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Spontaneously breaking space-time: Poincare, gravity, cosmology, and spinning objects

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...or how I learned to stop worrying and love the coset construction

1) SSB of space-time symmetries + coset construction

2) Gravity from the coset perspective

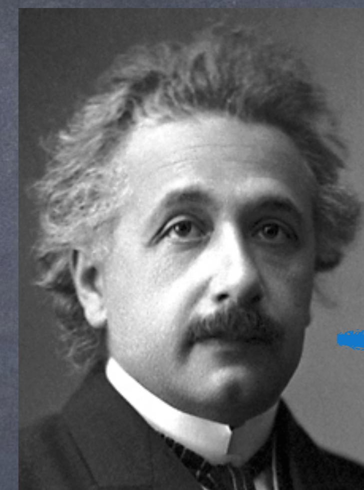
3) Example: Spinning object

4) Example: Cosmology

Act 1:

The Spontaneous breaking of space-time symmetries

Observation:

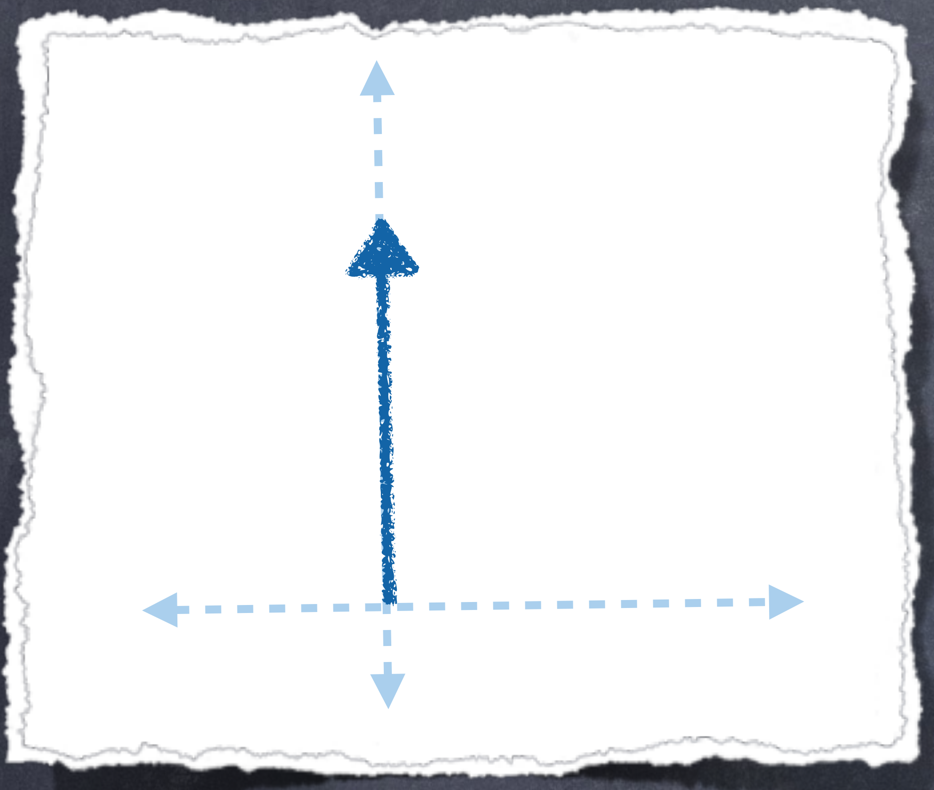
Experiments $\Rightarrow$ 

laws of physics
Poincaré invariant!

Experience $\Rightarrow$ “stuff” is not 

$\Rightarrow$ $|\psi\rangle$ spontaneously breaks Poincaré

Consider the (point-like) Lobster:

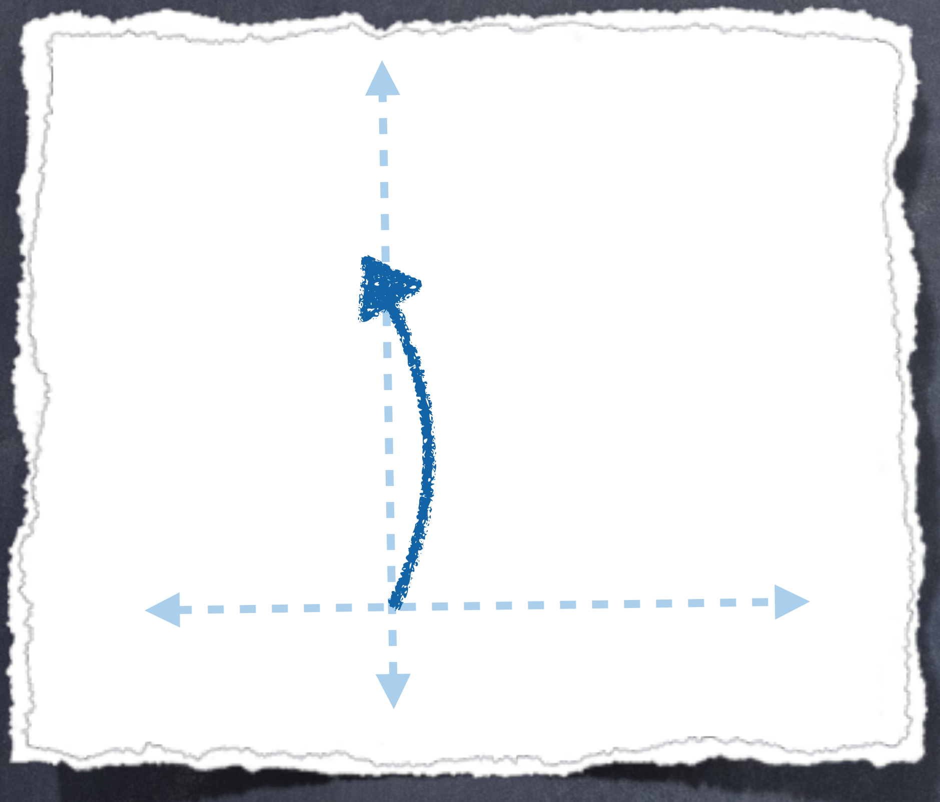


$$x^\mu = (t,\, 0,\, 0,\, 0)$$

$$\text{Unbroken} \quad = \quad \begin{cases} P_0 & \text{time translations} \\ J_{ij} & \text{spatial rotations} \end{cases}$$

$$\text{Broken} \quad = \quad \begin{cases} P_i & \text{spatial translations} \\ J_{0i} \equiv K_i & \text{boosts} \end{cases}$$

Consider the (point-like) Lobster:



usual treatment:

perturb w/ constraint:

$$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -1$$

$$-m\int d\tau = -m\int d\lambda\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$$

$$\Longrightarrow \int dt\left(-m+\frac{1}{2}mv^2+\ldots\right)$$

Consider the (point-like) Lobster:

Is there a way to construct the theory for the Goldstones (fluctuations) based on the symmetry breaking pattern alone?

Almost!*

*Ivanov and Ogievetsky 1970's

more recent literature:

- Neilson and Chadha (1976)
- Schäfer, Son, Stephanov, Toublan, and Verbaarschot hep-ph/0108210 (2001)
- Low and Manohar hep-th/0110285 (2002)
- Watanabe and Brauner 1109.6327 (2011)
- Nicolis, and Piazza 1112.5174 (2011)
- Watanabe and Murayama 1203.0609 (2012)
- Hidaka 1203.1494 (2012)
- Nicolis, Penco, Piazza, and Rosen 1306.1240 (2013)
- Endlich, Nicolis, and Penco 1310.2272 (2013)
- Endlich, Nicolis, and Penco 1311.6491 (2013)
- Brauner, Endlich, Monin, and Penco 1407.7730 (2014)
- Nicolis, Penco, Piazza, and Rattazzi 1501.03845 (2015)
- etc.

Why "almost" (classic interpretation)?

- one usually hears that:

$$N_{\text{Broken Generators}} \neq N_{\text{Goldstones}}$$

- $$\delta\mathcal{O}(x) = i\sum_I \pi_I(x)T_I\langle\mathcal{O}(x)\rangle$$

? $$\sum_I \pi_I(x)T_I\langle\mathcal{O}(x)\rangle = 0$$

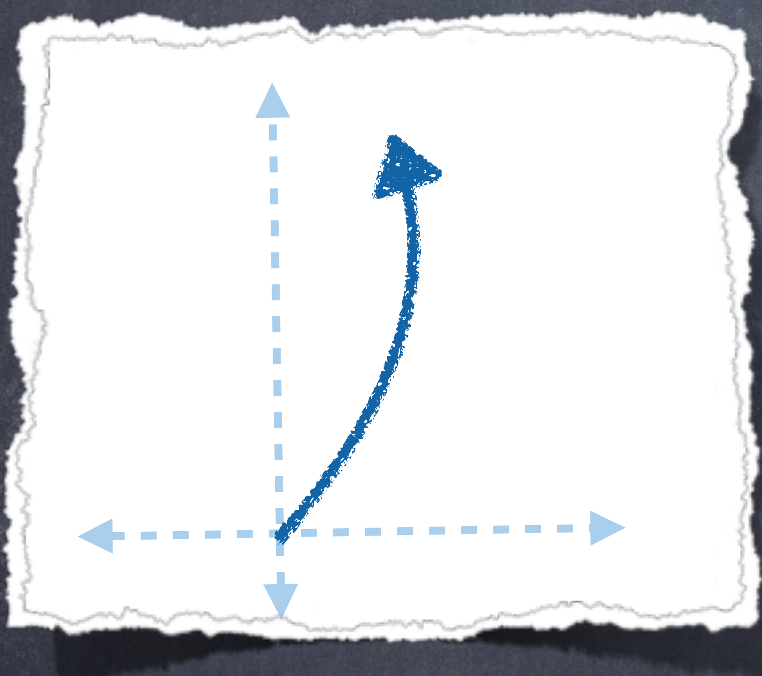
not independent fluctuations

eliminate redundancy via

"inverse Higgs constraint"

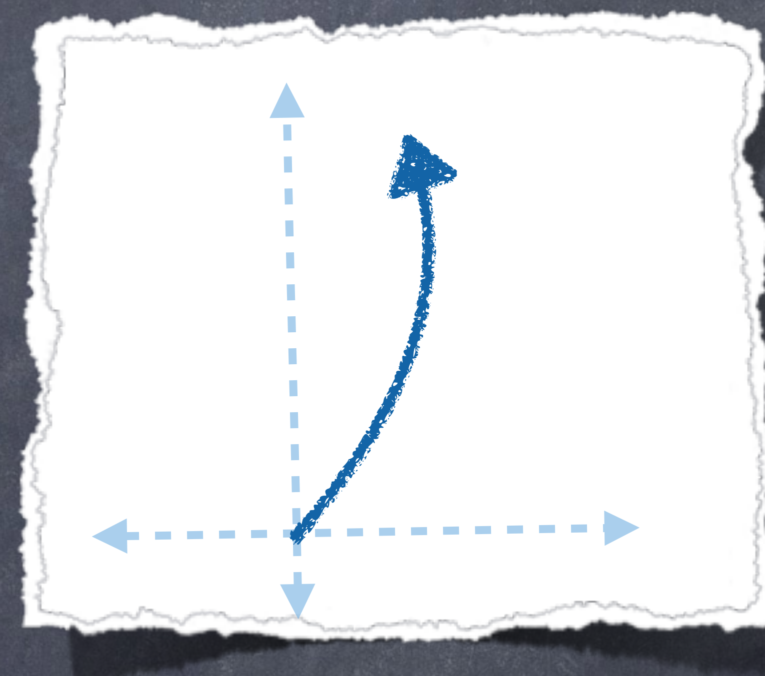
Ex.

Unbroken	=	P_0	time translations
		J_{ij}	spatial rotations
Broken	=	P_i	spatial translations
		$J_{0i} \equiv K_i$	boosts



time-dependent boost

Vs



time-dependent translation

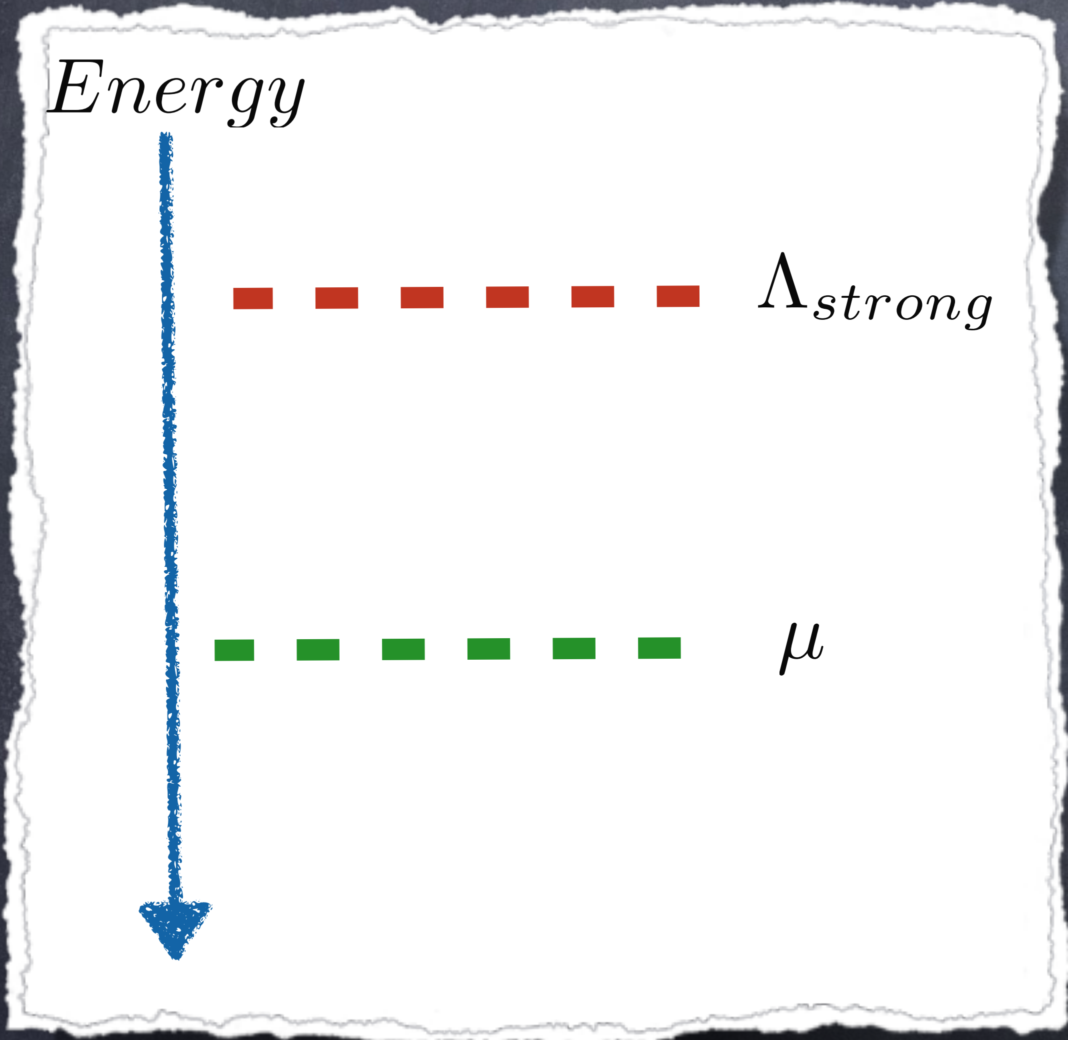
$$\pi_{boost} K + \pi_{trans} P = 0$$

$$\pi_{boost} \sim \frac{d}{dt} \pi_{trans}$$

⇨ same information

(however...can have no redundancy: choice?)

- $M_{\text{Goldstone}} \neq 0$ (read: gap)



⇦ work here...

⇦ ...or here

⇨ Mechanism dependent (sort of)

S. Endlich, A. Nicolis, and R. Penco (hep-th/1311.6491)
A. Nicolis, R. Penco, F. Piazza, and R. Rosen (hep-th/1306.1240)

basic construction: 1

1) identify symmetry breaking pattern

$$G \to H$$

2) build objects out of Goldstones that transform linearly*

$$\mathcal{D}\pi \to h(g,\pi)\mathcal{D}\pi$$

*along the lines of Callan, Coleman, Wess and Zumino + Volkov

basic construction: 2

3) Inverse Higgs constraints (algebra)

$$[\bar{P}, X] = X' + ...$$

=> structure of covariant derivatives

$$\mathcal{D}\pi = 0 \Longrightarrow \pi(\partial\pi')$$

can impose: should? necessary? depends...

4) build a Lagrangian out of H invariant objects made up of $\mathcal{D}\pi^a$

CCWZ construction generalized to space-time symmetries:

Expanding the magical step 2)

- organize $G \begin{cases} X_\alpha = \text{broken generators} \\ \bar{P}_a = \text{unbroken translations} \\ T_A = \text{other unbroken generators} \end{cases}$ $\Big\} \; H_0$

- parametrize the coset $$\Omega(y,\pi) \equiv e^{iy^a(x)\bar{P}_a} e^{i\pi^\alpha(x)X_\alpha}$$

transformation rules $$g\,\Omega(y,\pi) = \Omega(y',\pi')\,h(y,\pi,g)$$

Expanding the magical step 2) ... part 2

- Maurer-Cartan form $\Omega^{-1} d\Omega$

$$\Omega^{-1}\partial_\mu \Omega = E_\mu{}^a (\bar{P}_a + \nabla_a \pi^\alpha X_\alpha + A_a^B T_B)$$

transforms as a vierbein

transforms as a covariant derivative

transforms as a connection

⇨ $d^4x \det E$

$$\nabla_a \pi^\alpha(x) \quad \xrightarrow{g} \quad \nabla_a \pi'^\alpha(x) = h_a{}^b(y,\pi,g) h_\beta{}^\alpha(y,\pi,g) \nabla_b \pi^\beta(x)$$

⇨ $$\nabla_a^H \equiv [(E^{-1})_a{}^\mu \partial_\mu + i A_a^B T_B]$$

Act 2:

Gravity from the coset perspective

Gauge symmetries

rule of thumb (CCWZ):

$$\Omega^{-1}\partial_\mu\Omega \quad \rightarrow \quad \Omega^{-1}D_\mu\Omega \equiv \Omega^{-1}(\partial_\mu + i\tilde{A}_\mu^I V_I)\Omega.$$

can we do the same for space-time symmetries?

yes

Gravity as
"gauged ISO(3,1) with non-linearly realized translations"

1) non-linearly realized translations

2) $$\Omega \equiv e^{iy^a(x)P_a}$$

$$\Rightarrow \quad \Omega^{-1} D_\mu \Omega \equiv e^{-iy^a(x)P_a} \left(\partial_\mu + i\tilde{e}_\mu{}^a P_a + \frac{i}{2}\omega_\mu^{ab} J_{ab} \right) e^{iy^a(x)P_a}$$

$$= ie_\mu{}^a P_a + \frac{i}{2}\omega_\mu^{ab} J_{ab}$$

"spin" connection

$$e_\mu{}^a = \partial_\mu y^a + \tilde{e}_\mu{}^a + \omega_\mu^{ab} y_b$$ vierbein

⇨ all we have is:

$$\nabla^L_a \equiv (e^{-1})_a{}^\mu(\partial_\mu + \frac{i}{2}\omega^{bc}_\mu J_{bc})$$

4) $S = \int d^4x \det e\, \mathcal{L}(\nabla^L_a)$ GR?

- $[\nabla^L_a, \nabla^L_b]V^c = R^c{}_{dab}V^d - T_{ab}{}^d\nabla^L_d V^c$
- ⇨ $S = -\frac{1}{16\pi G}\int \det(e) d^4x \left[R^{ab}{}_{ab} + \sum_i^3 c_i T^2 + \cdots\right]$
- $\frac{\delta S}{\delta\omega^{ab}_\mu} = 0$ ⇨ $\omega(e) \sim e^{-1}\partial e + ...$

$T_{ab}{}^c = 0$ torsion free! GR! (as EFT)

Act 3:

Spinning object

Utilizing the coset

1) SSB pattern:

$$\text{Unbroken} \quad = \quad \begin{cases} P_0 & \text{time translations} \\ \bar{J}_{ij} = S_{ij} + J_{ij} & \text{residual rotations} \end{cases}$$

$$\text{Broken} \quad = \quad \begin{cases} P_i & \text{spacial translations} \\ J_{ab} & \text{rotations and boosts} \end{cases}$$

with $S \subseteq SO(d)$

2) with $\Omega = e^{iy^a P_a} e^{i\alpha_{ab} J^{ab}/2}$

$$\Rightarrow \quad \dot{x}^\mu \Omega^{-1} D_\mu \Omega = iE(P_0 + \nabla\pi^i P_i + \frac{1}{2}\nabla\alpha_{cd} J^{cd})$$

Utilizing the coset

$$E = \dot{x}^{\nu} e_{\nu}{}^{a} \Lambda_{a}{}^{0}$$

$$\nabla \pi^{i} = E^{-1} \dot{x}^{\nu} e_{\nu}{}^{a} \Lambda_{a}{}^{i}$$

$$\nabla \alpha^{ab} = E^{-1} \left(\Lambda_{c}{}^{a} \dot{\Lambda}^{cb} + \dot{x}^{\mu} \omega_{\mu}{}^{cd} \Lambda_{c}{}^{a} \Lambda_{d}{}^{b} \right)$$

$$= \frac{d\tau}{d\lambda}$$

with $\Lambda(\alpha) = \Lambda(\eta)\Lambda(\xi)$

3) as $[\bar{P}_0, K_i] \sim P_i$ $\Rightarrow$ $\nabla \pi^{i} = 0$

$\Rightarrow$ $u^{a} \Lambda_{a}{}^{i}(\eta) = 0$

boost into rest frame accompanying rotating object

Utilizing the coset

4) build the action

$$S = \int d\lambda \, E \left(-m + \frac{I_{ijkl}}{4} \nabla\alpha^{ij} \nabla\alpha^{kl} + \cdots \right)$$

with $\nabla\alpha = \Lambda^{-1} d_\tau \Lambda + u^\mu \Lambda^{-1} \omega_\mu \Lambda$

EFT describing rotating
object coupled to gravity
(as advertised)

Reality check

Turn OFF gravity... what do we get?

- $\int dt \, \frac{1}{2} I_{ij} \Omega^i \Omega^j + \cdots$ correct leading kinetic energy

- $\frac{\delta}{\delta \xi^i}(\cdots) = 0$ ⇨ Euler equations

- What about the $\cdot \; \cdot \; \cdot$'s?

Meaning of higher derivative terms

Real objects are NOT perfectly rigid... they distort under stress

Dimensionally $$S\,\nabla\alpha^4 \sim S\,\Omega^4 \sim \left(\frac{\Omega^2}{\omega_0^2}\right)(I\Omega^2)$$

For some "elastic" object EFT tells you ω_0 related to modes we have integrated out... the normal modes!

Generalized Euler (in 3 d):

$$\partial_t \Omega = I^{-1}\left((I\Omega)\times\Omega\right) + I^{-1}\left((S\Omega\Omega\Omega)\times\Omega\right) + I^{-1}(S\Omega\Omega)\left((I\Omega)\times\Omega\right) + \mathcal{O}\left(S\Omega^2/I\right)^2$$

Ex: Spherical chunk of cubic crystal rotating off axis

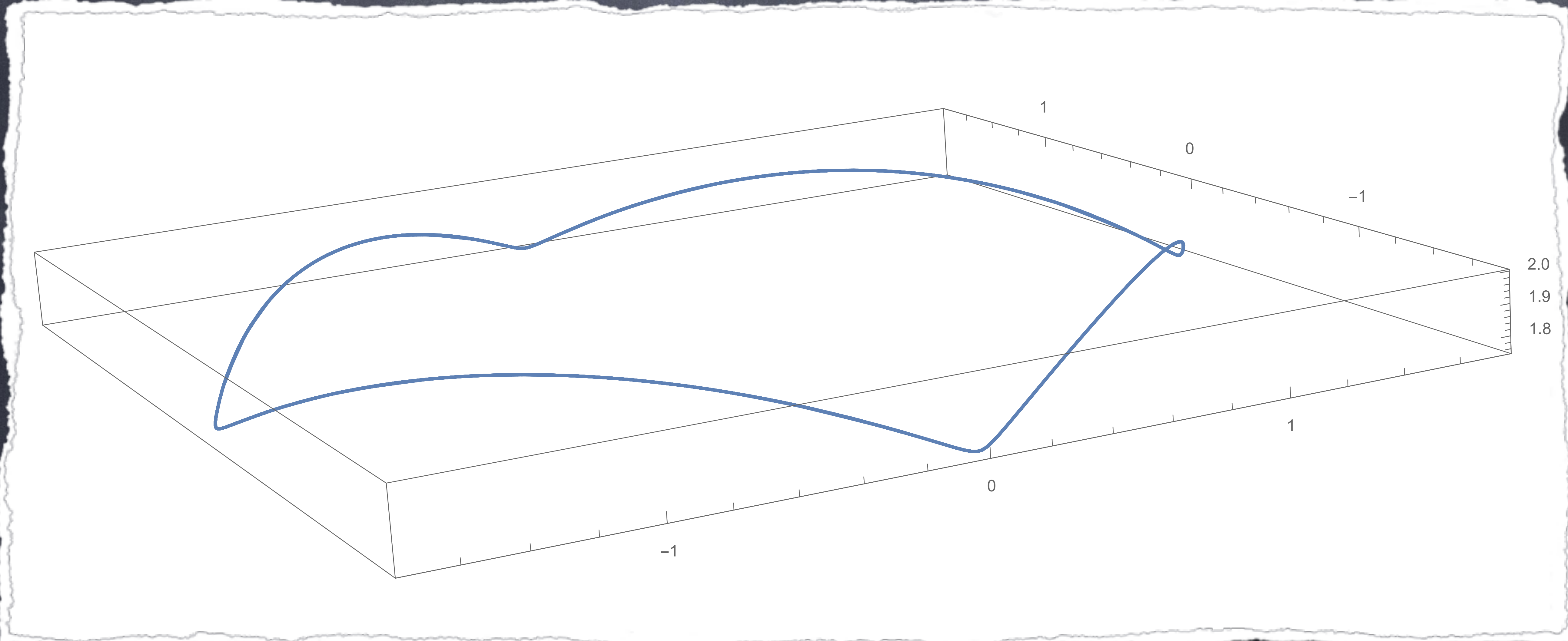
"spherical chunk" ⇨ $I \sim \delta$

"cubic crystal" ⇨ $S \sim \delta\delta + \mathrm{cubic}$

here ⇨ $\partial_t \Omega = I^{-1}\left((S\Omega\Omega\Omega)\times\Omega\right) + \mathcal{O}\left(S\Omega^2/I\right)^2$

Precession!

precession of angular velocity (as viewed in body frame)



... for some random input values

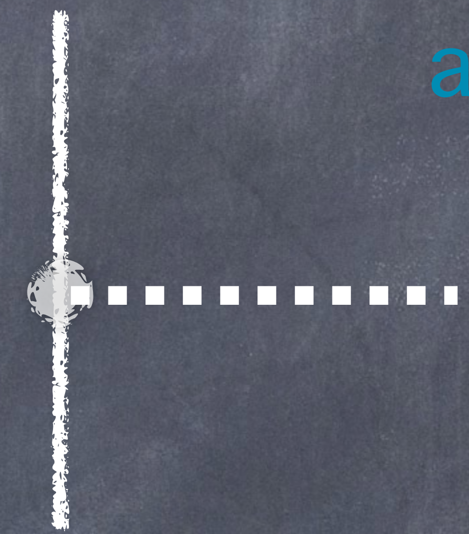
is this known?

Coupling to gravity

(À la Goldberger, Porto, Rothstein, and others)

Additional vertices, such as

- $\frac{1}{4M_{Pl}} L^k \epsilon^{kij} \partial^i {h_0}^j$



- $\sim \frac{1}{M_{Pl}^2} L^k \epsilon^{kij} \left[h^{i\lambda} \partial_\lambda {h_0}^j + \cdots \right]$

where $L = \Lambda(\xi) \left(I\Omega + S\Omega^3 + \cdots \right)$ is the angular momentum

⇨ Compute (spin-orbit, spin-spin, etc.)

Act 4:

cosmological theories

EFT of Inflation

- The early universe: homogenous, isotropic, and slightly time dependent

- Inflation driven by $\psi_a = \bar{\psi}_a(t)$

- Time-translations spontaneously broken

⇨ Goldstone boson = adiabatic perturbations

$$\psi_a = \bar{\psi}_a(t + \pi(x)) \simeq \bar{\psi}_a(t) + \partial_t \bar{\psi}_a(t) \cdot \pi(x)$$

- Construct a systematic effective field theory

(Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007)

Can we write down action for Goldstones directly?

- Proceed by gauging the space-time symmetries of de Sitter
- Couple to matter that spontaneously breaks some of these symmetries (+ internal)

Ex.

- spontaneously broken TIME + internal shifts — EFT of inflation
- spontaneously broken SPACIAL SHIFTS + internal shifts — Solid Inflation

- Construct a systematic effective field theory

Conclusions

- “stuff” non-linearly realizes space-time symmetries (usual procedures don’t work without being clever)
- in EFT one needs to identify 1) d.o.f. and 2) symmetries: coset helps with both
- general procedure to couple the Goldstones to gravity
- straightforward and exhaustive procedure
- SPIN: made simpler? new expansion parameter (ω/ω_0) to assist in computations
- straightforward construction of inflation
- Applications: cosmology, plasma physics, exotic condensed matter states, etc. all in a model independent fashion