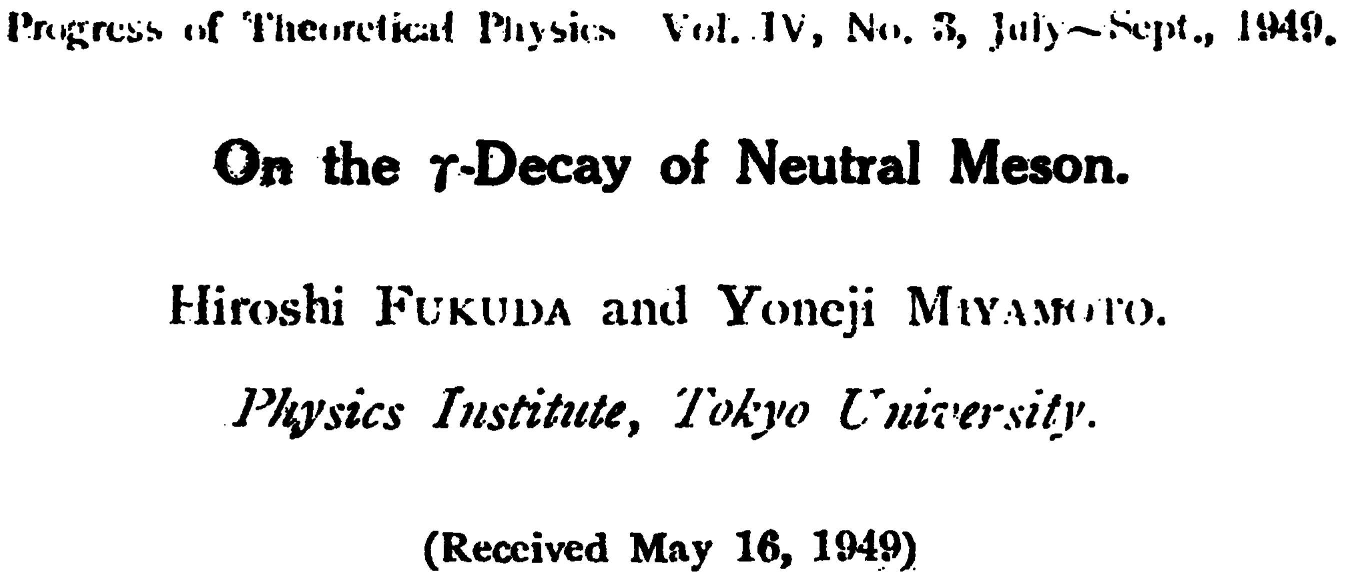
Holography: from black holes to condesed matter

Dam Thanh Son (University of Chicago)
27th Rencontres de Blois (2015)

Plan of the talk

- Anomalies
- Gauge/gravity duality
- Hydrodynamics with anomalies
- Magnetoresistance of Weyl semimetals

Puzzle of π^0 decay to 2γ



Progress of Theoretical Physics Vol. IV, No. 3, July~Sept., 1949.

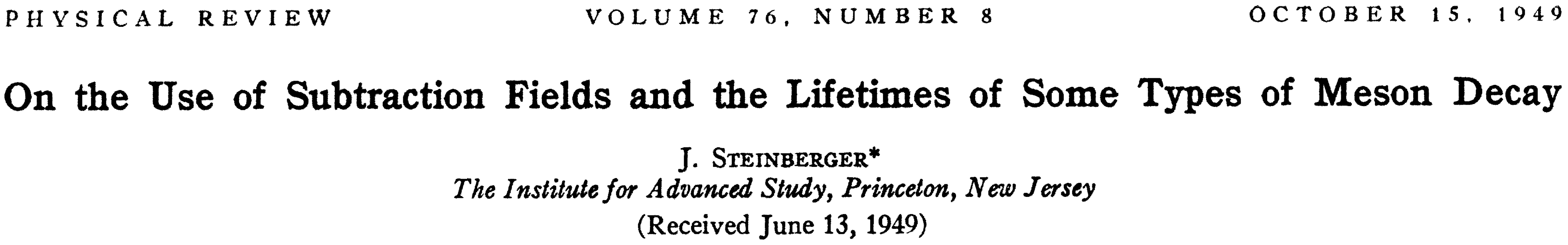
On the γ-Decay of Neutral Meson.

Hiroshi Fukuda and Yoneji Miyamoto.

Physics Institute, Tokyo University.

(Received May 16, 1949)

By using the method of evaluation which has been applied by Schwinger…we have obtained the convergent but non-gauge covariant result for the γ decay of neutral meson….Thus, in the present state of the field theory, we cannot give an unambiguous life-time for neutral meson.



PHYSICAL REVIEW VOLUME 76, NUMBER 8 OCTOBER 15, 1949

On the Use of Subtraction Fields and the Lifetimes of Some Types of Meson Decay

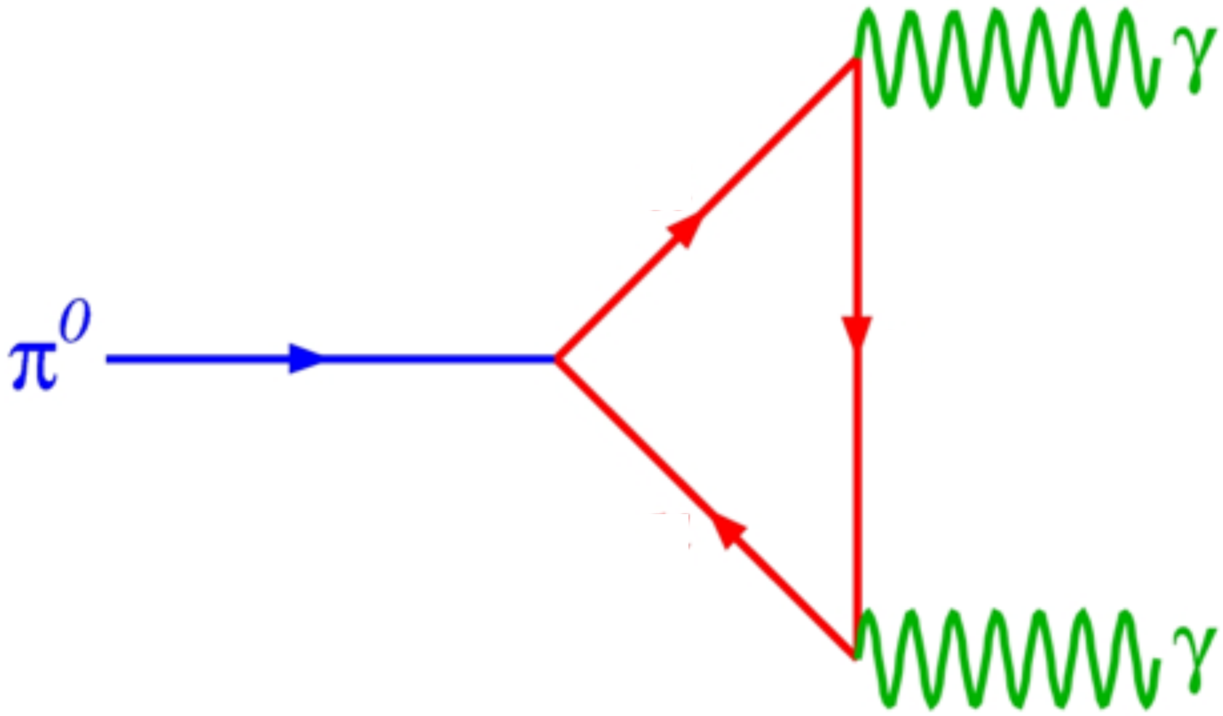
J. Steinberger*

The Institute for Advanced Study, Princeton, New Jersey

(Received June 13, 1949)

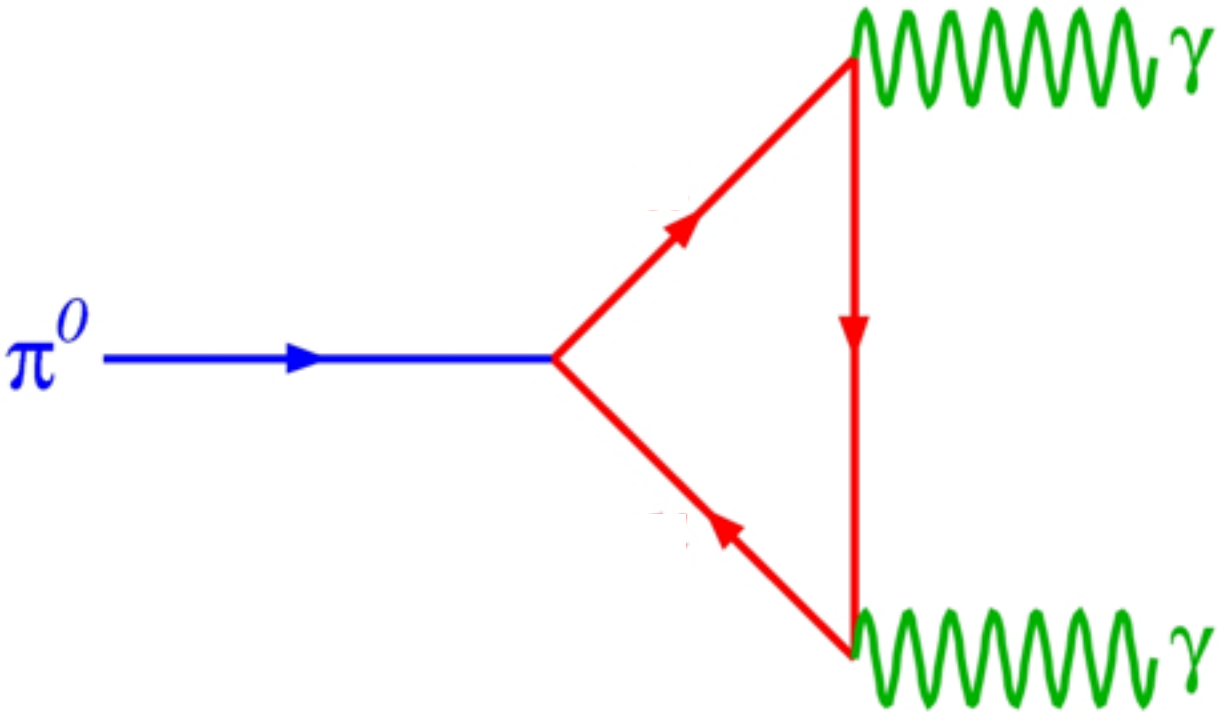
...The method [Pauli-Villars] is, however, not without ambiguity......

Puzzle of π^0 decay



$$\int_{-\infty}^{\infty} dx\,[f(x+a) - f(x)] = \int_{-\infty}^{\infty} dx\,[af'(x) + O(a^2)] = a[f(\infty) - f(-\infty)]$$

Puzzle of π⁰ decay



$$\begin{tikzcd} \int_{-\infty}^{\infty} dx\,[f(x+a) - f(x)] = \int_{-\infty}^{\infty} dx\,[af'(x) + O(a^2)] = a[f(\infty) - f(-\infty)] \arrow[d, "x \to x - a"] \\ \int_{-\infty}^{\infty} dx\,[f(x) - f(x)] = 0 \end{tikzcd}$$

Anomaly

- The key to the understanding of the pion decay puzzle was identified by Adler, Bell, Jackiw (1969)
- In massless electrodynamics, numbers of left- and right-handed electrons are not conserved separately in quantum theory

Chirality

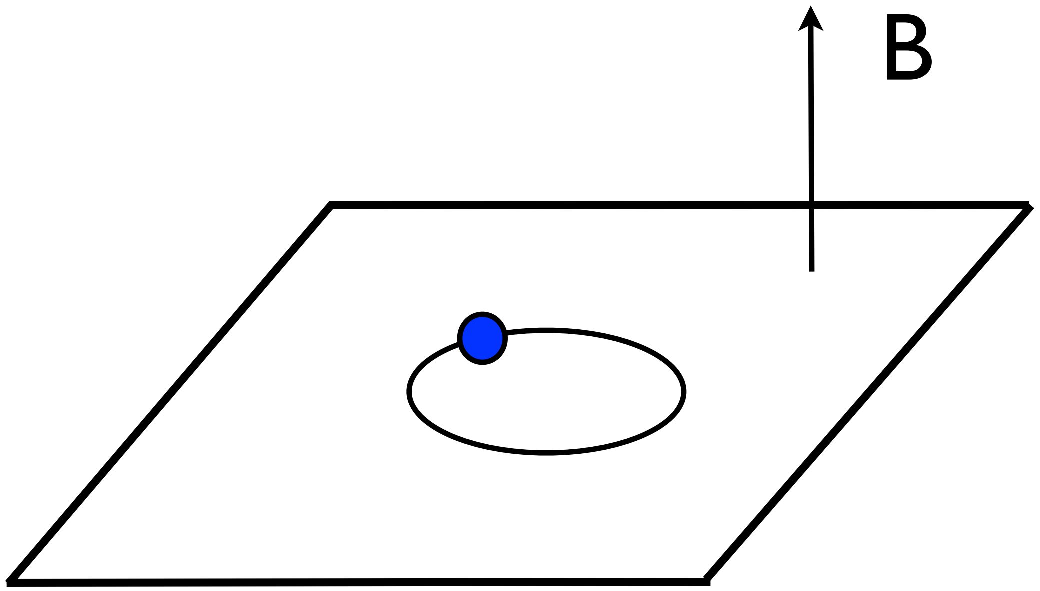
- Consider a massless spin-1/2 particle: 2 chiralities

Chirality does not change in when particle moves in EM field (classically)

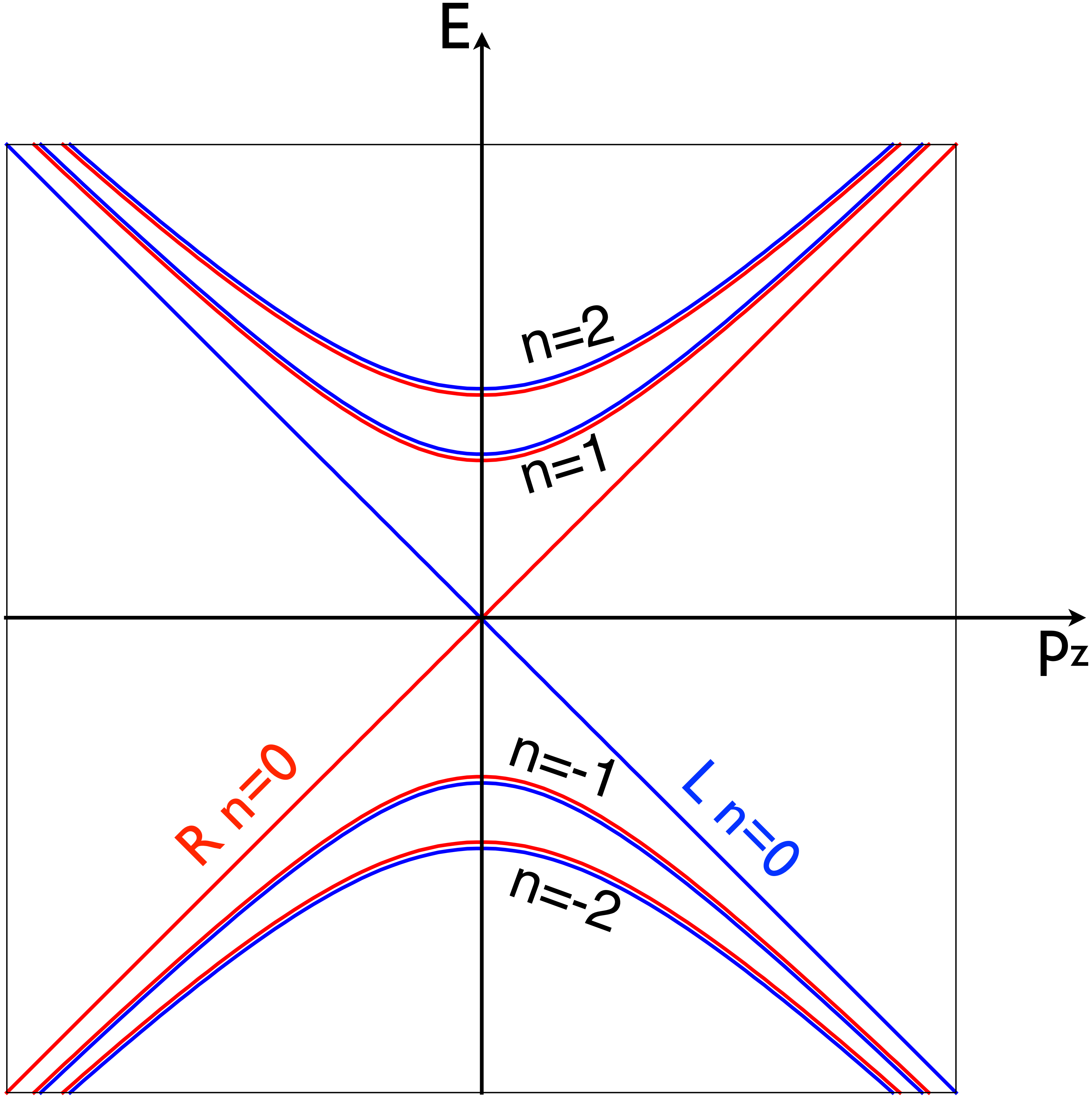
But chirality is not conserved in quantum theory: anomalies

Landau levels

- To understand anomalies, we start with quantum mechanics of a massless fermion in a magnetic field Nielsen Ninomiya



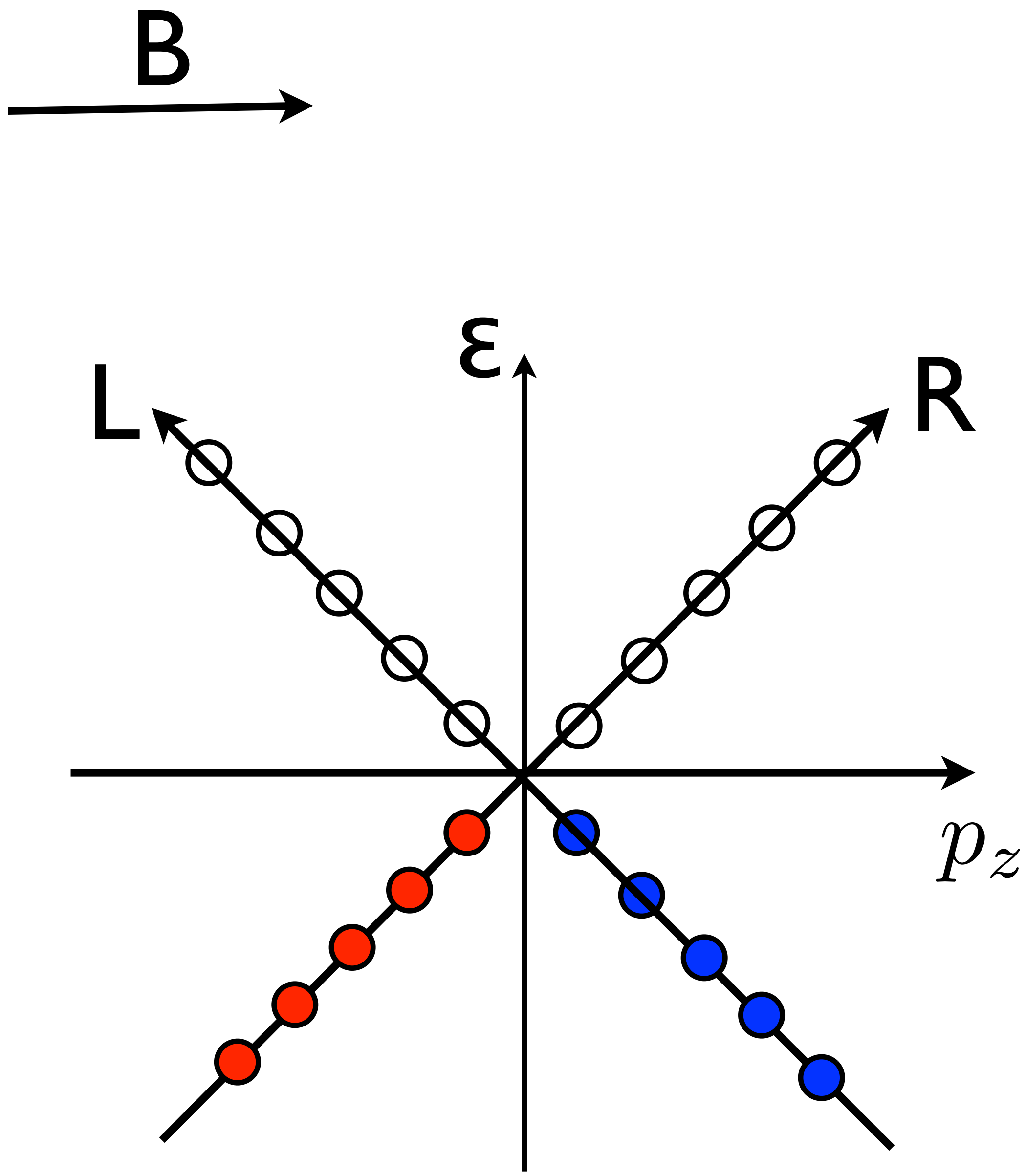
Massless fermion in a magnetic field



$$E^2 = p_z^2 + 2nB$$

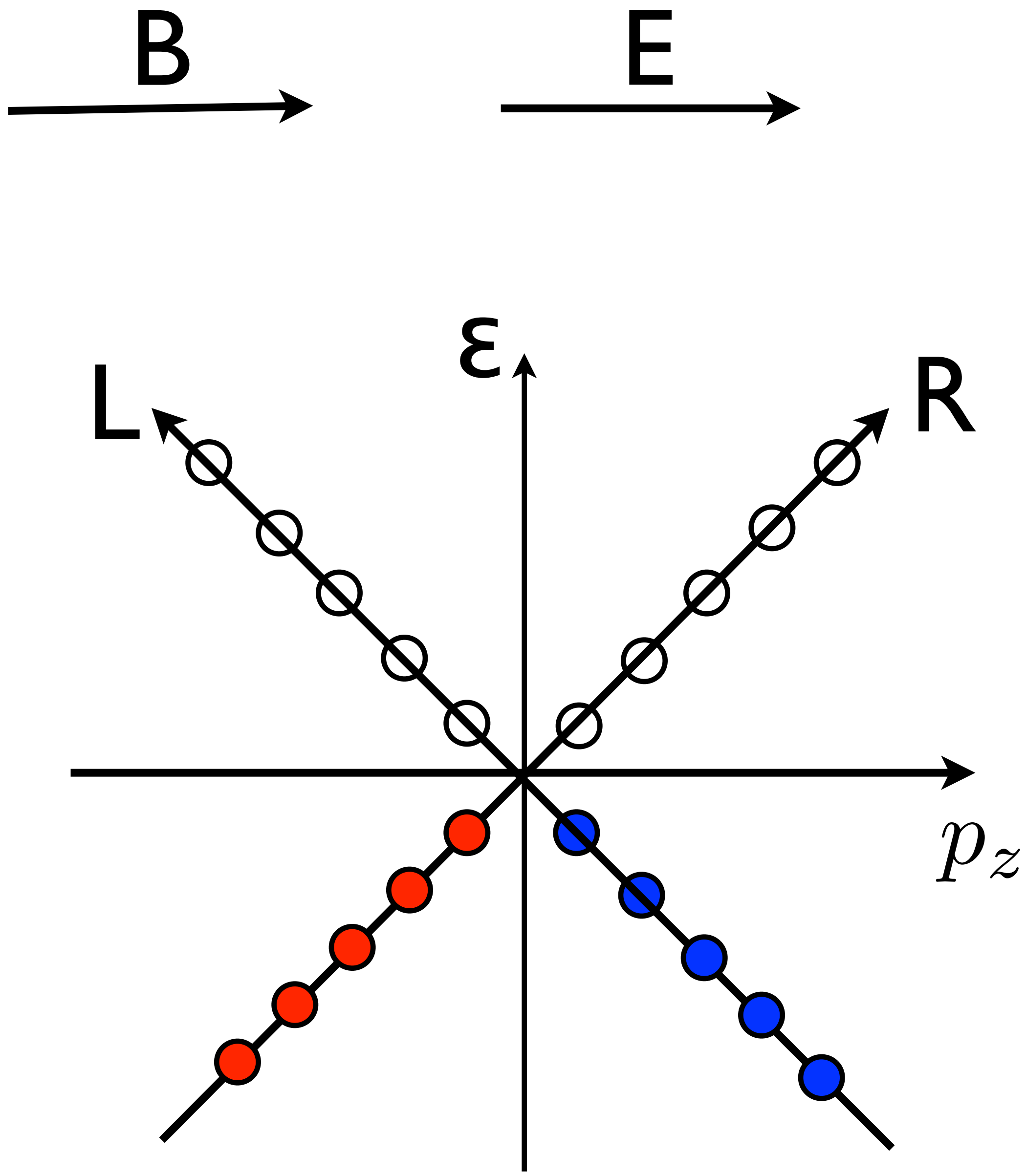
Anomalies

Turn on electric field for some duration of time



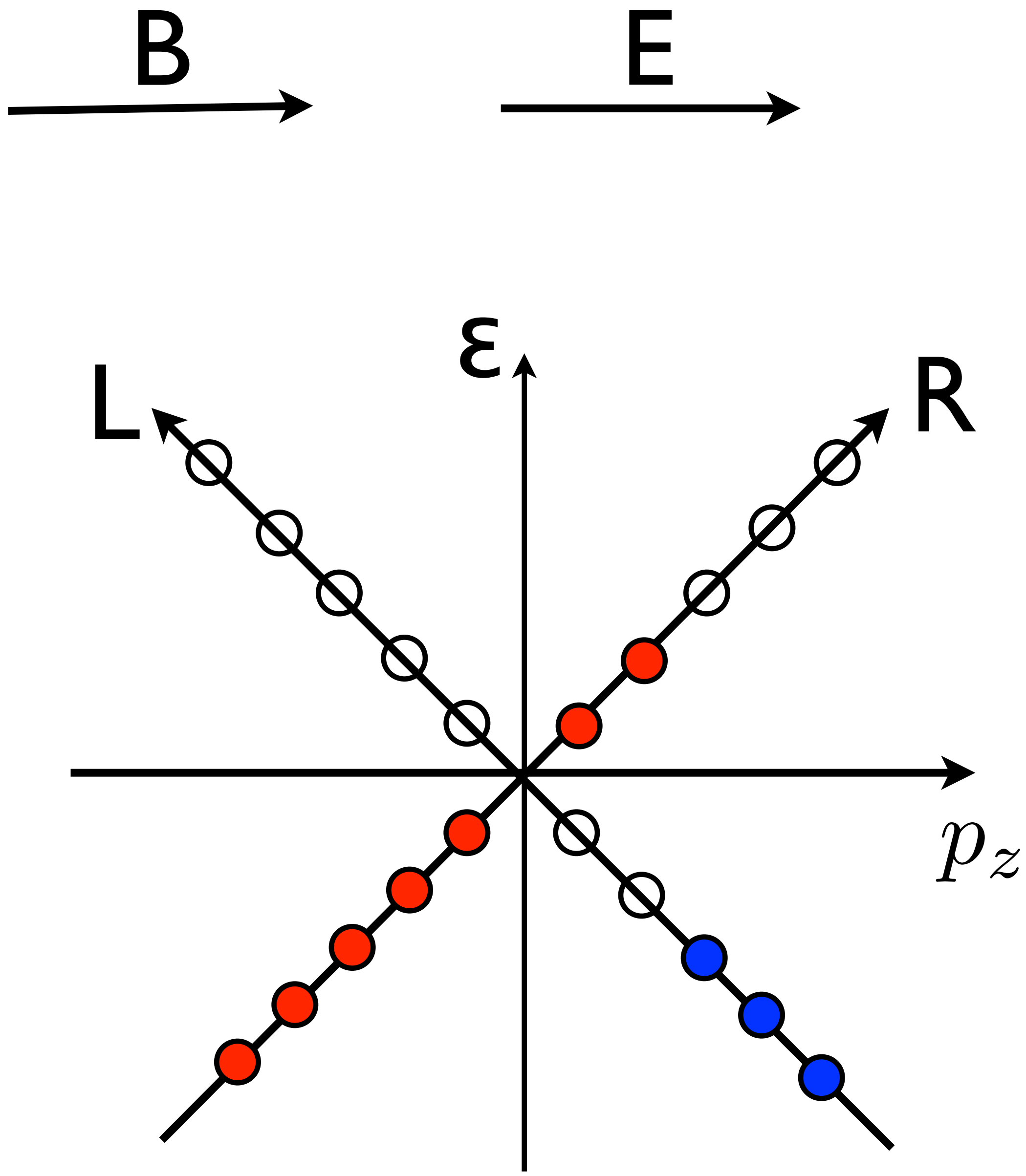
Anomalies

Turn on electric field for some duration of time



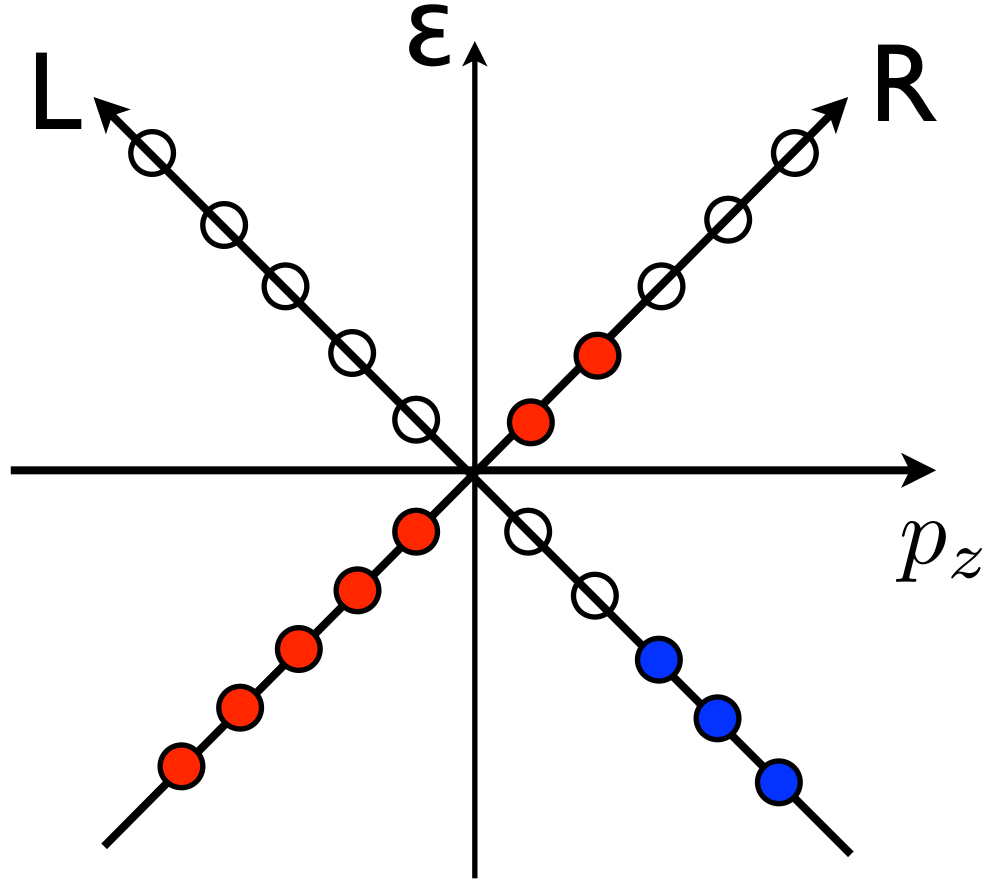
Anomalies

Turn on electric field for some duration of time



Anomalies

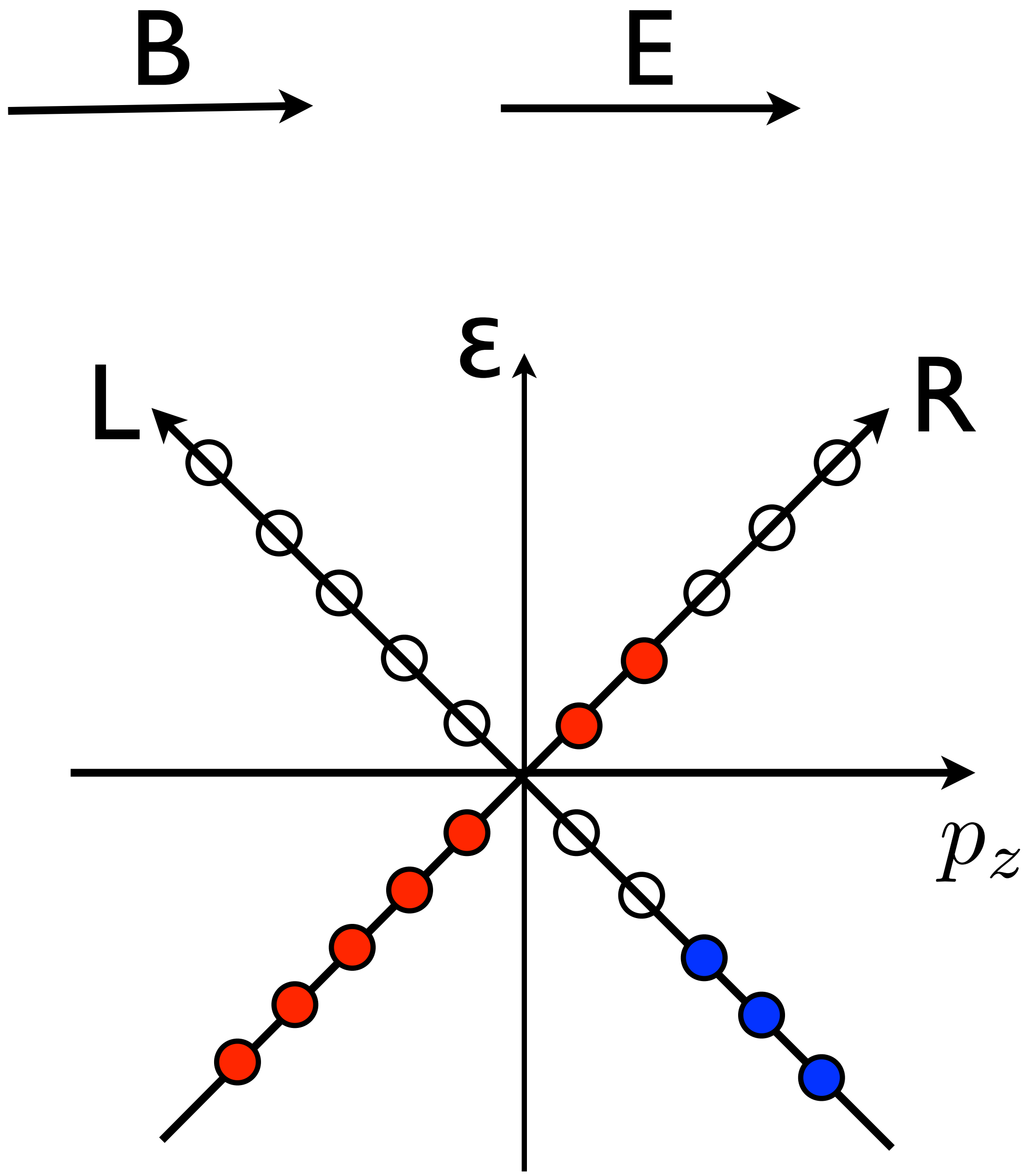
Turn on electric field for some duration of time



$$\frac{d}{dt}(N_R - N_L) \sim E \cdot B$$

Anomalies

Turn on electric field for some duration of time



$$\frac{d}{dt}(N_R - N_L) \sim E \cdot B \qquad \partial_\mu j^{5\mu} = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

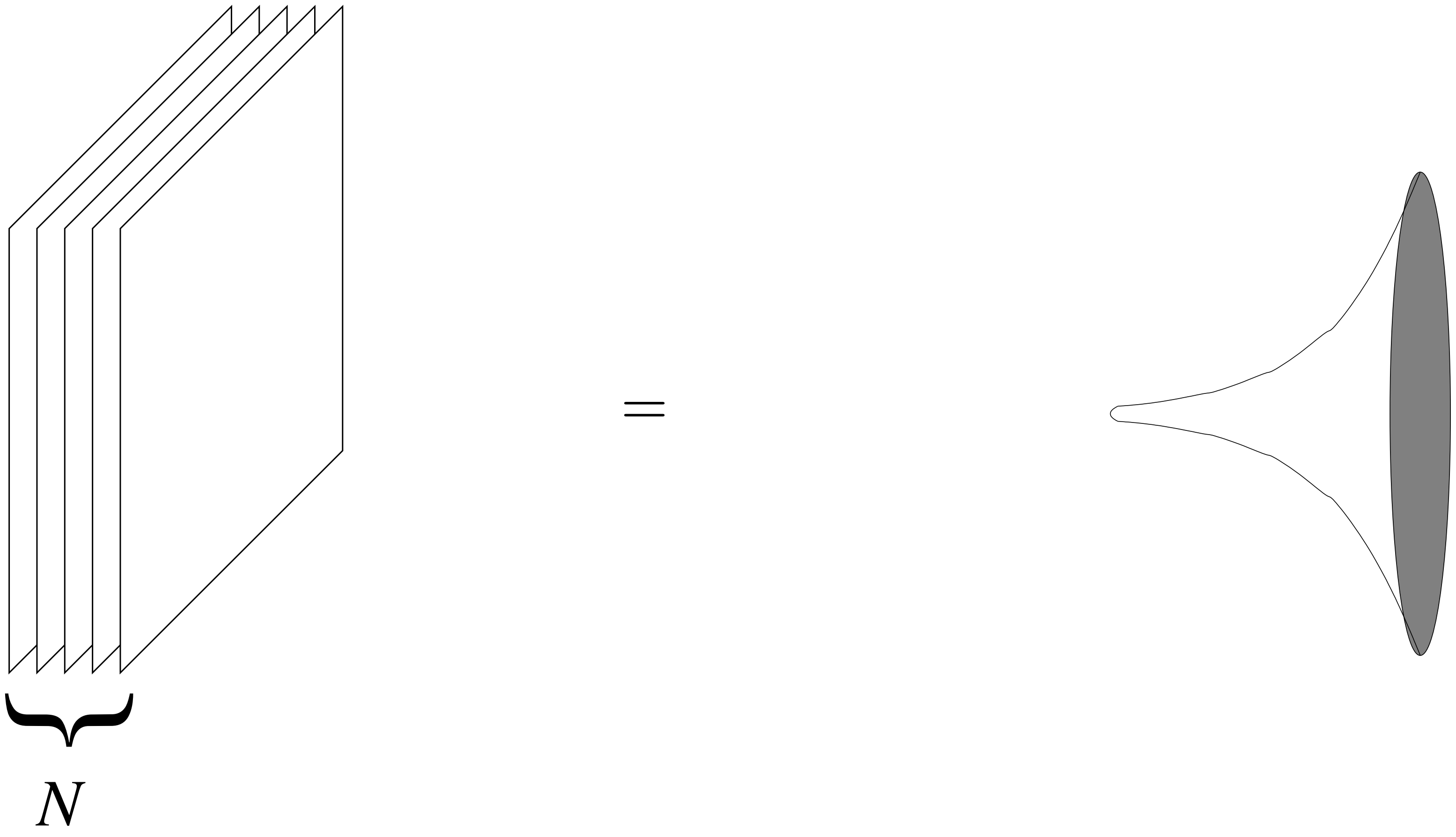
Anomalies and hydrodynamics

- A full understanding of anomalies in quantum field theory was achieved by 1980s
- But a possible connection between anomalies and hydrodynamics gone mostly unnoticed

  before a convenient technique for combining them arises: gauge-gravity duality

Gauge/gravity duality (“holography”)

Maldacena (1997): duality between QFT and string theory



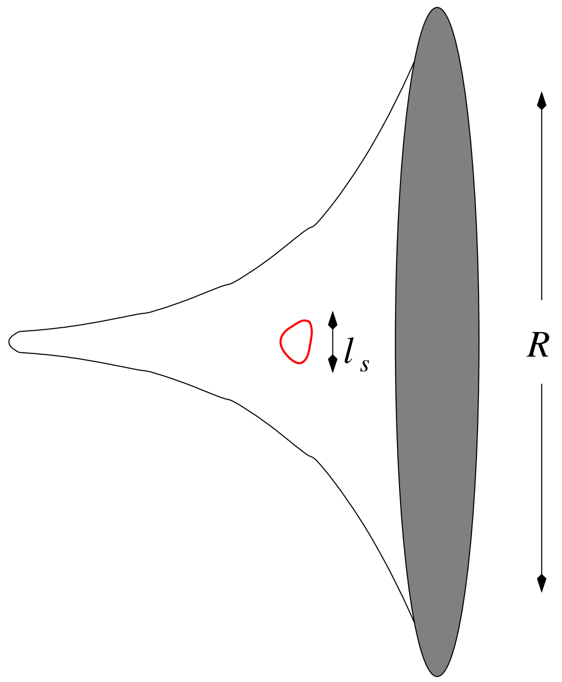
N=4 super Yang-Mills theory

string theory in AdS$_5$xS^5 space

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

Duality as a tool for QFT

- Gauge/gravity duality is particularly useful in the strong coupling regime of QFT



$$g^2 N_c = \frac{R^4}{\ell_s^4}$$

g^2N_c >>1: string theory becomes gravity

Difficult regime in field theory = easy in string theory

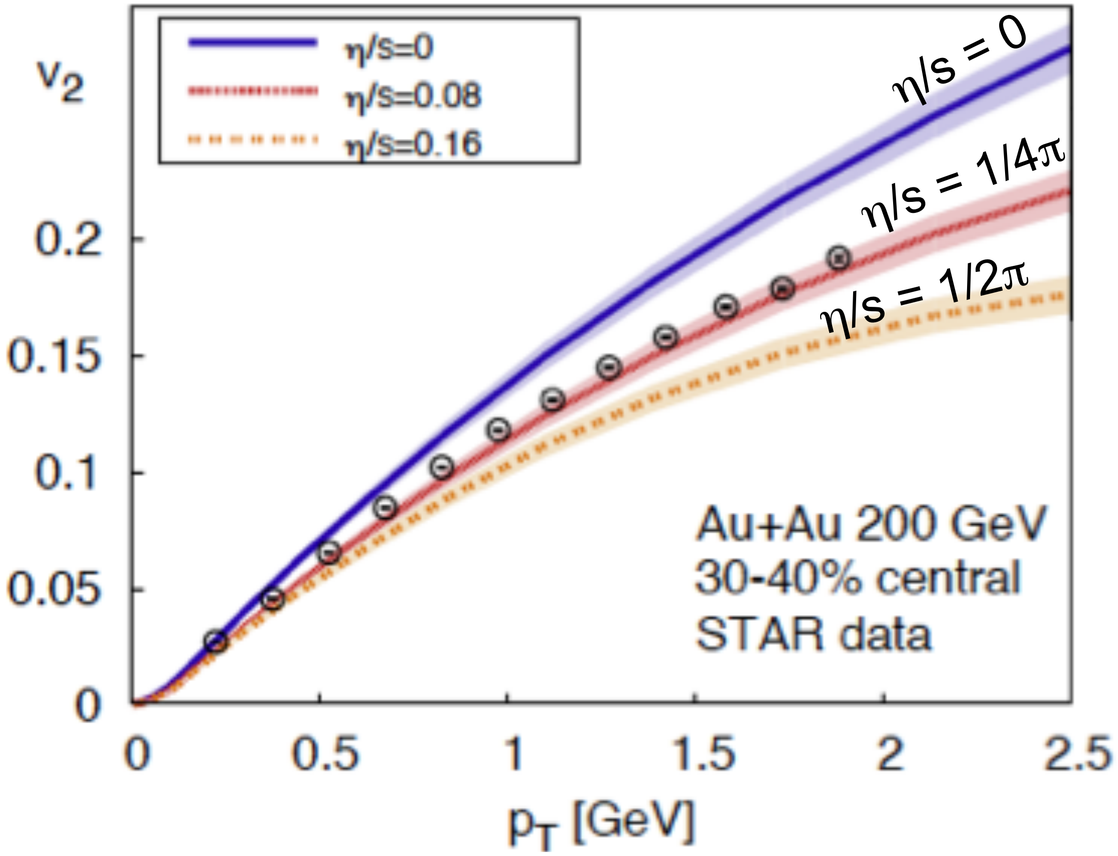
Gauge-gravity duality at finite temperature

- Soon gauge/gravity duality was generalized to finite temperature
- Quark-gluon plasma (in N=4 SYM theory) = black hole in AdS space
- Entropy of the quark gluon plasma = entropy of black hole

Hydrodynamics from BHs

- Around 2001 connection between black hole physics and hydrodynamics was found Kovtun, Policastro, Starinets, Son...
- Viscosity of QGP ~ absorption cross section of gravitational waves by black hole
- Universal ratio shear viscosity/ entropy density: $\eta/s=1/4\pi$
- Surprisingly close to observed value at RHIC

Viscosity of the quark gluon plasma



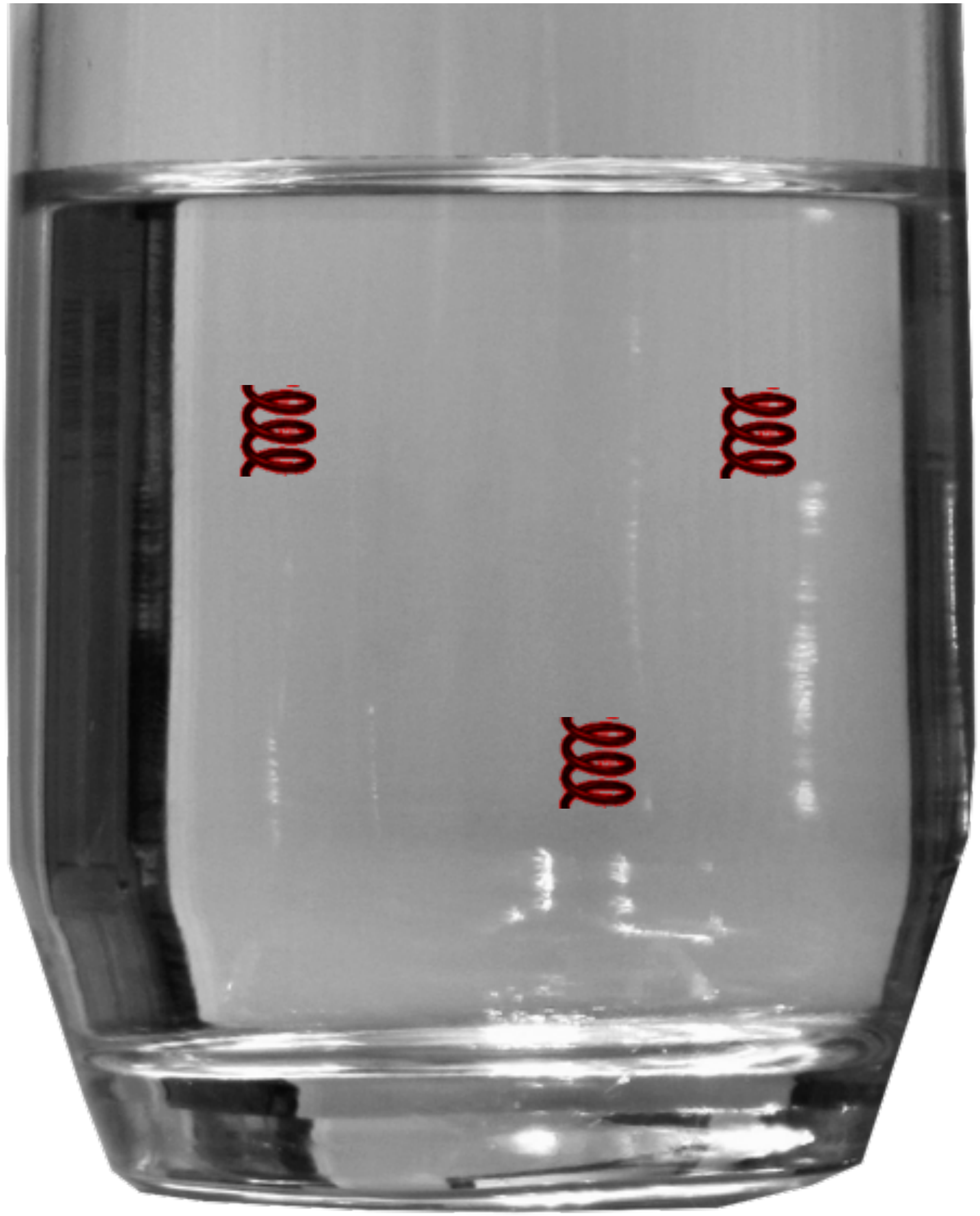
- Not only viscosity: the full theory hydrodynamics “emerges” from black hole physics
- Einstein equation -> Navier Stokes equation
- Furthermore, it has allowed investigation of liquids with chiral anomalies

Hydrodynamics with chirality?

Hydrodynamics with chirality?

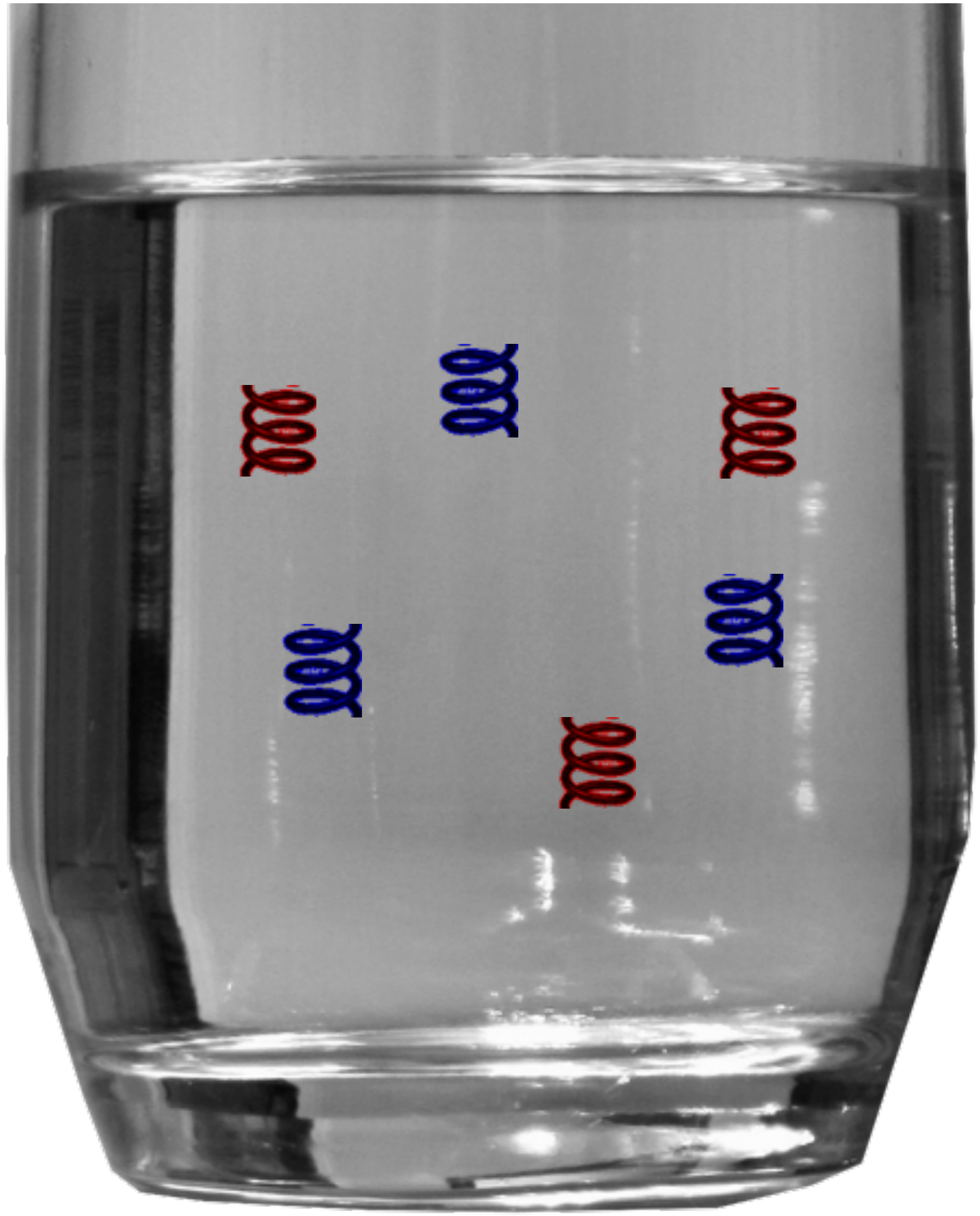
Hydrodynamics with chirality?

Hydrodynamics with chirality?

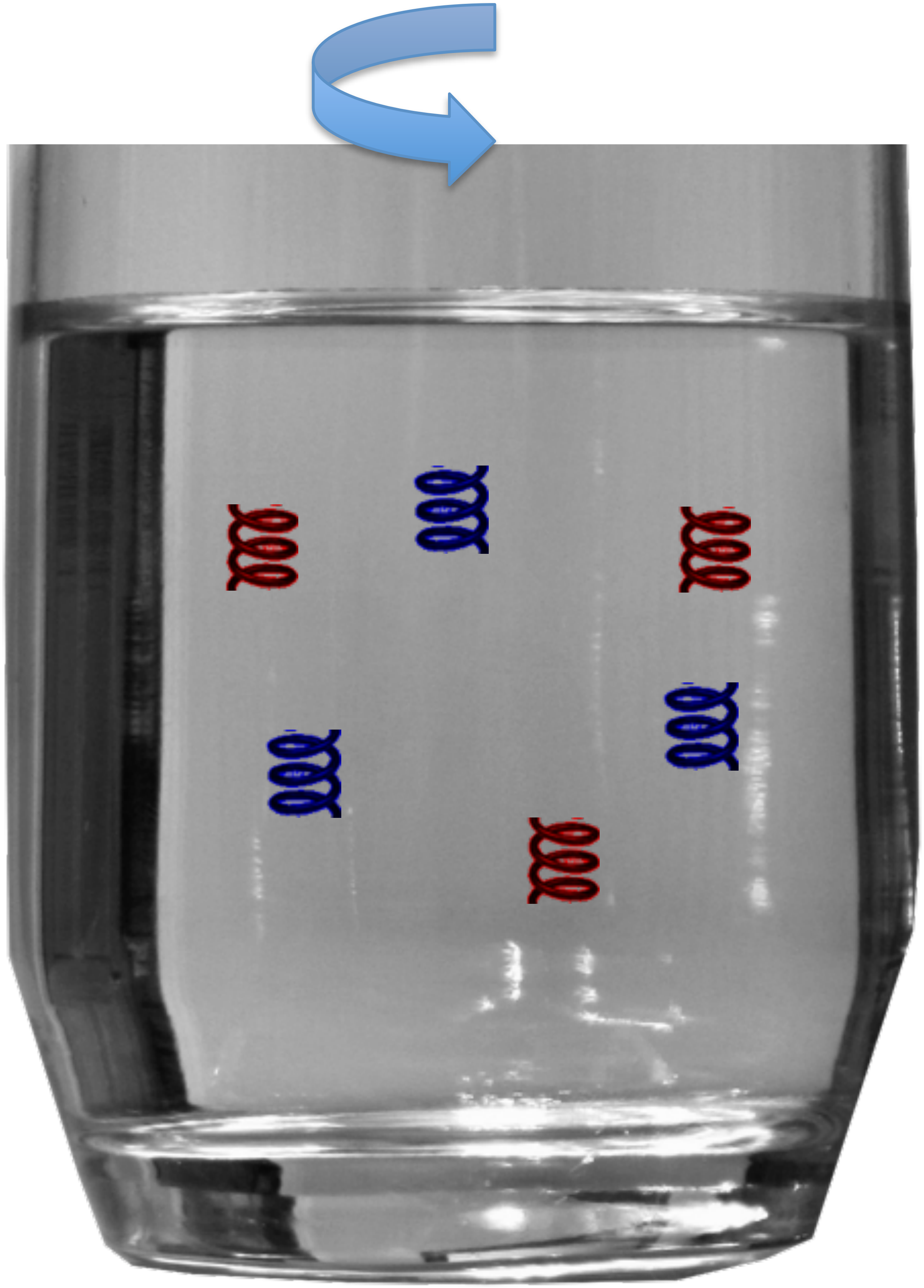


Hydrodynamics with chirality?

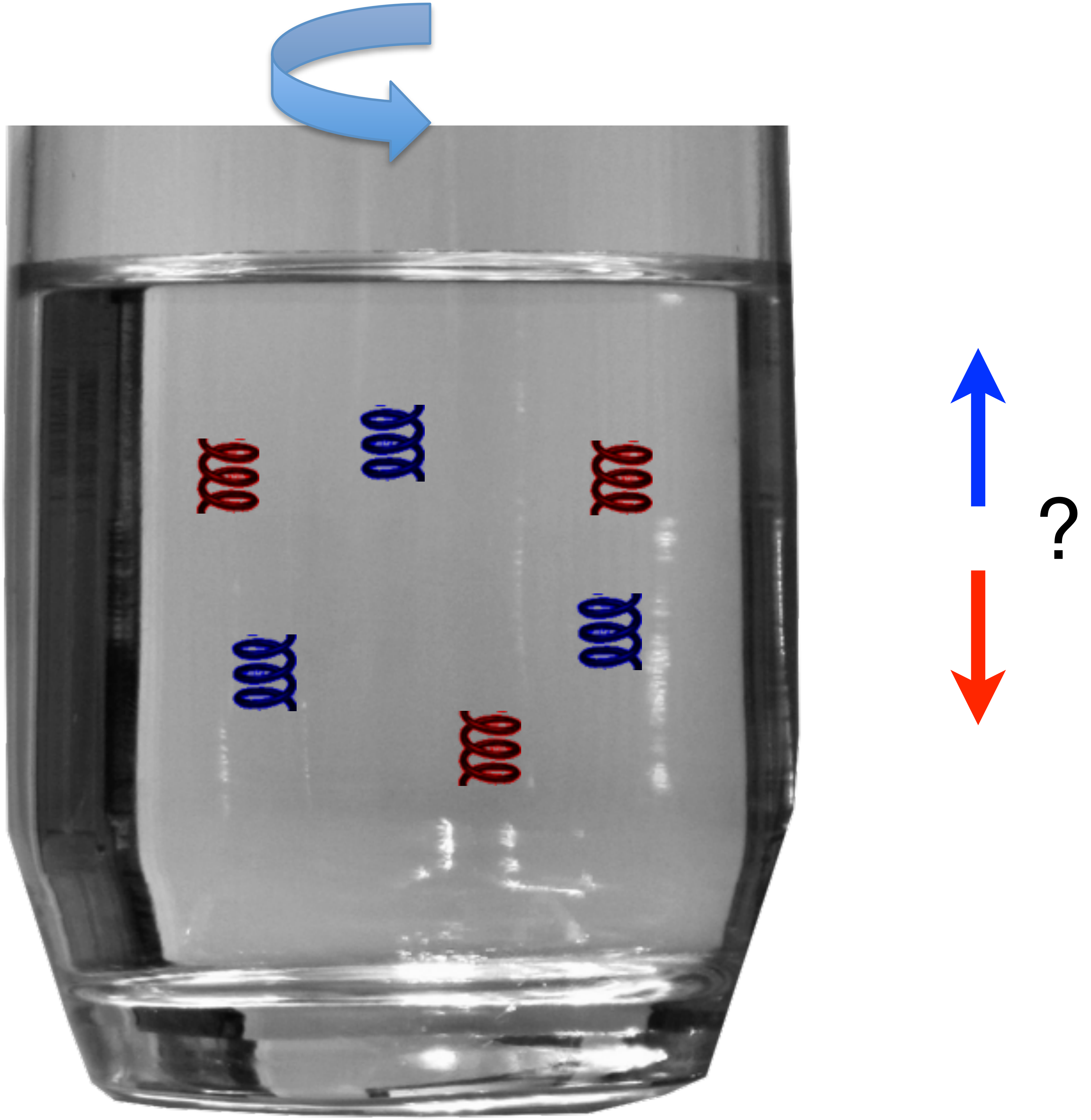
Hydrodynamics with chirality?



Hydrodynamics with chirality?



Hydrodynamics with chirality?



5D model of fluid with triangle anomaly

$$S = \frac{1}{8\pi G} \int d^5x \sqrt{-g} \left(R - 12 - \frac{1}{4} F_{AB}^2 + \frac{4\kappa}{3} \epsilon^{LABCD} A_L F_{AB} F_{CD} \right)$$

5D model of fluid with triangle anomaly

$$S = \frac{1}{8\pi G}\int d^5x\,\sqrt{-g}\left(R - 12 - \frac{1}{4}F_{AB}^2 + \frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB} F_{CD}\right)$$

- U(1) conserved charge modeled by a U(1) gauge field
- anomaly modeled by 5D Chern-Simons term
- rules to extract 4D physics from 5D equations

Two new effects

Erdmenger, Haack, Kaminski, Yarom

Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surówka

$$\vec{j}_5 = \rho_5 \vec{v} - D\vec{\nabla}\rho_5 + \xi_B \vec{B} + \xi \vec{\nabla} \times v$$

advection

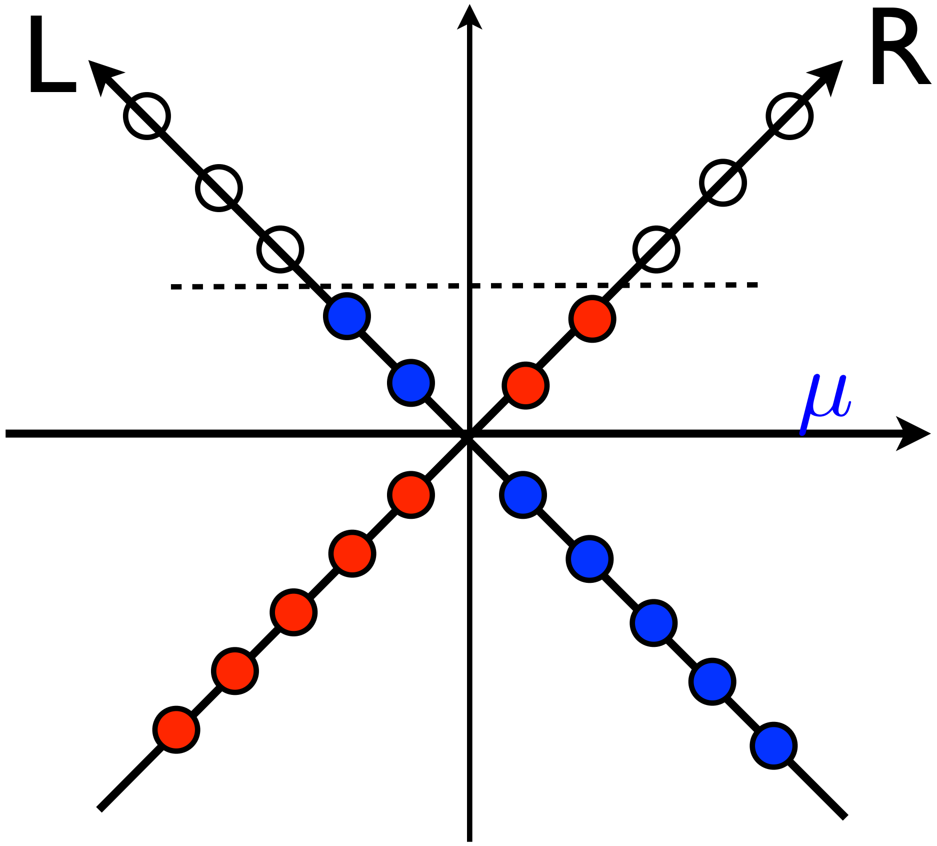
diffusion

Chiral magnetic effect (CME)

Chiral vortical effect (CVE)

Moreover, ξ and ξ_B completely determined by anomaly (required by 2nd law of thermodynamics: DTS, Surówka)

CME for free fermions

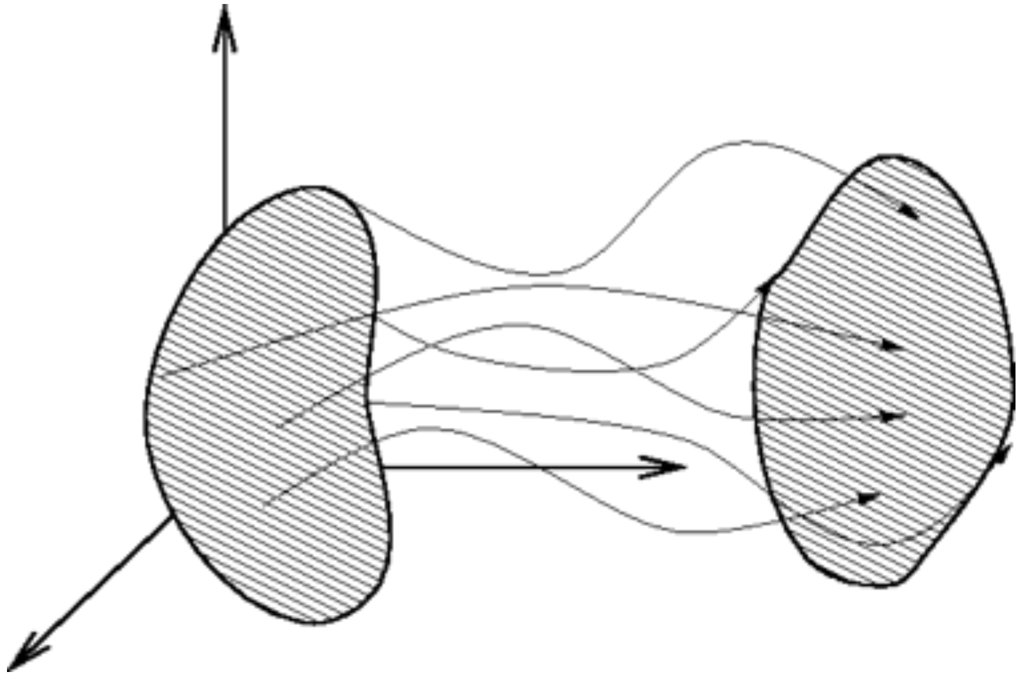


$$\vec{j}_5 = \frac{e^2}{2\pi^2}\mu\vec{B}$$

Role of collisions?

- The previous explanation of the CME relies on quantization of orbit
- This does not explain the CME when fermion has finite mean free path
- to see if CME is universal we turn to kinetic theory (Boltzmann equation)

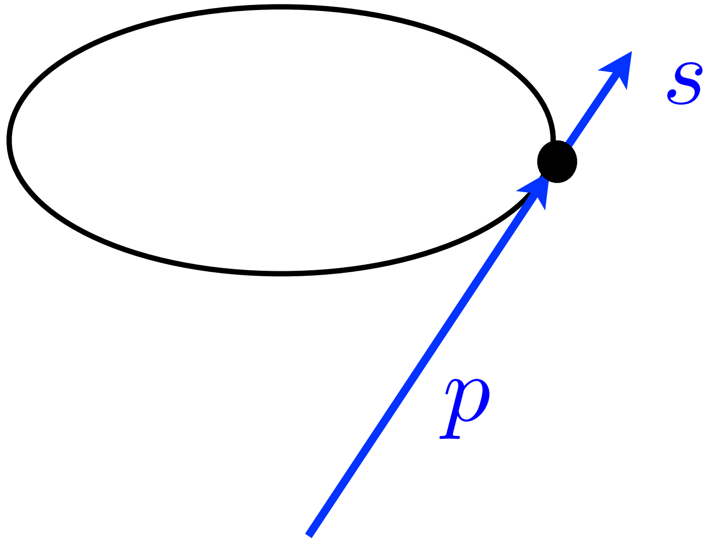
Can kinetic theory reproduce anomalies?



- In kinetic theory one follows the evolutions of particles in phase space
- By definition the particle number is conserved
- How can one get nonconservation?

Berry curvature in momentum space

- Spin and orbital motions are locked
- Momentum change → Berry phase
- modifies the equation of motion



Semiclassical equation

Chang, Niu

$$\dot{\mathbf{x}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{\Omega}$$

$$\dot{\mathbf{p}} = \mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}$$

right-handed

$$\mathbf{\Omega} = \pm \frac{\mathbf{p}}{2|\mathbf{p}|^3}$$

left-handed

Magnetic monopole

in momentum space

$$\mathbf{\Omega} = \mathbf{\nabla}_{\mathbf{p}} \times \mathcal{A}(\mathbf{p})$$

Berry curvature

Berry phase

Hamiltonian interpretation

$$\dot{\xi}^a = \{H,\, \xi^a\} \qquad \{\xi^a,\, \xi^b\} = \omega^{ab}$$

$$\{p_i,\, p_j\} = -\frac{\epsilon_{ijk} B_k}{1 + \mathbf{B} \cdot \boldsymbol{\Omega}} \qquad \{x_i,\, x_j\} = \frac{\epsilon_{ijk} \Omega_k}{1 + \mathbf{B} \cdot \boldsymbol{\Omega}}$$

$$\{p_i,\, x_j\} = \frac{\delta_{ij} + \Omega_i B_j}{1 + \mathbf{B} \cdot \boldsymbol{\Omega}}$$

Anomalies from Berry curvature

- Solving the equation of motion for a single particle

$$\dot{\mathbf{x}} = (1+\mathbf{\Omega}\cdot\mathbf{B})^{-1}\left[\mathbf{v}+\mathbf{E}\times\mathbf{\Omega}+(\mathbf{\Omega}\cdot\mathbf{v})\mathbf{B}\right]$$

$$\dot{\mathbf{p}} = (1+\mathbf{\Omega}\cdot\mathbf{B})^{-1}\left[\mathbf{E}+\mathbf{v}\times\mathbf{B}+(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}\right]$$

from this one derive the Liouville equation

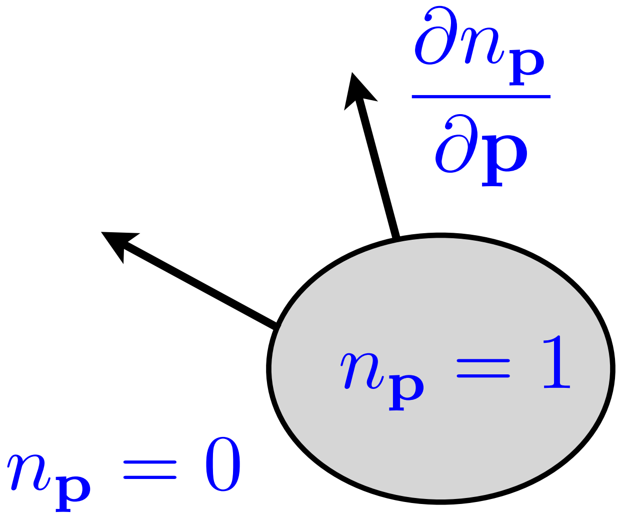
$$(1+\mathbf{\Omega}\cdot\mathbf{B})\frac{\partial n_{\mathbf{p}}}{\partial t}+\cdots+(\mathbf{E}\cdot\mathbf{B})\left(\mathbf{\Omega}\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\right)=0$$

Anomaly from kinetic theory

DTS, N. Yamamoto

$$n(t,\mathbf{x}) = \int \frac{d\mathbf{p}}{(2\pi)^3}\,(1+\mathbf{\Omega}\cdot\mathbf{B})\,n_{\mathbf{p}}(t,\mathbf{x})$$

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{j} = -(\mathbf{E}\cdot\mathbf{B})\int \frac{d\mathbf{p}}{(2\pi)^3}\,\mathbf{\Omega}\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}$$



flux of Ω through
the Fermi sphere

$$\partial_\mu j^\mu = \frac{1}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$$

anomaly is reproduced

Consequences of kinetic theory

- Anomalies exist in the presence of collisions
- Chiral magnetic effect (CME) also exists in the presence of collisions: no Landau level needed.

Anomaly in solid state physics

- Weyl or Dirac semimetals: solids in which low-energy electrons described by massless Dirac equations
- Dirac cones instead of Fermi surfaces
- Anomaly + chiral magnetic effect → negative magnetoresistance (DTS, Spivak)

Magnetoresitance from anomaly

- Consider a Weyl semimetal in external E and B fields

$$\frac{\partial n_5}{\partial t} = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B} - \frac{n_5}{\tau} \qquad\qquad n_5 \to \frac{e^2}{2\pi^2}EB\tau$$

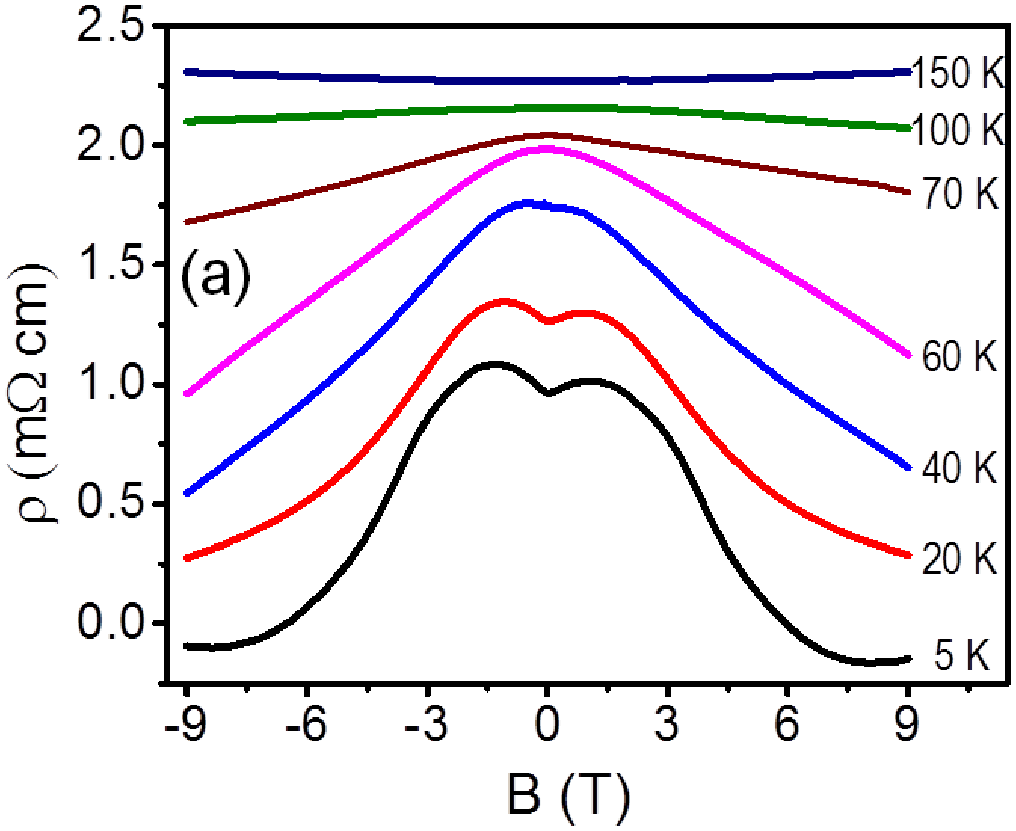
$$\mu_5 \to \frac{e^2}{2\pi^2\chi}EB\tau$$

$$j_5 = \frac{1}{2\pi^2}\mu_5 B = \underbrace{\frac{e^2}{(2\pi^2)^2\chi}B^2}\ E$$

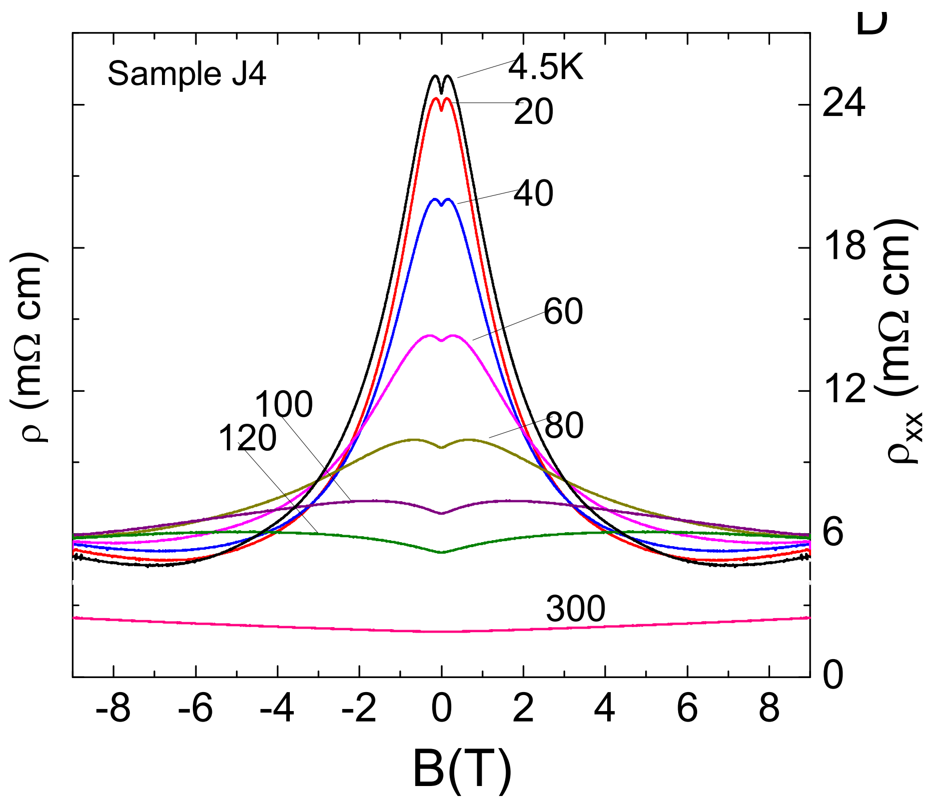
positive contrib to conductivity
negative magnetoresistance

Experimental observation of anomaly in solids

- Weyl semimetals found: Na_3Bi, $TaAs$, Ca_3As_2, $ZrTe_5$



1412.6543 ($ZrTe_5$)



1503.08179 (Na_3Bi)

Conclusion

- Effects of anomaly in hydrodynamics: first discovered within gauge/gravity duality
- Condensed matter physics: parallel developments: Berry curvature on Fermi surface
- Observed negative magnetoresistance of Weyl semimetal, suggested as a signature of anomaly