Asymmetric Chua's Circuit: Implementation, Modeling and Analysis

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БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ ИНСТИТУТ ПО ЕЛЕКТРОНИКА „АКАДЕМИК ЕМИЛ ДЖАКОВ"

BULGARIAN ACADEMY OF SCIENCES ACADEMICIAN EMIL DJAKOV INSTITUTE OF ELECTRONICS

Outline

- Introduction: Lorenz’s 1963 paper.
- Examples of simple continuous 3D chaotic systems.
- Asymmetric Chua’s circuit: experimental implementation
- Model of the asymmetric nonlinearity (Chua’s diode).
- Qualitative analysis: fixed points, stability, etc
- Numerical implementation
- Thinks to do.

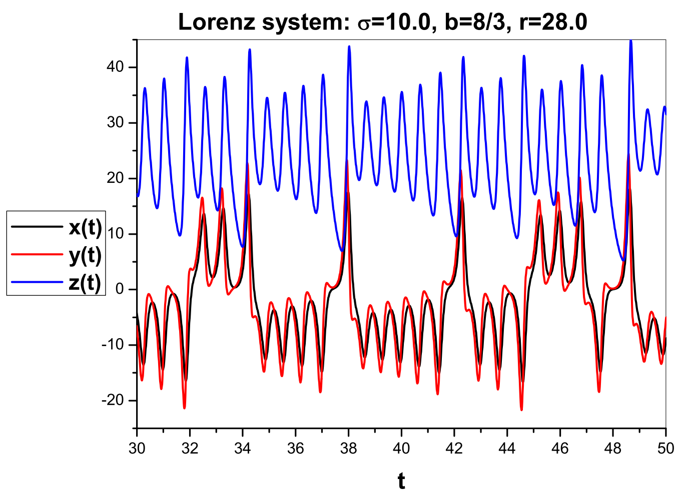
Lorenz’s 1963 paper

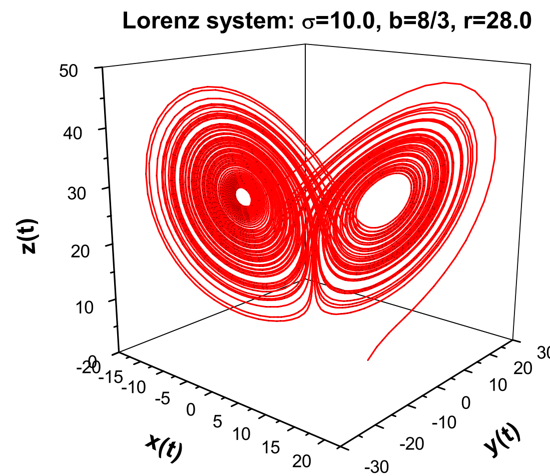
- Motivated to show that the “old meteo school” are wrong to employ linear regressions.
- In 1960: Lorenz’s first 12-variable dynamical model with aperiodic solutions obtained on Royal McBee LGP-30 computer (internal memory of 4096 32-bit words). Presented on a conference in Tokyo.
- Lorenz, Edward N., 1963: Deterministic Nonperiodic Flow, *J. Atmos. Sci.*, **20**, 130–141. Just a three-variable dynamical system! By around yesterday cited 14521 times! (12130)

Lorenz system: quasi-hyperbolic regime

$$\dot{x} = -\sigma\left(x - y\right), \qquad \dot{y} = rx - y - xz, \qquad \dot{z} = -bz + xy,$$

where σ, r, and b are parameters.

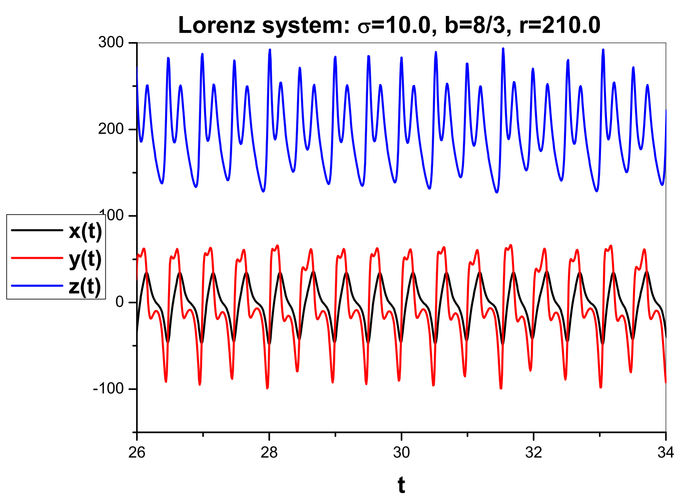


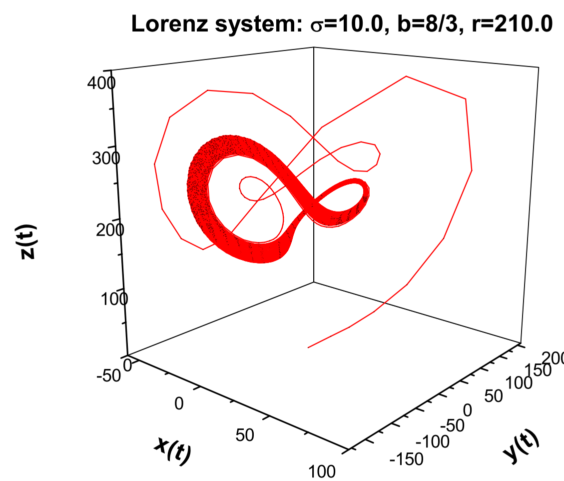


The system is dissipative: $\dfrac{dV}{dt} = \text{div}\vec{F}V = -\left(\sigma + b + 1\right)V$

Lorenz system: nonhyperbolic regime

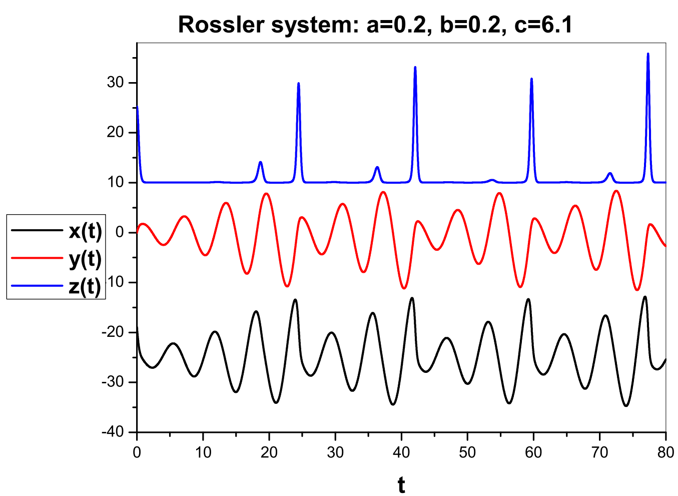
$$\dot{x} = -\sigma\,(x - y), \qquad \dot{y} = rx - y - xz, \qquad \dot{z} = -bz + xy$$

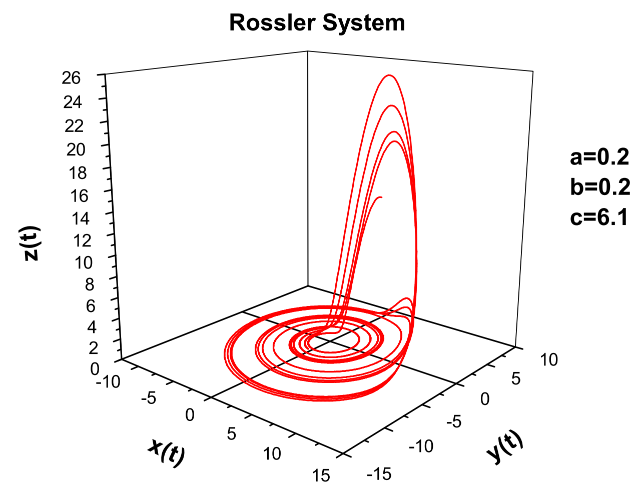




Rössler system: spiral regime

$$\dot{x} = -y - z, \qquad \dot{y} = x + ay, \qquad \dot{z} = b + z(x - c)$$



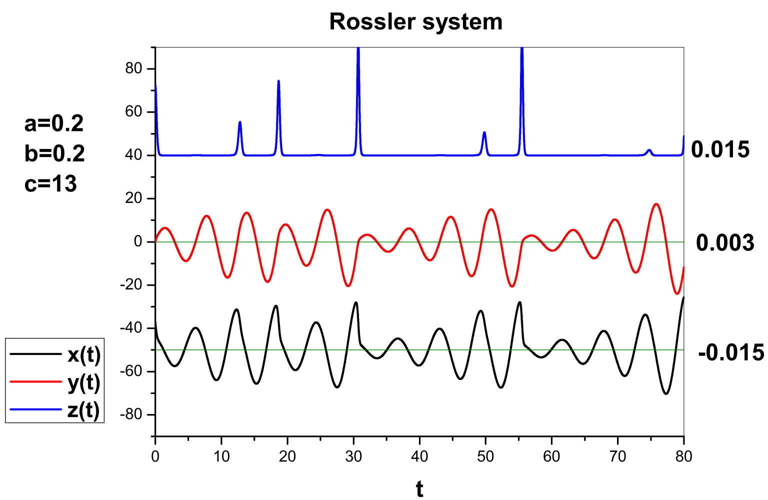


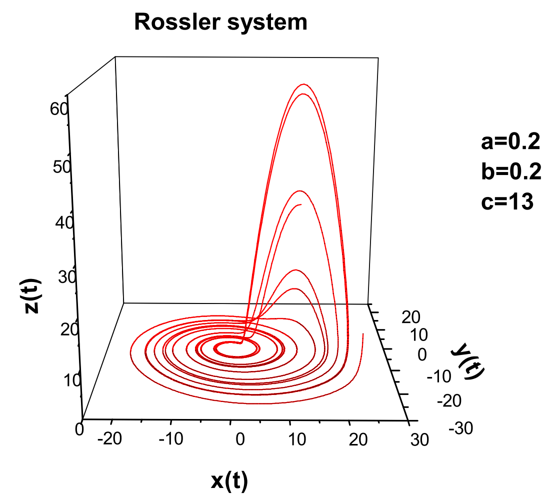
$$\frac{dV}{dt} = \text{div}\vec{F}V = (a + x - c)\,V$$

Rössler, O. E. (1976) An equation for continuous chaos, *Phys. Lett A*, **57**, 397.

Rössler system: funnel regime

$$\dot{x} = -y - z, \qquad \dot{y} = x + ay, \qquad \dot{z} = b + z(x - c)$$





J. Sprott's zoo of 3D chaotic attractors

Maximum quadratic nonlinearities, total of 30 parameters

$$\frac{dx_i}{dt} = a_i + \sum_j b_{i,j} x_j + \sum_{j,k} c_{i,jk} x_j x_k,$$

Computer search.

- Simplification: consider max 6 terms
- Each parameter varies within $[-5, 5]$ with a step 0.1, which gives 10^{20} cases.
- Of them 10^8 were randomly chosen for examination
- 19 chaotic systems were found.

J.C. Sprott, (1994) Some simple chaotic flows, *Phys. Rev E*, **50**, R647.

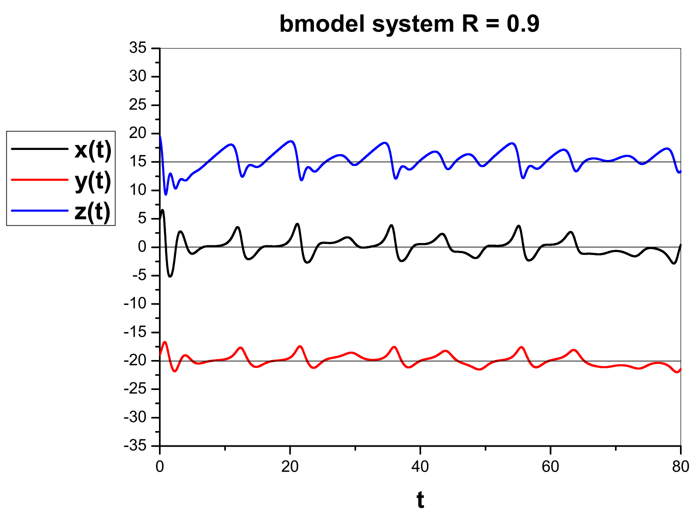
J. Sprott’s zoo of 19, 3D chaotic attractors

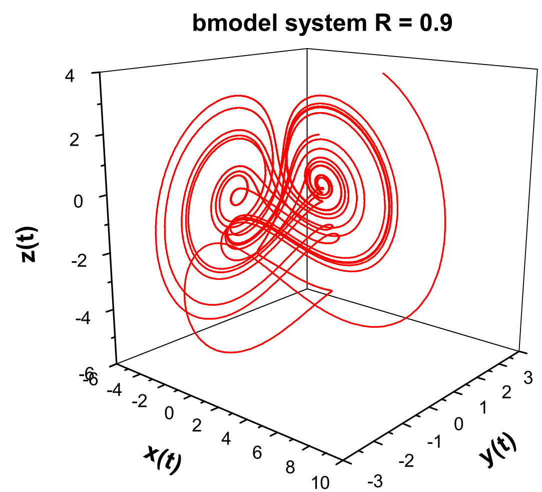
(A)	$\dot{x} = y$,	$\dot{y} = -x + yz$,	$\dot{z} = 1 - y^2$
(B)	$\dot{x} = yz$,	$\dot{y} = r(x - y)$,	$\dot{z} = 1 - xy$
(C)	$\dot{x} = yz$,	$\dot{y} = x - y$,	$\dot{z} = 1 - x^2$
	......	......	......
(N)	$\dot{x} = 2qy$,	$\dot{y} = x + z^2$,	$\dot{z} = 1 + y - 2ry$
	......	......	......
(S)	$\dot{x} = -x - 4y$,	$\dot{y} = x + z^2$,	$\dot{z} = 1 + x$
(U)	$\dot{x} = y$,	$\dot{y} = z$,	$\dot{z} = y^2 - x - az$
(V)	$\dot{x} = y$,	$\dot{y} = z$,	$\dot{z} = xy^2 - x - az$

Sprott's generalized B-system

$$\dot{x} = yz, \qquad \dot{y} = r(x-y), \qquad \dot{z} = 1 - xy$$

with two fixed points: $f_1 = (1,1,0)$ and $f_1 = (-1,-1,0)$





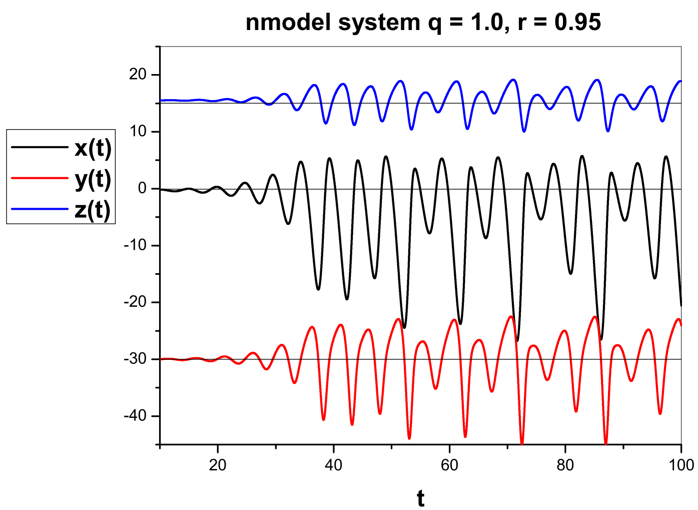
$$\frac{dV}{dt} = \text{div}\vec{F}V = (-r)\,V$$

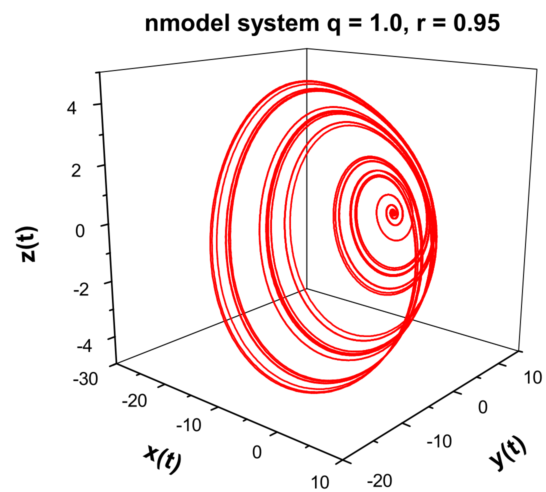
Sprott's generalized N-system

$$\dot{x} = -2qy, \qquad \dot{y} = x + z^2, \qquad \dot{z} = 1 + y - 2rz$$

For $q \neq 0$ the system has a single fixed point:

$$f_0 = (-1/4r^2, 0, 1/2r).$$





$$\frac{dV}{dt} = \text{div}\vec{F}V = (-2r)\,V$$

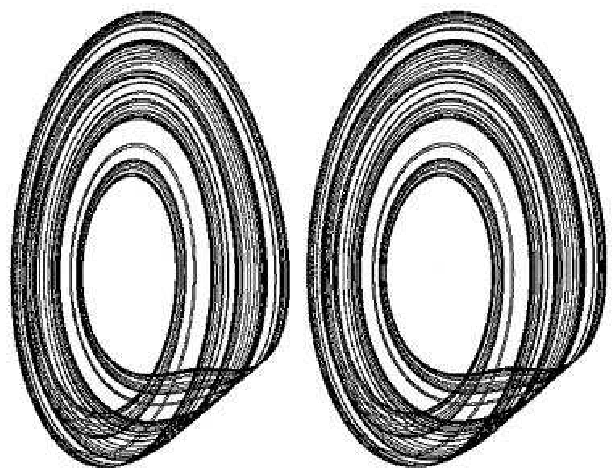
Sprott's generalized N-system and B-system

Dimitrova E. S. and Yordanov O. I., “Statistics of some low-dimensional chaotic flows”, *Intern. Journal of Chaos and Bifurcations*, vol. **11**, No. 10, pp. 2675-2682, (2001).

Piecewise linear chaotic systems

$$\dot{x} = y, \qquad \dot{y} = z, \qquad \dot{z} = -y - az + |x| - 1$$

Two fixed points: $f_{-1} = (-1, 0, 0)$ and $f_1 = (1, 0, 0)$



$$\frac{dV}{dt} = \text{div}\vec{F}V = (-a)\,V$$

Chua's oscillator

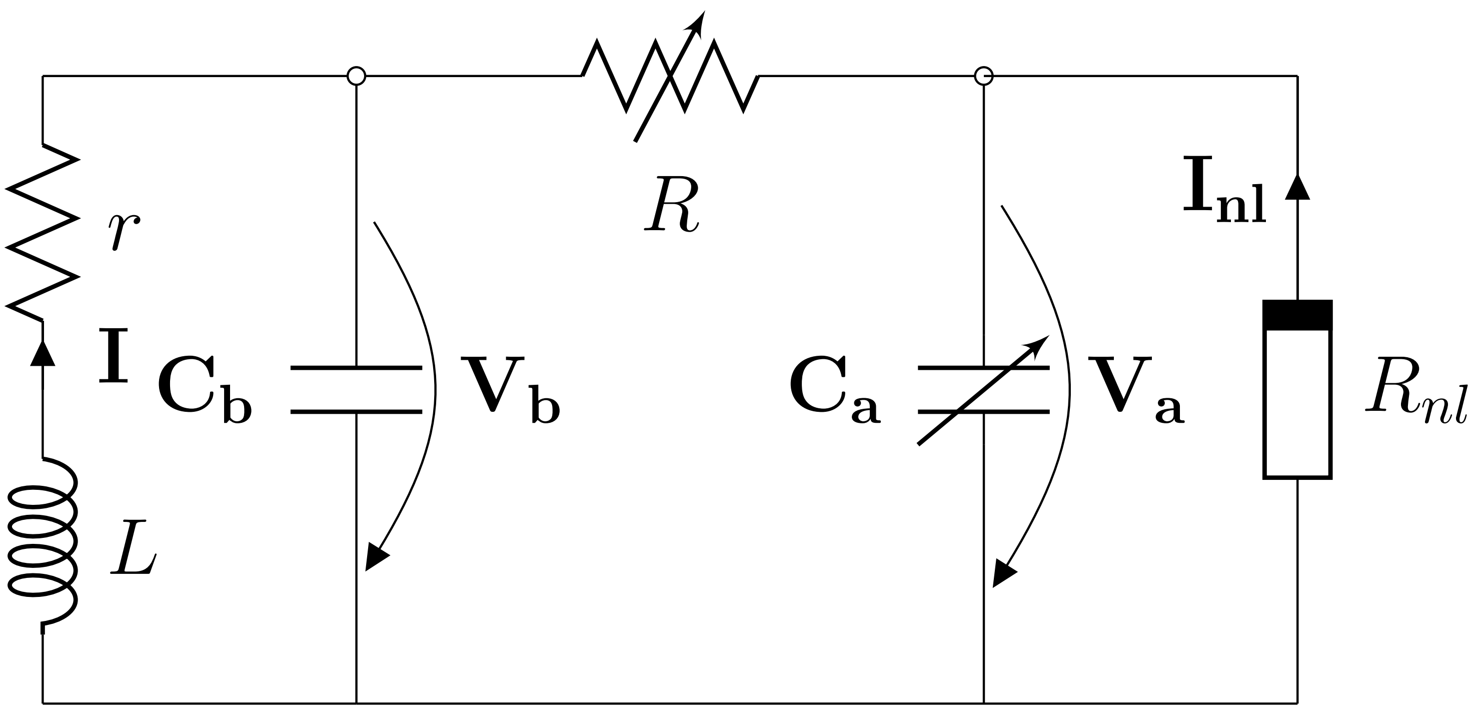


Figure 1: Basic Chua's circuit.

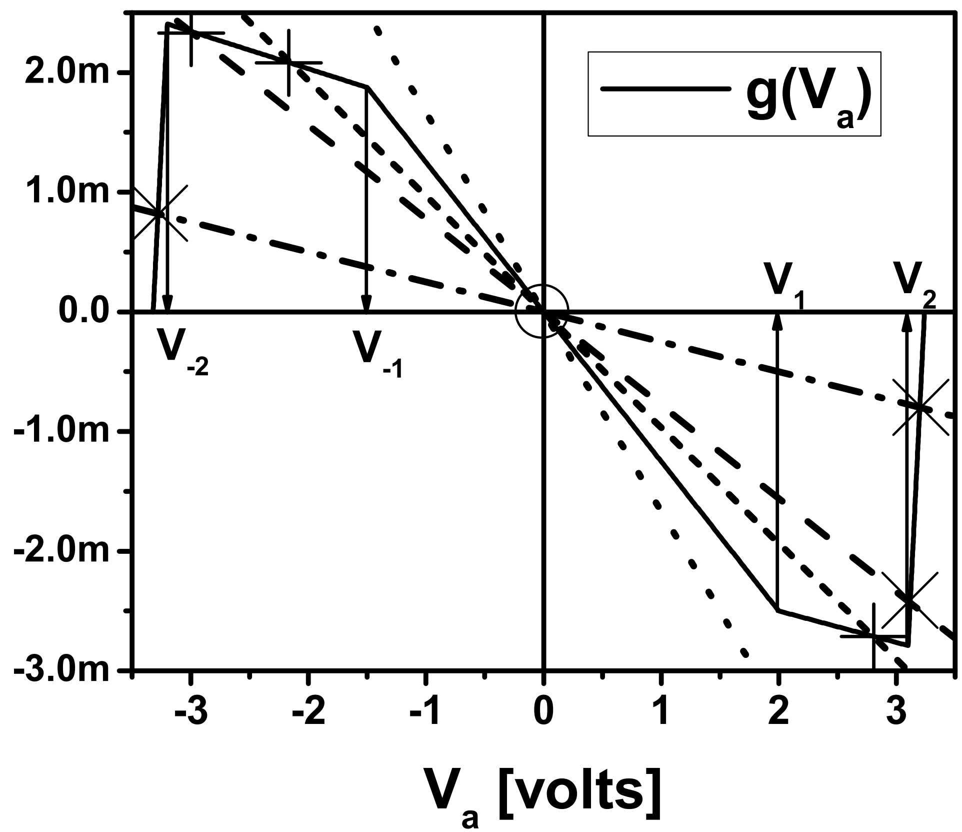
R_{nl} is a "negative" resistor, i.e a devise supplying energy to the system in the form of a nonlinear current, I_{nl}

Chua's system of differential equations

$$C_a \frac{dV_a}{dt} = R^{-1}\left(V_b - V_a\right) - g\left(V_a\right)$$
$$C_b \frac{dV_b}{dt} = -R^{-1}\left(V_b - V_a\right) + I$$
$$L \frac{dI}{dt} = -rI - V_b.$$

where $\boxed{I_{nl} = g(V_a)}$. Apart from $g(V_a)$, the system depends on 5 parameters: C_a, C_b, R, L and r.

Model for the nonlinear resistor



Parametrization of the asymmetric Chua's diode $g(V_a)$.

Model for the nonlinear resistor

$$g(V) = \begin{cases} V/R_{-2} + a_{-2} & \text{for} \quad V \le -V_{-2} \\ -V/R_{-1} + a_{-1} & \text{for} \ -V_{-2} \le V \le -V_{-1} \\ -V/R_0 & \text{for} \ -V_{-1} \le V \le V_1 \\ -V/R_1 + a_1 & \text{for} \quad V_1 \le V \le V_2 \\ V/R_2 + a_2 & \text{for} \quad V \ge V_2 \end{cases} \tag{1}$$

The “resistors” R_q, $q = \pm 1, \pm 2$, are effective and used to model the slopes of the respective V-I segments. The a-constants are determined by the requirement of continuity of $g(V)$.

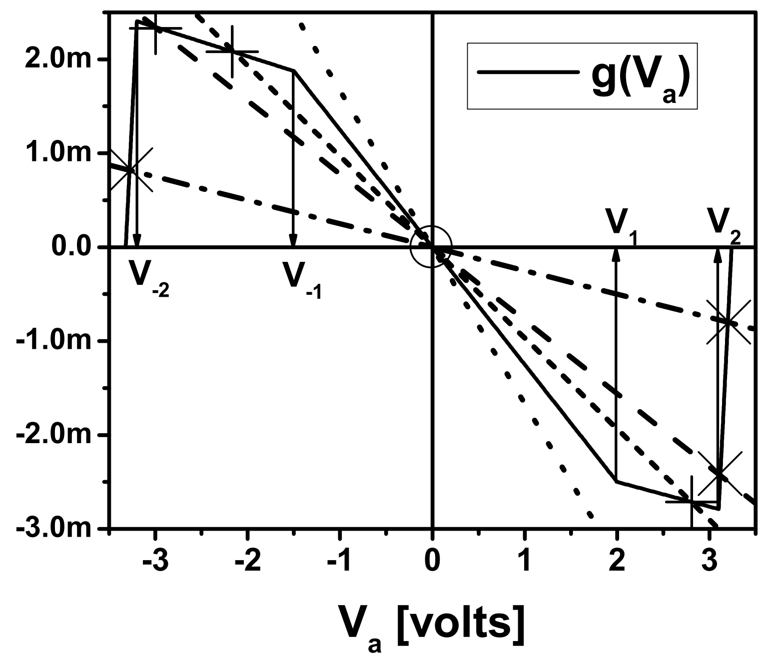
Fixed points

$$I = -V_b/r,$$

$$V_b = \frac{r}{R+r}V_a$$

Defining equation

$$-\frac{V_a}{(R+r)} = g\left(V_a\right)$$



Fixed points

1. $\mathcal{F}_{(0)}$: In the interval, $V_{-1} \leq V_a \leq V_1$, $\mathcal{F}_{(0)} = (0,0,0)$.
2. $\mathcal{F}_{(-1)}$: In the interval, $V_{-2} \leq V_a \leq V_{-1}$,
   $\mathcal{F}_{(-1)} = I_{(-1)}\left(-R-r,-r,1\right)$.
3. $\mathcal{F}_{(1)}$: In the interval, $V_1 \leq V_a \leq V_2$,
   $\mathcal{F}_{(1)} = I_{(1)}\left(R+r,+r,-1\right)$.
4. $\mathcal{F}_{(-2)}$: In the interval, $V_a < -V_{-2}$,
   $\mathcal{F}_{(-2)} = I_{(-2)}\left(-R-r,-r,1\right)$.
5. $\mathcal{F}_{(2)}$: In the interval, $V_a > V_2$,
   $\mathcal{F}_{(2)} = I_{(2)}\left(R+r,r,-1\right)$.

Conditions for existence

$\mathcal{F}_{(0)}$: The fixed point at zero always exists independent of the values of the parameters. In the exceptional case of $R + r = R_0$, all points in the interval are fixed.

A typical condition for existence, say for $\mathcal{F}_{(1)}$:

$$R_0 \leq (R + r) \leq \frac{R_1}{1 - (1 - R_1/R_0)(V_1/V_2)}$$

Divergencies

$$\frac{dV}{dt} = \mathsf{div}\vec{F}V = -\left(\frac{1}{C_a R_{-2}} + \frac{1}{C_a R} + \frac{1}{C_b R + \frac{r}{L}}\right)V$$

$$\frac{dV}{dt} = \mathsf{div}\vec{F}V = -\left(\frac{1}{C_a R_2} + \frac{1}{C_a R} + \frac{1}{C_b R + \frac{r}{L}}\right)V$$

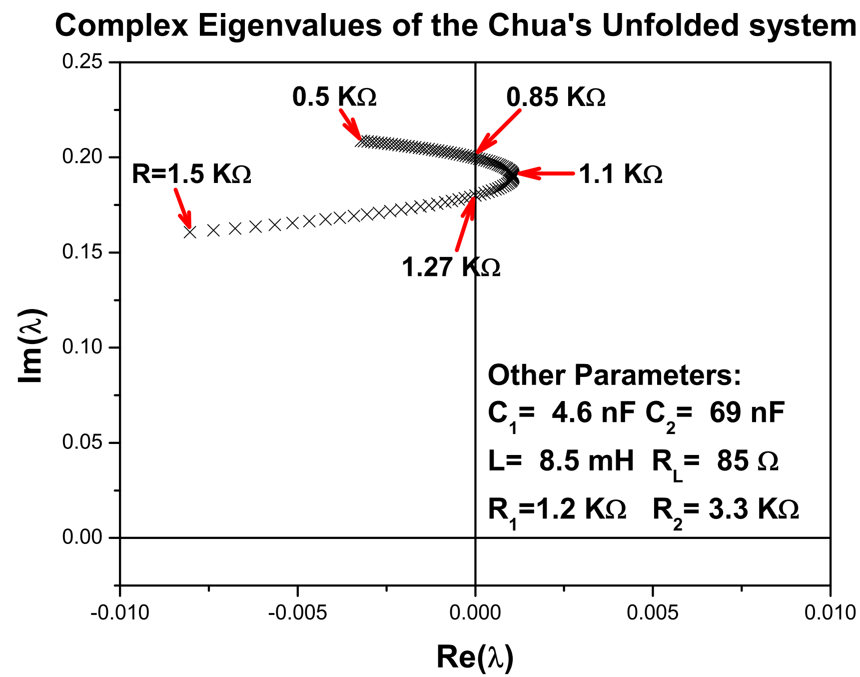
Re-scaled system

$V_a = V_{-1}x$, $V_b = V_{-1}y$, $I = (V_{-1}/R_0)\, z$, and $T = (C_a R_0)\, t$

$$\frac{dx}{dt} = a\,(y - x) - \widetilde{g}\,(x)$$
$$\frac{dy}{dt} = \sigma_c\left[-a\,(y - x) + z\right]$$
$$\frac{dz}{dt} = -c\left[y + \bar{r}z\right],$$

where $a = R_0/R$, $\sigma_c = C_a/C_b$, $c = C_a R_0^2/L$, $\bar{r} = r/R_0$ and $\widetilde{g}\,(x) = (R_0/V_{-1})\, g(V_{-1}X)$

Eigenvalues



λ_0 - real number

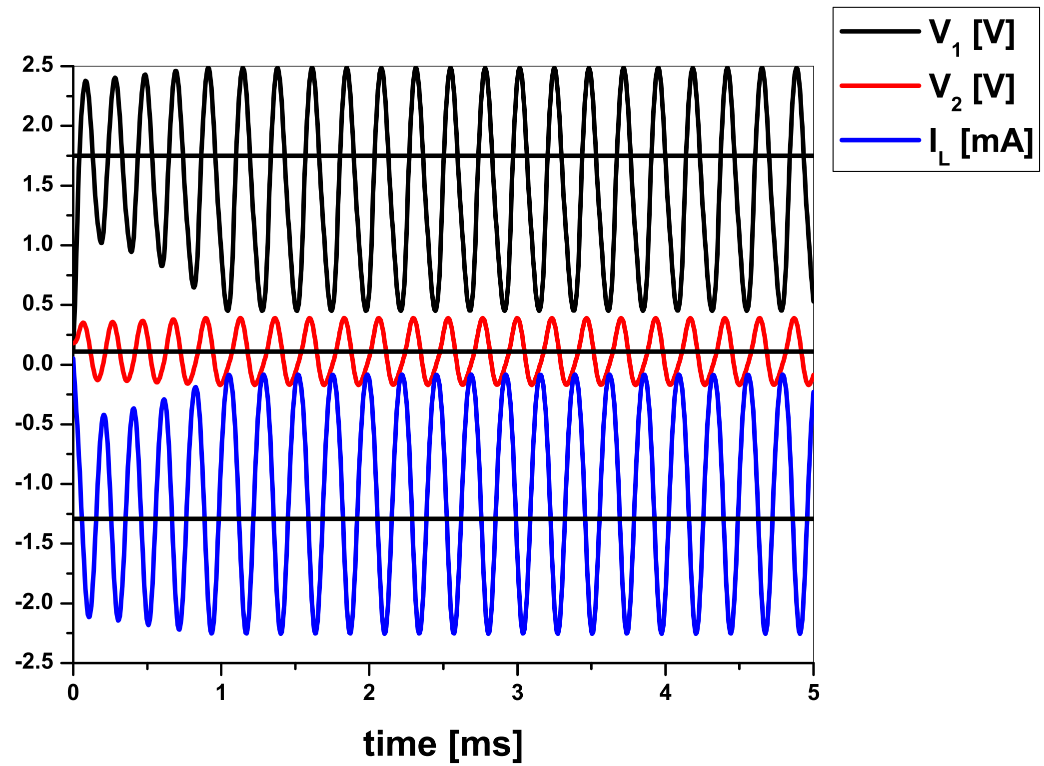
$\lambda_{1,2} = \lambda_r \pm i\lambda_i$;

$$v_1(\widetilde{t}) = v_{10} e^{\lambda_r \widetilde{t}} \cos(\lambda_i \widetilde{t} + \phi)$$

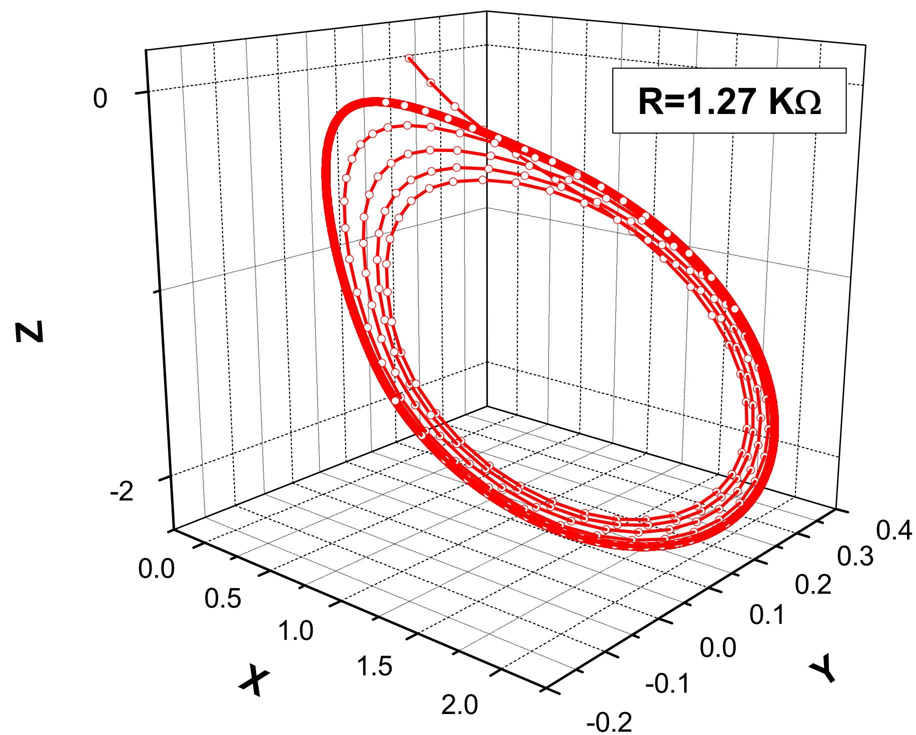
Dynamics of the Chua's circuit.

$R = 1.27$ KΩ
$C_1 = 4.6$ nF
$C_2 = 69$ nF
$L = 8.5$ mH
$r = 85\ \Omega$
$R_1 = 1.2$ KΩ
$R_2 = 3.3$ KΩ

$\lambda = 0.0 \pm i0.18$;

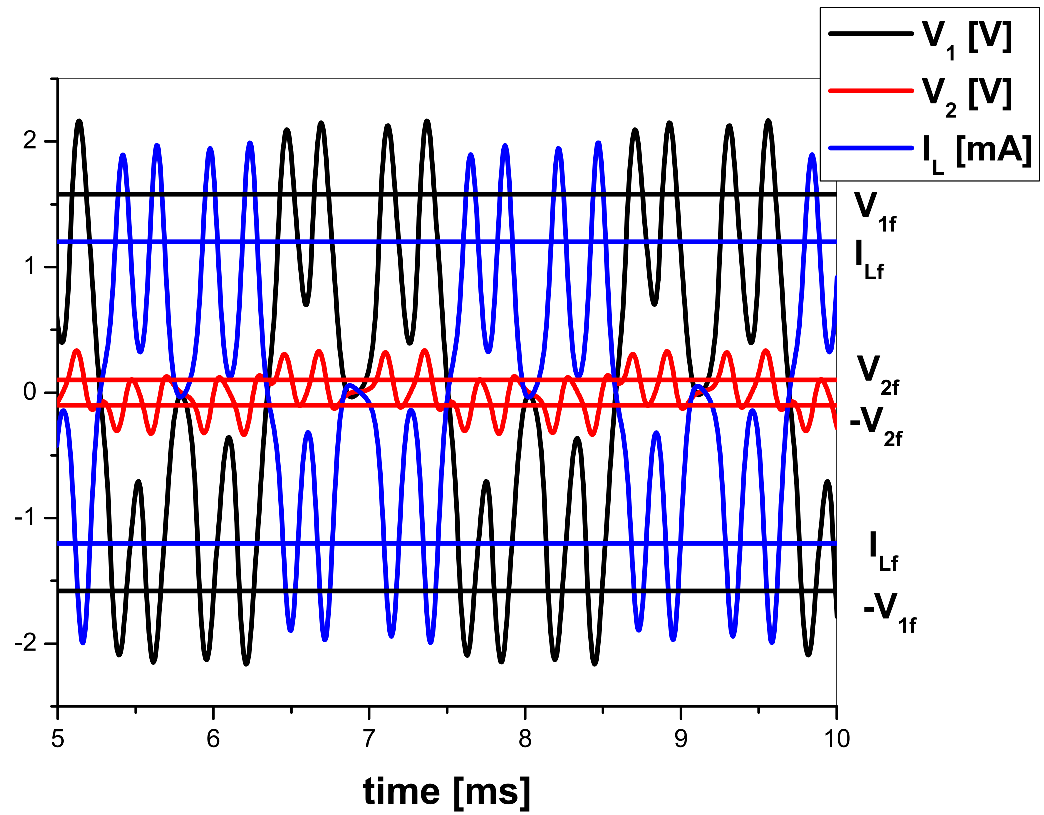


Chua’s circuit in the phase space



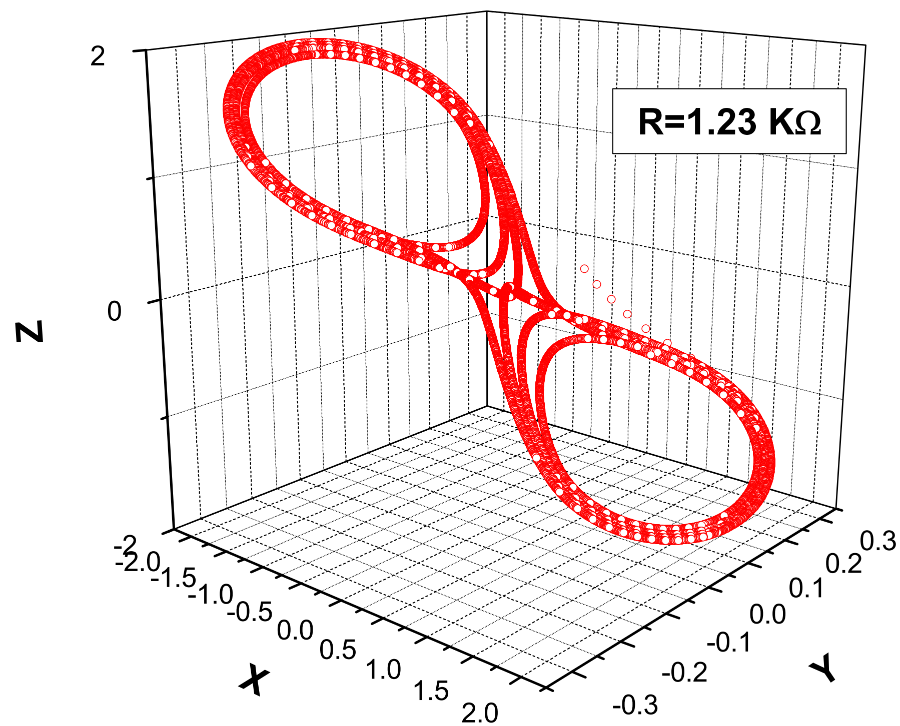
Dynamics of the Chua's circuit.

$R = 1.23$ KΩ
$C_1 = 4.6$ nF
$C_2 = 69$ nF
$L = 8.5$ mH
$R_L = 85\ \Omega$
$R_1 = 1.2$ KΩ
$R_2 = 3.3$ KΩ



$\lambda = 0.0047 \pm i0.183;$

Chua’s circuit in the phase space



Things to do

- Breaking the system on five linear systems and match the solutions in the passing between the segments
- Constructing the Poincaré map.
- Computing the Lyapunov exponents.
- Evaluating the statistical properties of the signals.

the thank-you-acknowledgment slide

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Thank you for your attention!

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