Spacetime curvature and Higgs stability during inflation

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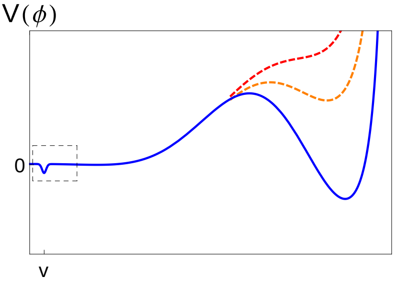
Outline

Standard Model Higgs potential

- $V(\phi)$ has a minimum at $\phi = v$
- Very sensitive to M_h and M_t
- A vacuum at $\phi \neq v$ incompatible with observations



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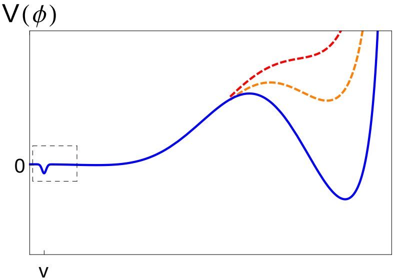

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    - Lifetime much longer than 13.8×10^9 years
  - Is this also true for the early Universe (inflation)?

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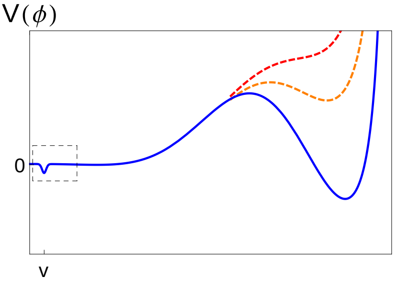

New physics needed to stabilize the vacuum?

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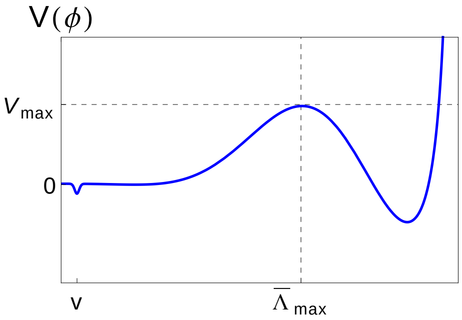
Inflation and the Standard Model

- In principle we can assume the SM to be valid
  - Energy-density is dominated by decoupled physics
- Inflation induces fluctuations to the Higgs field $\Delta\phi \sim H$
  - Important if $\overline{\Lambda}_{\mathrm{max}} \lesssim H$
  - State of the art calculations [2]: $\overline{\Lambda}_{\mathrm{max}} \sim 10^{11}\mathrm{GeV}$



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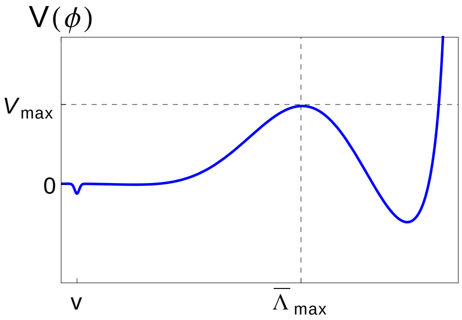

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$H \lesssim 10^{14}\text{GeV}$

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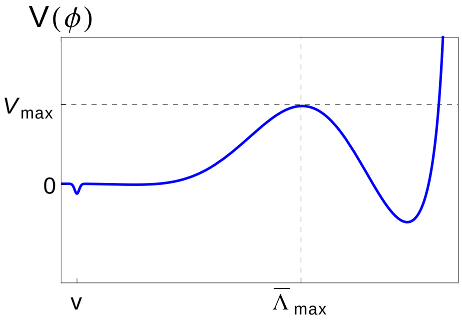

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Outline

Renormalization group improvement

- The perturbative $V_{\text{eff}}(\phi)$ suffers from large logarithms

Example: $V(\phi) = (1/2)m^2\phi^2 + (\lambda/4!)\phi^4$

$$V_{\text{eff}}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\overbrace{M(\phi)^4}^{M(\phi)^2 \equiv m^2 + \frac{\lambda}{2}\phi^2}}{64\pi^2}\left[\log\left(\frac{M(\phi)^2}{\mu^2}\right) - \frac{3}{2}\right]$$

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- The physical result must not depend on μ:

$$\frac{d}{d\mu}V_{\text{eff}}(\phi) = 0 \quad \Leftrightarrow \quad \left\{\mu\frac{\partial}{\partial\mu} + \beta_\lambda\frac{\partial}{\partial\lambda} + \gamma_\phi\phi\frac{\partial}{\partial\phi}\right\}V_{\text{eff}}(\phi) = 0$$

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$\Rightarrow$ can be used to improve the perturbative result [3]

optimal choice

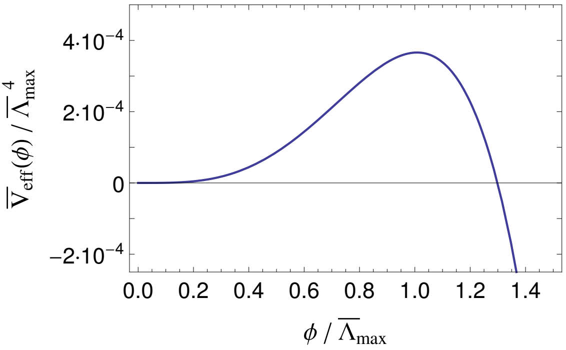
$\mu \sim \phi$

No large logarithms!

$$\Rightarrow \quad V_{\rm eff}(\phi) \approx \frac{\lambda(\phi)}{4}\phi^4$$

[3] Ford et. al. (1993)

Stability results (Minkowski)



- For large H $(\sim 10^3\overline{\Lambda}_{\rm max})$, the SM is not stable [4]

[4] Kobakhidze & Spencer-Smith (2014); Hook et. al. (2014); Fairbairn & Hogan (2014); Enqvist, Meriniemi & Nurmi (2014); Zurek et. al. (2015); Espinosa et. al. (2015)

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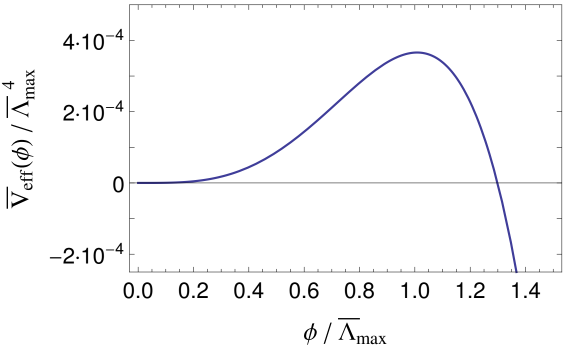

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Does including spacetime curvature in the quantum calculation change this?

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Curved space effective action

- What is usually meant by "curved space QFT"?
  - No loops with $g^{\mu\nu}$
- A "curvature mass" is generated for SM

$$\Rightarrow \quad V_{\text{eff}}(\phi) \approx \frac{\lambda(\mu)}{4}\phi^4 + \frac{\xi(\mu)}{2}R\phi^2$$

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- Solve modes in FRW space

$$\Big[-\Box + M(\phi)^2 + \xi R\Big]\hat{\phi} = 0; \qquad \hat{\phi} = \int \frac{d^3k}{a(t)^{3/2}}\left[\hat{a}_{\mathbf{k}}u_{\mathbf{k}} + \hat{a}^{\dagger}_{\mathbf{k}}u^{*}_{\mathbf{k}}\right],$$

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$$u_{\mathbf{k}} = \frac{1}{\sqrt{W}}e^{-i\int^t W dt'}e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$W^2 = \frac{k^2}{a(t)^2} + M(\phi)^2 + \left(\xi - \frac{1}{6}\right)R + \mathcal{O}(k^{-2})$$

in curved space

$$\mu^2 \sim \phi^2 + R$$

1-loop Effective potential in curved space

$$V_{\text{eff}}(\phi, R) = -\frac{1}{2}m^2(\mu)\phi^2 + \frac{1}{2}\xi(\mu)R\phi^2 + \frac{1}{4}\lambda(\mu)\phi^4$$

$$+\sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{|M_i^2(\phi)|}{\mu^2} - c_i\right] \quad ; \quad \begin{array}{l} M_i^2(\phi) = \kappa_i\phi^2 - \kappa_i' + \theta_i R \\ \mu^2 = \phi^2 + R \end{array}$$

Φ	i	n_i	κ_i	κ_i'	θ_i	c_i
$W^\pm$	1	2	$g^2/4$	0	$1/12$	$3/2$
	2	6	$g^2/4$	0	$-1/6$	$5/6$
	3	-2	$g^2/4$	0	$-1/6$	$3/2$
Z^0	4	1	$(g^2+g'^2)/4$	0	$1/12$	$3/2$
	5	3	$(g^2+g'^2)/4$	0	$-1/6$	$5/6$
	6	-1	$(g^2+g'^2)/4$	0	$-1/6$	$3/2$
t	7	-12	$y_t^2/2$	0	$1/12$	$3/2$
ϕ	8	1	3λ	m^2	$\xi - 1/6$	$3/2$
χ_i	9	3	λ	m^2	$\xi - 1/6$	$3/2$

Stability results (curved space) I

- First attempt, set $\xi_{\rm EW} = 0$ and $H \sim 10^{10}{\rm GeV}$ ($\sim 10^3 \overline{\Lambda}_{\rm max}$)

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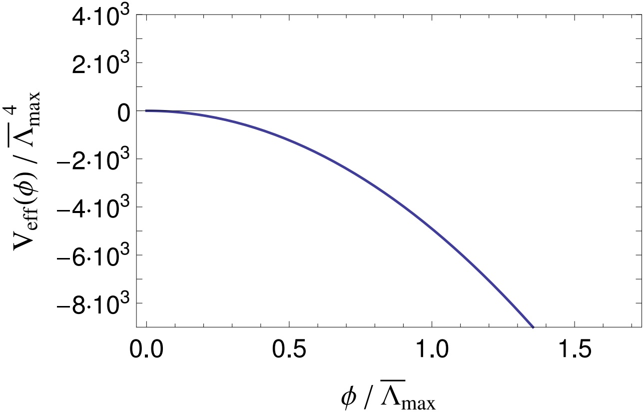

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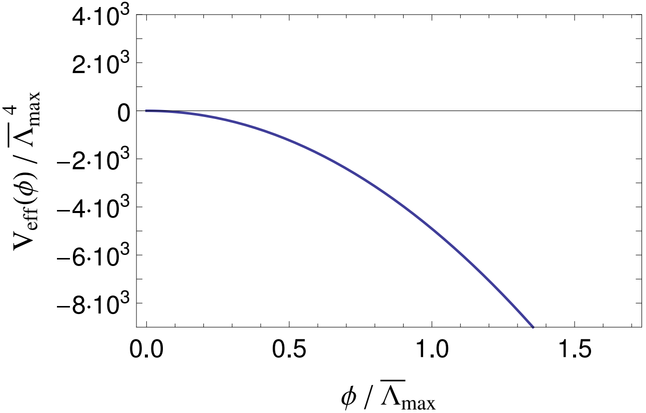

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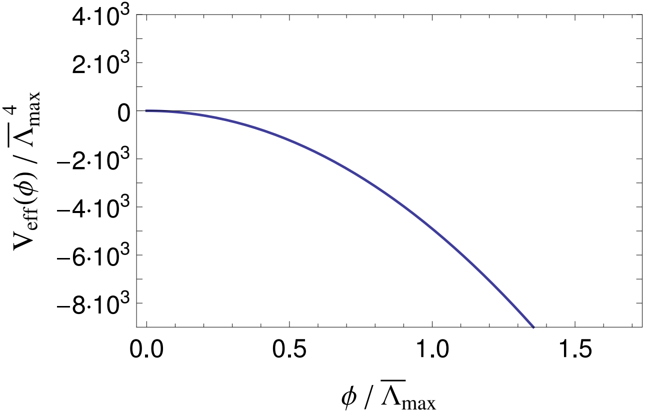

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- Now choosing $\xi_{\rm EW} = 0.1$ [5]

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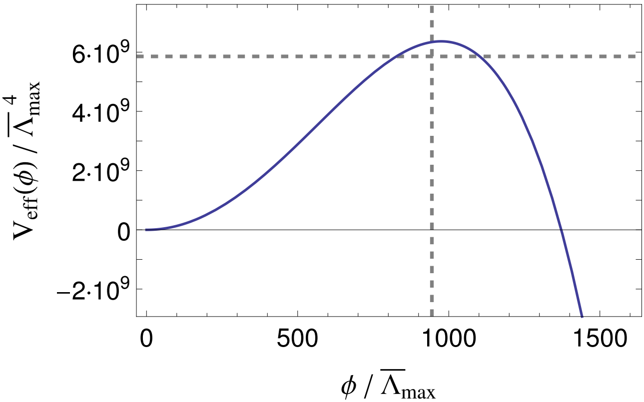

$$\Lambda_{\rm max} \simeq H \left(\frac{12\xi}{|\lambda|}\right)^{1/2}$$

$$V_{\rm max}^{1/4} \simeq H \frac{(6\xi)^{1/2}}{|\lambda|^{1/4}}$$

- $V_{\rm max}(\text{curved}) \gg V_{\rm max}(\text{flat})$ (and at a higher scale)

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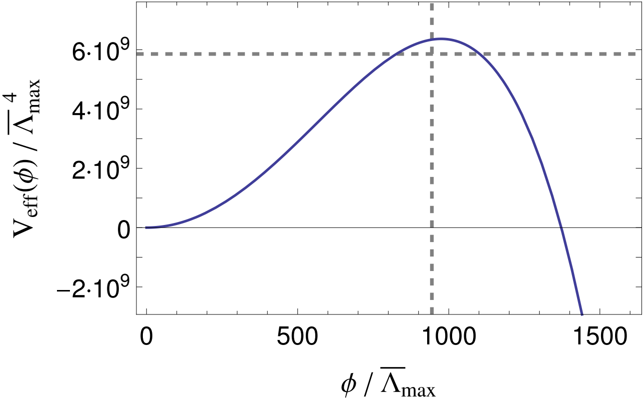

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$$P \sim \exp\left[-8\pi^2\,(V_{\rm max}/3H^4)\right] \quad \Rightarrow \quad \text{Stable!}$$

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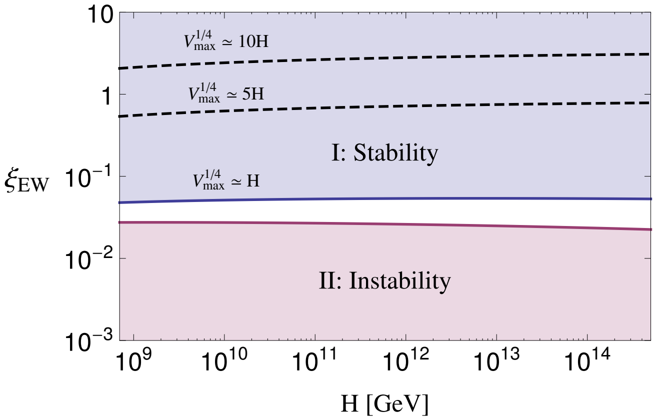
(in)stability from ξ

- The stability/instability of the potential is determined by ξ_{EW}



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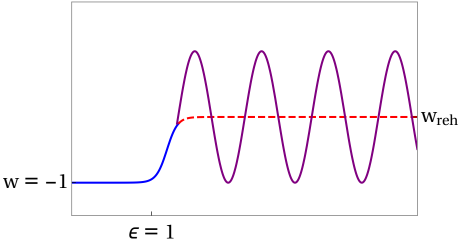
Reheating

- Energy of inflation is transferred to SM degrees of freedom, $T = 0 \rightarrow T_{\rm reh}$
- Equation of state $w = p/\rho$ changes



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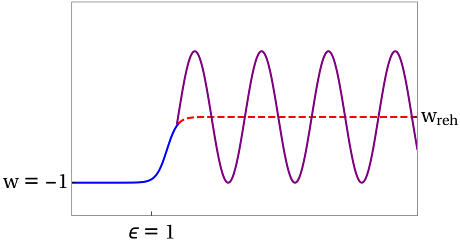

- ϕ feels the dynamics of inflation via ξ !

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- More loops and smaller H
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Thank You!

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Running of ξ

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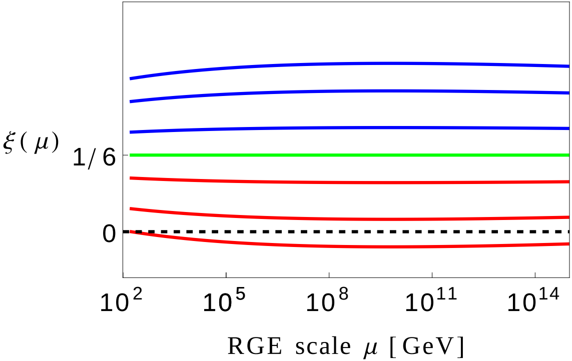

ξ_{EW}

0, 0.05, 0.12, 1/6,
0.22, 0.28, 0.33

Sensitivity to the choice of μ

- A loop calculation is never fully scale invariant
- How dependent is the result on the choice $\mu(t)^2 = \phi(t)^2 + R$?

$$\mu(t)^2 = \alpha\phi(t)^2 + \beta R \qquad \alpha, \beta \in \{0.1 \cdots 10\}$$