Metric-Independent Volume-Forms in Gravity and Cosmology

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Background material:

- E. Guendelman, E.N., S. Pacheva and M. Vasihoun, in "VIII-th Mathematical Physics Meeting", B. Dragovic and Z. Rakic (eds.), Belgrade Inst. Phys. Press, 2015 (*arxiv:1501.05518* [hep-th]);
  E. Guendelman, E.N., S. Pacheva and M. Vasihoun, *Bulg. J. Phys.* 41 (2014) 123-129 (*arxiv:1404.4733* [hep-th]).
- E. Guendelman, R. Herrera, P. Labrana, E.N. and S. Pacheva, *General Relativity and Gravitation* 47 (2015) art.10 (*arxiv:1408.5344v4* [gr-qc]).
- E. Guendelman, E.N. and S. Pacheva, *arxiv:1504.01031* [gr-qc].

Introduction - Overview of Talk

Alternative spacetime volume-forms (generally-covariant integration measure densitites) independent on the Riemannian metric on the pertinent spacetime manifold have profound impact in (field theory) models with general coordinate reparametrization invariance – general relativity and its extensions, strings and (higher-dimensional) membranes. Although formally appearing as “pure-gauge” dynamical degrees of freedom the non-Riemannian volume-form fields trigger a number of remarkable physically important phenomena.

Introduction - Overview of Talk

Among the principal new phenomena are:

- (i) new mechanism of dynamical generation of cosmological constant;
- (ii) new mechanism of dynamical spontaneous breakdown of supersymmetry in supergravity;
- (iii) new type of "quintessential inflation" scenario in cosmology;
- (iv) gravitational electrovacuum "bags".

Modified-Measure Theories

In a series of previous papers [E.Guendelman *et.al.*] a new class of generally-covariant (non-supersymmetric) field theory models including gravity – called “two-measure theories” (TMT) was proposed.

- TMT appear to be promising candidates for resolution of various problems in modern cosmology: the *dark energy* and *dark matter* problems, the fifth force problem, etc.
- Principal idea – employ an alternative volume form (volume element or generally-covariant integration measure) on the spacetime manifold in the pertinent Lagrangian action.

Modified-Measure Theories

In standard generally-covariant theories (with action $S = \int d^D x \sqrt{-g}\mathcal{L}$) the Riemannian spacetime volume-form, *i.e.*, the integration measure density is given by $\sqrt{-g}$, where $g \equiv \det \|g_{\mu\nu}\|$ is the determinant of the corresponding Riemannian metric $g_{\mu\nu}$.

$\sqrt{-g}$ transforms as scalar density under general coordinate reparametrizations.

There is NO *a priori* any obstacle to employ insted of $\sqrt{-g}$ another alternative non-Riemannian volume element given by the following ***non-Riemannian*** integration measure density:

$$\Phi(B) \equiv \frac{1}{(D-1)!}\varepsilon^{\mu_1 \ldots \mu_D}\, \partial_{\mu_1} B_{\mu_2 \ldots \mu_D} \;. \qquad (1)$$

Modified-Measure Theories

Here $B_{\mu_1 \ldots \mu_{D-1}}$ is an auxiliary rank $(D-1)$ antisymmetric tensor gauge field, which will turn out to be pure-gauge degree of freedom. $\Phi(B)$ similarly transforms as scalar density under general coordinate reparametrizations.

In particular, $B_{\mu_1 \ldots \mu_{D-1}}$ can also be parametrized in terms of D auxiliary scalar fields:

$$B_{\mu_1 \ldots \mu_{D-1}} = \frac{1}{D} \varepsilon_{I J_1 \ldots J_{D-1}} \phi^I \partial_{\mu_1} \phi^{J_1} \ldots \partial_{\mu_{D-1}} \phi^{J_{D-1}},$$

so that:

$$\Phi(B) = \frac{1}{D!} \varepsilon^{\mu_1 \ldots \mu_D} \varepsilon_{I_1 \ldots I_D} \partial_{\mu_1} \phi^{I_1} \ldots \partial_{\mu_D} \phi^{I_D}.$$

To illustrate the TMT formalism let us consider the following action:

$$S = c_1 \int d^D x \, \Phi(B) \Big[L^{(1)} + \frac{\varepsilon^{\mu_1 \ldots \mu_D}}{(D-1)!\sqrt{-g}} \partial_{\mu_1} H_{\mu_2 \ldots \mu_D} \Big] + c_2 \int d^D x \sqrt{-g}\, L^{(2)} \qquad (2)$$

with the following notations:

- The Lagrangians $L^{(1,2)} \equiv \frac{1}{2\kappa^2} R + L^{(1,2)}_{\rm matter}$ include both standard Einstein-Hilbert gravity action as well as matter/gauge-field parts. Here $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ is the scalar curvature within the first-order (Palatini) formalism and $R_{\mu\nu}(\Gamma)$ is the Ricci tensor in terms of the independent affine connection $\Gamma^{\mu}_{\lambda\nu}$.

Modified-Measure Theories

- In general, second Lagrangian $L^{(2)}$ might contain also higher curvature terms like R^2.
- In the first ***modified-measure term*** of the action (2) we have included an additional term containing another auxiliary rank $(D-1)$ antisymmetric tensor gauge field $H_{\mu_1\ldots\mu_{D-1}}$. Such term would be purely topological (total divergence) one if included in standard Riemannian integration measure action like the second term with $L^{(2)}$ on the r.h.s. of (2).

$H_{\mu_1\ldots\mu_{D-1}}$ similarly will turn out to be pure-gauge degree of freedom, however, both auxiliary tensor gauge fields (B and H) will nevertheless play crucial role in the sequel.

Varying (2) w.r.t. H and B tensor gauge fields we get:

$$\partial_\mu\Big(\frac{\Phi(B)}{\sqrt{-g}}\Big) = 0 \;\;\to\;\; \frac{\Phi(B)}{\sqrt{-g}} \equiv \chi = \mathrm{const}\;, \tag{3}$$

$$L^{(1)} + \frac{\varepsilon^{\mu_1\ldots\mu_D}}{(D-1)!\sqrt{-g}}\partial_{\mu_1} H_{\mu_2\ldots\mu_D} = M = \mathrm{const}\;, \tag{4}$$

where χ (ratio of the two measure densities) and M are ***arbitrary integration constants***.

Performing canonical Hamiltonian analysis of (2) we find that the above integration constants M and χ are in fact ***constrained a'la Dirac canonical momenta*** of B and H.

Modified-Measure Theories

Now, varying (2) w.r.t. $g^{\mu\nu}$ and taking into account (3)–(4) we arrive at the following effective Einstein equations (in the first-order formalism):

$$R_{\mu\nu}(\Gamma) - \frac{1}{2} g_{\mu\nu} R + \Lambda_{\rm eff}\, g_{\mu\nu} = \kappa^2 T^{\rm eff}_{\mu\nu} , \qquad (5)$$

with effective energy-momentum tensor:

$$T^{\rm eff}_{\mu\nu} = g_{\mu\nu} L^{\rm eff}_{\rm matter} - 2 \frac{\partial L^{\rm eff}_{\rm matter}}{\partial g^{\mu\nu}} \quad , \quad L^{\rm eff}_{\rm matter} \equiv \frac{1}{c_1 \chi + c_2} \Bigl[c_1 L^{(1)}_{\rm matter} + c_2 L^{(2)}_{\rm matter} \Bigr] \, , \qquad (6)$$

and with a ***dynamically generated effective cosmological constant*** thanks to the non-zero integration constants

$$\Lambda_{\rm eff} = \kappa^2 \left(c_1 \chi + c_2 \right)^{-1} \chi M \; . \qquad (7)$$

Supersymmetric Higgs Effect in Supergravity

Let us now apply the above TMT formalism to construct a modified-measure version of $N = 1$ supergravity in $D = 4$.
Recall the standard component-field action of $D = 4$ (minimal) $N = 1$ supergravity:

$$S_{\rm SG} = \frac{1}{2\kappa^2}\int d^4x\, e\Big[R(\omega,e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda\Big] \; , \tag{8}$$

$$e = \det \| e^a_\mu \| \;\; , \;\; R(\omega,e) = e^{a\mu} e^{b\nu} R_{ab\mu\nu}(\omega) \; . \tag{9}$$

$$R_{ab\mu\nu}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega^c_{\mu a}\omega_{\nu cb} - \omega^c_{\nu a}\omega_{\mu cb} \; . \tag{10}$$

$$D_\nu \psi_\lambda = \partial_\nu \psi_\lambda + \frac{1}{4}\omega_{\nu ab}\gamma^{ab}\psi_\lambda \;\; , \;\; \gamma^{\mu\nu\lambda} = e^\mu_a e^\nu_b e^\lambda_c \gamma^{abc} \; , \tag{11}$$

where all objects belong to the first-order “vierbein” (frame-bundle) formalism.

Supersymmetric Higgs Effect in Supergravity

The vierbeins e^a_μ (describing the graviton) and the spin-connection $\omega_{\mu ab}$ ($SO(1,3)$ gauge field acting on the gravitino ψ_μ) are *a priori* independent fields (their relation arises subsequently on-shell); $\gamma^{ab} \equiv \frac{1}{2}\left(\gamma^a\gamma^b - \gamma^b\gamma^a\right)$ *etc.* with γ^a denoting the ordinary Dirac gamma-matrices. The invariance of the action (8) under local supersymmetry transformations:

$$\delta_\epsilon e^a_\mu = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu \;\;,\;\; \delta_\epsilon\psi_\mu = D_\mu\varepsilon \qquad (12)$$

follows from the invariance of the pertinent Lagrangian density up to a total derivative:

$$\delta_\epsilon\Big(e\big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda\big]\Big) = \partial_\mu\big[e\big(\bar{\varepsilon}\zeta^\mu\big)\big]\;, \qquad (13)$$

where ζ^μ functionally depends on the gravitino field ψ_μ.

Supersymmetric Higgs Effect in Supergravity

We now propose a modification of (8) by replacing the standard generally-covariant measure density $e = \sqrt{-g}$ by the alternative measure density $\Phi(B)$ (Eq.(1) for $D = 4$):

$$\Phi(B) \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\,\partial_\mu B_{\nu\kappa\lambda}\ , \tag{14}$$

and we will use the general framework described above. The modified supergravity action reads:

$$S_{\mathrm{mSG}} = \frac{1}{2\kappa^2}\int d^4x\,\Phi(B)\left[R(\omega,e)-\bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda+\frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\,\partial_\mu H_{\nu\kappa\lambda}\right], \tag{15}$$

where a new term containing the field-strength of a 3-index antisymmetric tensor gauge field $H_{\nu\kappa\lambda}$ has been added.

Supersymmetric Higgs Effect in Supergravity

The equations of motion w.r.t. $H_{\nu\kappa\lambda}$ and $B_{\nu\kappa\lambda}$ yield:

$$\partial_\mu\Big(\frac{\Phi(B)}{e}\Big) = 0 \quad \rightarrow \quad \frac{\Phi(B)}{e} \equiv \chi = \text{const} \ , \tag{16}$$

$$R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\, e} \partial_\mu H_{\nu\kappa\lambda} = 2M \ , \tag{17}$$

where χ and M are arbitrary integration constants.
The action (15) is invariant under local supersymmetry transformations (12) supplemented by transformation laws for $H_{\mu\nu\lambda}$ and $\Phi(B)$:

$$\delta_\epsilon H_{\mu\nu\lambda} = -e\, \varepsilon_{\mu\nu\lambda\kappa}\big(\bar{\varepsilon}\zeta^\kappa\big) \quad , \quad \delta_\epsilon \Phi(B) = \frac{\Phi(B)}{e}\, \delta_\epsilon e \ , \tag{18}$$

which algebraically close.

Supersymmetric Higgs Effect in Supergravity

The appearance of the integration constant M represents a ***dynamically generated cosmological constant*** in the pertinent gravitational equations of motion and, thus, it signifies a *spontaneous (dynamical) breaking of supersymmetry*.

Indeed, varying (15) w.r.t. e^a_μ:

$$e^{b\nu} R^a_{b\mu\nu} - \frac{1}{2}\bar{\psi}_\mu \gamma^{a\nu\lambda} D_\nu \psi_\lambda + \frac{1}{2}\bar{\psi}_\nu \gamma^{a\nu\lambda} D_\mu \psi_\lambda + \frac{1}{2}\bar{\psi}_\lambda \gamma^{a\nu\lambda} D_\nu \psi_\mu + \frac{e^a_\mu}{2}\,\frac{\varepsilon^{\rho\nu\kappa\lambda}}{3!\,e}\,\partial_\rho H_{\nu\kappa\lambda} = 0 \tag{19}$$

and using Eq.(17) (containing the arbitrary integration constant M) to replace the last H-term on the l.h.s. of (19), the results is:

Supersymmetric Higgs Effect in Supergravity

We obtain the vierbein counterparts of the Einstein equations including a dynamically generated ***floating*** cosmological constant term $e^a_\mu M$:

$$e^{b\nu} R^a_{b\mu\nu} - \frac{1}{2} e^a_\mu R(\omega, e) + e^a_\mu M = \kappa^2 T^a_\mu \ ,$$

$$\kappa^2 T^a_\mu \equiv \frac{1}{2} \bar{\psi}_\mu \gamma^{a\nu\lambda} D_\nu \psi_\lambda - \frac{1}{2} e^a_\mu \bar{\psi}_\rho \gamma^{\rho\nu\lambda} D_\nu \psi_\lambda - \frac{1}{2} \bar{\psi}_\nu \gamma^{a\nu\lambda} D_\mu \psi_\lambda - \frac{1}{2} \bar{\psi}_\lambda \gamma^{a\nu\lambda} D_\nu \psi_\mu \ .$$

Recall: according to the classic paper *[Deser-Zumino, 78]* the sole presence of a cosmological constant in supergravity, even in the absence of manifest mass term for the gravitino, implies that the gravitino becomes ***massive***, i.e., it absorbs the Goldstone fermion of spontaneous supersymmetry breakdown – a ***supersymmetric Higgs effect***.

AdS Supergravity

More interesting scenario: let us start with anti-de Sitter (AdS) supergravity:

$$S_{\mathrm{AdS-SG}} = \frac{1}{2\kappa^2}\int d^4x\, e\Big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda - m\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu - 2\Lambda_0\Big]\,, \qquad (21)$$

$$m \equiv \frac{1}{L} \quad , \quad \Lambda_0 \equiv -\frac{3}{L^2}\,. \qquad (22)$$

The action (21) contains additional explicit mass term for the gravitino as well as a bare cosmological constant Λ_0 balanced in a precise way $|\Lambda_0| = 3m^2$ so as to maintain local supersymmetry invariance and, in particular, keeping the ***physical gravitino mass zero***!

AdS Supergravity

Note: Here we have AdS spacetime as a background with curvature radius L (unlike Minkowski background in the absence of a bare cosmological constant).

Therefore, the notions of “mass” and “spin” are given in terms of the Casimir eigenvalues of the UIR’s (discrete series) of the group of motion of AdS space $SO(2,3) \sim Sp(4,\mathbb{R})$ (for $D = 4$) instead of the Poincare group ($SO(1,3) \ltimes R^4$) Casimirs.

Identification (correspondence) of AdS ($SO(2,3) \sim Sp(4,\mathbb{R})$) Casimirs with Minkowski (Poincare) "mass" and "spin" Casimirs proceeds only in the limit of $\Lambda_0 \to 0$ (very small cosmological constant). Otherwise "massless" within AdS means having only two "helicities".

Modified AdS Supergravity

Now, let us apply the above TMT-formalism to construct a modified-measure AdS supergravity:

$$S_{\mathrm{mod-AdS-SG}} = \frac{1}{2\kappa^2}\int d^4x\,\Phi(B)\Big[R(\omega,e) - \bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda \\ -m\,\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu - 2\Lambda_0 + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!\,e}\partial_\mu H_{\nu\kappa\lambda}\Big] \; , \tag{23}$$

with $\Phi(B)$ as in (14) and m,Λ_0 as in (21). The action (23) is invariant under local supersymmetry transformations:

$$\delta_\epsilon e^a_\mu = \frac{1}{2}\bar{\varepsilon}\gamma^a\psi_\mu \quad , \quad \delta_\epsilon\psi_\mu = \Big(D_\mu - \frac{1}{2L}\gamma_\mu\Big)\varepsilon \; , \\ \delta_\epsilon H_{\mu\nu\lambda} = -e\,\varepsilon_{\mu\nu\lambda\kappa}\big(\bar{\varepsilon}\zeta^\kappa\big) \quad , \quad \delta_\epsilon\Phi(B) = \frac{\Phi(B)}{e}\,\delta_\epsilon e \; . \tag{24}$$

Modified AdS Supergravity - Principal Result

The modified AdS supergravity action (23) will trigger dynamical spontaneous supersymmetry breaking resulting in the appearance of the dynamically generated floating cosmological constant M which will add to the bare Λ_0.

Thus, we can achieve via appropriate choice of $M \simeq |\Lambda_0|$ a ***very small effective observable cosmological constant*** $\Lambda_{\rm eff} = M + \Lambda_0 = M - 3m^2 << |\Lambda_0|$ and, simultaneously, a ***large physical gravitino mass*** $m_{\rm eff}$ which will be very close to the gravitino mass parameter $m = \sqrt{|\Lambda_0|/3}$ since now background spacetime geometry becomes almost flat.

This is precisely what is required by modern cosmological scenarios for slowly expanding universe of today [A. Riess *et.al.*, S. Perlmutter *et.al.*].

Generalized Modified-Measure Theories

Let us now consider modified-measure gravity-matter theories constructed in terms of two different non-Riemannian volume-forms (employing again Palatini formalism, and using units where $G_{\rm Newton} = 1/16\pi$):

$$S = \int d^4 x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4 x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big] \; . \tag{25}$$

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; , \tag{26}$$

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda} \;\; \big(\text{as in (15) above}\big) \; . \tag{27}$$

Generalized Modified-Measure Theories

- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field of the form:

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \quad , \quad V(\varphi) = f_1 \exp\{-\alpha\varphi\} \; , \tag{28}$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) \quad , \quad U(\varphi) = f_2 \exp\{-2\alpha\varphi\} \; , \tag{29}$$

where α, f_1, f_2 are dimensionful positive parameters, whereas b is a dimensionless one.

- Global Weyl-scale invariance of the action (25):

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \; , \; \Gamma^\mu_{\nu\lambda} \to \Gamma^\mu_{\nu\lambda} \; , \; \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda \; ,$$

$$A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \; , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \; , \; H_{\mu\nu\kappa} \to H_{\mu\nu\kappa} \; .$$

Generalized Modified-Measure Theories

Eqs. of motion w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$ yield a solution for the latter as a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2}\bar{g}^{\mu\kappa}\left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}\right) \; , \tag{30}$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R) g_{\mu\nu} \;\; , \;\; \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} \;\; , \;\; \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}} \; . \tag{31}$$

Transition from original metric $g_{\mu\nu}$ to $\bar{g}_{\mu\nu}$: **"Einstein-frame"**, where the gravity eqs. of motion are written in the standard form of Einstein's equations: $R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\mathrm{eff}}_{\mu\nu}$ with an appropriate **effective** energy-momentum tensor given in terms of an Einstein-frame scalar Lagrangian L_{eff} (see (34) below).

Generalized Modified-Measure Theories

Variation of the action (25) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Big[R + L^{(1)} \Big] = 0 \; , \; \partial_\mu \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \Big] = 0 \; , \; \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}} \Big) = 0 \; , \tag{32}$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} \quad , \quad R + L^{(1)} = -M_1 = \text{const} \; ,$$
$$L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} \; . \tag{33}$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants.

Generalized Modified-Measure Theories

The first integration constant χ_2 in (33) preserves global Weyl-scale invariance whereas the appearance of the second and third integration constants M_1, M_2 signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance due to the scale non-invariant solutions (second and third ones) in (33).

It is very instructive to elucidate the physical meaning of the three arbitrary integration constants M_1, M_2, χ_2 from the point of view of the canonical Hamiltonian formalism: M_1, M_2, χ_2 are identified as conserved Dirac-constrained canonical momenta conjugated to (certain components of) the auxiliary maximal rank antisymmetric tensor gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ entering the original non-Riemannian volume-form action (25).

Generalized Modified-Measure Theories

Performing transition to the Einstein frame yields the following effective scalar Lagrangian of non-canonical "k-essence" (kinetic quintessence) type ($X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ – scalar kinetic term):

$$L_{\rm eff} = A(\varphi)X + B(\varphi)X^2 - U_{\rm eff}(\varphi) \;, \tag{34}$$

where (recall $V = f_1 e^{-\alpha\varphi}$ and $U = f_2 e^{-2\alpha\varphi}$):

$$A(\varphi) \equiv 1 + \Bigl[\frac{1}{2}be^{-\alpha\varphi} - \epsilon(V - M_1)\Bigr]\frac{V - M_1}{U + M_2 + \epsilon(V - M_1)^2} \;, \tag{35}$$

$$B(\varphi) \equiv \chi_2 \frac{\epsilon\Bigl[U + M_2 + (V - M_1)be^{-\alpha\varphi}\Bigr] - \frac{1}{4}b^2 e^{-2\alpha\varphi}}{U + M_2 + \epsilon(V - M_1)^2} \;, \tag{36}$$

$$U_{\rm eff}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2\Bigl[U + M_2 + \epsilon(V - M_1)^2\Bigr]} \;. \tag{37}$$

Generalized Modified-Measure Theories

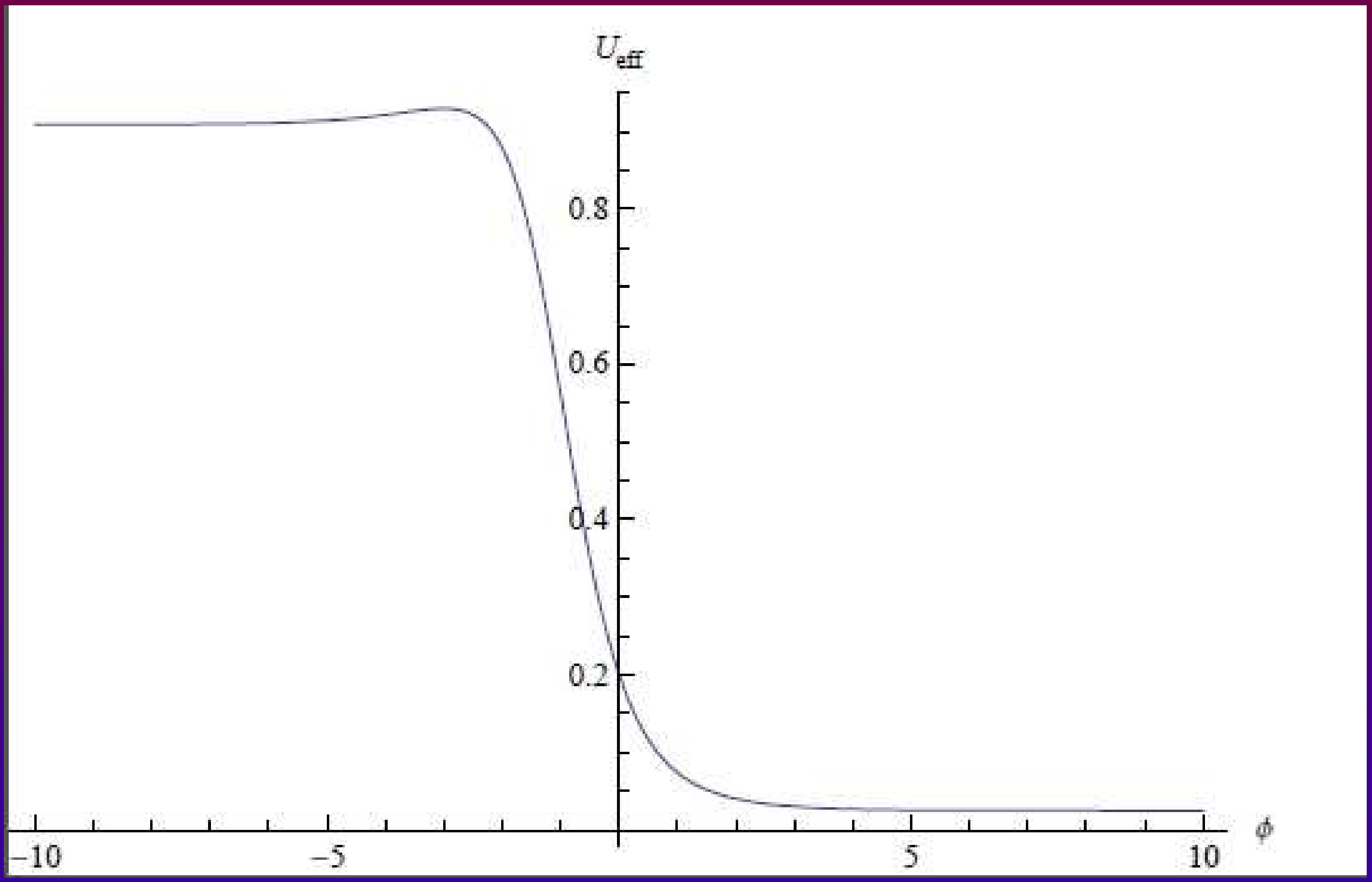
Most remarkable feature of the effective scalar potential $U_{\rm eff}(\varphi)$ (37) – two **infinitely large flat regions**:

- **(-) flat region** – for large negative values of φ:

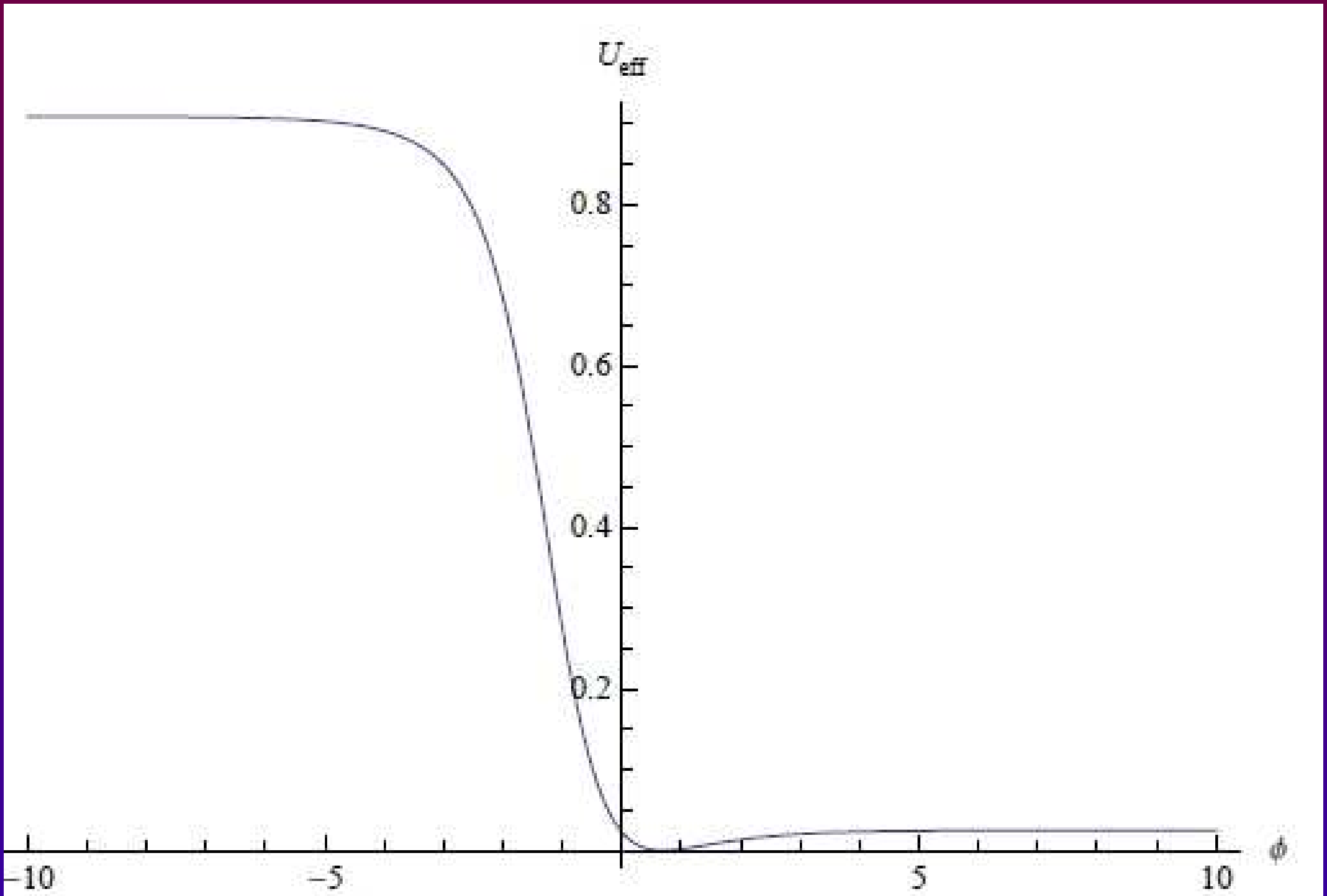
$$U_{\rm eff}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1+\epsilon f_1^2/f_2)} \;, \tag{38}$$

- **(+) flat region** – for large positive values of φ:

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)} \;, \tag{39}$$



Qualitative shape of the effective scalar potential U_{eff} (37) as function of φ for $M_1 < 0$.



Qualitative shape of the effective scalar potential U_{eff} (37) as function of φ for $M_1 > 0$.

Generalized Modified-Measure Theories

From the expression for $U_{\rm eff}(\varphi)$ (37) and the figures 1 and 2 we deduce that we have an **explicit realization of quintessential inflation scenario** (continuously connecting an inflationary phase to a slowly accelerating “present-day” universe through the evolution of a single scalar field).

The flat regions (38) and (39) correspond to the evolution of the **early** and the **late** universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{f_1^2/f_2}{1+\epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1+\epsilon M_1^2/M_2}\,, \qquad (40)$$

which makes the **vacuum energy density of the early universe** $U_{(-)}$ **much bigger than that of the late universe** $U_{(+)}$.

The inequality (40) is equivalent to the requirements:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2} \quad , \quad |\epsilon| \frac{M_1^2}{M_2} \ll 1 \; . \tag{41}$$

If we choose the scales $|M_1| \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where $M_{EW}, \, M_{Pl}$ are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density:

$$U_{(+)} \sim M_{EW}^8 / M_{Pl}^4 \sim 10^{-120} M_{Pl}^4 \; , \tag{42}$$

which is the right order of magnitude for the present epoche's vacuum energy density.

Generalized Modified-Measure Theories

On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2} M_{Pl})^4$, then the order of magnitude of the vacuum energy density of the early universe becomes:

$$U_{(-)} \sim f_1^2/f_2 \sim 10^{-8} M_{Pl}^4 \,, \tag{43}$$

which conforms to the Planck Collaboration data (also BICEP2) implying the energy scale of inflation of order $10^{-2} M_{Pl}$.

“Emergent universe”

There exists explicit cosmological solution of the Einstein-frame system (34)-(37) describing an epoch of a non-singular creation of the universe – “emergent universe”, preceding the inflationary phase. The starting point are the Friedman eqs.:

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \; , \qquad (44)$$

describing the universe’ evolution. Here:

$$\rho = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{3}{4}B(\varphi)\,\dot{\varphi}^4 + U_{\rm eff}(\varphi) \; , \qquad (45)$$

$$p = \frac{1}{2}A(\varphi)\,\dot{\varphi}^2 + \frac{1}{4}B(\varphi)\,\dot{\varphi}^4 - U_{\rm eff}(\varphi) \qquad (46)$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$.

"Emergent universe"

"Emergent universe" is defined as a solution of the Friedman eqs.(44) subject to the condition on the Hubble parameter H:

$$H = 0 \quad \to \quad a(t) = a_0 = \mathrm{const}\,, \quad \rho + 3p = 0 \;\;, \quad \frac{K}{a_0^2} = \frac{1}{6}\rho \; (= \mathrm{const}) \;, \tag{47}$$

with ρ and p as in (45)-(46). Here $K = 1$ ("Einstein universe"). The "emergent universe" condition (47) implies that the φ-velocity $\dot{\varphi} \equiv \dot{\varphi}_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2} B_{(-)} \, \dot{\varphi}_0^{\,4} + 2 A_{(-)} \, \dot{\varphi}_0^{\,2} - 2 U_{(-)} = 0 \;, \tag{48}$$

where $A_{(-)}$, $B_{(-)}$, $U_{(-)}$ are the limiting values on the $(-)$ flat region of $A(\varphi)$, $B(\varphi)$, $U_{\mathrm{eff}}(\varphi)$ (35)-(37).

“Emergent universe”

The solution of Eq.(48) reads:

$$\dot{\varphi}_0^{\,2} = -\frac{2}{3B_{(-)}}\left[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\right] . \qquad (49)$$

and, thus, the “emergent universe” is characterized with **finite initial** Friedman factor and density:

$$a_0^2 = \frac{6K}{\rho_0} \quad , \quad \rho_0 = \frac{1}{2}A_{(-)}\,\dot{\varphi}_0^{\,2} + \frac{3}{4}B_{(-)}\,\dot{\varphi}_0^{\,4} + U_{(-)} \, , \qquad (50)$$

with $\dot{\varphi}_0^{\,2}$ as in (49).

“Emergent universe”

Analysis of stability of the “emergent universe” solution (50) yields a harmonic oscillator type equation for the perturbation of the Friedman factor δa:

$$\delta\,\ddot{a} + \omega^2 \delta a = 0 \quad , \quad \omega^2 \equiv \frac{2}{3}\rho_0 \frac{\sqrt{A^2_{(-)} + 3B_{(-)}U_{(-)}}}{A_{(-)} - 2\sqrt{A^2_{(-)} + 3B_{(-)}U_{(-)}}} \; . \tag{51}$$

Thus stability condition $\omega^2 > 0$ yields the following constraint on the coupling parameters:

$$\max\Big\{-2\, ,\, -8\big(1+3\epsilon f_1^2/f_2\big)\Big[1-\sqrt{1-\frac{1}{4\big(1+3\epsilon f_1^2/f_2\big)}}\Big]\Big\} < b\frac{f_1}{f_2} < -1 \; . \tag{52}$$

"Emergent universe"

Since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the $(-)$ flat region of the effective scalar potential, *i.e.*, the vacuum energy density in the early universe, must be large (cf. (40)), we find that the lower end of the interval in (52) is very close to the upper end, *i.e.*, $b\frac{f_1}{f_2} \simeq -1$.

From Eqs.(49)-(50) we obtain an inequality satisfied by the initial energy density ρ_0 in the emergent universe: $U_{(-)} < \rho_0 < 2U_{(-)}$, which together with the estimate of the order of magnitude for $U_{(-)}$ (43) implies order of magnitude for $a_0^2 \sim 10^{-8} K M_{Pl}^{-2}$, where K is the Gaussian curvature of the spacial section.

Coupling to Charge-Confining Gauge Field

G. ‘t Hooft phenomenological confinement proposal: the energy density of electrostatic field configurations in the low-energy description of confining quantum gauge theories must be a linear function of the electric displacement field in the infrared region (the latter appearing as a quantum “infrared counterterm”).

Explicit realization of ‘t Hoofts idea [Guendelman et.al.]:

$$S = \int d^4x \sqrt{-g}\Big[L(F^2) + A_\mu J^\mu\Big] \quad , \quad L(F^2) = -\frac{1}{4}F^2 - \frac{f_0}{2}\sqrt{-F^2}\,, \tag{53}$$

where $F^2 \equiv F^2(g) = F_{\kappa\lambda}F_{\mu\nu}g^{\kappa\mu}g^{\lambda\nu}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The square root of the Maxwell term naturally arises as a result of **spontaneous breakdown of scale symmetry** of the original scale-invariant Maxwell action with f_0 appearing as an integration constant responsible for the spontaneous breakdown.

Coupling to Charge-Confining Gauge Field

The nonlinear gauge field action (53) yields eqs. of motion:

$$\partial_\nu\Big(\sqrt{-g}4L'(F^2)F^{\mu\nu}\Big)+\sqrt{-g}J^\mu = 0 \quad , \quad L'(F^2) = -\frac{1}{4}\Big(1-\frac{f_0}{\sqrt{-F^2}}\Big) \ , \tag{54}$$

whose $\mu = 0$ component – the nonlinear “Gauss law” constraint equation reads:

$$\frac{1}{\sqrt{-g}}\partial_i\big(\sqrt{-g}D^i\big) = J^0 \quad , \quad D^i = \Big(1 - \frac{f_0}{\sqrt{-F^2}}\Big)F^{0i} \ , \tag{55}$$

with $\vec{D} \equiv (D^i)$ denoting the electric displacement field nonlinearly related to the electric field $\vec{E} \equiv (F^{0i})$ as in the last relation (55).

Coupling to Charge-Confining Gauge Field

In the nonlinear gauge field theory (53) there exists a nontrivial vacuum solution $\sqrt{-F^2_{\rm vac}} = f_0$, which implies simultaneous vanishing of the electric displacement field, $\vec{D} = 0$ meaning zero observed charge, and at the same time nontrivial electric field. In particular, for static spherically symmetric fields in static spherically symmetric metric the only surviving component of $F_{\mu\nu}$ is the nonvanishing radial component of the electric field $E^r = -F_{0r}$, so that $\sqrt{-F^2_{\rm vac}} = \sqrt{2}|\vec{E}| = f_0$. This can be viewed as the simplest classical manifestation of charge confinement:

$\vec{D} = 0$ **and nontrivial** $\vec{E}$.

Canonically quantizing the spherically symmetric restriction of (53) we are able to show that the effective potential between two oppositely charged fermions is of the “Cornell”-type:

$$V_{\rm eff}(L) = -\frac{e_0^2}{2\pi}\frac{1}{L} + e_0 f_0 \sqrt{2}\, L + \big(L\text{−independent const}\big).$$

Coupling to Charge-Confining Gauge Field

Let us now consider the gravity-matter model with two different non-Riemannian volume-forms (25) coupled to the charge-confining (53):

$$S = \int d^4x\, \Phi_1(A)\Big[R + L^{(1)} - \frac{f_0}{2}\sqrt{-F^2(g)}\Big]$$
$$+ \int d^4x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 - \frac{1}{4e^2}F^2(g) + \frac{\Phi(H)}{\sqrt{-g}}\Big]\,. \tag{56}$$

Repeating the same steps as with (25) above, the Einstein-frame effective matter/gauge field Lagrangian takes the generalized “k-essence” form as (34) with the same “k-essence” coefficient functions $A(\varphi),\, B(\varphi)$ (35)-(36) and effective scalar potential $U_{\rm eff}(\varphi)$ (37) possessing two infinitely large $(-)$ and $(+)$ flat regions, however, now it contains additional gauge field dependent terms:

Coupling to Charge-Confining Gauge Field

$$L_{\rm eff} = A(\varphi)X + B(\varphi)X^2 - U_{\rm eff}(\varphi)$$
$$- \frac{F^2(\bar{g})}{4e^2_{\rm eff}(\varphi)} - \frac{f_{\rm eff}(\varphi)}{2}\sqrt{-F^2(\bar{g})} - \epsilon\chi_2 f_0 A(\varphi) X \sqrt{-F^2(\bar{g})}\;, \qquad (57)$$

where now the gauge coupling constants are "running" with the "dilaton" φ:

$$f_{\rm eff}(\varphi) = f_0 \frac{f_2 e^{-2\alpha\varphi} + M_2}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2}\;, \qquad (58)$$

$$\frac{1}{e^2_{\rm eff}(\varphi)} = \chi_2 \Big[\frac{1}{e^2} + \epsilon\, f_0^2 \frac{f_2 e^{-2\alpha\varphi} + M_2}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2}\Big]\;. \qquad (59)$$

(recall $V = f_1 e^{-\alpha\varphi}$ and $U = f_2 e^{-2\alpha\varphi}$),

“Vacuum” Configurations

The eqs. motion resulting from Einstein-frame Lagrangian (57):

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu\varphi\frac{\partial L_{\rm eff}}{\partial X}\right)-\frac{\partial L_{\rm eff}}{\partial\varphi}=0 \quad , \quad \partial_\nu\left(\sqrt{-\bar{g}}F^{\mu\nu}\frac{\partial L_{\rm eff}}{\partial F^2}\right)=0 \tag{60}$$

allow for the following two classes of nontrivial “vacuum” solutions:

- (i) “Standard vacuum” containing standard constant “dilaton” vacuum plus nontrivial gauge field vacuum:

$$\varphi = \mathrm{const} \quad \to \quad X = 0 \quad , \quad \frac{\partial L_{\rm eff}}{\partial\varphi} = 0 \quad , \quad \frac{\partial L_{\rm eff}}{\partial F^2} = 0 \, . \tag{61}$$

Here the value $\varphi = \mathrm{const}$ belongs to either the $(-)$ flat region (38) or the $(+)$ flat region (39) of the effective scalar potential.

- (ii) “Kinetic vacuum” (this type of “vacuum” exists thanks to the nonlinear w.r.t. X “k-essence” nature of the effective Lagrangian (57)):

$$\frac{\partial L_{\rm eff}}{\partial X} = 0 \quad , \quad \frac{\partial L_{\rm eff}}{\partial \varphi} = 0 \quad , \quad \frac{\partial L_{\rm eff}}{\partial F^2} = 0 \ . \tag{62}$$

  Here the “dilaton” $\varphi = \varphi(x)$ will be slightly space-varying but its values again will belong to either the $(-)$ flat region (38) or the $(+)$ flat region (39).

Because of the presence of the two flat regions of the effective scalar potential, the scalar dilation second-order eqs. of motion are automatically (approximately) satisfied.

“Standard Vacuums” = de Sitter + Confinement

In the first class of “standard vacuum” solutions the last equation (61) yields the following non-trivial “vacuum” value for the gauge field:

$$\sqrt{-F^2_{(\pm)}} = e^2_{(\pm)} f_{(\pm)} \; . \tag{63}$$

Here and below the subscripts $(\pm)$ indicate limiting values of $e^2_{\rm eff}(\varphi),\; f_{\rm eff}(\varphi)$ (59)-(58) on the $(\pm)$ flat regions of effective scalar potential. For the associated matter energy-momentum tensor we get:

$$T^{\rm eff}_{\mu\nu} = \bar{g}_{\mu\nu} L_{\rm eff}\, \Big|_{X=0,\, \frac{\partial L_{\rm eff}}{\partial F^2}=0} = -\bar{g}_{\mu\nu} U^{(\rm standard)}_{(\pm)} \tag{64}$$

where $U^{(\rm standard)}_{(\pm)}$ is the total effective scalar potential in the “standard vacuums” (61):

“Standard Vacuums” = de Sitter + Confinement

$$U^{(\mathrm{standard})}_{(-)} = U_{(-)} + \frac{1}{4} e^2_{(-)} f^2_{(-)} = \frac{1}{4\epsilon\chi_2}\Big[1 - \frac{1}{1 + \epsilon f_1^2/f_2 + \epsilon e^2 f_0^2}\Big] \; , \quad (65)$$

$$U^{(\mathrm{standard})}_{(+)} = U_{(+)} + \frac{1}{4} e^2_{(+)} f^2_{(+)} = \frac{1}{4\epsilon\chi_2}\Big[1 - \frac{1}{1 + \epsilon M_1^2/M_2 + \epsilon e^2 f_0^2}\Big] \; . \quad (66)$$

Therefore, according to (64) the solutions of the Einstein-frame $\bar{g}_{\mu\nu}$-equations are of de Sitter type (dS or Schw-dS):

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -\mathcal{A}(r) dt^2 + \frac{dr^2}{\mathcal{A}(r)} + r^2 \left(d\theta^2 + \sin^2\theta d\phi\right) \; , \quad (67)$$

$$\mathcal{A}(r) = 1 - \frac{\Lambda_{(\pm)}}{3} r^2 \quad , \quad \mathrm{or} \;\; \mathcal{A}(r) = 1 - \frac{2m}{r} - \frac{\Lambda_{(\pm)}}{3} r^2 \quad (68)$$

in static spherically symmetric coordinate chart, with effective **dynamically induced** cosmological constants $\Lambda_{(\pm)}$ given by:

$\Lambda_{(-)} = \frac{1}{2} U^{(\mathrm{standard})}_{(-)} \quad , \quad \Lambda_{(+)} = \frac{1}{2} U^{(\mathrm{standard})}_{(+)}$.

“Standard Vacuums” = de Sitter + Confinement

Let us recall that as demonstrated above, the strength of charge confinement is proportional to the non-zero vacuum value of the nonlinear gauge field $\sqrt{-F^2_{(\pm)}}$ (63).

Now, from the above analysis of the “standard vacuum” solutions – given by $\varphi = \mathrm{const}$ belonging to the $(\pm)$ flat regions of the effective scalar potential (38)-(39) and possessing **non-zero gauge field vacuum values** (63) and non-zero vacuum energy densities (65)-(66) – we conclude that these “standard vacuum” solutions describe **charge confining** phases of different confining strength and with different **dynamically generated** cosmological constants. The latter property is analogous to the above cosmological scenario context where the evolution of the early and late universe was related to two flat regions of the effective scalar potential (two different vacuum energy densities).

"Kinetic vacuum"

The "kinetic vacuum" eqs. $\frac{\partial L_{\rm eff}}{\partial X} = 0$ and $\frac{\partial L_{\rm eff}}{\partial F^2} = 0$ yield:

$$X_{\rm kin} = -\frac{A}{2B}\,\frac{1 - \epsilon\chi_2 f_0 f_{\rm eff} e^2_{\rm eff}}{1 - \epsilon^2\chi^2_2 f^2_0 e^2_{\rm eff} A^2/B}\;, \tag{69}$$

$$\sqrt{-F^2_{\rm kin}} = e^2_{\rm eff}\,\frac{f_{\rm eff} - \epsilon\chi_2 f_0 A^2/B}{1 - \epsilon^2\chi^2_2 f^2_0 e^2_{\rm eff} A^2/B}\;, \tag{70}$$

$$T^{\rm eff}_{\mu\nu} = \bar{g}_{\mu\nu} L_{\rm eff}\,\Big|_{\frac{\partial L_{\rm eff}}{\partial X}=0,\,\frac{\partial L_{\rm eff}}{\partial F^2}=0} = -\bar{g}_{\mu\nu} U^{(\rm kinetic)}_{\rm total}\;, \tag{71}$$

where $U^{(\rm kinetic)}_{\rm total}$ is the total effective scalar potential in the "kinetic vacuum" (62):

$$U^{(\rm kinetic)}_{\rm total} = U_{\rm eff} + \frac{A^2}{4B} + \frac{1}{4} e^2_{\rm eff}\,\frac{\left(f_{\rm eff} - \epsilon\chi_2 f_0 \frac{A^2}{B}\right)^2}{1 - e^2_{\rm eff}\epsilon^2\chi^2_2 f^2_0 \frac{A^2}{B}}\;. \tag{72}$$

“Kinetic vacuum” on the $(+)$ Flat Region

From (72) we deduce that in the “kinetic vacuum” the effective gauge coupling constants become:

$$\widetilde{f}_{\rm eff} = f_{\rm eff} - \epsilon\chi_2 f_0 \frac{A^2}{B} \quad , \quad \widetilde{e}^2_{\rm eff} = \frac{e^2_{\rm eff}}{1 - e^2_{\rm eff}\epsilon^2\chi_2^2 f_0^2 \frac{A^2}{B}} \tag{73}$$

Inserting in (69)-(72) the values of the respective parameters for the $(+)$ flat region of the effective scalar potential yields:

$$\sqrt{-F^2_{\rm kin}}\,\Big|_{(+)} = 0 \quad , \quad X_{\rm kin} \simeq X_{(+)} = -\frac{A_{(+)}}{2B_{(+)}} = -\frac{1}{2\epsilon\chi_2} \tag{74}$$

$$U^{(\rm kinetic)}_{\rm total} \simeq U^{(\rm kinetic)}_{(+)} = \frac{1}{4\epsilon\chi_2} \quad \to \quad T^{\rm eff}_{\mu\nu} = -\bar{g}_{\mu\nu}\frac{1}{4\epsilon\chi_2} \; , \tag{75}$$

i.e., we have here an effective cosmological constant:

$$\Lambda_{(+)} \equiv \Lambda^{(\rm kinetic)}_{(+)} = \frac{1}{8\epsilon\chi_2} \; . \tag{76}$$

“Kinetic vacuum” on the $(+)$ Flat Region

Remarkable feature: the first relation in (74) – $\sqrt{-F^2_{\rm kin}}\,\big|_{(+)} = 0$, *i.e.*, the zero vacuum value for the nonlinear gauge field, which is due to the vanishing of the effective coupling constant of the “square-root” Maxwell term (73) on the $(+)$ flat region.

In accordance with ‘t Hooft’s phenomenological confinement proposal and as demonstrated explicitly in [GNP, 2015], the latter implies **absence of confinement of charged particles**, *i.e.*, the “kinetic vacuum” (74)-(75) describes a **deconfinement** phase.

According to (71) and (75)-(76) the solutions of the Einstein-frame $\bar{g}_{\mu\nu}$-equations in the “kinetic vacuum” are again of de Sitter type (67)-(68) with $\Lambda_{(+)}$ given by (76).

“Kinetic vacuum” on the $(+)$ Flat Region

The equation for the “dilaton” “kinetic vacuum” (second Eq.(74)) reads explicitly:

$$\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{\epsilon\chi_2} = 0 \ . \tag{77}$$

It has precisely the form of Hamilton-Jacobi equation for the Hamilton-Jacobi action:

$$S(x) \equiv \varphi(x) = \frac{1}{\sqrt{\epsilon\chi_2}} \int_{\lambda_{\rm in}}^{\lambda_{\rm out}} d\lambda \sqrt{g_{\mu\nu}(x(\lambda))\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}} \tag{78}$$

corresponding to spacelike geodesics $x^\mu(\lambda)$ starting from some fixed point $x_{(0)}$ (*e.g.*, $x_{(0)} = 0$) at a fixed value of the affine parameter $\lambda_{\rm in}$ and passing through $x = x(\lambda_{\rm out})$ at $\lambda_{\rm out}$. This Hamilton-Jacobi action (78) measures the proper distance between the points $x_{(0)}$ and x on the manifold modulo the numerical factor $1/\sqrt{\epsilon\chi_2}$.

“Kinetic vacuum” on the $(+)$ Flat Region

A static spherically symmetric solution for $\varphi(x)$ is given by:

$$\left(\frac{\partial\varphi}{\partial r}\right)^2 = \frac{1}{\epsilon\chi_2\,\mathcal{A}(r)} \quad \to \quad \varphi(r) = \varphi_{(+)} + \frac{1}{\sqrt{\epsilon\chi_2}}\int^r \frac{dr'}{\sqrt{\mathcal{A}(r')}} \;, \qquad (79)$$

where the initial value $\varphi_{(+)}$ must belong to the $(+)$ flat region (large positive φ).

In the case of pure de Sitter metric (67)-(68) the solution $\varphi(r)$ (79), measuring the proper radial distance between 0 and r, is clearly defined only for r in the interval $r \in (0, r_{(+)})$, where $r_{(+)} = \sqrt{24\epsilon\chi_2}$ is the de Sitter horizon radius.

The solution $\varphi(r)$ reads explicitly:

$$\varphi(r) = \varphi_{(+)} + \sqrt{24}\,\mathrm{arcsin}\Big(\frac{r}{r_{(+)}}\Big) \;, \qquad (80)$$

where the initial value $\varphi_{(+)}$ belongs to the $(+)$ flat region of the effective scalar potential.

“Kinetic vacuum” on the $(+)$ Flat Region

Since the “kinetic vacuum” corresponding to the $(+)$ flat region described by (74)-(80) is defined only within the finite-volume space region below the de Sitter horizon, in order to be extended to the whole space it must be matched to another spherically symmetric configuration with the standard constant “dilaton” vacuum defined in the outer region beyond the de Sitter horizon with:

$$\varphi = \varphi(r_{(+)}) = \varphi_{(+)} + \sqrt{6}\pi = \mathrm{const} \quad \mathrm{for}\ r > r_{(+)}\ , \tag{81}$$

where the latter is the limiting value of (80) at the horizon. The corresponding construction yields a gravitational bag-like solution mimicking both some of the features of the MIT bags in QCD phenomenology as well as some of the features of the “constituent quark” model.

Gravitational Bag-like Solutions

Here we construct matching of the “kinetic vacuum” in $(+)$ flat region of the effective scalar potential given by de Sitter metric (67)-(68) in the interior region ($r < r_{(+)}$) below the de Sitter horizon $r_{(+)} = \sqrt{24\epsilon\chi_2}$ with effective cosmological constant (76) and by Eqs.(74)-(80), to a static spherically symmetric configuration containing the standard constant “dilaton” vacuum (81) in the outer region ($r > r_{(+)}$) beyond the de Sitter horizon. The “matching” specifically means that the “dilaton” field, the gauge field strength and the metric with its first derivatives must be continuous across the horizon, in particular, the de Sitter horizon of the interior metric must coincide with a horizon of the exterior metric.

Gravitational Bag-like Solutions

Previously we have already explicitly derived static spherically symmetric solutions of the coupled gravity/nonlinear gauge field/scalar "dilaton" system (57) with a **generalized Reissner-Nordström-(anti)de Sitter** geometry carrying a **non-vanishing background constant radial electric field** in addition to the standard Coulomb field. We will use this type of solution in the outer region beyond the de Sitter horizon to be matched with the "kinetic vacuum" (74)-(80) in the interior region.

Specifically, for $r > r_{(+)} = \sqrt{24\epsilon\chi_2}$ the solution reads:

$$ds^2 = -\mathcal{A}_{\rm out}(r)dt^2 + \frac{dr^2}{\mathcal{A}_{\rm out}(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \; , \quad (82)$$

$$\mathcal{A}_{\rm out}(r) = 1 + \frac{1}{16\pi}\left[-\sqrt{8\pi}|Q|f_{(+)} - \frac{2m}{r} + \frac{Q^2}{e^2_{(+)}\, r^2}\right] - \frac{\Lambda_{\rm out}}{3}r^2 \; , \quad (83)$$

Gravitational Bag-like Solutions

$$\Lambda_{\rm out} = \frac{1}{2} U^{(\rm standard)}_{(+)} = \frac{1}{8\epsilon\chi_2}\Big[1 - \frac{1}{1+\epsilon M_1^2/M_2 + \epsilon e^2 f_0^2}\Big] \; ,$$

$$\sqrt{-F^2_{\rm out}(r)} = \sqrt{2}|\vec{E}_{\rm out}(r)| = e^2_{(+)} f_{(+)} - \frac{|Q|}{\sqrt{2\pi}\, r^2} \; , \qquad (84)$$

$$\varphi = \varphi(r_{(+)}) = \mathrm{const} \;\; (\text{as in } (81)) \; , \qquad (85)$$

where $e^2_{(+)}, \, f_{(+)}$ are the limiting values of the "running" gauge coupling constants on the $(+)$ flat region of the effective scalar potential:

$$e^2_{\rm eff}(\varphi) \simeq e_{(+)} \equiv \frac{e^2}{\chi_2} \frac{1+\epsilon M_1^2/M_2}{1+\epsilon M_1^2/M_2 + e^2\epsilon f_0^2} \; ,$$

$$f_{\rm eff}(\varphi) \simeq f_{(+)} \equiv \frac{f_0}{1+\epsilon M_1^2/M_2} \; . \qquad (86)$$

Gravitational Bag-like Solutions

The matching of the exterior solution (82)-(85) with the interior region "kinetic vacuum" (74)-(80) at the de Sitter horizon uniquely determines m, Q parameters:

$$m = 0 \quad , \quad |Q| = \sqrt{2\pi} e^2_{(+)} f_{(+)}\, 24\epsilon\chi_2 = \frac{\sqrt{2\pi} 24\epsilon e^2 f_0}{1 + \epsilon M_1^2/M_2 + e^2\epsilon f_0^2} \; , \tag{87}$$

with the following additional relation between the integration constants $M_{1,2}$ and the initial coupling constants ϵ, e, f_0:

$$1 + \epsilon \frac{M_1^2}{M_2} - 3\epsilon f_0^2 e^2 = 0 \; . \tag{88}$$

Gravitational Bag-like Solutions

To recapitulate, we have obtained the following “vacuum-like” solution:

- In the inner space region $r < r_{(+)} = \sqrt{24\epsilon\chi_2}$ we have an interior de Sitter region below the de Sitter horizon at $r = r_{(+)}$ with effective cosmological constant $\Lambda_{\rm in} = \frac{1}{2}U^{(\rm kinetic)}_{(+)} = 1/8\epsilon\chi_2$, with vanishing vacuum gauge field (first Eq.(74)), “kinetic vacuum” scalar “dilaton” according to (80) and vacuum energy density (75):

$$\rho_{\rm in} \simeq U^{(\rm kinetic)}_{(+)} = \frac{1}{4\epsilon\chi_2}\ . \tag{89}$$

- In the outer space region $r > r_{(+)} = \sqrt{24\epsilon\chi_2}$ we have static spherically symmetric metric (83) with:

$$\mathcal{A}_{\rm out}(r) = 1 - \frac{1}{2\epsilon e^2 f_0^2} + \frac{6\chi_2}{e^2 f_0^2}\frac{1}{r^2} - \frac{r^2}{24\epsilon\chi_2}\left(1 - \frac{1}{4\epsilon e^2 f_0^2}\right) . \tag{90}$$

Gravitational Bag-like Solutions

- The outside nonlinear gauge field (84) is a static radial electric field of the explicit form:

$$\sqrt{-F^2_{\rm out}(r)} = \sqrt{2}\,|E^r_{\rm out}(r)| = \frac{1}{4\epsilon\chi_2 f_0}\Bigl(1 - \frac{24\epsilon\chi_2}{r^2}\Bigr)\;, \qquad (91)$$

where again we have used (86) and (88). In (91) there is a Coulomb piece in addition to a non-zero background constant radial electric field:

$$|E^r_{\rm background}| = \frac{1}{\sqrt{2}} e^2_{(+)} f_{(+)} = \frac{1}{\sqrt{2}\,4\epsilon\chi_2 f_0}\;. \qquad (92)$$

Thanks to the latter the Coulomb field is completely cancelled at the horizon.

- The outside scalar "dilaton" is constant (85) and the energy density ($\rho = -T^0_0$) reads (using again (86), (66) and (88)):

$$\rho_{\rm out}(r) \simeq U^{(\rm standard)}_{(+)} - e^2_{(+)} f^2_{(+)} \left(\frac{r^2_{(+)}}{2r^2} - \frac{r^4_{(+)}}{4r^4}\right)$$
$$= \frac{1}{4\epsilon\chi_2}\left(1 - \frac{1}{4\epsilon e^2 f_0^2}\right) - \frac{1}{\epsilon e^2 f_0^2\, r^2}\left(1 - \frac{24\epsilon\chi_2}{r^2}\right) . \qquad (93)$$

Obviously (recall in (93) $r > r_{(+)} \equiv \sqrt{24\epsilon\chi_2}$):

$$\rho_{\rm out}(r) \leq U^{(\rm standard)}_{(+)} = \frac{1}{4\epsilon\chi_2}\left[1 - \frac{1}{1 + \epsilon M_1^2/M_2 + \epsilon e^2 f_0^2}\right] < \rho_{\rm in} = \frac{1}{4\epsilon\chi_2} . \qquad (94)$$

Gravitational Bag-like Solutions

The above solution (82)-(94) is a **electrovacuum gravitational bag-like** configuration on the $(+)$ flat region of the effective scalar potential which mimics some of the properties of the MIT bag. Indeed:

(i) In the inner finite volume space region below the horizon ($r < r_{(+)}$) the vanishing vacuum value of the gauge field (first Eq.(74)) implies **absence of confinement of charged particles**.

(ii) According to (94) the vacuum energy density ρ_{in} in the inner finite volume space region (for $r < r_{(+)}$) is larger than the energy density ρ_{out} in the outside region.

Gravitational Bag-like Solutions

There are, however, other properties of the present electrovacuum gravitational "bag" solution which are substantially different from those of the MIT bag and which rather resemble some of the properties of the solitonic "constituent quark" model:

(a) It is charged (the overall charge Q is non-zero (87)).

(b) It carries non-zero "color" flux to infinity – because of the non-zero background constant radial electric field (92).

Conclusions

- Non-Riemannian volume-form formalism in gravity/matter theories (*i.e.*, employing alternative non-Riemannian reparametrization covariant integration measure densities on the spacetime manifold) naturally generates a ***dynamical cosmological constant*** as an arbitrary dimensionful integration constant.
- Within non-Riemannian-modified-measure minimal $N = 1$ supergravity the dynamically generated cosmological constant triggers spontaneous supersymmetry breaking and mass generation for the gravitino (supersymmetric Brout-Englert-Higgs effect).

Conclusions

- Within modified-measure anti-de Sitter supergravity we can fine-tune the dynamically generated cosmological integration constant in order to achieve simultaneously a very small physical observable cosmological constant and a very large physical observable gravitino mass – a paradigm of modern cosmological scenarios for slowly expanding universe of today.
- Employing two different non-Riemannian volume-forms leads to the construction of a new class of gravity-matter models, which produce an effective scalar potential with two infinitely large flat regions. This allows for a unified description of both early universe inflation as well as of present dark energy epoch.

Conclusions

- For a definite parameter range the above model with the two different non-Riemannian volume-forms possesses a *non-singular "emergent universe"* solution which describes an initial phase of evolution that precedes the inflationary phase. For a reasonable choice of the parameters this model conforms to the Planck Collaboration data.
- Adding interaction with a special nonlinear ("square-root" Maxwell) gauge field (known to describe charge confinement in flat spacetime) produces various phases with different strength of confinement and/or with deconfinement, as well as gravitational electrovacuum "bags" partially mimicking the properties of MIT bags and solitonic constituent quark models.

Conclusions

THANK YOU – Merci beaucoup

Vielen Dank – Tack – Multumesc

Gracias – Obrigado – Grazie

хвала – hvala – спасибо – благодаря

спасибi – дзякуй – dziękuję

ありがとう – شكرا – תודה לך – ευχαριστώ