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# Searching for links between General Relativity and Quantum Mechanics

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# Contents

# 1- Introduction

- ***Objective***: problem of ***consistency*** between ***QM*** and ***GR***.

  ***Motivated by*: Extra-dimensions**: Kaluza and Klein [1,2]

  ***Semi-classical approach to QM*** de Broglie & Bohm [3,4].

  (However: *Violation of Bell inequalities* in [5,6]).

- ***Recent works on EDs***: they are mostly with *space-like EDs.*

  *We are interested in approach* ***with time-like EDs****:*

- ***Anti-de Sitter*** geometry: Maldacena [7]: AdS/CFT; Randall [8]: (hierarchy).
- ***Induced matter*** models: Wesson [9,10]; Koch [11,12].
- **Our study** based on ***space-time symmetry*** [13,14]: following ***Induced matter models where quantum mechanical equations*** is identical to a micro ***gravitational geodesic description of curved time-like EDs.***

# 2- Symmetry with time-like extra-dimensions (1)

- Considering two orthogonal sub-spaces 3D-time and 3D-space,

we construct an ideal ***6D flat time-space*** $\{t_1, t_2, t_3 | x_1, x_2, x_3\}$:

With the quadratic form: $\boldsymbol{dS^2 = dt_k^2 - dx_l^2}$ ; $Summation$: $k, l = 1 \div 3$.

- We are working further at its ***symmetrical "light-cone"*** :

$$\boldsymbol{d\vec{k}^2 = d\vec{l}^2} \quad \text{or} \quad \sum dt_k{}^2 = \sum dx_l{}^2;\ Summation\text{: } k, l = 1 \div 3 \qquad (1)$$

Natural units ($\hbar = c = 1$) used unless it needs an explicit quantum dimensions.

For transformation to 4D space-time let's postulate:

- ***Conservation of linear Translation (CLT principle) in transformation from higher dimensional geometries to 4D space-time for all linear translational elements of more general geometries***. (***the Eq*** (1) ***of linear time & space intervals*** *(* $\boldsymbol{d\vec{k}^2 = d\vec{l}^2}$*) is to be conserved not only for flat Euclidean/ Minkowski geometries).*

It bases on: Lorentz invariance-homogeneity-isotropy of 4D space-time.

# 2- Symmetry with time-like extra-dimensions (2)

- Introducing a 6D isotropic plane wave equation:

$$\frac{\partial^2 f(t_k,x_l)}{\partial t_k^2} = \frac{\partial^2 f(t_k,x_l)}{\partial x_l^2} \quad ; \qquad (2)$$

- Assuming that the wave transmission (2) serves a ***primitive energy-momentum formation***, *which serves a* ***source to form a time-like vacuum potential*** $V_T$ which generates strong quantum fluctuations with ***circular polarization*** about $t_3$ , keeping ***evolution to the future***, ***constrained by a time-like cylindrical condition.***

- Simultaneously, it leads to ***a violation of the primary space-time symmetry*** (***similar to the Higgs mechanism)***.

- We use for cylinder in 3D time polar coordinates $\{\psi(t_0), \varphi(t_0), t_3\}$ :

$$dt^2 = d\psi(t_0)^2 + \psi(t_0)^2 d\varphi(t_0)^2 + dt_3^2 = ds^2 + dt_3^2 \; ; \quad (3)$$

***linear time*** $dt_3$ in (3) is ***identical*** to $d\vec{k}$ in (1) in according to CLT principle.

as $dt_3$ orthogonal to $dt_0$ : $\Omega dt = \Omega_0 dt_0 + \Omega_3 \, dt_3$

# 2- Symmetry with time-like extra-dimensions (3)

- **We use in 3D-space for description of spin $\vec{s}$ spherical coordinates:** $\{\psi(x_n), \theta(x_n), \varphi(x_n)\}$ :

$$d\lambda^2 = d\psi(x_n)^2 + \psi(x_n)^2[d\theta^2 + sin^2\theta\, d\varphi(x_n)^2] + dx_l^2$$
$$= d\sigma_{ev}{}^2 + d\sigma_L{}^2 + dl^2; \qquad (4)$$

Where: $d\sigma_{ev}$ local interval characterizing ***P-even spinning contribution***;

$d\sigma_L$ ***P-odd contribution*** **of intrinsic space-like curvature**.

$s_L$ // $x_l$ (*left-handed helicity*) local rotation in orthogonal plane $P_n$ →local proper $x_n \in P_n$ serves an ***affine parameter*** to describe a weak curvature in 3D-space.

***EDs turn into the dynamical depending on other 4D space-time dimensions:***

$$\psi = \psi(t_0, t_3, x_n, x_l) \text{ and } \varphi = \Omega t - k_j x_j = \Omega_0 t_0 + \Omega_3 t_3 - k_n x_n - k_l x_l.$$

- In the result, the 6D time-space (1) being generalized with curvature gets a new quadratic form:

$$dt^2 - ds^2 = d\lambda^2 - d\sigma_{ev}{}^2 - d\sigma_L{}^2 ; \quad (5)$$

# 2- Symmetry with time-like extra-dimensions (4)

- It leads to a ***more general 4D Minkowski space-time*** of a spinning particle with both motions: linear translation and rotation:

$$\boldsymbol{d\Sigma^2 = ds^2 - d\sigma_{ev}{}^2 - d\sigma_L{}^2 = dt^2 - d\lambda^2} \; ; \qquad (6)$$

Where $\boldsymbol{d\Sigma}$ is the total space-time interval.

- In a lab−frame $\boldsymbol{d\sigma_{ev}}$ can be compensated by the curved space component of $\boldsymbol{d\lambda}$, and the quadratic form (6) transforms into a 4D Minkowski space-time with a slightly adjusted interval, which is inconsistency with special relativity:

$$\boldsymbol{ds^2 - d\sigma_L{}^2 = dt^2 - dl^2} \; ; \qquad (7)$$

# 3- Geodesic deviation of micro time and space (1)

- Let's assume that any *deviation from the linear translation in 3D-time should be compensated by a deviation in 3D-space* for conserving space-time symmetry (1):

$$\boldsymbol{Du(t_0) = Du(x_n)} \;; \qquad (8) \qquad \text{with velocity } \boldsymbol{u(s)} = \frac{\partial\psi}{\partial s}.$$

- We derive a symmetrical equation of geodesic acceleration of the deviation $\boldsymbol{\psi}$:

$$\frac{\partial^2\psi}{\partial {t_0}^2} + \Gamma^{\psi}_{\alpha\beta}\left(\frac{\partial t_\alpha}{\partial t_0}\right)\left(\frac{\partial t_\beta}{\partial t_0}\right) = \frac{\partial^2\psi}{\partial x_n^2} + \Gamma^{\psi}_{\gamma\,\sigma}\left(\frac{\partial x_\gamma}{\partial x_n}\right)\left(\frac{\partial x_\sigma}{\partial x_n}\right); \quad (9)$$

- Where: $\boldsymbol{t_\alpha, t_\beta} \in \{\boldsymbol{\psi(t_0), \varphi(t_0), t_3}\}; \boldsymbol{x_\gamma, x_\sigma} \in \{\boldsymbol{\psi(x_n), \varphi(x_n), x_l}\}$.
- In both sides: $\Gamma^{\psi}_{\varphi(t_0)\,\varphi(t_0)} = -\psi$ and $\Gamma^{\psi}_{\varphi(x_n)\,\varphi(x_n)} = -\psi.\sin^2\theta$ ;
- Other terms with $\Gamma^{\psi}_{\alpha\beta}$ and $\Gamma^{\psi}_{\gamma\ \sigma}$ are vanished due to orthogonality of $\boldsymbol{dt_\alpha}$ (or $\boldsymbol{dt_\beta}$) to $\boldsymbol{dt_0}$ and of $\boldsymbol{dx_\gamma}$ (or $\boldsymbol{dx_\sigma}$) to $\boldsymbol{dx_n}$.

Adding to (9) differential ***equation of linear elements***: $\frac{\partial^2\psi}{\partial t_3^2} = \frac{\partial^2\psi}{\partial x_l^2}$ ; (10)

- *We obtain* ***equation including rotation and linear translation****:*

$$\frac{\partial^2\psi}{\partial {t_0}^2} - \psi\left(\frac{\partial\varphi}{\partial t_0}\right)^2 + \frac{\partial^2\psi}{\partial {t_3}^2} = \frac{\partial^2\psi}{\partial {x_n}^2} - \psi\,\sin^2\theta\left(\frac{\partial\varphi}{\partial x_n}\right)^2 + \frac{\partial^2\psi}{\partial {x_l}^2} \;; \qquad (11)$$

# 3- Geodesic deviation of micro time and space (2)

- Due to orthogonality of each pair of differentials $(\boldsymbol{dt_3}\ \&\ \boldsymbol{dt_0})$ and $(\boldsymbol{dx_l}\ \&\ \boldsymbol{dx_n})$ their second derivatives are combined together:

$$\frac{\partial^2\psi}{\partial {t_0^+}^2}+\frac{\partial^2\psi}{\partial {t_3}^2}=\frac{\partial^2\psi}{\partial t^2} \quad ; \quad (12) \qquad \text{and} \qquad \frac{\partial^2\psi}{\partial {x_n}^2}+\frac{\partial^2\psi}{\partial {x_l}^2}=\frac{\partial^2\psi}{\partial {x_j}^2} \quad ; \quad (13)$$

- In the result of two operations:
  - Defining $\boldsymbol{\psi}$ as a deviation parameter;
  - The unification of time-like dimensions (12)

→ ***6D manifold reduces into a 4D space-time***

- Finally, from (11) we obtain the geodesic equation as follows:

$$-\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^2\psi}{\partial {x_j}^2}=-\left[\Lambda_{\ T}-(k_n.s_l)^2_{even}-\Lambda_{\ L}\right]\psi; \qquad \textbf{(14)}$$

Where : Effective potentials $\mathbf{V}_{\ T}$ of a time-like "cosmological constant" $\boldsymbol{\Lambda}_{\ T}$ and an odd component of the space-like $\boldsymbol{\Lambda}$: $[\Lambda_{\ T}-\Lambda_{\ L}]\psi=\left[\left(\frac{\partial\varphi}{\partial t_0^+}\right)^2-\left(\frac{\partial\varphi}{\partial x_n^L}\right)^2\right]\psi.$

# 3- Geodesic deviation of micro time and space (3)

$$-\frac{\partial^2\psi}{\partial t^2}+\frac{\partial^2\psi}{\partial {x_j}^2}=-[\Lambda_T-(k_n.s_l)^2_{even}-\Lambda_L]\psi; \qquad (14^*)$$

- During transformation from 6D time-space to 4D space-time, ***the time-space symmetry is to be broken***: the time-like curvature is dominant, while the space-like ones are hidden in 3D-space, leaving a small PNC effect.

- As $\psi$- function characterizes a strong time-like curvature → Equation (14) is an ***emission law of micro gravitational waves in time-space*** from the source $V_T$.

- In Laboratory frame without polarization analyzer it is able to follow only linear translation in 3D-space, because the intrinsic P-even spinning is compensated by the local 3D-space geodesic condition:

$$\frac{\partial^2\psi}{\partial {x_n}^2}=\psi\, sin^2\theta\left(\frac{\partial\varphi}{\partial x_n}\right)^2=\psi\,(k_n.s_l)^2_{even} \qquad ; \qquad (15)$$

# 4- Quantum equations and indeterminism (1)

- For formulation of quantum mechanical equations adopting the quantum operators, such as:

$$\frac{\partial}{\partial t} \rightarrow i.\hbar\frac{\partial}{\partial t} = \hat{E} \quad \text{and} \quad \frac{\partial}{\partial x_j} \rightarrow -i.\hbar\frac{\partial}{\partial x_j} = \hat{p}_j$$

→ Equation (19) leads to the basic quantum equation of motion:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2\frac{\partial^2\psi}{\partial {x_j}^2} - m^2\psi = 0 \;; \quad (16)$$

Where : $m^2 = m_0^2 - \delta m^2 = m_0^2 - m_S^2 - m_L^2$

- $m_0$ is the conventional rest mass, defined by $\Lambda_T$;
- $m_S$ as a P-even contribution links with an external rotational curvature in 3D-space which vanishes due to the geodesic condition (15);
- $m_L \ll m_S$ is a tiny mass factor generated by $\Lambda_L$, related to a P-odd effect of parity non-conservation (PNC).

# 4- Quantum equations and indeterminism (2)

***Based on local geodesic deviation acceleration conditions, we can understand some QM phenomena:***

- ***Bohm quantum Potential***: for the exact condition of geodesic deviation, with the even spinning, Equation (15) leads to:

$$\left(\frac{\partial S}{\partial x_n}\right)^2 = (\hbar.\boldsymbol{k_n}.\boldsymbol{s_l})^2_{even} = \frac{\hbar^2}{\psi}\frac{\partial^2\psi}{\partial {x_n}^2} = -2mQ_B; \quad (17)$$

which is ***proportional to Bohm's quantum potential $Q_B$*** assumed in [4].

- ***Schrödinger's Zitterbewegung***:

  - The existence of ***the spin term*** in (16) is reminiscent of ***ZBW of free electron*** [15].

  - When we ***describe a linear translation*** of the freely moving particle by Equation (16), the ***ZBW term is almost compensated by the condition*** (15) except a tiny P-odd term. However the latter is hard to observe.

- In the case of a non-polarized field, $m \rightarrow m_0$, Equation (16) is identified as the traditional ***Klein-Gordon-Fock equation***:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + \hbar^2\frac{\partial^2\psi}{\partial {x_l}^2} - {m_0}^2\psi = 0 \quad ; \qquad (18)$$

# 4- Quantum equations and indeterminism (3)

- ***In particular, this shed light on Heisenberg Indeterminism:***

***(I)*** - ***Coordinate-momentum inequality***:

- In according to (8), the local geodesic condition (15) leads to an equation:

$$\frac{1}{\psi} d\left(\frac{\partial \psi}{\partial x_n}\right) . dx_n = sin^2 \theta \, d\varphi^2 \geq 0 \; ; \qquad (19)$$

- the relation for space-momentum indetermination reads:

$$|\Delta p| . |\Delta x| \geq |\Delta p_n| . |\Delta x_n| > \psi^{-1} \left| d\left(i . \hbar \frac{\partial \psi}{\partial x_n}\right)\right| . |dx_n| =$$

$$= |i . \hbar| . sin^2 \theta \, d\varphi^2 \geq 0; \qquad (20)$$

Suggesting the conditions:

i/ As ***a spatial quantization*** by the cylindrical condition:

$sin^2 \theta = 1$ i.e. $\theta$ =(n+1/2)π ;

ii/ For a Poisson distribution with $< \varphi >$= 2π and $d\varphi \approx < \sigma_\varphi >= \sqrt{2\pi}$.

# 4- Quantum equations and indeterminism (4)

- ***(II)*** - ***Time-energy inequality***:
- Similarly, following the local geodesic condition in 3D-time:

$$\frac{1}{\psi} d\left(\frac{\partial\psi}{\partial t_0}\right).dt_0 = d\varphi^2 \geq 0 \;; \qquad (21)$$

- Accordingly, we got the relation for time-energy indetermination:

$$|\Delta E|.|\Delta t| \geq |\Delta E_0|.|\Delta t_0| > \psi^{-1}\left|d\left(i.\hbar\frac{\partial\psi}{\partial t_0}\right)\right|.|dt_0| =$$

$$= |i.\hbar|.d\varphi^2 \geq 0 \;; \qquad (22)$$

→ The inequality (20) as well as relation (22), can turn to an equality only for flat time-space Euclidean geometry.

In the result, we come to Heisenberg inequalities: $|\Delta p|.|\Delta x| > 2\pi\,\hbar$

and $|\Delta E|.|\Delta t| > 2\pi\,\hbar$.

# 6- Charged lepton generations (1)

- In 4D space-time assuming that all leptons, as a material points, are to involve in a common basic time-like cylindrical geodesic evolution with a internal 1D circular curvature of the time-like circle $\boldsymbol{S_1}(\varphi^+)$, where $\varphi^+$is azimuth rotation in the plane $\{t_1, t_2\}$ about $t_3$ and its sign "+" means a fixed time-like polarization to the future;

  Developing more generalized 3D spherical system, described by ***nautical angles*** $\{\varphi^+, \theta_T, \gamma_T\}$, where $\theta_T$ is a zenith in the plane $\{t_1, t_3\}$ and $\gamma_T$ is another zenith in the orthogonal plane $\{t_2, t_3\}$ .

- For *n*-hyper spherical surfaces their highest order curvatures $\boldsymbol{C_n}$ is inversely proportional to *n*-power of time-like radius:

$$\boldsymbol{C_n \sim \psi^{-n}} \quad ; \quad n = 1,3 \quad ;$$

→ In according to general relativity, the energy density $\boldsymbol{\rho_n}$ correlates with the scalar curvature:

$$\boldsymbol{\rho_n = \frac{\epsilon_0}{\psi^n}} \quad ; \qquad (23)$$

Where the factor $\boldsymbol{\epsilon_0}$ is assumed a universal lepton energy parameter

(probably, because all 3 generations are involved in cylindrical condition).

# 6- Charged lepton generations (2)

- The 4D observers (coexisting in the same time-like cylindrical curved evolution $\varphi^+$) see electron oscillating on a line-segment of the time-like amplitude $\boldsymbol{\Phi}$, formulating a 1D proper (or comoving) "volume":

$$V_1(\varphi^+) = \Phi = \psi T;$$

where $\boldsymbol{T}$ is the 1D time-like Lagrange radius.

- For instance, $\boldsymbol{\Phi}$ plays a role of the time-like micro Hubble radius and the wave function $\boldsymbol{\psi}$ plays a role of the scale factor. They are probably changeable during the expansion of the Universe.
- The mass of electron defined by 1D Lagrange "volume" will be:

$$m_1 = \rho_1 V_1 = \rho_1 \Phi = \frac{\epsilon_0}{\psi} \psi T = \epsilon_0 T \quad ; \quad (24)$$

For muon and tauon except the basic time-like cylindrical curved evolution $\varphi^+$, the 4D-observers can see some more additional ED curvatures come from simplest configurations of hyperspheres $S_1\ (\theta_T)$ & $S_1\ (\gamma_T)$ or $S_2(\theta_T, \gamma_T)$.

(the additional curvatures are external, as the observers are not involved in).

# 6- Charged lepton generations (3)

- The "comoving volumes" $V_n(\Phi)$ with fixed $\Phi$ are calculated as:

$$V_n(\Phi) = \int_0^{\Phi} S_{n-1}(v)dv = S_{n-1}(\Phi)\int_0^{\Phi} dv = S_{n-1}\Phi = V_1 S_{n-1}$$

- For "homogeneity condition" of the motion equation of a particle at rest, the simplest "2D-rotational comoving volume" is:

$$V_2(\varphi^+, "\theta_T + \gamma_T") = V_1(\varphi^+)[S_1(\theta_T) + S_1(\gamma_T)] = \Phi.2S_1 = 4\pi\Phi^2$$

- Accordingly, the lepton mass of 2D time-like curved particle is:

$$m_2 = \rho_2 V_2 = \frac{\epsilon_0}{\psi^2} 4\pi\Phi^2 = \epsilon_0 4\pi T^2; \qquad (25)$$

- The next simplest "3D-rotational comoving volume" is:

$$V_3(\varphi^+, "\theta_T * \gamma_T") = V_1(\varphi^+)S_2(\theta_T, \gamma_T) = \Phi.S_2 = 4\pi\Phi^3$$

- Accordingly, the lepton mass of 3D time-like curved particle is:

$$m_3 = \rho_3 V_3 = \frac{\epsilon_0}{\psi^3} 4\pi\Phi^3 = \epsilon_0 4\pi T^3; \qquad (26)$$

# 6- Charged lepton generations (4)

***Assumption (qualitative) for estimation of Lagrange radius $T$*** :

- During the Big-Bang inflation, we suggest, the following a ***scenario similar to the standard cosmological model***: micro factor $\boldsymbol{\psi}$ increases exponentially ( time-like Hubble constant $\boldsymbol{H_T} = \sqrt{\Lambda_T}$ ) and (the instant of inflation typical for E-M interaction: $\Delta t_1 = (1.1 \div 1.7)10^{-18}$ sec after 1 sec from the Big-Bang). For the next time-life of the Universe 13.7 Bill. years assuming: $\boldsymbol{\psi} \sim t^{1/2}$ or $t^{2/3}$ .

→ The time-like Lagrange radius $\boldsymbol{T}$ decreases from $\boldsymbol{T}_0 = \frac{\boldsymbol{\Phi}}{\boldsymbol{\psi_0}}$=1 for $\Delta t_1$ then steps up to the present value $\boldsymbol{T} = \frac{\boldsymbol{\Phi}}{\boldsymbol{\psi}} \approx 16.5$.

- For leptons born after the inflation era, assuming following anthropic principle (very *qualitatively)* that the Hubble radius of any quantum fluctuations should adapt the contemporary value $\boldsymbol{\Phi}$, while the scale factor $\boldsymbol{\psi}$ being governed by a contemporary chaotic Higgs-like potential in such a way (e.g. contracting) that is to meet the contemporary time-like Lagrange radius $\boldsymbol{T}$ (for today, $\boldsymbol{T}$ =16.5).
- Using $\boldsymbol{T = 16.5}$, and the energy parameter $\boldsymbol{\epsilon_0 = 31.0}\ keV$ calibrated to $\boldsymbol{m_e}$, we come to mass ratios of all three charged lepton generations:

$$\boldsymbol{m_e : m_\mu : m_\tau = m_1 : m_2 : m_3} = 1 : 207.4 : 3421.5 =$$

$$= \mathbf{0.511 : 106.0 : 1748.4} \text{ (in MeV) ; } \quad (27)$$

# 6- Charged lepton generations (5)

The result (as for the 1rst order of approximation) is resumed in the Table:

| n-Lepton | 1-electron | 2-muon | 3-tau lepton |
|---|---|---|---|
| Density, $\rho_n$ | $\frac{\epsilon_0}{\psi}$ | $\frac{\epsilon_0}{\psi^2}$ | $\frac{\epsilon_0}{\psi^3}$ |
| Comoving volume, $\boldsymbol{V}_n$ | $\boldsymbol{\Phi}$ | $4\pi\boldsymbol{\Phi}^2$ | $4\pi\boldsymbol{\Phi}^3$ |
| Formulas of mass, $\boldsymbol{m_n}$ | $\epsilon_0 T$ | $\epsilon_0 4\pi T^2$ | $\epsilon_0 4\pi T^3$ |
| Calculated mass ratio ($\boldsymbol{T \approx 16.5}$) | **1** | **207.4** | **3421.5** |
| Experimental leton mass, $\boldsymbol{m_n}$ $(MeV)$ ** | 0.510998928(11) | 105.6583715(35) | 1776.82(16) |
| Calculated lepton mass, $\boldsymbol{m_n}$ $(MeV)$ | **0.511*** | **106.0** | **1748.4** |

**) Same value $\boldsymbol{m_e}$ for calibration.*
***) J. Beringer et al. (Particle Data Group), PR D86 (2012) 010001 .*

- The deviation from masses of muon and tau-lepton < +1% and - 2%.
- This may be ***a solution to the problem of charged lepton mass hierarchy and to the puzzle why there are exactly 3 (three) generations.***
- In opposite, ***this fact is a promising argument for adopting the 3D-time geometry (not less nor higher dimensional than 3D)***.

# 6- Charged lepton generations (6)

- ❑ ***For illustration*** from (7), a reminiscence of de-Sitter (dS) geometry :

$$dt^2 - \boldsymbol{dl^2} - ds^2 = -d\sigma_L{}^2; \quad (7^*)$$

When $s = s(\psi, \varphi)$ is a combination of ED variables (not an invariant) we got:

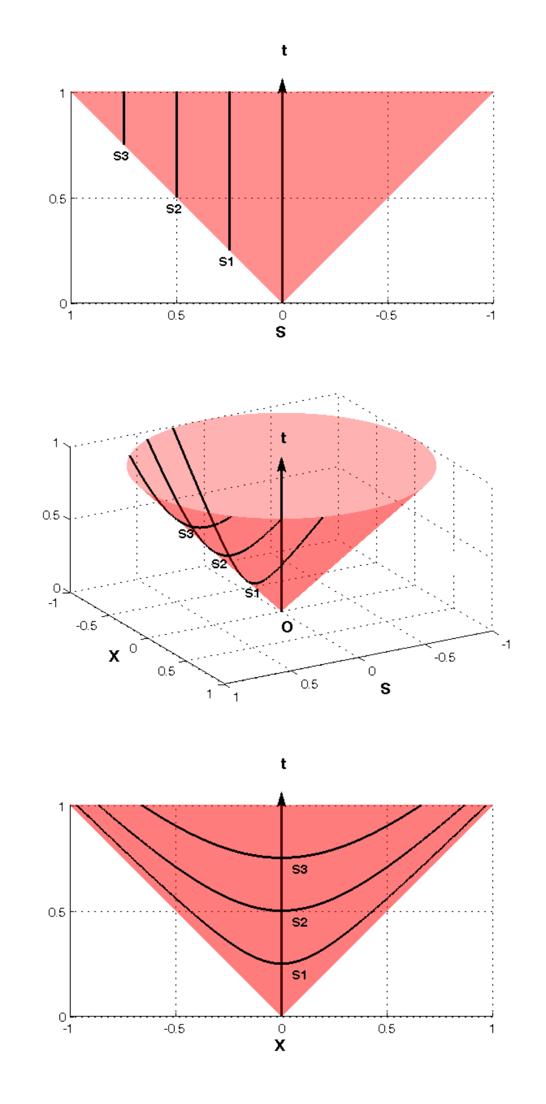
$$t^2 - \boldsymbol{l}(\boldsymbol{X})^2 - s(\psi, \varphi)^2 = -\sigma_L{}^2 \; ; \qquad (28)$$

As $\sigma_L{}^2 << s^2$, this hyperboloid is getting to its "light-cone"(see Figs. a,b,c).

- ❑ Because $s(\psi, \varphi)$ is not a space-like ED (i.e. not as for dS manifold),, representing for curved time-like EDs, then:
- ➢ The physical time *t* can be parametrized as $dt^2 = dt_3{}^2 + ds^2$
- ➢ Each hyperbola (28) as an intersection of the "light-cone" with a flat plane at $s = s_n$ (see Fig.a). Such a hyperbola is the world-line of lepton *n*, e.g. e,μ,τ (evolving at the levels $s_1$, $s_2$ and $s_3$, correspondingly).

- ❑ The "anthropic principle" is made in according to this:
- ➢ Being coexisting at level $s_1$ of electrons do observe the physics world from their "electron" position, the 4D observers can not see any change of electron mass during the cosmological expansion. However, they can measure the changeable masses of μ and τ with Big-Bang expansion: **it would be a window for experiments**.

- ➢ Being constructed at a flat plane (*X*-*t*) 4D-Minkowski at the origin O on which all hyperbolas of different $s_n$ are projected (see Fig.c) , Quantum mechanics serves as an 4D effective holography for restoration of physics on the extended "light-cone" of 6D time-space.



# 7- Conclusions

- ❑ **There are *strong arguments for existence*** of ***time-like EDs in terms of the wave function ψ and the proper time*** $t_0$ .
- ➢ The ***curvature are revealing in emission of micro-gravitational waves*** which is ***described by the quantum Klein-Gordon-Fock equation***.

- ❑ The ***geodesic equation of deviation ψ*** shed light on:
- ➢ ***Bohm's quantum potential***;
- ➢ ***Zitterbewegung*** (Schrödinger's ZBW) of a spinning free electron;
- ➢ ***Heisenberg inequalities***.
- → In particular, ***triumph of Heisenberg indeterminism serves a strong argument for the curvature of micro time-space***.

- ❑ Number of charged lepton generations is assumed equal to the maximal time-like dimension (3D):
- ➢ Based on the common cylindrical 1D-mode: extending the curvature to additional 2D and 3D time-like hyper-spherical configurations to estimate the ***mass ratios of all charged lepton***: quantitatively satisfactury.

→It would serve a ***solution of problems of number "3" of lepton generations and lepton mass hierarchy***.

Finally, we have shown ***more evidence of a deep consistency*** between:

***Quantum Mechanics*** and ***General Relativity***.

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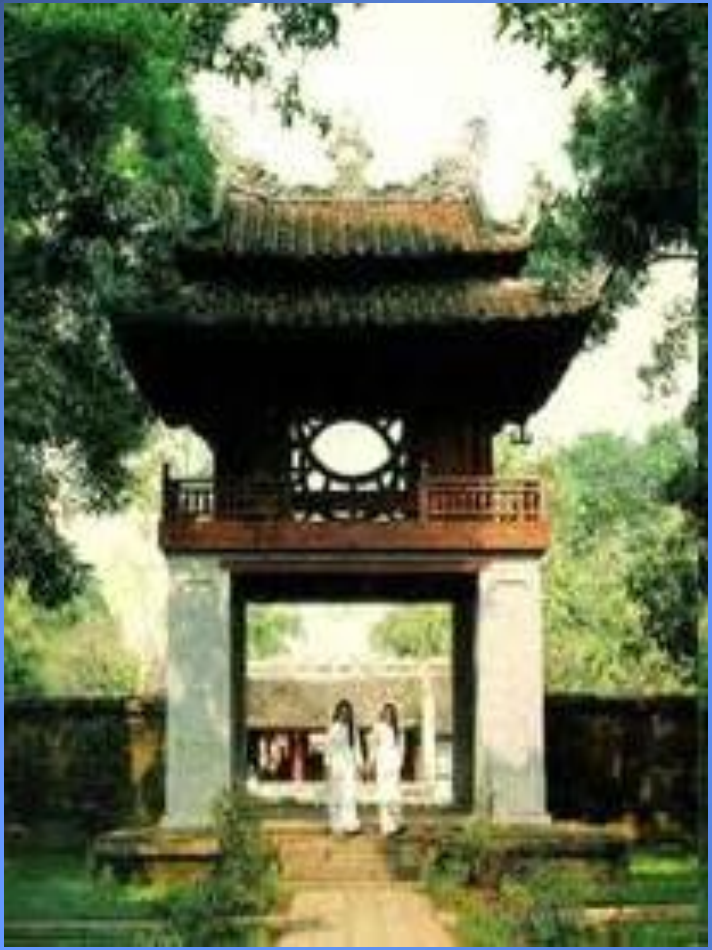

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# Thank You for Your Attention!