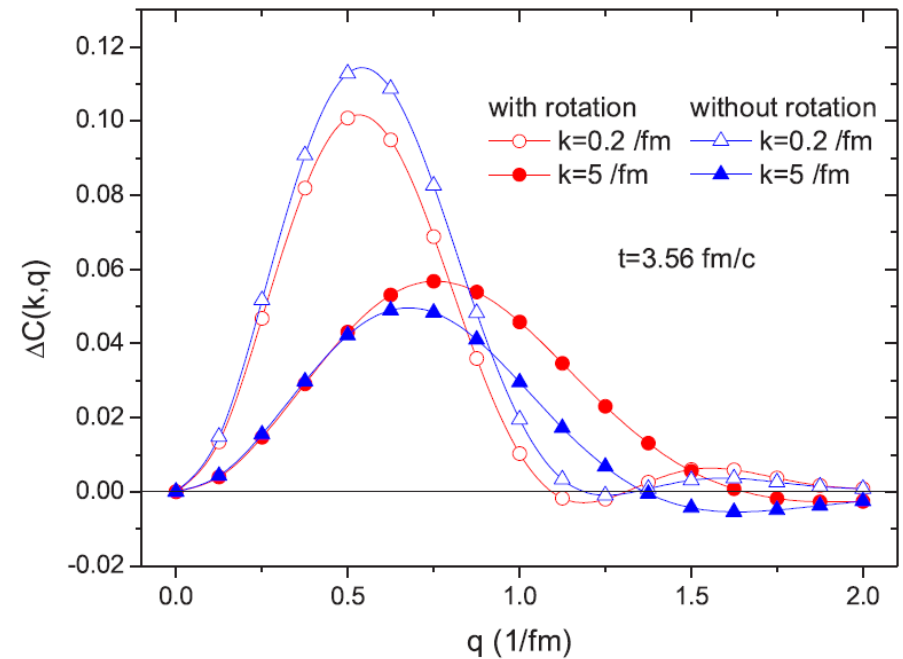
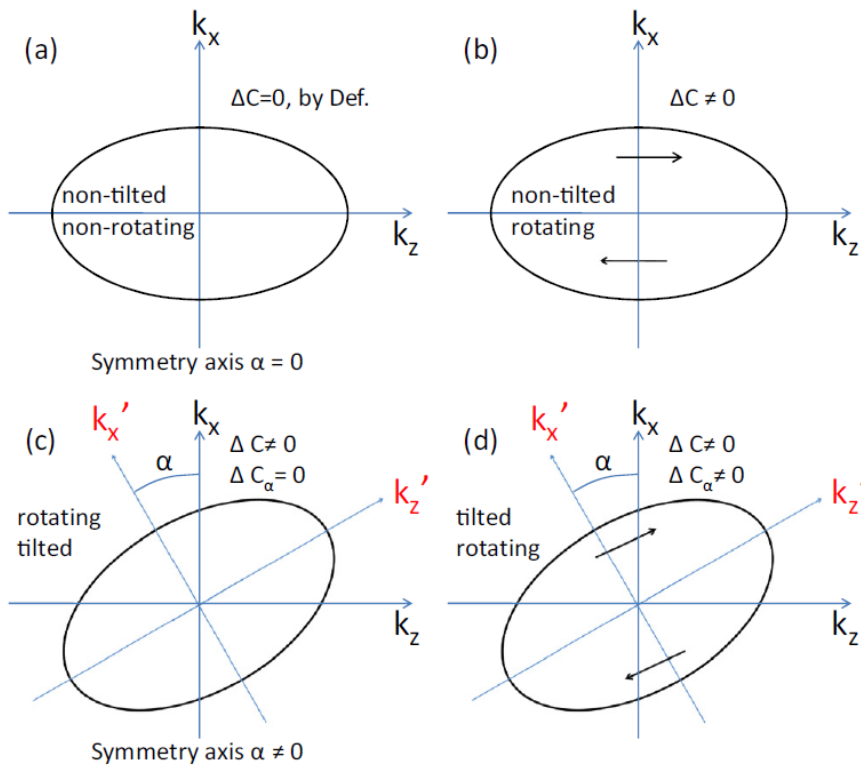
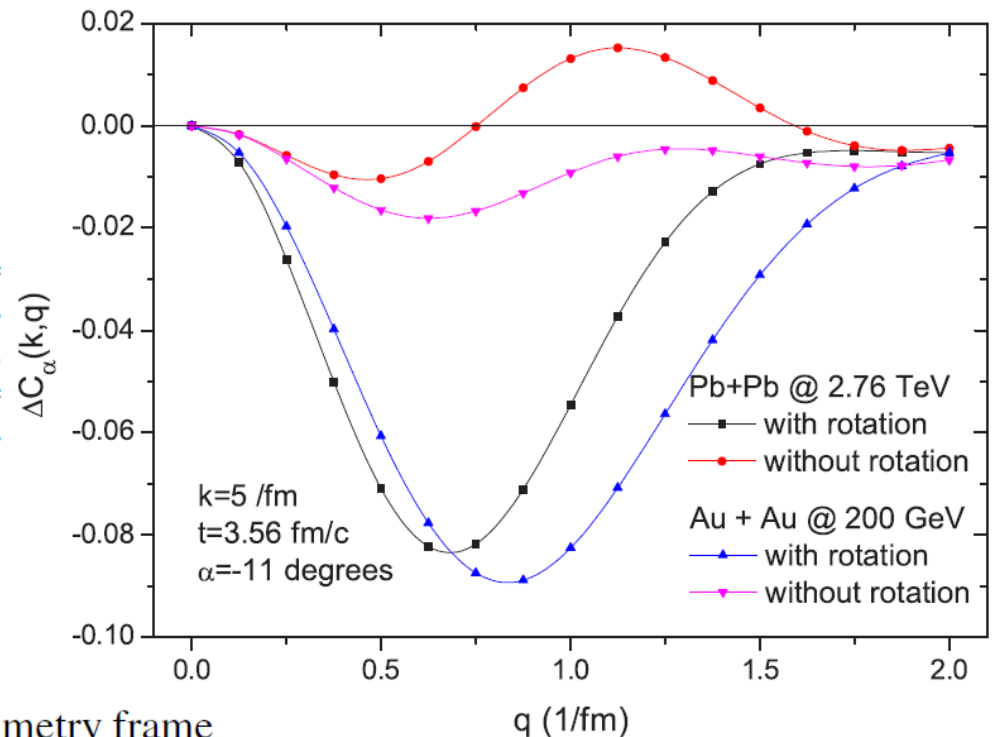


FIG. 3. (Color online) Sketch of the configuration in different reference frames, with and without rotation of the flow. The nonrotating configurations may have radial flow velocity components only. The DCF,  $\Delta C_\alpha(k, q)$ , is evaluated in a  $K'$  reference frame rotated by an angle  $\alpha$  in the  $x, z$ , reaction plane. We search for the angle  $\alpha$ , where the nonrotating configuration is “symmetric,” so that it has a “minimal” DCF as shown in Fig. 4.

Measure  $k, q$  ( $p_1, p_2, p_3, p_4$ ) in the **out & long** directions, in the R.P.

# Signs of rotation

FIG. 5. (Color online) The DCF with and without rotation in the reference frames, deflected by the angle  $\alpha$ , where the rotationless DCF is vanishing or minimal. In this frame the DCF of the original, rotating configuration indicates the effect of the rotation only. The amplitude of the DCF of the original rotating configuration doubles for the higher energy (higher angular momentum) collision.



To perform the analysis in the rotationless symmetry frame one can find the symmetry axis the best with the azimuthal HBT method, which provides even the transverse momentum dependence of this axis [20]. It is also important to determine the precise event-by-event c.m. position of the participants [21] and minimize the effect of fluctuations to be able to measure the emission angles accurately, which is crucial in the present  $\Delta C(k,q)$  studies.

[LP. Csernai, S. Velle, DJ. Wang, *Phys. Rev. C* **89** (2014) 034916]

[LP. Csernai, S. Velle, *Int. J. Mod. Phys. E* **23** (2014) 1450043]

[ **Sindre Velle: Talk at WPCF 2014** ]





## Summary

- We have shown how to split Collective flow & Fluctuations
- When Collective Flow is identified: *New patterns*
- Small viscosity ( $\rightarrow$  fluctuations & instabilities)
- Rotation
- Kelvin-Helmholtz Instability (KHI)  $\sim$  turbulence
- These are observable in polarizations and in HBT

