

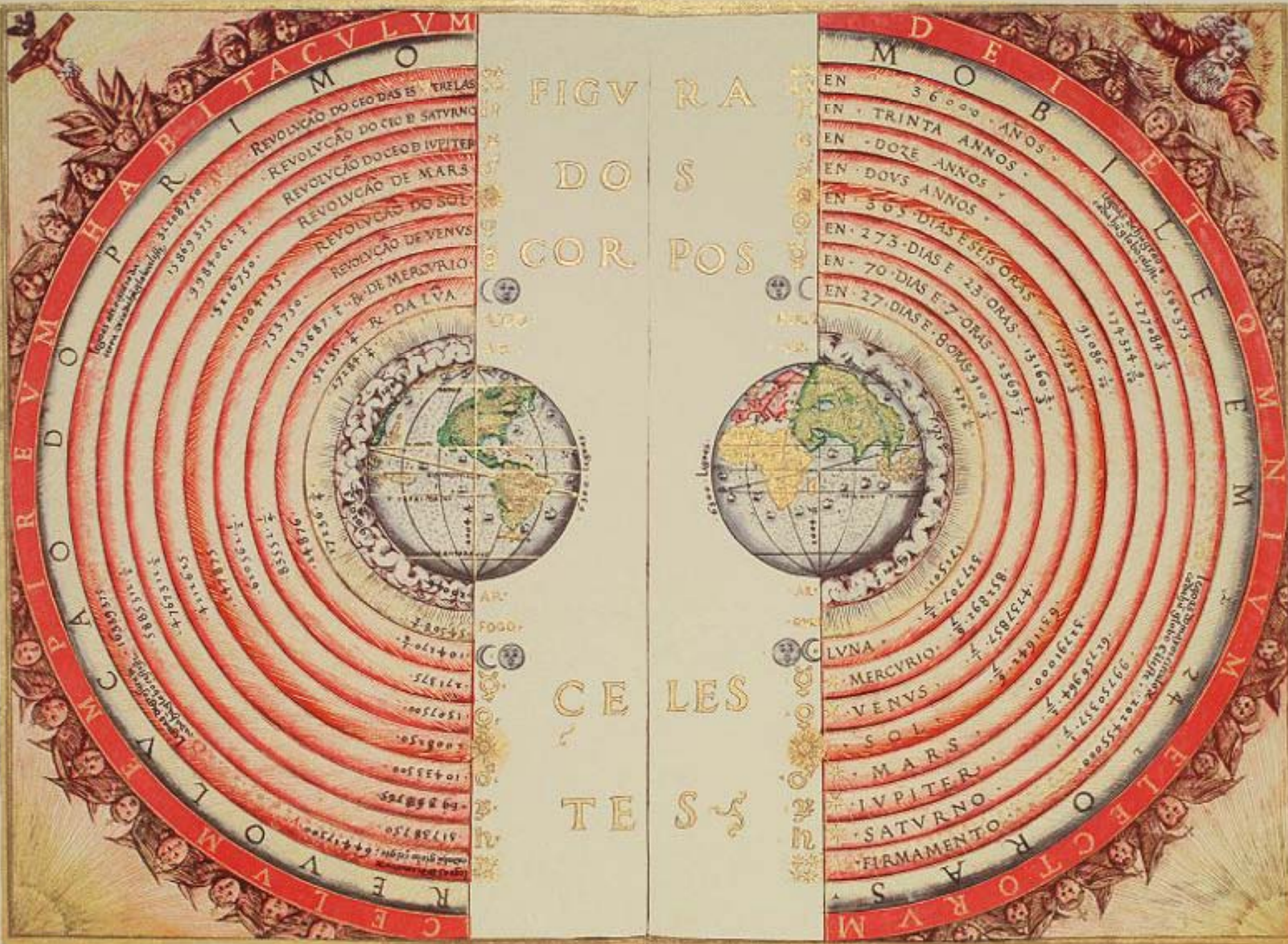
Introduction to Cosmology

II. Modeling the Universe

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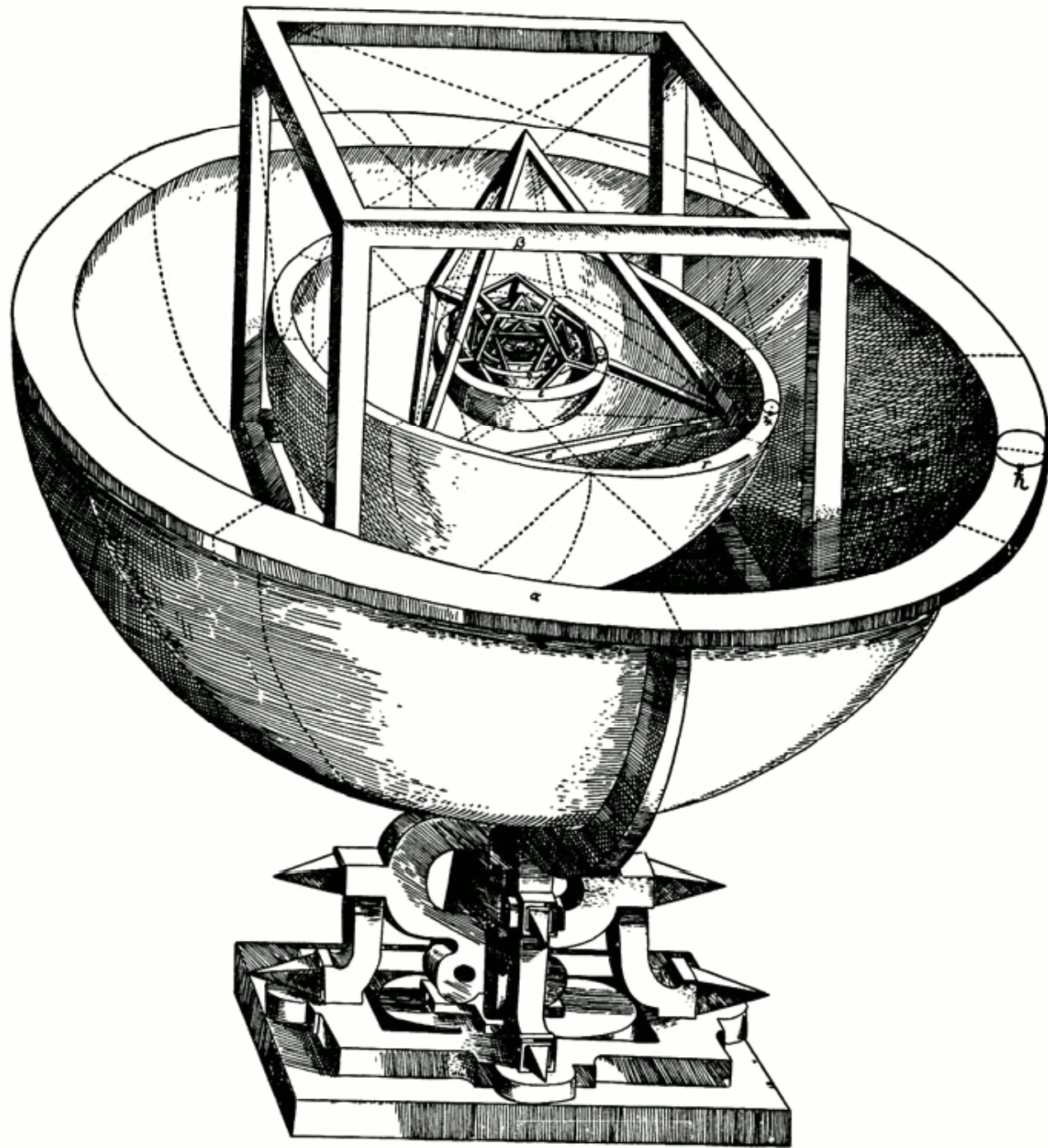
ratiōne salua manente, immo eū cōmmentatiōe allegabit
 q̄ ut magnitudinē orbium multitudine ip̄is notetur, op̄da spha-
 rarum sequitur in hunc modū: a sūmo capientes incensum.

1 Stellarū fixarū sphaera immobilis
 sequitur omniū est stellarum
 xarum sphaera separata
 et omnia continens
 Idcirco immobilis



Idcirco immobilis
 ut tempore
 moti locus
 ad quē
 motus
 us
 et
 p̄

aliqui:
 in deducione motus terrestris assignabimus causam. Sequitur
 errantium primus Saturnus: qui xxx annos suū complet circa
 itā post hunc Iovis duodecimā h̄ revolutione mobilis. Deinde
 Martis vobis qui biennio circūit. Quartū in ordine annū revolu-
 tio locum optinet: in quo terra cum orbe Lunari hanc epicyclo
 continens ducimus. Quinto loco Venus nono mense rediatur







$$\Delta\varphi - \Lambda\varphi = 4\pi G\rho$$

when $\rho = \rho_0 = \text{const}$

$$\varphi = -\frac{4\pi G}{\Lambda}\rho_0$$

$$ds^2 = c^2 dt^2 - \left[\delta_{ab} + \frac{x_a x_b}{R^2 - (x_1^2 + x_2^2 + x_3^2)} \right] dx^a dx^b$$

where $a, b = 1, 2, 3$.

$$G_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu},$$

where $\kappa = \frac{8\pi G}{c^2}$.

$$-\frac{2}{R^2} + \Lambda = -\frac{\kappa \rho}{2}, \quad \Lambda = \frac{\kappa \rho}{2}$$

$$\Lambda = \frac{\kappa \rho}{2} = \frac{1}{R^2}$$

$$M = 2\pi^2 R^3 \rho = 4\pi^2 \frac{R}{\kappa} = \frac{32\pi^2}{\sqrt{\kappa^3 \rho}}$$

Let us calculate Λ and R assuming that

$$\rho = \rho_{crit} = 0.946 \cdot 10^{-29} \text{g/cm}^3$$

$$\Lambda = 8.87 \cdot 10^{-57} \text{cm}^{-2}$$

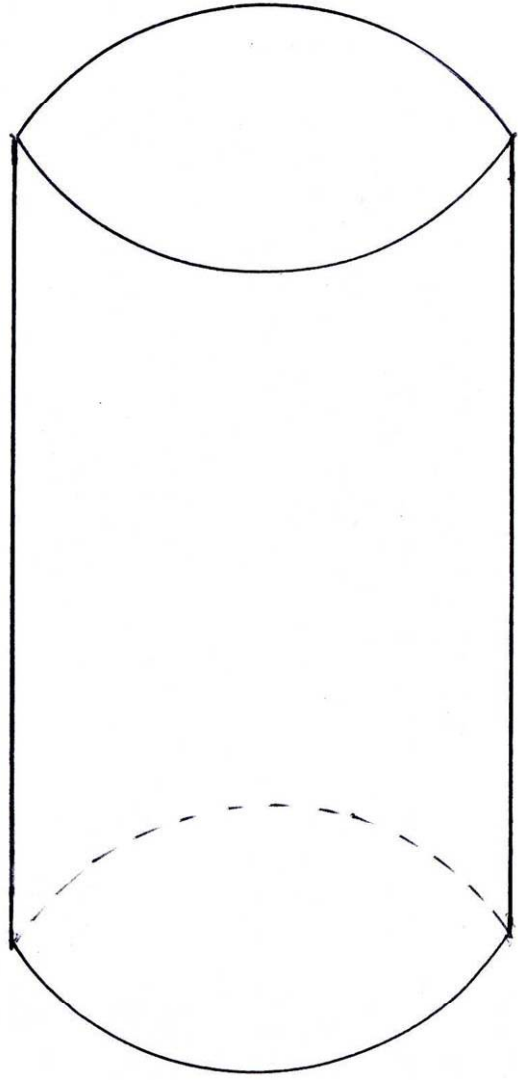
$$R = \frac{1}{\sqrt{\Lambda}} = 1.06 \cdot 10^{28} \text{cm} = 3440 \text{Mpc}$$

Present data

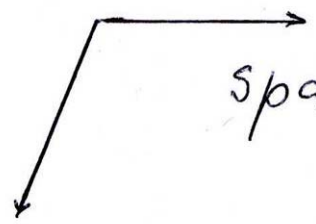
$$R = R_H = 1.2 \cdot 10^{28} \text{cm} = 3896 \text{Mpc}$$

$$\Lambda = 1.46 \cdot 10^{-56} \text{cm}^{-2}$$

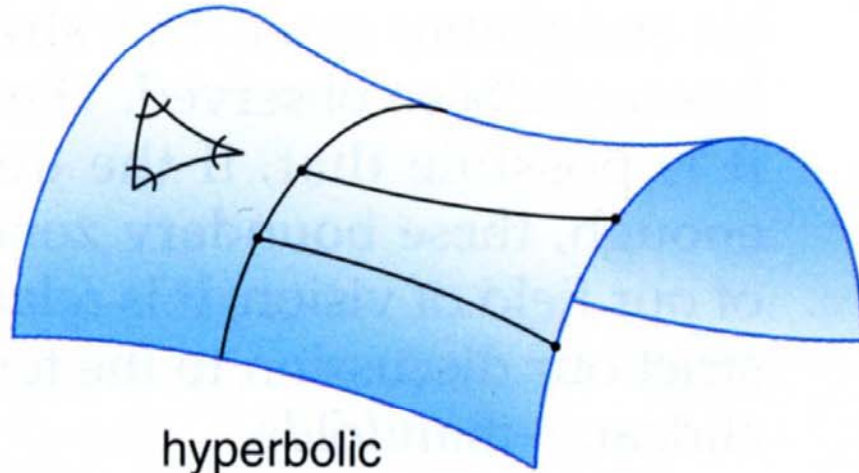
time



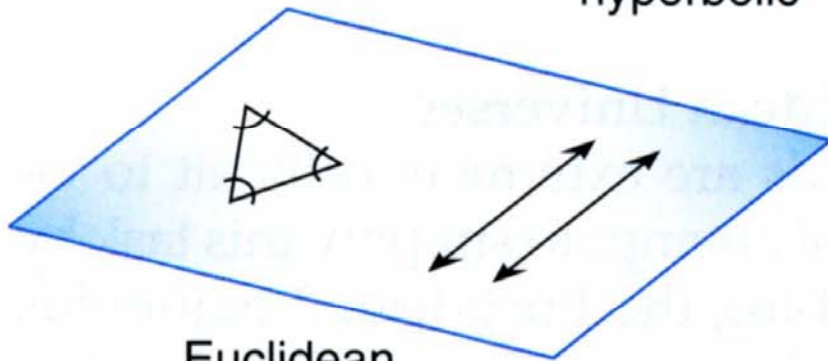
space



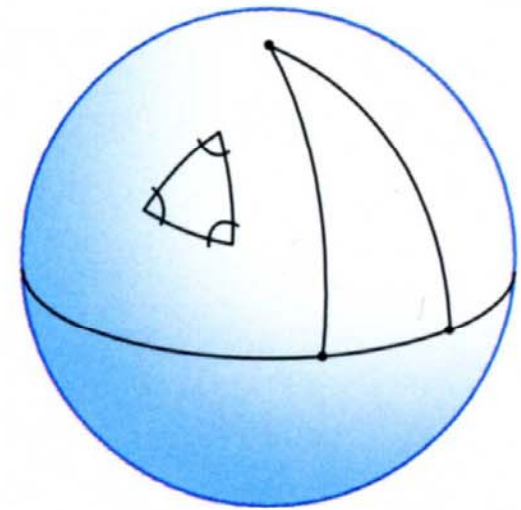




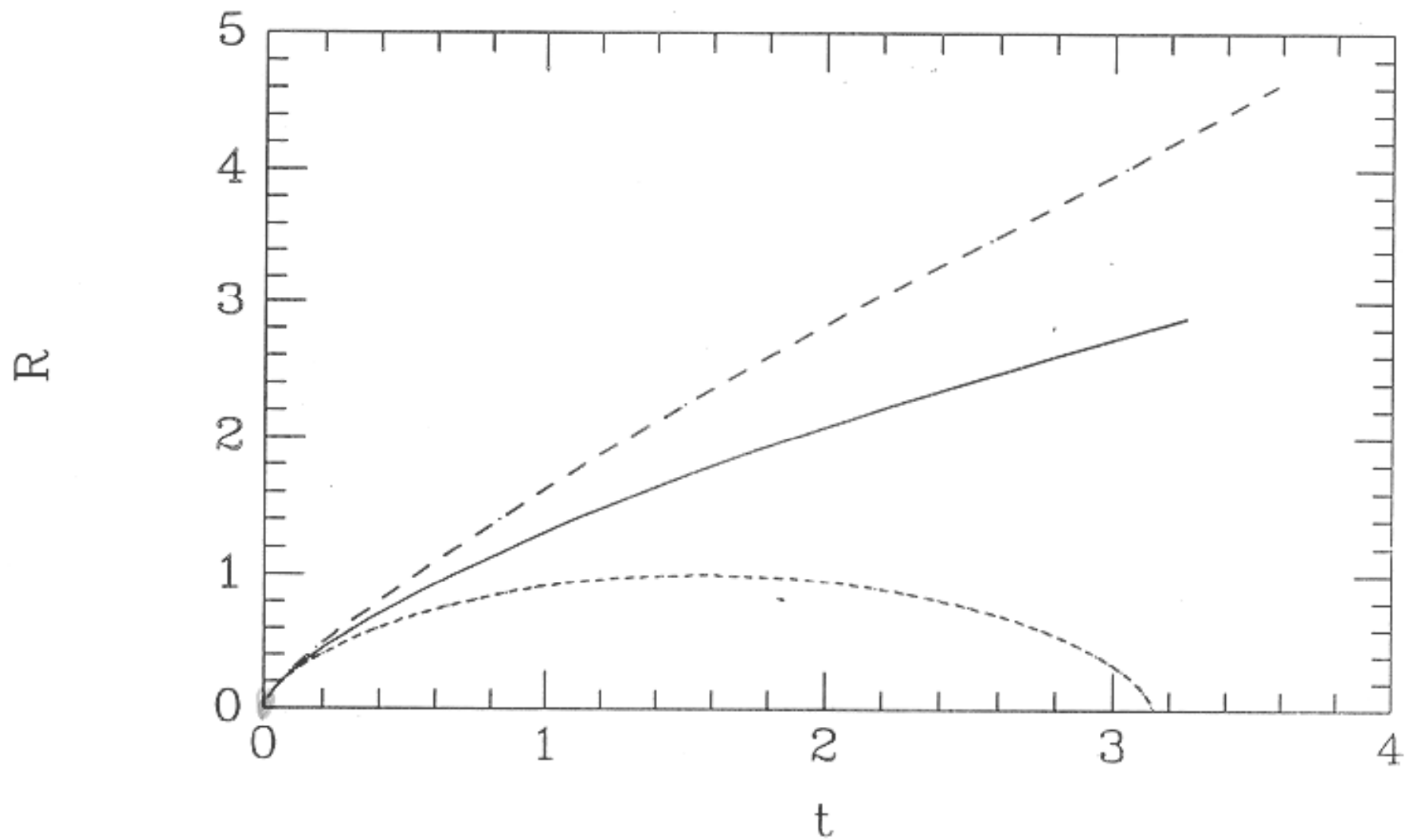
hyperbolic

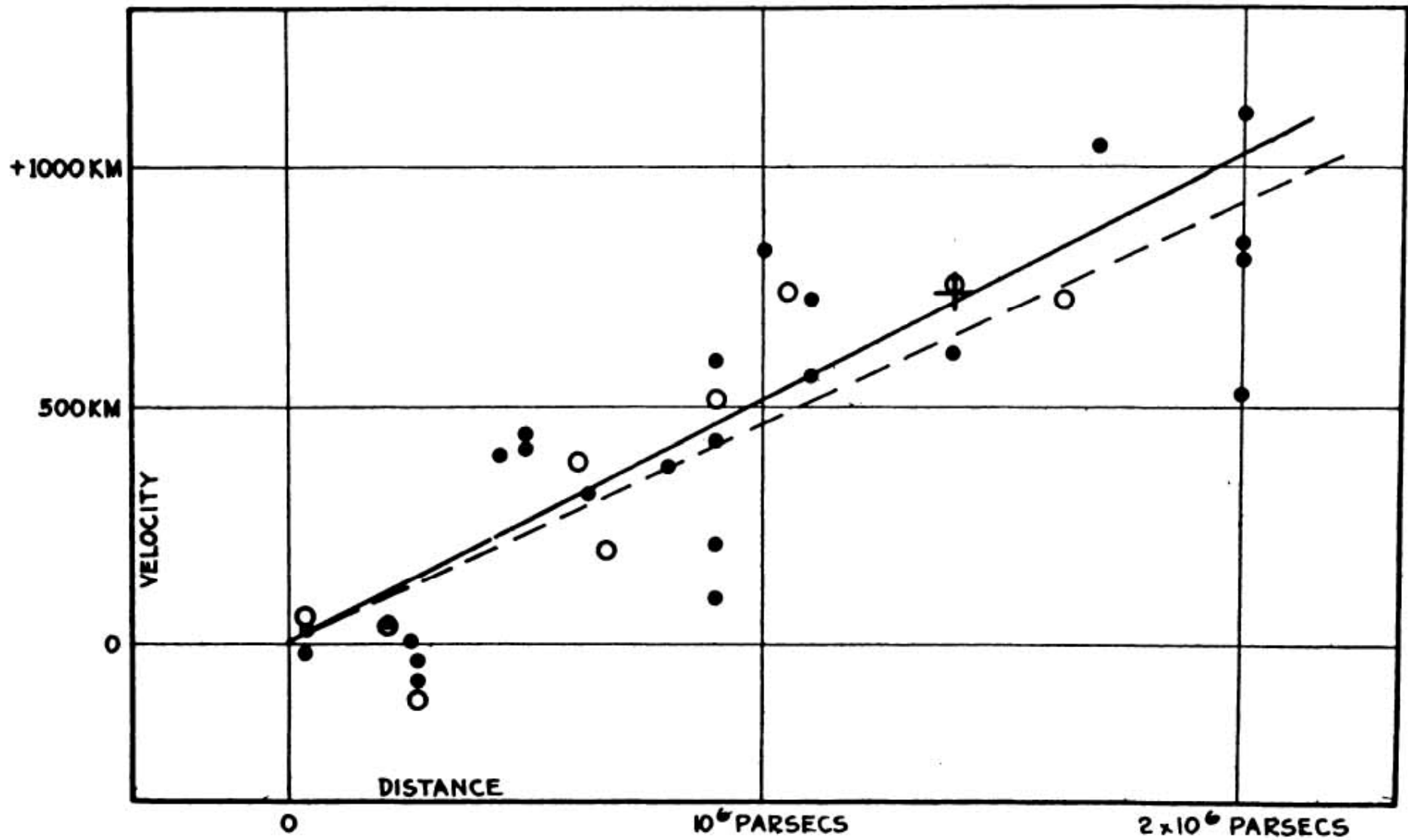


Euclidean

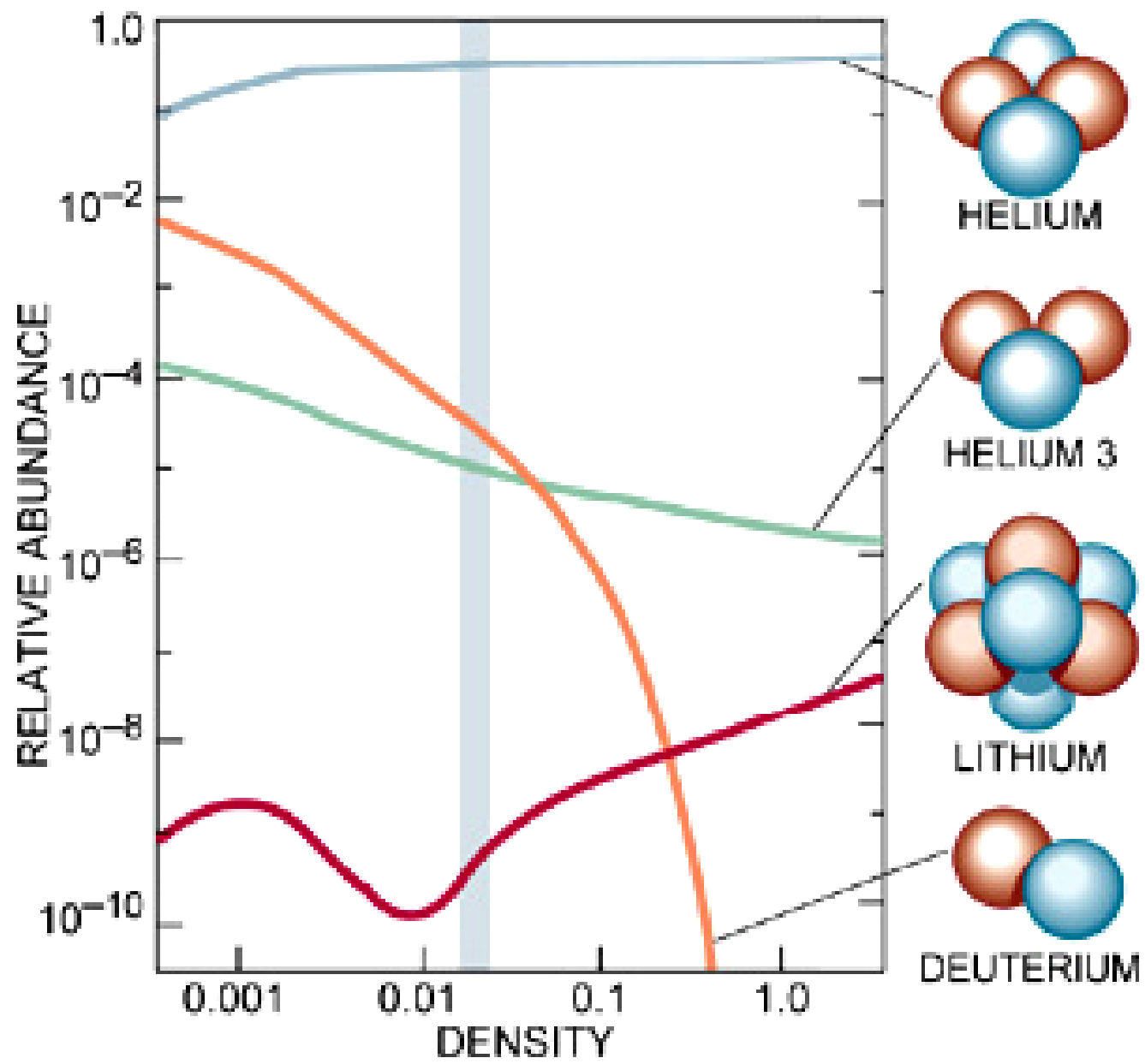


spherical









Similarly, noticing that, in the present era, the radiation-density must vary in inverse proportion to the fourth power of the time (because $\rho \sim T^4$, $T \sim t^{-1}$, and $l \sim t$), we find,

$$\rho_{\text{rad. (present)}} = \frac{3.1 \times 10^{37}}{t^4} \text{ gm per cm}^3. \quad \dots(10)$$

For the present density of residual radiation we obtain 6×10^{-32} , corresponding to about 6°K. Thus we may conclude that the residual heat found at present in the Universe is comparable with the heat provided by nuclear transformations in stars.

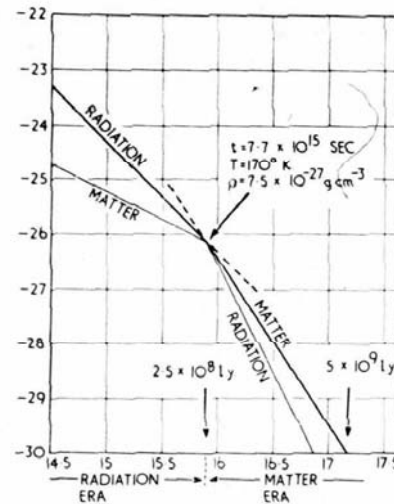
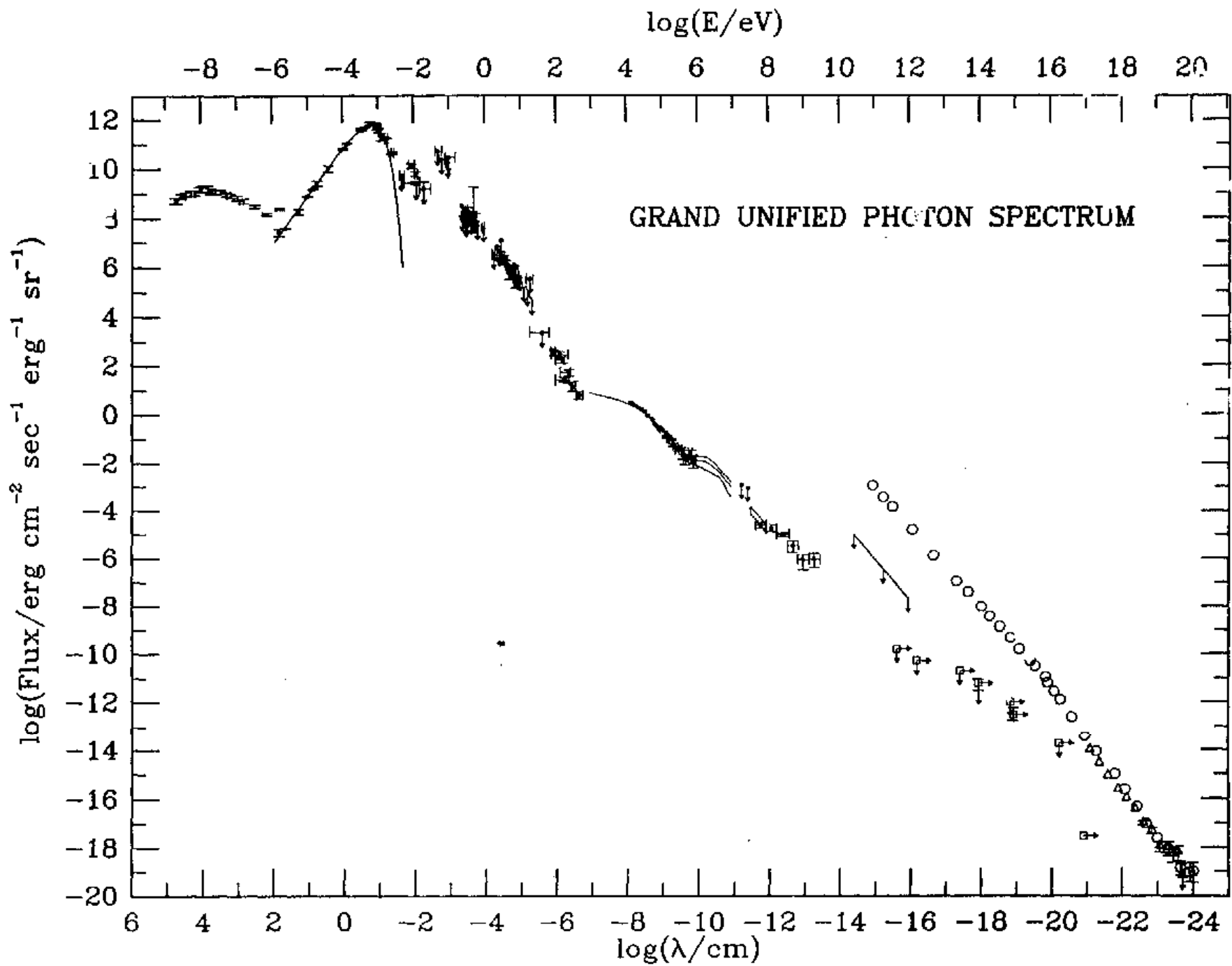


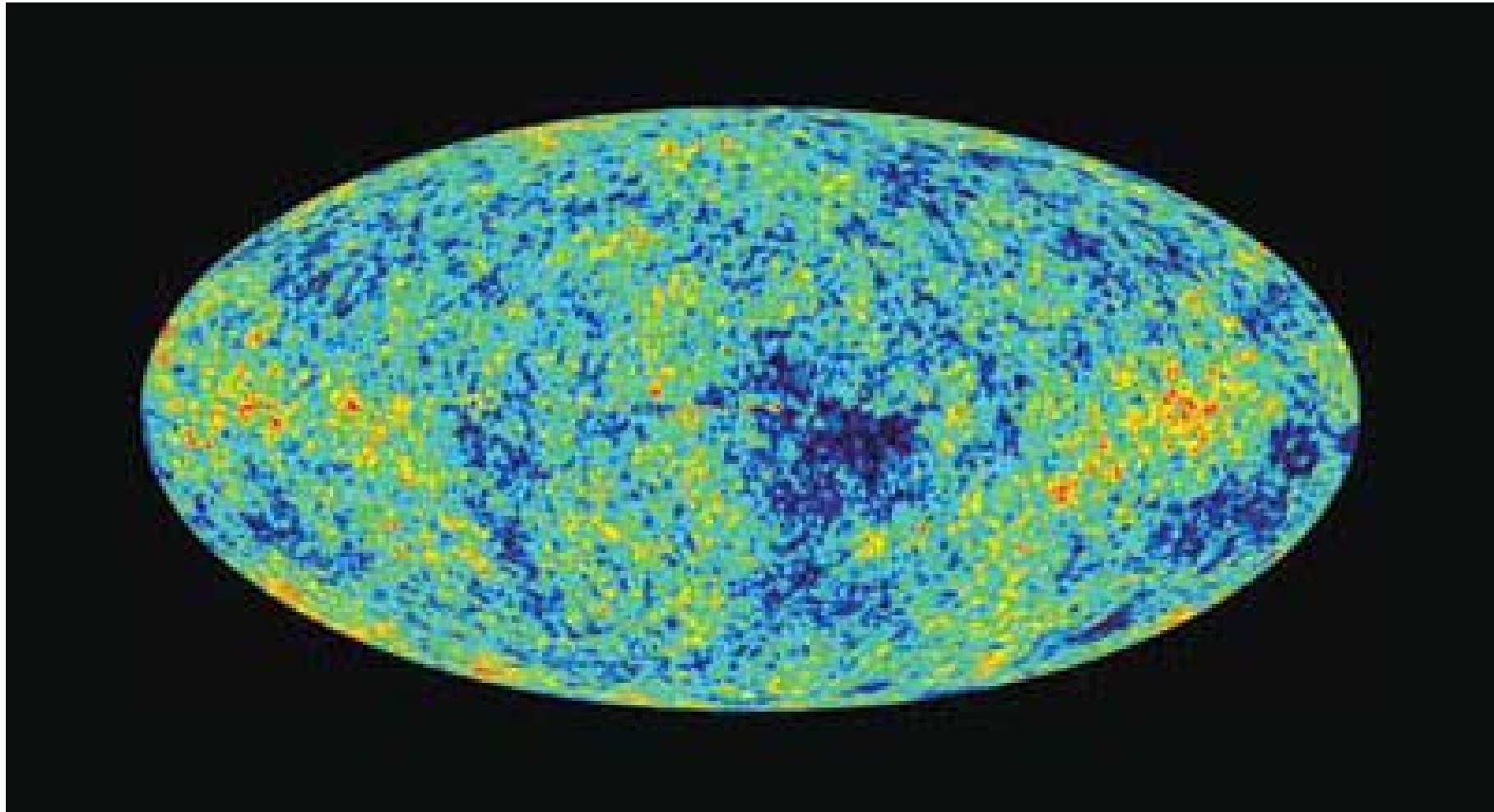
Fig. 1. The densities, in gm per cm³, of matter and radiation (ordinates) plotted against time in seconds (abscissae): logarithmic scale

4. FORMATION OF CHEMICAL ELEMENTS AND ORIGIN OF GALAXIES

The above considerations give us a general picture of changing physical conditions characteristic of the evolutionary history of our Universe. We will indicate here only quite briefly how this information can be used for the explanation of various characteristic properties of the Universe as we know it to-day. First of all, it may be suggested that, at least partially, the relative abundances of the atoms of various chemical elements were conditioned by thermonuclear reactions which took place at high speed during the very early stages of expansion while the temperature of the Universe was exceedingly high. And, in fact, the calculations in that direction, carried out by the present writer*, and later in some more detail by FERMI and TURKEVICH,† lead to a value of the H/He ratio which is in good agreement with observational data. However, there are still some difficulties to be overcome in understanding the abundances of heavier elements, and there is a possibility that the original distribution was partially modified by various processes during the later stages of the evolution.







Old Universe – **New** Numbers

$$\Omega_{\text{tot}} = 1.02^{+0.02}_{-0.02}$$

$$w < -0.78 \text{ (95\% CL)}$$

$$\Omega_{\Lambda} = 0.73^{+0.04}_{-0.04}$$

$$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$$

$$\Omega_b = 0.044^{+0.004}_{-0.004}$$

$$n_b = 2.5 \times 10^{-7+0.1 \times 10^{-7}}_{-0.1 \times 10^{-7}} \text{ cm}^{-3}$$

$$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$$

$$\Omega_m = 0.27^{+0.04}_{-0.04}$$

$$\Omega_v h^2 < 0.0076 \text{ (95\% CL)}$$

$$m_\nu < 0.23 \text{ eV (95\% CL)}$$

$$T_{\text{cmb}} = 2.725^{+0.002}_{-0.002} \text{ K}$$

$$n_\gamma = 410.4^{+0.9}_{-0.9} \text{ cm}^{-3}$$

$$\eta = 6.1 \times 10^{-10+0.3 \times 10^{-10}}_{-0.2 \times 10^{-10}}$$

$$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$$

$$\sigma_8 = 0.84^{+0.04}_{-0.04} \text{ Mpc}$$

$$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$$

$$A = 0.833^{+0.086}_{-0.083}$$

$$n_s = 0.93^{+0.03}_{-0.03}$$

$$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$$

$$r < 0.71 \text{ (95\% CL)}$$

$$z_{\text{dec}} = 1089^{+1}_{-1}$$

$$\Delta z_{\text{dec}} = 195^{+2}_{-2}$$

$$h = 0.71^{+0.04}_{-0.03}$$

$$t_0 = 13.7^{+0.2}_{-0.2} \text{ Gyr}$$

$$t_{\text{dec}} = 379^{+8}_{-7} \text{ kyr}$$

$$t_r = 180^{+220}_{-80} \text{ Myr (95\% CL)}$$

$$\Delta t_{\text{dec}} = 118^{+3}_{-2} \text{ kyr}$$

$$z_{\text{eq}} = 3233^{+194}_{-210}$$

$$\tau = 0.17^{+0.04}_{-0.04}$$

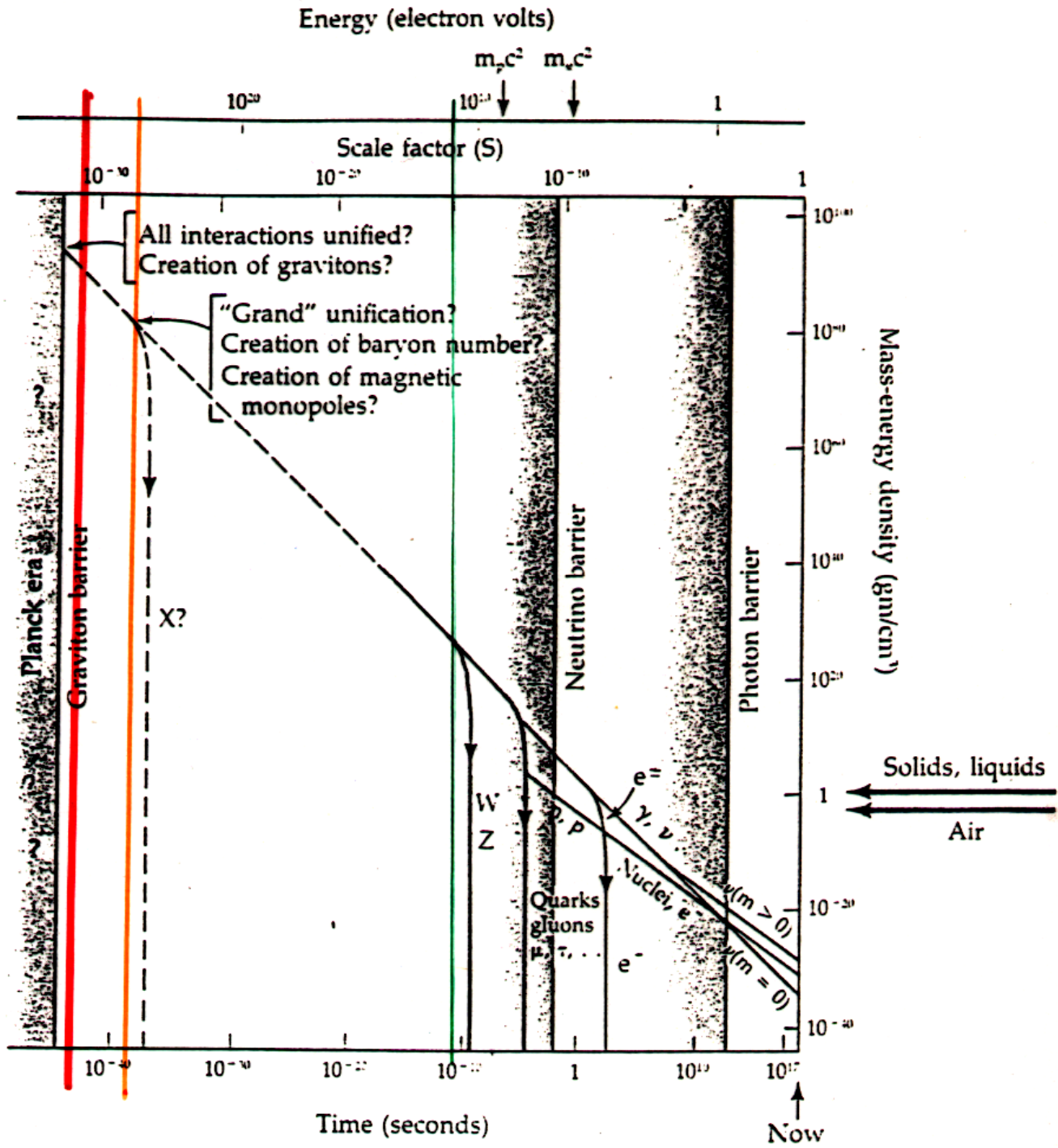
$$z_r = 20^{+10}_{-9} \text{ (95\% CL)}$$

$$\theta_A = 0.598^{+0.002}_{-0.002}$$

$$d_A = 14.0^{+0.2}_{-0.3} \text{ Gpc}$$

$$l_A = 301^{+1}_{-1}$$

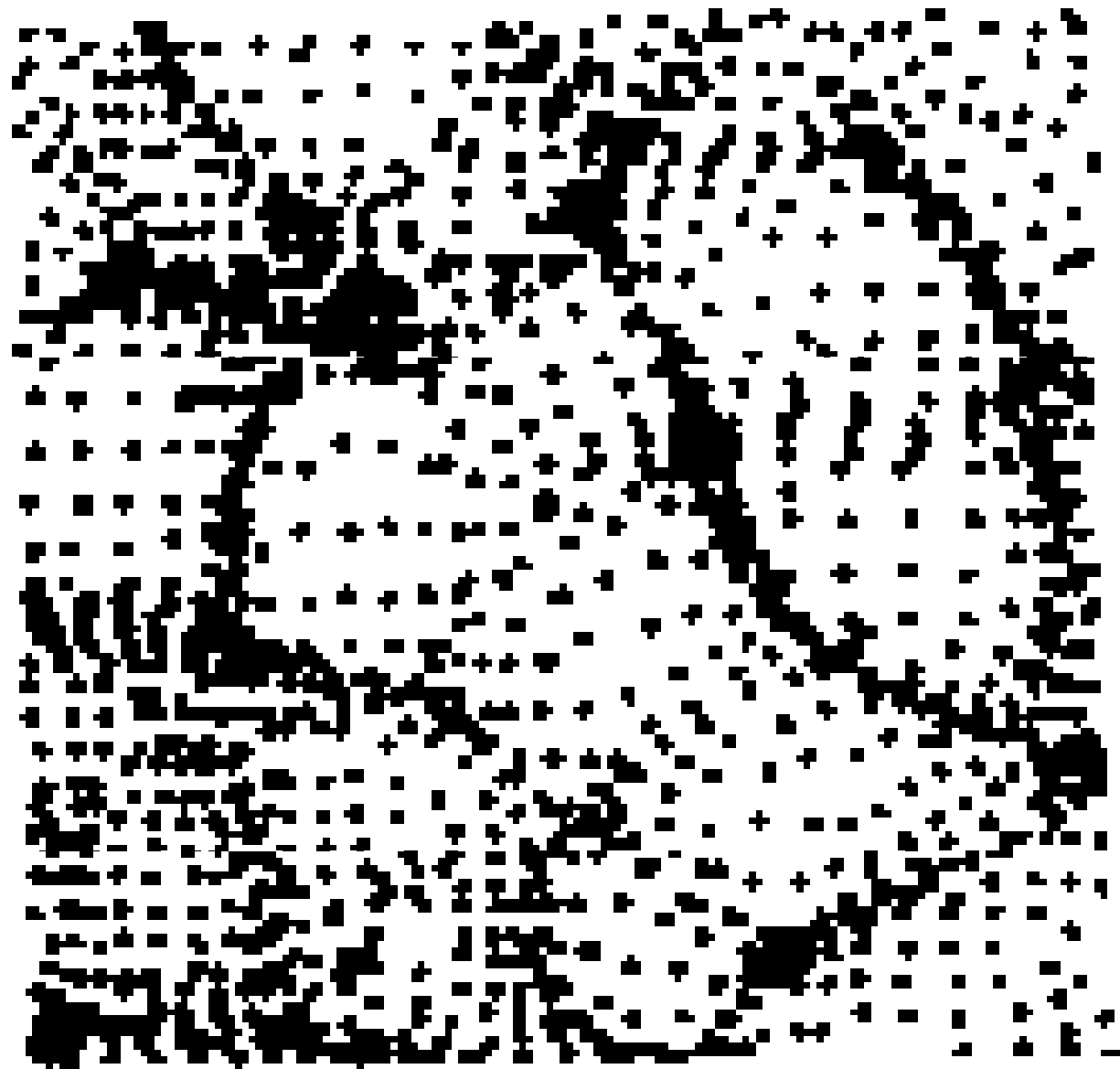
$$r_s = 147^{+2}_{-2} \text{ Mpc}$$



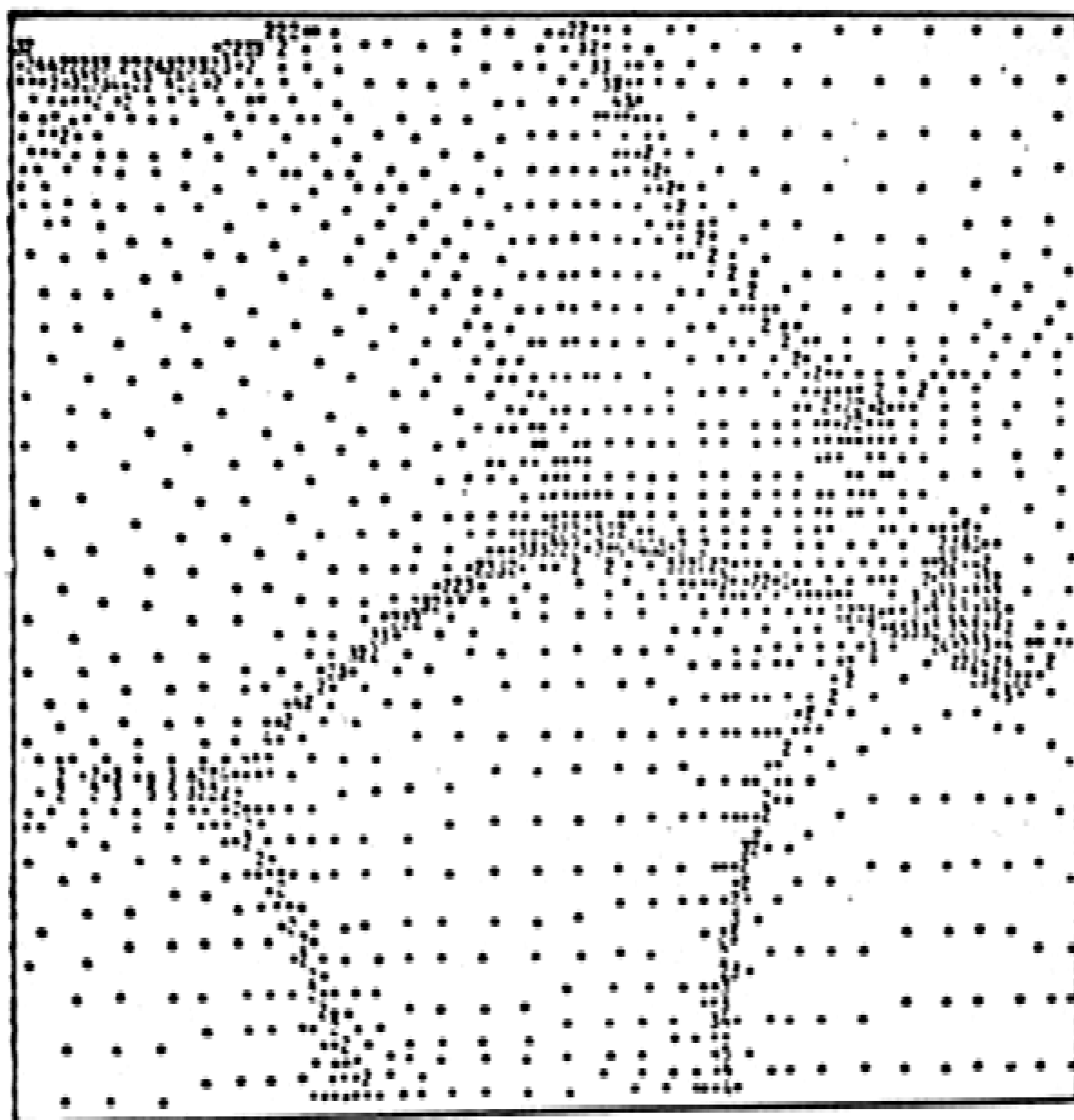
Temperature anisotropy (adiabatic mode)

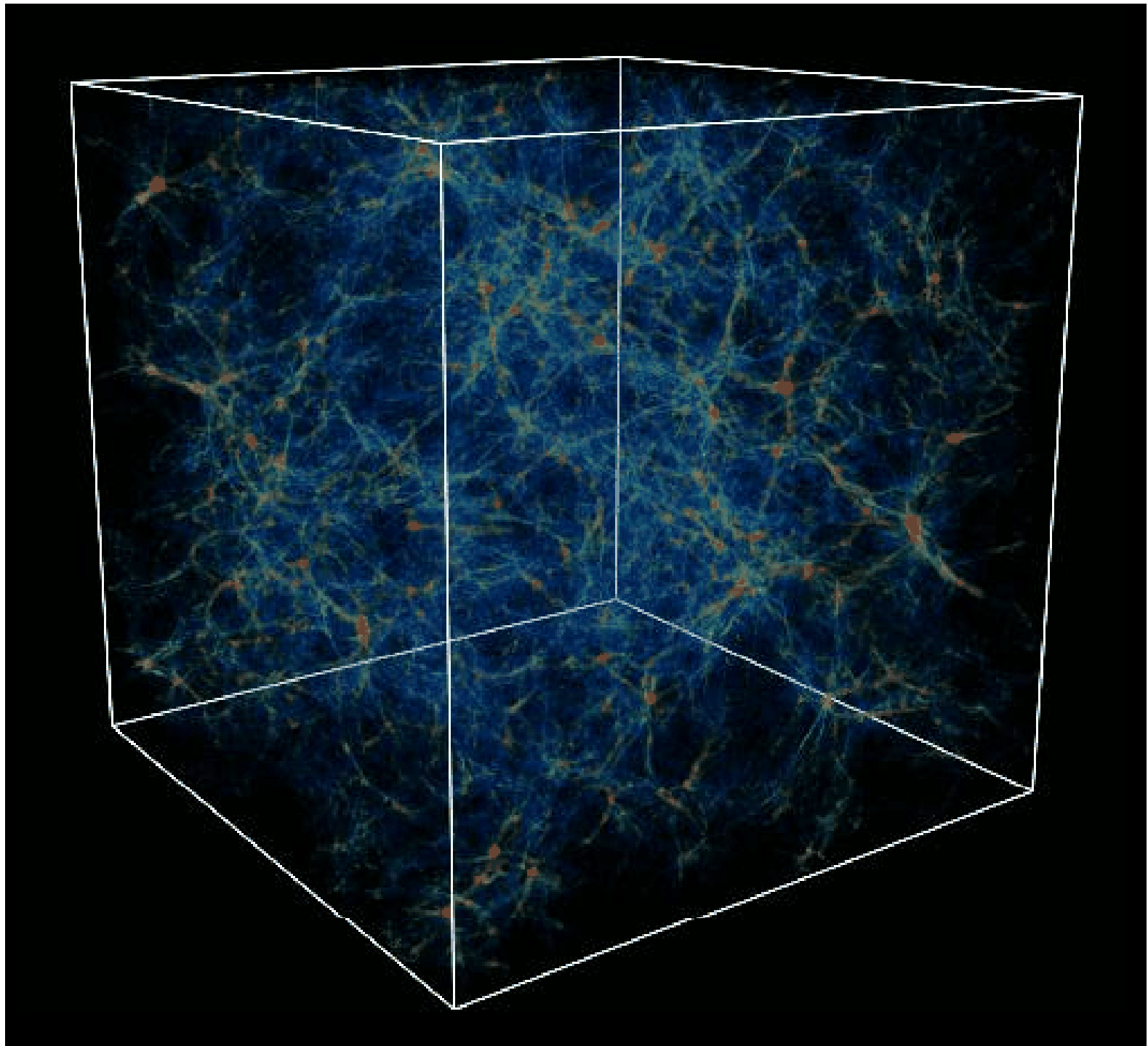
$$\left(\frac{\delta T}{T}\right)_{obs} \sim 10^{-5}$$

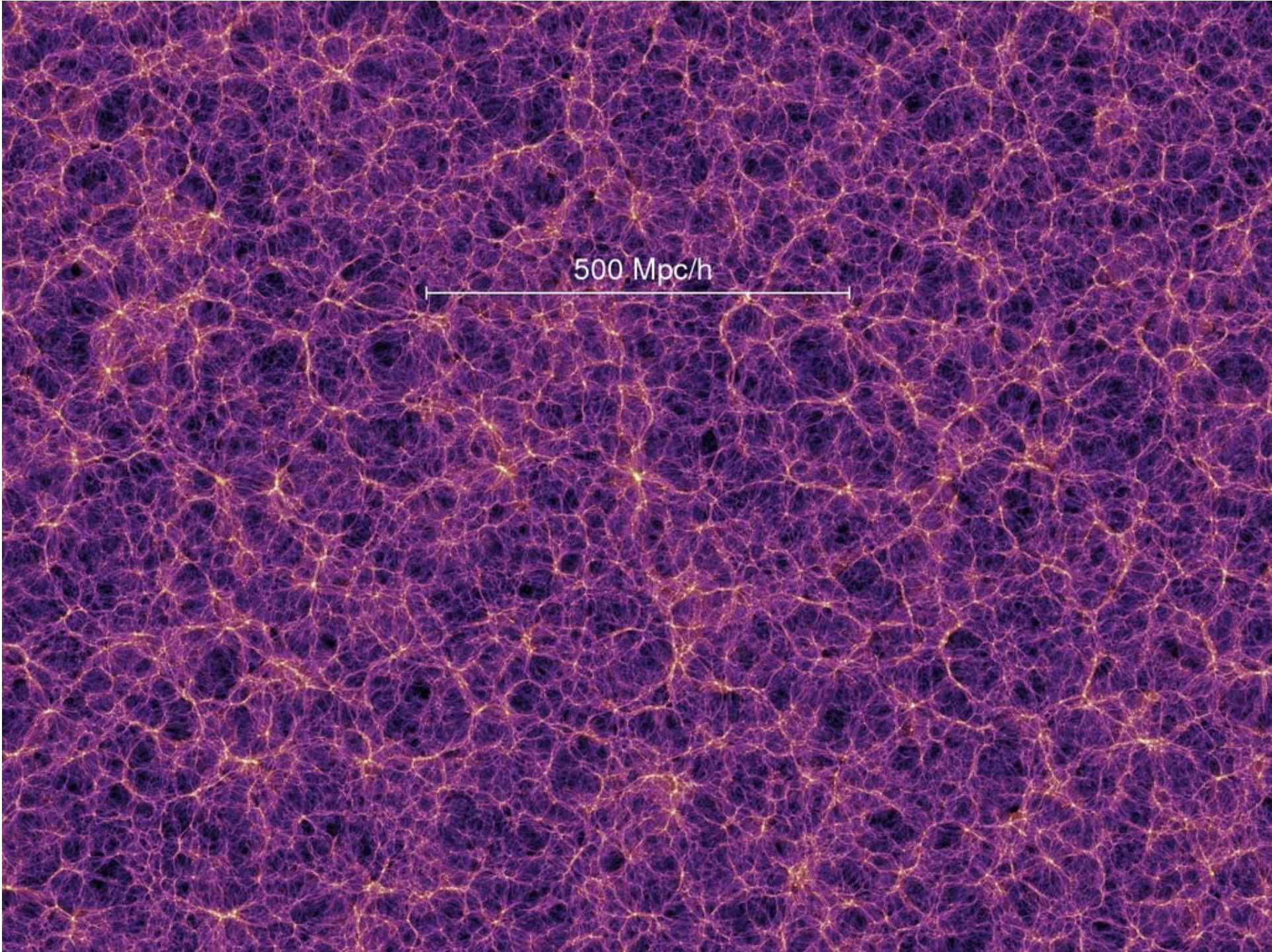
$$\frac{\delta \varrho}{\varrho} \sim 3 \frac{\delta T}{T}$$

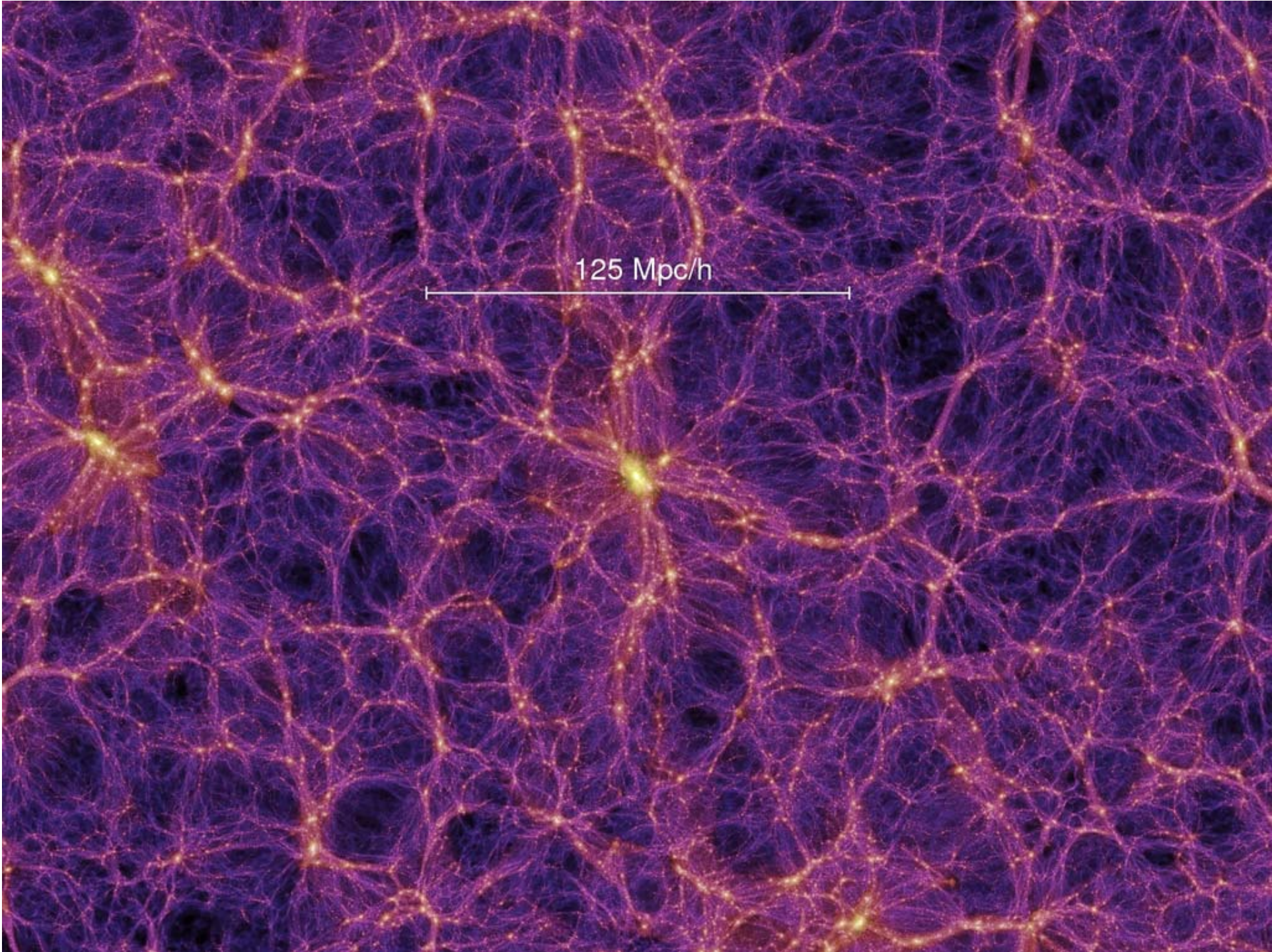


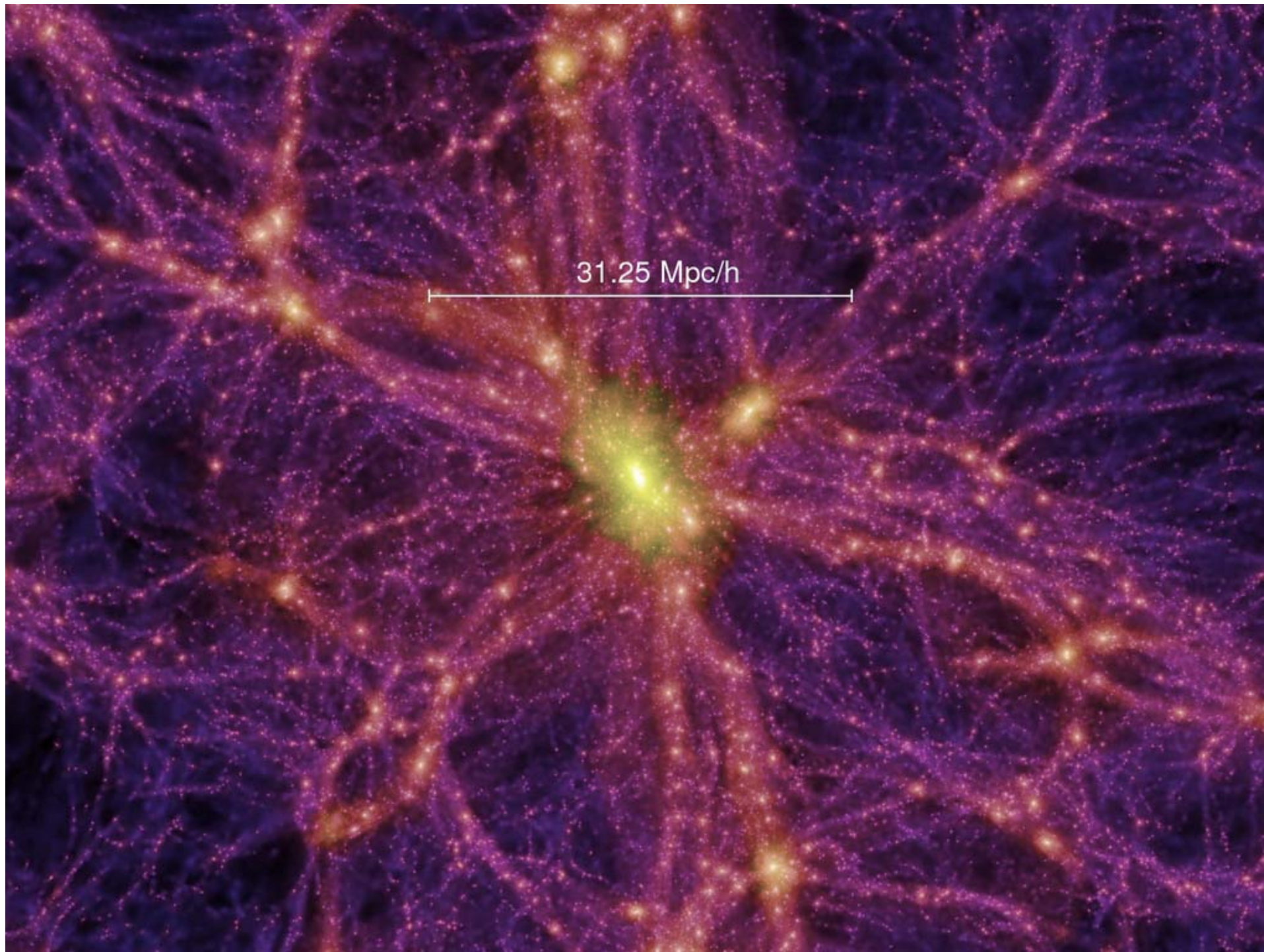
(a)

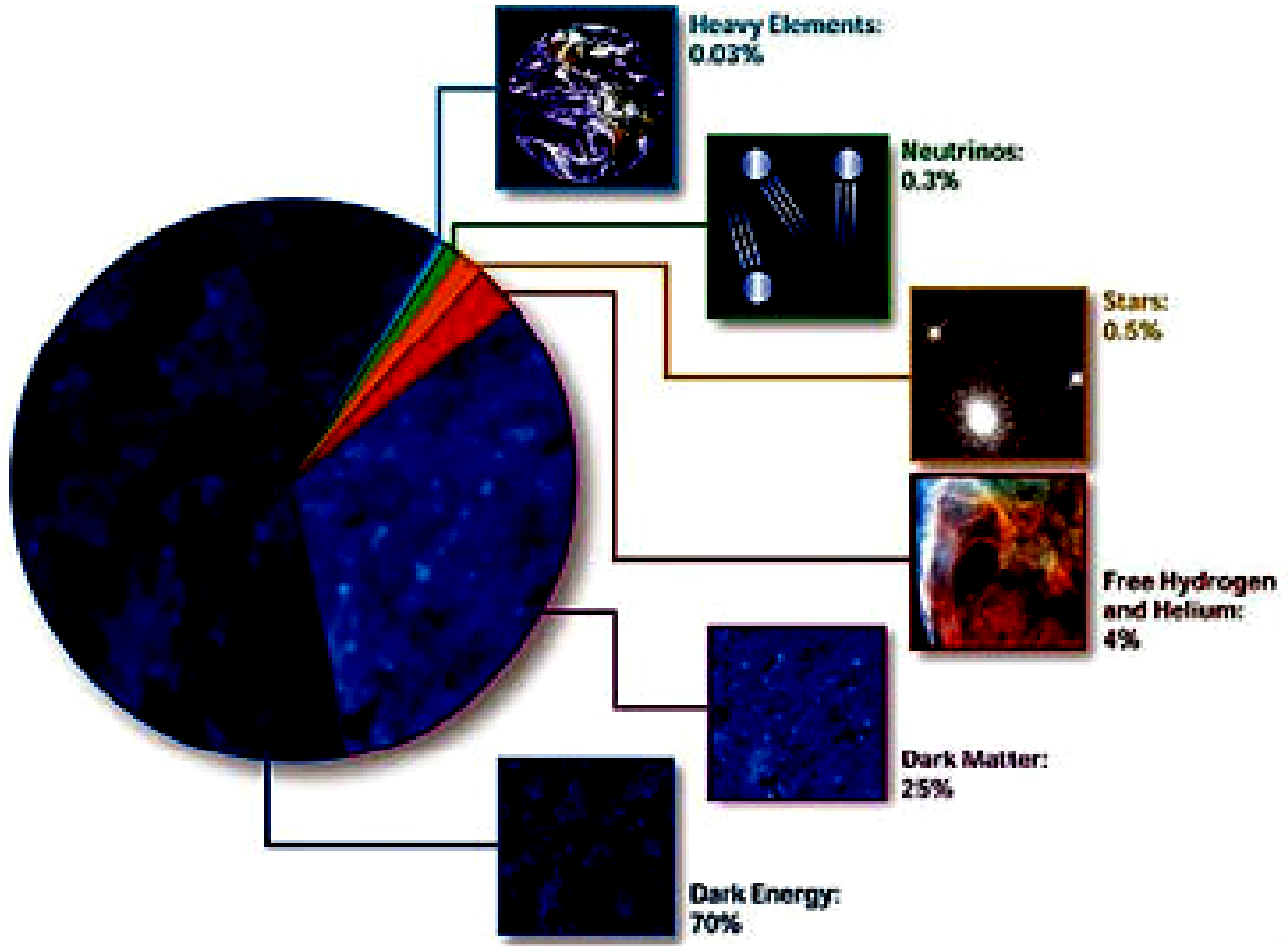


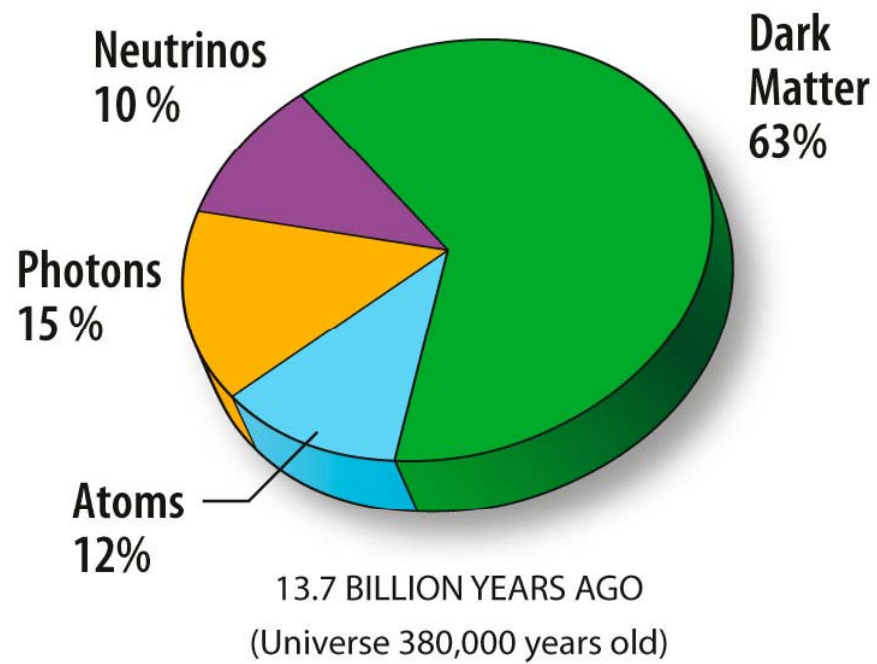
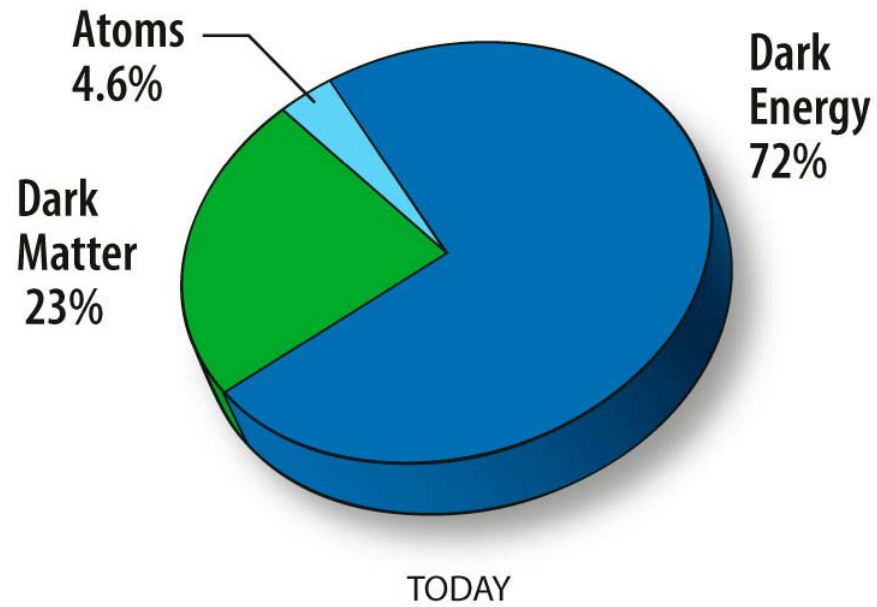












Possible dark matter particles

neutralino, $m_N \sim 50$ GeV,

gravitino,

axions,

other WIMPs

Dark energy

cosmological constant,

vacuum energy,

quintessence (potential
energy of some scalar field)

????

