Vortices and string/gauge theory correspondence

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Low energy limit of string theory

- Closed strings
- Open strings

AdS/CFT correspondence

- Branes and their sources
- Gauge theories on Dp brane
- AdS/CFT conjecture

Non-abelian vortices

- Relation to Toda theories
  - Abelian case
  - Non-abelian generalization

Final comments

String(Gravity)/Gauge theory duality

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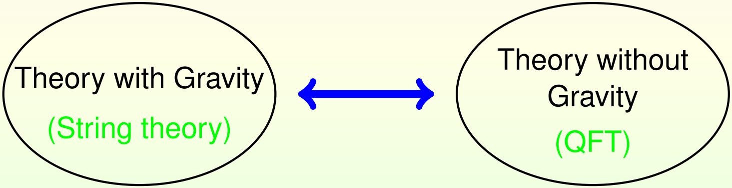

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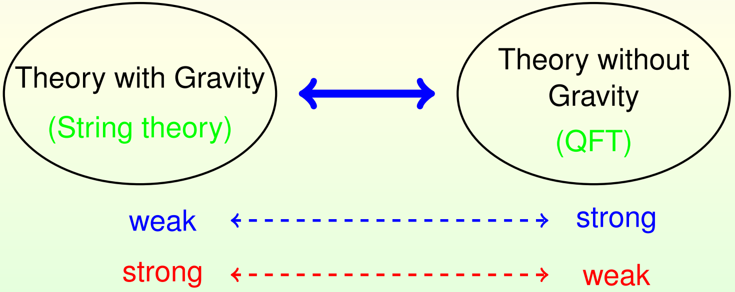

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- Φ-coupling

$$S_\Phi = -\frac{1}{4\pi}\int d^2\sigma\sqrt{g}\,\Phi(X^\mu)R^{(2)}.$$

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- The energy-momentum tensor is:

$$T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu - \frac{1}{2}\eta_{\alpha\beta}\partial^\rho X^\sigma \partial_\rho X_\sigma = 0.$$

- The conservation conditions

$$\partial^\alpha T_{\alpha\beta} = 0, \quad , T^\alpha{}_\alpha = 0.$$

Define $\Theta(z) = T^{00} + T^{01}$ and $\bar{\Theta}(\bar{z}) = T^{00} - T^{01}$. The algebra closed by $\Theta(z)$ is the so-called Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}, \quad \Theta(z) = \sum_n \frac{L_n}{z^{n+2}}.$$

- Requiring above symmetries of the σ-model string action, one ends up with the conditions ensuring the vanishing of the corresponding β-functions:

$$\beta^G_{\mu\nu}: \quad R_{\mu\nu} - \frac{1}{4} H_{\mu\sigma\lambda} H_\nu^{\sigma\lambda} + 3 D_\mu \partial_\nu \Phi = 0,$$

$$\beta^B_{\mu\nu}: \quad -\frac{1}{2} D^\sigma H_{\sigma\mu\nu} + H_{\sigma\mu\nu} D^\sigma \Phi = 0,$$

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The effective 10d action from closed strings

$$S = \frac{1}{2\kappa} \int d^{10}X \sqrt{|G|} e^{-2\Phi} \left(R + 4(\partial\Phi)^2 - \frac{1}{12} H^2\right)$$

√ should be understood as a universal one, i.e. any superstring background must satisfy the above equations
√ the equations of motion following from this action coincide with the conditions ensuring vanishing of the β-functions.

Open strings

- adding open string sector
  In the case of boundaries of the world sheet one can write the action as

$$S = \frac{1}{1\pi\alpha'}\left(\int_\Sigma d^2\sigma \frac{1}{2}(\partial_\alpha X_\mu \partial^\alpha X^\mu \right.$$
$$\left. + \varepsilon^{\alpha\beta} B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu) + \int_{\partial\Sigma} d\sigma^1 A_\mu \partial_1 X^\mu \right)$$

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- boundary conditions:

$$(\mathbf{NN}):\ \partial_\sigma X_{|\sigma=0,\pi} = 0 \implies$$

$$X(\tau,\sigma) = q + 2\alpha' p\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\cos n\sigma e^{-in\tau}$$

$$(\mathbf{DD}): \; X_{|\sigma=0} = q_i, \; X_{|\sigma=\pi} = q_f \implies$$

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- expanding the above action we get model dependent terms

$$S^{II}_{open} = -\frac{1}{2\kappa^2}\int d^{10}x \sum_p \frac{1}{2(p+2)!}F^2_{p+2},$$

where F_{p+2} is the field strength of a $p+1$ form gauge field. The couplings A_μ get promoted to gauge fields on the subspace where string endpoints live.

Brane degrees of freedom

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- Dirichlet boundary conditions: determine a subspace called D-brane
- Stack of N parallel D branes

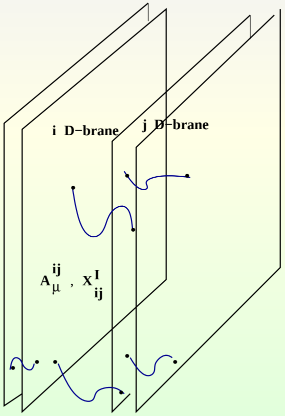

Figure: The degrees of freedom N parallel D-branes.

Effective open string action sources

To complete this discussion we list the various sources that arise in string theory in Table 1.

type	form	electric source	magnetic source
IIA	$F_{[2]}$	D0	D6
IIA	$F_{[4]}$	D2	D4
IIB	$F_{[1]}$	D(-1)	D7
IIB	$F_{[3]}$	D1	D5
IIB	$F_{[5]}$	D3	D3

Table: Field strengths and sources in type II strings

The main conclusion one can draw is that the $(p+1)$-dimensional world-volume serves as source of $(p+2)$-form field $F_{[p+2]}$.

Gauge theories on Dp brane

$$S_{Dp} = S_{DBI} + S_{WZ} ,$$

$$S_{DBI} = -T_p \int_{Dp} d^{p+1}\xi \, \mathsf{STr}\sqrt{-\det\left(\mathcal{P}_{ab}[G_{\mu\nu} + B_{\mu\nu}] + 2\pi\alpha' F_{ab}\right)} ,$$

$$S_{WZ} = T_p \int_{Dp} \sum_i \mathsf{STr}\, \mathcal{P}[C_{(i)}] \wedge e^{\mathcal{P}[B] + 2\pi\alpha' F} ,$$

where T_p is the brane tension.

$$T_p = \frac{1}{g_S\,(2\pi)^p \alpha'^{(p+1)/2}} .$$

The pull-back of the background metric $G_{\mu\nu}$ and Kalb-Ramond-field $B_{\mu\nu}$ is denoted by $\mathcal{P}$. STr is the symmetrized trace.

One can expand the brane action and obtain the leading contribution

$$S = T_p \int d^{p+1}\xi \sqrt{g} e^{-\phi} (2\pi\alpha')^2 \frac{1}{2} \mathrm{tr} \left(F_{\alpha\beta} F^{\alpha\beta} \right) + T_p \int \sum_r C^{(r)} \wedge \mathrm{tr} e^{2\pi\alpha' F} + \cdots , \quad (1)$$

where we have not written terms involving fermions and scalars. This action is the sum of

- a Yang-Mills term,
- a Wess-Zumino term,
- an infinite number of corrections at higher orders in α' indicated by $\cdots$ in (1)

- The string endpoints on the same D_p branes transform under adjoint representation of the gauge group. The large number of these branes gives the background geometry.
- The string endpoints ending on different $D_p - D_{p'}$ branes transform in the fundamental. This is the way we introduce flavors in the theory.

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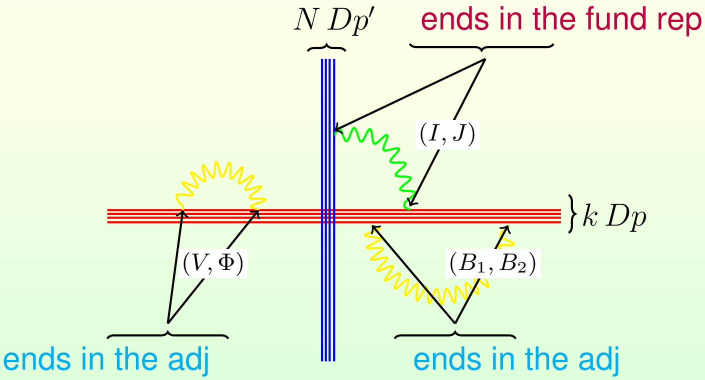

The solution for the $D3$ branes can be obtained under simple assumptions of Lorentz invariance, rotational symmetry in the transverse space and supersymmetry. It arises from solving the combined system

$$S = \frac{1}{2\kappa^2} \int\limits_M d^{10} \mathcal{L}_{(10)} + \int\limits_\Sigma d^4 z \mathcal{L}_{D3} + S_{int}. \qquad (2)$$

The explicit form of the $D3$-brane solution is

$$D3\text{-brane} = \begin{cases} ds^2 = H^{-\frac{1}{2}} dx^2_{(4)} + H^{\frac{1}{2}} \left(dy^2 + y^2 d\Omega^2_{(5)} \right), \\ H(y) = 1 + \left(\frac{R}{y} \right)^4, \\ F_{(5)} = d^4x \wedge dH^{-1} + \star d^4x \wedge dH^{-1}, \\ e^{\Phi} = g_s, \quad R^4 = 4\pi g_s N_c (\alpha')^2. \end{cases}$$

We note that all the elementary brane solutions for small y have the warp factor which on the near horizon limit determines the geometry of the background.

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- Defining $\varphi = x_0^{\Delta - d}\phi$ one can state the correspondence

$$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle = \frac{\delta^n S^{ren}_{Grav}[\phi]}{\delta\varphi(x_1)\cdots\delta\varphi(x_n)}_{|\varphi=0}.$$

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- There are cases where the SUSY is broken down to $\mathcal{N} = 1$ and those theories are much closer to Quantum ChromoDynamics (QCD).
- There are cases where holography is established for lower dimensions - $AdS_4/CFT_3,\ AdS_3/CFT_2$.

• We will be interested mainly in AdS_4/CFT_3, or so-called ABJM theory. The ABJM model represents the IR limit of the theory of N coincident M2-branes moving in $\mathbb{R}_{2,1} \times \mathbb{C}^4/\mathbb{Z}_k$ background. It is a $\mathcal{N} = 6$ supersymmetric $U(N)_k \times U(N)_{-k}$ CS gauge theory, with bi-fundamental scalars, fermions in the fundamental of the $SU(4)_R$.
• Consistent *Abelization* ansätz in bottom-up approach to ABJM $\Rightarrow$ Abelian vortices and Toda system[1].

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Non-abelian vortices

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Non-abelian vortices

The Field content:

- Gauge fields A_μ, $\mu = 0, 1, \ldots, d-1$
- Two Higgs fields: $N \times N$ matrices H^1, H^2
- Adjoint scalars coming from dimensional reduction ϕ^r, $r = 1, \ldots, 6-d$.

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The Lagrangian density. We give below the lagrangian in 6d. The other cases can be obtained by trivial dimensional reduction[2].

$$\mathcal{L}_{6d} = \mathrm{tr}\left[-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + D^\mu H^i (D_\mu H^i)^\dagger\right] - V, \qquad (3)$$

where (we include the covariant derivative for eventual scalars ϕ^r coming from the dimensional reduction)

$$D_\mu \phi^r = \partial_\mu \phi^r + i[A_\mu, \phi^r], \quad D_\mu H = (\partial_\mu + iA_\mu)H,$$
$$F_{\mu\nu} = -i[D_\mu, D_\nu]. \qquad (4)$$

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The potential V is given by

$$V = \frac{g^2}{4}\mathrm{tr}\left[\left(H^1H^{1\dagger} - H^2H^{2\dagger} - c\mathbf{1}\right)^2 + 4H^2H^{1\dagger}H^1H^{2\dagger}\right]. \quad (5)$$

• Dimensional reduction ⇒ additional terms:

$$\frac{1}{2g^2}[\phi^r, \phi^s]^2 + (H^2H^{2\dagger} + H^1H^{1\dagger})\phi^r\phi^r. \quad (6)$$

The triplet of Fayet-Iliopoulos parameters is chosen to the third direction: $(0, 0, c)$.

• The SUSY requires vanishing of the vacuum energy. This means that the following equations must be satisfied

$$H^1H^{1\dagger} - H^2H^{2\dagger} = c\mathbf{1} \quad (7)$$

$$H^2H^{1\dagger} = 0. \quad (8)$$

The additional terms coming from the dimensional reduction force $\phi^r = 0$, $r = 1, 2, 3$.

Symmetries:

- Gauge symmetry: $U(N_C)$
- Flavor symmetry: $SU(N_F)$.

The Higgs fields: H^1 transforms as $(\mathbf{N}, \bar{\mathbf{N}})$ while H^2 transforms as $(\bar{\mathbf{N}}, \mathbf{N})$. The explicit matrix structure of these field is as follows:

$$H^1 \equiv H^{1a}_{\ \ i}, \; H^2 \equiv H^{2i}_{\ \ a}, \quad a = 1, \ldots, N_C, \quad , i = 1, \ldots, N_F$$

where a are $U(N_C)$ index and i is $SU(N_F)$ index. *In either dimension, the vacuum is the so-called color-flavor locking phase, i.e. the ground state develop a gap.* This phase is characterized by

$$H^1 = \sqrt{c}\mathbf{1}_N, \quad \& \quad H^2 = 0. \tag{9}$$

In this phase the symmetry is broken

$$U(N_C) \times SU(N_F) \xrightarrow{c>0} SU(N)_{(C+F)}. \tag{10}$$

The symmetry is further broken by the presence of vortex:

$$SU(N)_{(C+F)} \xrightarrow{\text{vortex}} U(1)^N.$$

Vortex equations. To obtain the vortex equations we use the Bogomol'nyi completion trick. Namely, we write that energy (hamiltonian density) as complete square

$$\mathcal{E} = \mathrm{tr}\left[\frac{1}{g^2}\Big(B_3 + \frac{g^2}{2}(c\mathbf{1}_N - HH^\dagger)\Big)^2 + (D_1H + iD_2H)\cdot \right.$$
$$\left. \cdot(D_1H + iD_2H)^\dagger\right] + \mathrm{tr}\left[-cB_3 + 2i\partial_{[1}HD_{2]}H^\dagger\right], \quad (11)$$

where we set $H^1 = H$, $F_{12} = B_3$ is the third component of the magnetic field. All others are vanishing in order to satisfy the vacuum conditions.
We conclude that the Bogomol'nyi bound is saturated iff

$$D_1H + iD_2H = 0, \quad (12)$$

$$F_{12} = \frac{g^2}{2}(HH^\dagger - c\mathbf{1}_N). \quad (13)$$

Tension: The tension of the vortex string measures the winding number k of the $U(1)$ part of the broken $U(N_C)$ gauge symmetry

$$T = -c \int dx \mathrm{tr} F_{12} = 2\pi c k, \quad k \in \mathbb{Z}_{\geq 0}. \tag{14}$$

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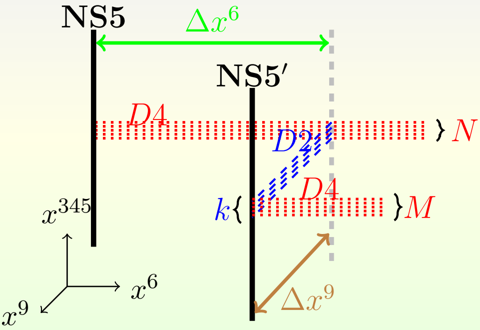
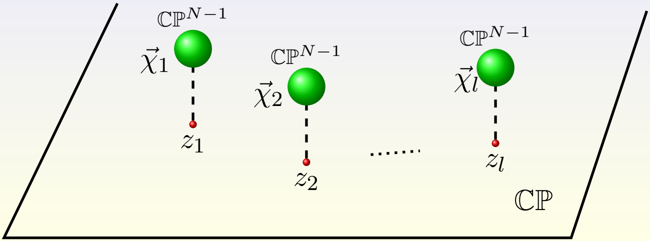


the gauge coupling g: $g = \frac{\Delta x^6 l_s}{2 g_s}$; FI parameter: $(c =) v^2 = \frac{\Delta x^9}{(2\pi)^3 g_s l_s^3}$

Figure: The brane construction of non-abelian vortices and the corresponding parameters (Hanany-Tong, Tong).

The moduli space for $k = 1$ vortices is:
$\mathcal{M}_{N,k=1} = \mathbb{C} \times \mathbb{CP}^{N-1}$, and for l copies of them looks like



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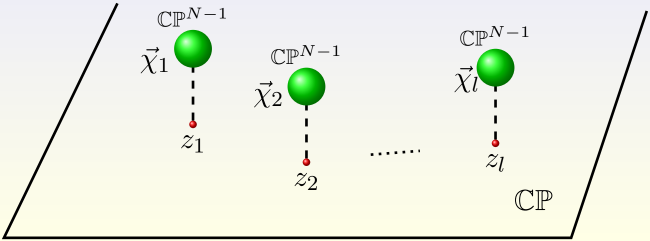


Figure: The moduli space of $k = 1$ vortex.

In the case of arbitrary k the moduli space is more complicated. For instance in the case of $N_C = N_F$

$$\mathcal{M}_{N,k} = \frac{\{H_0(z)\,|\,H_0(z) \in M_N,\ \deg(\det(H_0)) = k\}}{\{V(z)\,|\,\tilde{V}(z) \in M_N,\ \det\tilde{V}(z) = 1\}}, \quad (15)$$

where $M_N = N \times N$ *are matrices of polynomials of* z:

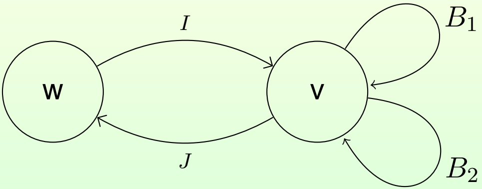
- One can start with the moduli space of instantons which can be described by the following quotient

$$\mathcal{M}_{k,N} = \{(B,I)\,|\,[B,B^\dagger] + II^\dagger = \zeta\mathbf{1}\}/U(k)$$
$$= \{(B_{\mathbb{C}}, I_{\mathbb{C}})\}/GL(k;\mathbb{C}), \qquad (16)$$

where we have defined the $k \times N$ matrix $(I = (I_{k_1}, I_{k_2}, \ldots, I_{k_N}))$ (each I_{k_i} is a k-column).

- Where and how $(B_{\mathbb{C}}, I_{\mathbb{C}})$ act $\Rightarrow$

$$B_{\mathbb{C}} \in \mathrm{Hom}(V,V); \quad B_{\mathbb{C}}:\ V \longrightarrow V \quad (\dim_{\mathbb{C}} V = k); \qquad (17)$$
$$I_{\mathbb{C}} \in \mathrm{Hom}(W,V); \quad I_{\mathbb{C}}:\ W \longrightarrow V \quad (\dim_{\mathbb{C}} W = N). \qquad (18)$$



The ADHM construction uses the following data:

- complex vector spaces V and W of dim k and N,
- $k \times k$ complex matrices B_1, B_2, a $k \times N$ complex matrix I and a $N \times k$ complex matrix J,
- a real moment map $\mu_r = [B_1, B_1^\dagger] + [B_2, B_2^\dagger] + II^\dagger - J^\dagger J$,
- a complex moment map $\mu_c = [B_1, B_2] + IJ$.

- Given B_1, B_2, I, J such that $\mu_r = \mu_c = \zeta \mathbf{1}$, an anti-self-dual instanton in a $SU(N)$ gauge theory with instanton number k can be constructed,
- All anti-self-dual instantons can be obtained in this way and are in one-to-one correspondence with solutions up to a $U(k)$ rotation which acts on each B_i in the adjoint representation and on I and J via the fundamental and antifundamental representations
- The metric on the moduli space of instantons is that inherited from the flat metric on B_i, I and J.

Noncommutative instantons: (N. Nekrasov, A. Schwarz, 98') In a noncommutative gauge theory, the ADHM construction - the procedure is identical to thye commutative case, moment map $\vec{\mu}$ is set equal to the self-dual projection of the noncommutativity matrix of the spacetime times identity matrix.

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Vortices: Setting B_2 and J to zero, one obtains the classical moduli space of nonabelian vortices in a supersymmetric gauge theory with an equal number of colors and flavors. The Fayet-Iliopoulos term, which determines a squark condensate, plays the role of the noncommutativity parameter in the real moment map.

- **Abelian case:**

The setup:

- ϕ is a complex smooth scalar field on $\mathbb{R}^2$ ($\equiv H^1$ above).
- $A_i, \;\; i = 1, 2$ is a smooth real vector field.

We assume that the above two fields satisfy

$$D_\pm \phi := (\partial_1 \pm i\partial_2)\phi - i(A_1 \pm iA_2)\phi = 0.$$

For $\omega \in \mathbb{R}^2$ smooth and real, we have the invariance

$$\phi \;\to\; e^\omega \phi; \quad A_i \;\to\; A_i + \partial_i \omega,$$

- We assume that $\mathcal{A}$ is specified according to the Coulomb gauge, i.e. it is divergence-free

$$\partial_1 A_1 + \partial_2 A_2 = 0. \tag{19}$$

- We choose a real function, ξ, satisfying

$$\nabla \xi = \pm(-A_2, A_1), \tag{20}$$

$$\psi := e^{-\xi}\phi \quad \Rightarrow \quad (\partial_1 \pm i\partial_2)\psi = 0. \tag{21}$$

Moduli matrix approach.

To solve votex eqs $\Rightarrow$ ansatz:

$$H = S^{-1}(z,\bar{z})H_0(z), \tag{22}$$

$S^{-1}(z,\bar{z}) \in GL(N_c,\mathbb{C})$; $H_0(z)$ is an arbitrary (so far) holomorphic matrix.

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- Vortex eqs give:

$$A_1 + iA_2 = -2iS^{-1}(z,\bar{z})\partial_{\bar{z}}S(z,\bar{z}). \tag{23}$$

The matrix $\mathbf{H_0}(z,\bar{z})$ is called *moduli matrix*.
It is convenient to introduce a matrix $\Omega(z,\bar{z})$ as

$$\Omega(z,\bar{z}) := S(z,\bar{z})S^{\dagger}(z,\bar{z}). \tag{24}$$

In terms of $\Omega(z,\bar{z})$ (13) takes the form

$$\partial_z(\Omega^{-1}\partial_{\bar{z}}\Omega) = \frac{g^2}{2}(c\mathbf{1}_{N_c} - \Omega^{-1}H_0H_0^{\dagger}). \tag{25}$$

In terms of $\Omega(z,\bar{z})$ the energy density (11) is

$$\mathcal{E}_{BPS} = 2c\bar{\partial}\partial\left(1-\frac{4}{cg^2}\bar{\partial}\partial\right)\log\det\Omega. \qquad (26)$$

The equation (25): asymptotically $\Omega_{z\to\infty} \to \frac{1}{c}H_0H_0^\dagger$. The vorticity k:

$$T = 2\pi ck = -\frac{c}{2}i\oint dz\partial_z \log\det(H_0H_0^\dagger) + c.c. \qquad (27)$$

which leads to boundary conditions for H_0 on S'_∞:

$$\det(H_0) \sim z^k, \quad \text{for } z\to\infty. \qquad (28)$$

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- Orientational moduli:

$$H_0(z_i)\vec{\chi}_i = 0 \quad \Longleftrightarrow \quad H(z = z_i, \bar{z} = \bar{z}_i)\vec{\chi} = 0. \qquad (29)$$

A different approach:

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- Let $\{z_1, \ldots, z_N\}$ be the set of zeroes of ϕ (with multiplicities). It is useful to explicitly separate zeroes

$$\hat{\phi}(z, \bar{z}) := e^{\xi(z,\bar{z})} \prod_{j=1}^{N} (z - z_j)\, \hat{h}(z), \qquad (30)$$

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Now we can change the gauge as

$$\hat{\phi} \longrightarrow |\hat{h}|\hat{h}^{-1}\hat{\phi}. \tag{31}$$

Then one can define the field ϕ using ξ and $\hat{h}$ as

$$\phi(z) = |\phi(z)| e^{\pm i \sum\limits_{j=1}^{N} \varphi_j}, \quad |\phi(z)| = e^{\xi} |\hat{h}(z)| \prod_{j=1}^{N} |z - z_j|,$$

$$|\hat{h}| = |\det \hat{h}(z)|, \quad \varphi_j = \mathrm{Arg}(z - z_j).$$

The equation (19) ($D_{\pm}\phi = 0$) becomes

$$\partial_2 \log|\phi| + \partial_1 \sum_{j=1}^{N} \varphi_j = \pm A_1,$$
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- Having that $\nabla \xi = (-A_2, A_1)$, one easily finds that $\log |h|$ is a harmonic.
- For the laplacian of $\log |\phi|$ we find

$$-\Delta \log |\phi|^2 = -2\Delta\xi - 2\Delta \log |h|^2 - 4\pi \sum_{j=1}^{N} \delta(z - z_j)$$

$$= \pm F_{12} - 4\pi \sum_{j=1}^{N} \delta(z - z_j). \tag{33}$$

• In the abelian case $\phi(z)$ defines the Higgs (matter) sector. To make connection with Liouville theory let us define

$$\rho := \log|\phi|^2, \quad \& \text{ set of zeroes } \{z_1, \ldots, z_N\}. \tag{34}$$

Then the field $\phi(z)$ takes the form familiar from Liouville theory

$$\phi(z) = e^{\frac{1}{2}\rho(z) \pm i \sum_{j=1}^{N} \varphi_j}. \tag{35}$$

The equations (32) in terms of ρ are

$$A_1 = \pm\left(\frac{1}{2}\partial_2\rho + \partial_1 \sum_{j=1}^{N} \varphi_j\right) \tag{36}$$

$$A_2 = \mp\left(\frac{1}{2}\partial_1\rho - \partial_2 \sum_{j=1}^{N} \varphi_j\right). \tag{37}$$

- The equation for *Higgs-Maxwell vortex* in terms of φ becomes that of Liouville field, namely

$$-\Delta\rho = 1 - e^{\rho} - 4\sum_{j=1}^{N}\delta(z - z_j), \tag{38}$$

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- Any solution for $\rho(z)$ has the structure

$$\rho(z) = \sum_{j=1}^{N}\log|z - z_j|^2 + \text{smooth function} \tag{41}$$

• **Non-abelian case:** Consider non-abelian Higgs-CS theory defined with the combined lagrangians

$$\begin{aligned} \mathcal{L}(\mathcal{A},\phi) &= k\mathcal{L}_{CS} + \mathrm{tr}\left[D_a\phi(D^a\phi)^\dagger\right] - V \\ \mathcal{A}_{CS} &= \frac{\epsilon^{\mu\nu\alpha}}{2}\mathrm{tr}\left(A_\mu\partial_\nu A_\alpha + \frac{2}{3}A_\mu A_\nu A_\alpha\right) \\ V &= \frac{1}{k^2}\left|[[\phi,\phi^\dagger],\phi] - v^2\phi\right|^2. \end{aligned} \tag{42}$$

Expanding explicitly on the generators of the gauge group, $\phi = \phi^a E_a$ we find

$$V = \frac{1}{k^2}\left|\phi^a(v^2 - C_{ba}|\phi^b|^2)\right|^2, \quad D_a D^a\phi = \frac{\partial V}{\partial\phi^\dagger}$$

$$\frac{k}{2}\epsilon^{\mu\alpha\beta}F_{\alpha\beta} = -iJ^\mu, \qquad J^\mu = i\left([D^\mu\phi,\phi^\dagger] - [\phi,(D^\mu\phi)^\dagger]\right).$$

The setup in the non-abelian case uses

$$\mathcal{A} = A_\alpha dx^\alpha; \quad A_\alpha = -iA^a_\alpha T_a; \quad \phi = \phi^a E_a, \tag{43}$$

where T_a:the generators of the gauge group, E_a: the simple roots generators, $A^a_\alpha \in \mathbb{R}$, $\phi^a \in \mathbb{C}$, a set of zeroes of ϕ, $\{z_1, \ldots, z_{N_a}\}$ where $N_a \in \mathbb{N}$. For the non-abelian generalization:

$$\phi^a := e^{\frac{1}{2}\rho_a \pm i \sum_{j=1}^{N} \varphi^a_j}, \quad a = 1, \ldots, r; \;\; \varphi^a_j = \mathrm{Arg}(z - z^a_j).$$

$$C_{ba} A^b_1 = \pm \left(\frac{1}{2}\partial_2 \rho_a + \partial_1 \sum_{j=1}^{N} \varphi^a_j \right)$$

$$C_{ba} A^b_2 = \mp \left(\frac{1}{2}\partial_1 \rho_a - \partial_2 \sum_{j=1}^{N} \varphi^a_j \right)$$

$$C_{ab} A^b_0 = \pm \frac{1}{k} \left(v^2 - C_{ba} e^{\rho_b} \right).$$

In these notations the self-dual vortex equations become

$$-\Delta\rho^a = \frac{4}{k^2}\left(v^2 C_{ba} e^{\rho_b} - C_{ba} e^{\rho_b} C_{cb} e^{\rho_c}\right) - 4\pi \sum_{j=1}^{N} \delta(z - z_j^a).$$

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THANK
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