Vortices in axion condensate dark halos in cosmological context

M. Szydłowski[1,3] and A. Krawiec[2,3]

[1] Astronomical Observatory, Jagiellonian University
[2] Institute of Economics and Management, Jagiellonian University
[3] Mark Kac Complex Systems Research Centre, Jagiellonian University

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abstract

We consider the possibility of the vortex formation in axion condensates in galactic halos to explain the dark matter problem in astrophysics. We assume vortices as a result of global rotation of the early universe. We study the Bose-Einstein condensate of axion using the non-relativistic Gross-Pitaevski equation. We reconstruct the galaxy rotation curves and show that they are in good agreement with observational data.

We also considered the cosmological model with dark matter with mass of dark particle changing during the cosmic evolution. We show that such model is not in a good agreement with rotation curves.

Introduction

Challenge 1 in the modern astrophysics and astroparticle physics

dark energy problem.

Recent astronomical measurements of cosmic microwave background (CMB) radiation by WMAP, or distant supernova type Ia (SNIa) indicate that the Universe, apart from baryonic matter, is fulfilled with a phenomenological fluid with negative pressure (called dark energy) which could be responsible for the current acceleration of the universe.

Challenge 2 in the modern astrophysics and astroparticle physics

dark matter problem.

Its existence has been inferred only through its gravity clearly influences the galactic scale dynamics. Its abundance is given in terms of the density parameter $\Omega_{\rm dm} = \rho_{\rm dm}/\rho_c$, where ρ_c is the critical energy density.

For the "concordance" flat ΛCDM model we have total matter $\Omega_m = 0.28$; in turn from emission and absorption of photons visible matter is roughly $\Omega_{vis} \simeq 0.04$ and it gives us that dark matter amounts to $\Omega_{dm} \simeq 0.24$. The nature of dark matter is unknown although some candidates have been proposed like neutralino and axions.

The rotation curves of spiral galaxies give us the strongest evidence for dark matter. Dark matter is necessary to explain why at large distance r from the centre of a given galaxy, we would find circular velocity $v_c^2 \simeq GM_{\rm vis}/r$, since visible matter is concentrated around its centre (Salucci, 1996). However, observations show that v_c is independent of r at large distances ($v_c \sim 200$ km s^{-1} is its typical value). From numerical simulations of the halo formation we obtain density profiles for both small and large values $\rho_{halo} \propto r^{-\alpha}$ with $\alpha \in (1; 1.5)$ for small and $\alpha = 3$ for large distances.

There are two main candidates for cold dark matter, namely the axion and neutralino (Lazarides, 2007). We concentrate on the axion, which were originally proposed to solve the strong CP problem in the QCD. We investigate the possibility of explanation of flat velocity curves of spiral galaxies in terms of the Bose-Einstein condensate of axions which can be present in the Universe since the Peccei-Quinn phase transition.

We use the Gross-Pitaevski equation in an expanding FRW universe

$$i\hbar\left(\frac{\partial}{\partial t}+\frac{3}{2}\frac{\dot{a}(t)}{a(t)}\right)\phi(\vec{r},t)= \\ =\left(-\frac{\hbar^2}{2m}\frac{1}{a^2(t)}\nabla^2+U(\vec{r})+g^2|\phi(\vec{r},t)|^2\right)\phi(\vec{r},t), \quad (1)$$

where $U(\vec{r})$ is the external potential, g^2 is the coupling constant between axions and $a(t)$ is the scale factor. Here we describe the condensate by one particle wave function $\phi(\vec{r},t)$. The circulation in condensate is given by

$$\Gamma=\oint_C \vec{v}\cdot d\vec{r}=\frac{\hbar}{m}2\pi l, \quad (2)$$

where l is an integer called topological charge. C denotes any contour around a vortex.

When there is no vortex, $l = 0$ and the circulation vanishes. When the condensate is inside a rotating environment the vortex is formed. In the interacting condensate vortices with $l > 1$ are unstable and decay to the vortices with $l = 1$. So in a realistic situation we have a net of elementary vortices ($l = 1$) which are stable.

Let us consider the possibility that a galactic halo is just such a vortex in the axion condensate which was created in the early universe. Namely, in the presence of global rotation, the proposed mechanism yields a huge amount of small vortices whose topological charges equal 1.

Free axion condensate

Because the interaction between axions is very week, in the first approximation we can omit nonlinear term and external potential in eq. (1) and we consider, in fact, a free particle equation

$$i\hbar\left(\frac{\partial}{\partial t}+\frac{3}{2}\frac{\dot{a}(t)}{a(t)}\right)\phi(\vec{r},t)= \\ =-\frac{\hbar^2}{2m}\frac{1}{a^2(t)}\left[\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{l(l+1)}{r^2}\right]\phi(\vec{r},t). \quad (3)$$

In case of the de Sitter evolution of universe ($a(t)=\exp(Ht)$) the solution, density distribution in the vortex, has the form

$$\phi(\vec{r},t)=\frac{C}{a^{3/2}}\exp\left\{-\frac{i}{\hbar}\int\frac{\mu}{a^2(t)}\,dt\right\}j_l(\sqrt{\lambda}r)\,Y_l^n(\theta,\varphi), \quad (4)$$

where C is the normalization constant and $\lambda=2m\mu/\hbar^2$. $E=\mu/a^2$ is the energy of the particle, $j_l(x)$ is the spherical Bessel function related to the ordinary one with $j_l(x)=\sqrt{\pi/2x}J_{l+\frac{1}{2}}(x)$ and Y_l^n are the spherical harmonics.

Let small vortices are created in the early universe (being quantum objects) and then grow during the expansion of the universe (becoming a classical object). We assume that during this transition the shape of the vortex is conserved. To calculate how much the vortex has grown we need to know when its creation took place. To obtain an upper bound for this value we use the relation

$$m_a \simeq 6\mu\ eV \left(\frac{10^{12}\mathrm{GeV}}{f_a} \right), \qquad (5)$$

where f_a is the energy of $U_{PQ}(1)$ symmetry breaking and creation of axions. There is some evidence that $m_a = 10^{-3}$ eV, for this value of energy is

$$f_a \simeq 6 \cdot 10^9 \mathrm{GeV}. \qquad (6)$$

Taking the scale factor $a_0 = 1$ (at the time of the creation of condensate) we have today

$$a = \frac{f_a}{3.7\mathrm{K}} = \frac{6 \cdot 10^9 \mathrm{GeV}}{3.2 \cdot 10^{-13} \mathrm{GeV}} \simeq 2 \cdot 10^{22}. \qquad (7)$$

So we see that the vortex cannot grow more than $\simeq 10^{22}$ times.

Now we need to calculate the value of the scale factor a inferred from our solutions. Let us examine first the case without the vortex $l = 0$. The solution is of the form

$$\phi(\vec{r}, t) = \mathrm{C}(t) \cdot \frac{\sin(\sqrt{\lambda} r)}{\sqrt{\lambda} r}. \tag{8}$$

This solution describes a spherically symmetrical halo with the density distribution $\rho(r, t) \sim |\phi(\vec{r}, t)|^2$. We must remember that the coordinate r in not a physical distance, which is $R = a \cdot r$. For a spherical distribution the velocity rotation curve is from the relation

$$v(R) = \sqrt{\frac{G\mathcal{M}(R)}{R}}, \tag{9}$$

where $\mathcal{M}(R)$ is the mass function and is expressed as

$$\mathcal{M}(R) = 4\pi \int_0^R R'^2 \rho(R') dR'. \tag{10}$$

Based on this, we find

$$v(R) = v_0\sqrt{1 - \frac{\sin(R/R_0)}{r/r_0}}, \tag{11}$$

where

$$v_0 = \sqrt{2\pi G\rho_0\frac{a^2}{\lambda}}, \quad R_0 = \frac{a}{\sqrt{\lambda}}.$$

Figure 1 shows the velocity curve for axion condensate with assumed $\sqrt{\lambda}/a = 0.2$ kpc.

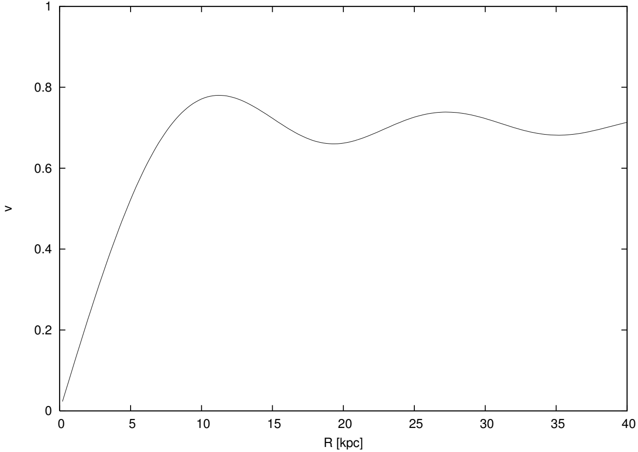


Figure: The contribution to the galactic velocity curve from the axion condensate with assumed $\sqrt{\lambda}/a = 0.2$ kpc.

Vortex in the free axion condensate

Similar results can be obtained by considering the vortex solution with $l = 1$. The velocity curves also exhibit a plateau, like the solution with $l = 0$. The only difference is that we now have the angle dependence $\rho \sim \cos^2\theta$. This produces, like for each vertex solution, distortions from the spherical shape of a dark matter halo. The $l = 1$ is a stable vortex configuration.
With the use of the expression on R (without the factor 4π, since we have already integrated over the solid angle) and (9) we find

$$v(R) = v_0 \frac{R_0}{R} \sqrt{4\left[\cos\left(\frac{R}{R_0}\right) - 1\right] + \left(\frac{R}{R_0}\right)^2 + \left(\frac{R}{R_0}\right)\sin\left(\frac{R}{R_0}\right)} \tag{12}$$

where v_0 and R_0 are defined similarly to the $l = 0$ case.

In Fig. 2 we compare expression (12) with the analogous function previously found in the $l = 0$ case. As we see there is no qualitative difference between the velocity curves for $l = 1$ and $l = 0$. In both cases, the velocity curves become flat, $v(R) \approx v_0$, for $R \gg R_0$.

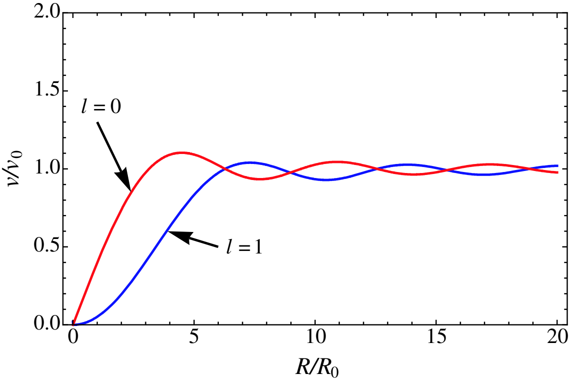


Figure: A comparison of the radial velocity curves for $l = 1$ and $l = 0$. In both cases, the velocity curves are flat, $v(R) \approx v_0$, for $R \gg R_0$.

In Fig. 3 we compare the velocity curve (12) with the exemplary galactic velocity curve.

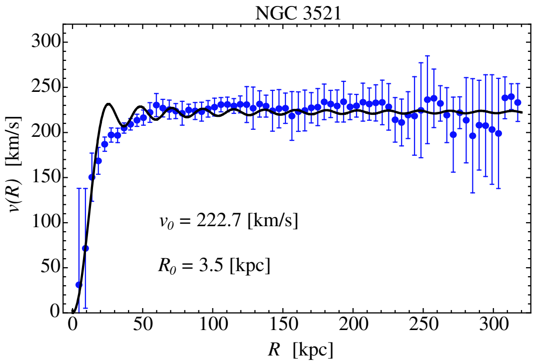


Figure: Radial velocity curve (12) for the axion condensate with $l = 1$ compared with the observational data for the galaxy NGC 3521. In general, we observe that there is a good agreement between the theoretical curve and the observational data. The main difference is the first peak of the theoretical curve. The predicted radial velocity is at this point about 30 km/s higher than the observed value.

Dark matter in cosmological context

The problem considered:
Is mass of dark particles constanst during the cosmic evolution?
Let consider flat homogenouos and isotropic cosmological model.
We additionally assume that between sectors of dark matter and dark energy exist an interaction.
Let $\delta(a) = d(\ln m)/d(\ln a)$ represents the rate of change of dark matter mass during cosmic evolution (Amendola et al., 2007), where a is a scale factor which satisfies Friedman equation

$$3H^2 = 3\frac{\dot{a}^2}{a^2} = \rho_{\rm b} + \rho_{\rm dm} + \rho_{\rm de}.$$

where $H = \frac{d}{dt}(\ln a)$ and t is the cosmological time.

From observation of galaxy rotation curves of relatively not so distant galaxies we have the almost today mass of dark matter particles is rather big. But what was the mass of dark matter particles in the early Universe?
We consider cosmological model to answer this question.
Both dark matter and barionic matter are pressureless ($p_{\mathrm{b}} = 0$ and $p_{\mathrm{dm}} = 0$. We assume that equation of state for dark energy is

$$p_X = w_X \rho_X, \quad \text{where } w_X = \text{const}$$

When $w_X = -1$ we have model with the cosmological constant.

The conservation condition $T^{\mu\nu}{}_{;\mu} = 0$ leads to

$$\dot{\rho}_{\rm b} = -3H\rho_{\rm b} \Rightarrow \rho_{\rm b} = \rho_{\rm b,0} a^{-3}$$

and energy density of dark matter and dark energy satisfy equations

$$\dot{\rho}_{\rm dm} + 3H\rho_{\rm dm} - \delta H\rho_{\rm dm} = 0$$
$$\dot{\rho}_{\rm de} + 3H\rho_{\rm de}(1 + w_X) + \delta H\rho_{\rm de}\rho_{\rm dm} = 0$$

and $-\delta H\rho_{\rm dm}$ is an interacting term.
For simplicity we assume that

$$\frac{1}{H}\frac{\dot{\rho}_{\rm dm}}{\rho_{\rm dm}} = \frac{d(\ln \rho_{\rm dm})}{d(\ln a)} = \delta = \text{const}$$

i.e. the constant rate of change of mass of dark particles during the cosmic evolution.

Therefore

$$\rho_{\rm dm} = \rho_{\rm dm,0} a^{-3+\delta}$$

and $\rho_{\rm de}$ satisfies equation

$$\frac{d\rho_{\rm de}}{da} + \frac{3}{a}\rho_{\rm de}(1 + w_X) + \delta\rho_{\rm dm,0} a^{-4+\delta} = 0.$$

To test this model we need $H(z)$ formula

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{\rm b,0}(1+z)^3 + \Omega_{\rm dm,0}(1+z)^{3-\delta} + (1-\Omega_{\rm b,0} - \Omega_{\rm dm,0}(1+z)^{3(1+w_X)}.$$

The interaction causes $\rho_{\rm dm}$ to deviate from the standard scaling like a^{-3}. Therefore the matter is not "conserved" or equivalently, mass m of dark matter $\rho_{\rm dm} = m(a)n$, where n is their number density, varies with time such that $d\ln(m)/dln(a) = \delta$ that is δ can be interpreted as the rate of change of particle mass per Hubble time. We assume that $\delta = \text{const}$ ($\lessgtr 0$) but in reality dark matter (energy) transfer $\delta(a)$ need to be varied.

The evolution of the Universe can be represented as the dynamical system of a Newtonian type

$$\ddot{a} = -\frac{\partial \bar{V}_{\text{eff}}}{\partial a}$$

where

$$\bar{V}_{\text{eff}} = -\frac{1}{6}\rho_{\text{eff}} a^2 = -\frac{1}{6}\Omega_{\text{eff}} a^2.$$

Therefore

$$\bar{V}_{\text{eff}} = -\frac{1}{6}H_0^2 \left[\Omega_{\text{b},0} a^{-1} + \Omega_{\text{dm}} a^{-1+\delta} + (1 - \Omega_{\text{b},0} - \Omega_{\text{dm}}) a^2\right]$$

If we reparametrize the original time variable t

$$t \to \tau : |H_0| dt = d\tau$$

we obtain

$$\frac{d^2 a}{d\tau^2} = -\frac{\partial V_{\text{eff}}}{\partial a}.$$

If we replace the scale factor by the radial coordinate r (where $r = a\chi$ in the comoving with galaxy coordinate system) we obtain effective potential for the problem of motion of a mass particle in the Schwarzschild spherical metric

$$\bar{V}_{\text{eff}} = -\frac{1}{6}\left[\Omega_{\text{b},0} r^{-1} + \Omega_{\text{dm},0} r^{-1+\delta} + \Omega_{\Lambda,0} r^{2}\right].$$

Let us try to evaluate the rotation curve in the Newtonian limit of gravity

$$\frac{m v_c^2(r)}{r} = -\frac{\partial V_{\text{eff}}}{\partial r}$$

where

$$V_{\text{eff}}(r) \propto -\frac{1}{r}\left[1 + \left(\frac{r}{r_c}\right)^{\delta}\right]$$

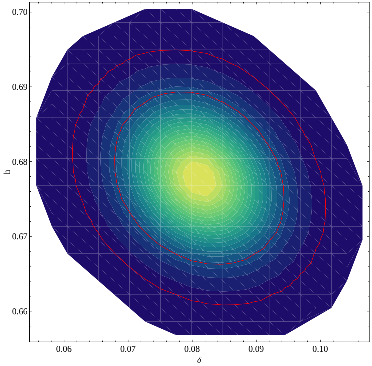
Here the effect of $\Omega_\Lambda, 0$ is neglected in a galactic scale because the system is gravitationally bounded.

From cosmological model estimation using all available astronomical data (Union 2.1 SNia, CMB, BAO, $h(z)$) we obtain

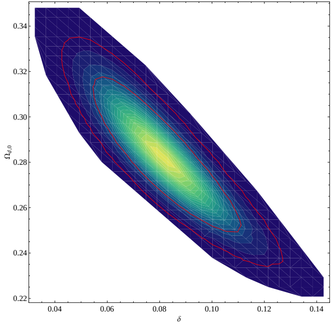
parameter	model with $w_X = -1$ (95% CL)	model with any w_X
w_X	fixed	-1.08
δ	$0.081^{0.050}_{-0.042}$	0.066
$\Omega_{\mathrm{dm},0}$	$0.282^{+0.063}_{-0.056}$	0.293
H_0	$67.8^{+2.0}_{-2.0}$	67.8

with the assumption of $\Omega_{\mathrm{b}} = 0.02226h^{-2} = 0.048468$ where $h = H_0/100$.

$h(\delta)$



$\Omega_{\mathrm{dm}}(\delta)$



Conclusions

1. At 1σ confidence level we found $\delta > 0$ which means that energy transfer preferable occuring from dark energy to dark matter sector.

2. While the mass of dark matter can growth with constant rate during the cosmic evolution this effect cannot reproduce flat rotation curves. because the delta is too small, e.g. for NGC5023 (estimated $\delta = 0.714$).

Acknowledgments

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