Electron-Muon Ranger (EMR)
Step I Paper

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Structure of the EMR Step I paper

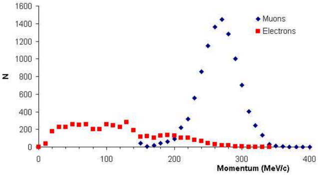
1. Introduction
   - Ionization Cooling, MICE
   - Purpose of the EMR
2. Electron-Muon Ranger
   - Structure of the detector
3. Performance in the MICE Beam
   - TOF selection and particle tagging
   - Correction for the energy loss in TOF2 and KL
   - Useful variables for PID
   - Efficiency of a simple test statistic
   - Momentum reconstruction from the range
4. Conclusions

NB: This paper demonstrates the **capability of the EMR (+ App. A)**
NB': **Appendices** at the end of the slides for **additional information**

1. & 2. Electron-Muon Ranger

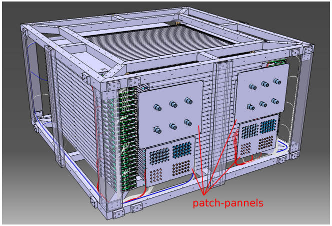
Pupose of the EMR in MICE:

- Reject the muons that decayed inside the cooling channel and their decay products
- Redundant measurements of the trajectories and momenta



The EMR is fully active scintillator tracker calorimeter

- 48 planes of 59 triangular scintillator bars
- Readout by multi-anode and single-anode PMTs



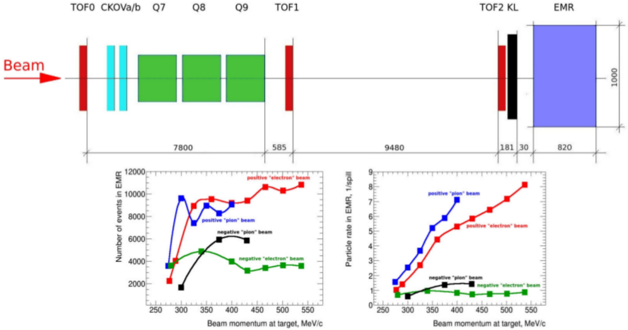
→ Final version of these sections at

`http://micewww.pp.rl.ac.uk/issues/1472`

3. Performance in the MICE beam

- One month of data taking in the MICE beam at Step I
- Array of beam settings ($e^{\pm}$, $\pi^{\pm}$) with momenta ranging from 250 to 550 MeV/c "at target" (setting in the magic spreadsheet)

$\rightarrow$ Rates and accumulated data are shown as a function of the setting



3.1 TOF selection and particle tagging (+ App. B)

- Only the tracks with one TOF spacepoint are selected (single tracks)
- For a given beam setting, we fit the distribution of TOFs with a 3-peaks Gaussian ($\rightarrow \mu_\alpha\,, \sigma_\alpha$)
- For each particle trigger, the probability of belonging to each peak is computed and a particle tag is associated to it
- The momentum of muons and pions is reconstructed from the TOF measurement:

$$p_\alpha = \frac{m_\alpha c}{\sqrt{\left(\frac{c\mathrm{TOF}}{\Delta z_{12}}\right)^2 - 1}} \qquad (1)$$

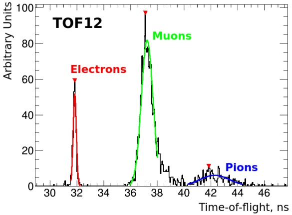
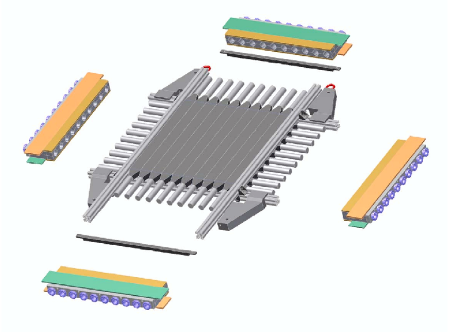


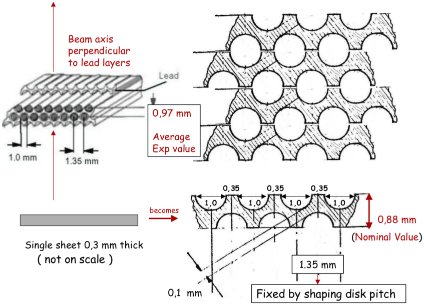
Figure: TOFs of a positron beam of 230 MeV/c @D2

3.2 Energy loss before entering the EMR

After tagging, a particle goes through TOF2 and KL before the EMR:

- Composition of TOF2:
  - 2" ($\sim$ 5cm) of PVT (Polyvinyl Toluene) scintillator bars
- Composition of KL:
  - 4 cm calorimeter made of Pb and PS scintillating fibres
  - $V_{PS}/V_{Pb} \simeq 2$
  - On average $\sim 3X_0$ and $\sim 0.1\lambda_I$



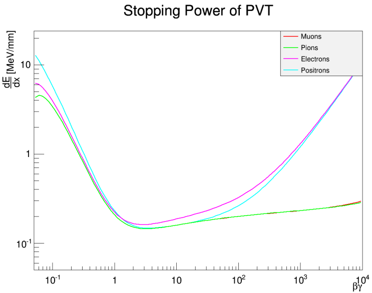


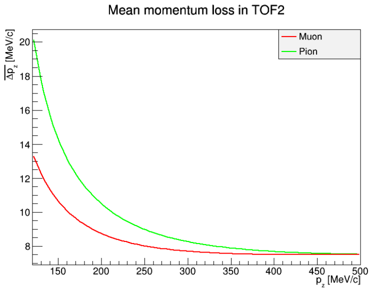
Energy loss in TOF2

Characteristics of the energy loss:

- $X_0 \simeq 42.6$ cm in PVT
- **MIP particles** loose ~ 10 MeV/c in TOF (muons and pions with $p_z > 2m_i c$)
- **Low energy muons and pions** ($p_z < m_i c$) will experience higher energy loss. At 120 MeV/c, a pion looses 20 MeV/c on average
- The **electrons** are all ultra-relativistic ($\beta\gamma > 100$). Due to the high X_0, they are unlikely to shower in TOF2 (0.1 X_0)

$\rightarrow$ For μ and π, the shift in energy is only significant at low energies

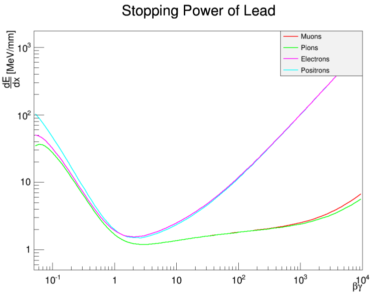


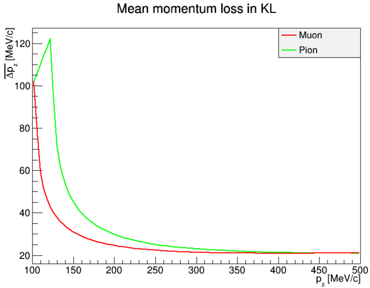


Energy loss in KL (1) (+ App. B)

Characteristics of the energy loss:

- $X_0 \simeq$ 0.5 cm in Pb
- **MIP particles** loose $\sim$ 20 MeV/c in KL (muons and pions with $p_z > 2m_i c$)
- **Low energy muons and pions** ($p_z < m_i c$) can potentially stop in the detector if $p_\mu < 100 MeV/c$ or $p_\pi < 120 MeV/c$
- The **electrons** are all ultra-relativistic ($\beta\gamma > 100$) and will shower in the lead of KL ($3X_0$)
- **Pions** can hadronize in KL and loose substantially more energy on occasions ($0.1\lambda_I$)

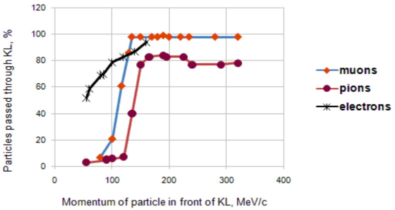




Energy loss in KL (2)

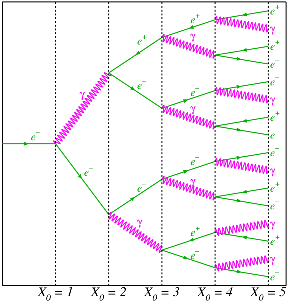
Survival of **muons** and **pions** after KL:

- In the 2010 PID detectors run, TAG counters were placed behind KL to see what comes out of it
- The theoretical suspicions are confirmed, muons and pions are killed under a certain threshold

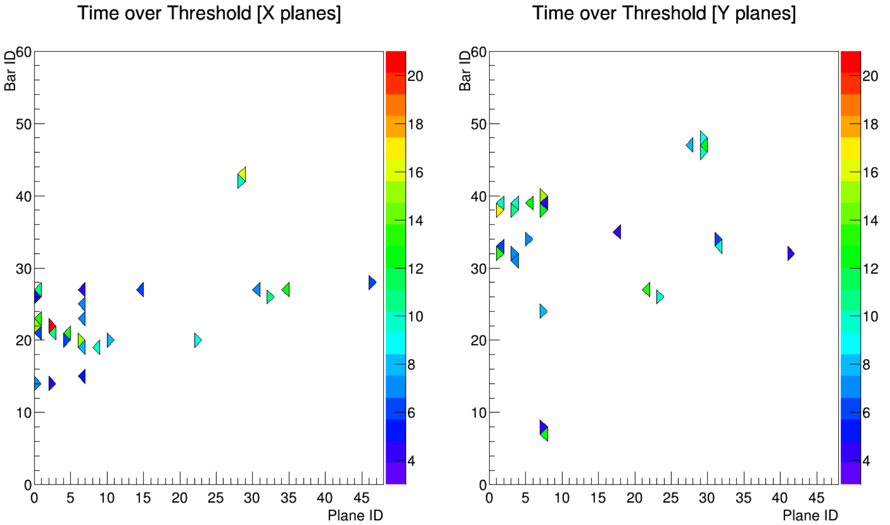


Electromagnetic **showers**:

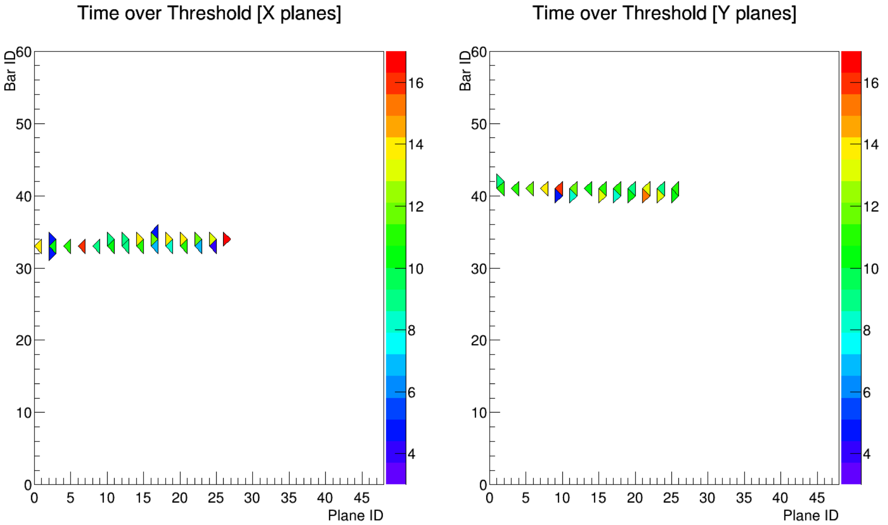
- Electrons radiate in KL and create several secondary γ, e^- and e^+
- e^- and e^+ come out with very low momentum ($\sim p_z/2^3$)
- Photons go through the EMR with low probability of interacting with the scintillating material (hits rare)



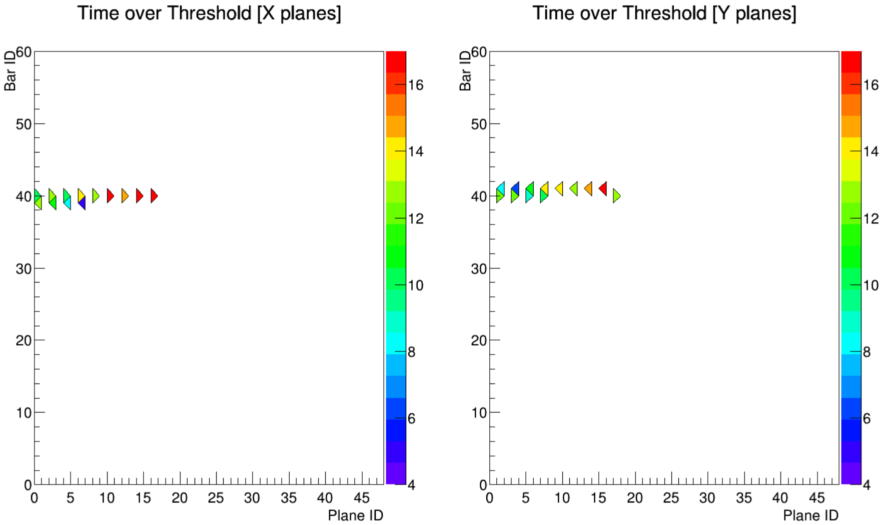
Electron event (shower in KL, no clear track)



Muon event (~ 250 MeV/c)



Pion event (~ 250 MeV/c)



3.3 Useful variables to discriminate electrons

For each beam setting (i.e. **momentum**) and each **event**, we measure:

1. Plane **density** ρ_p

   $\rightarrow$ Measurement of the hit density in the active volume

2. **Spread** in terms of χ^2 in the two projections

   $\rightarrow$ Track / Shower spread of a particle

3. **Range** R

   $\rightarrow$ Penetration of the particle in the EMR detector

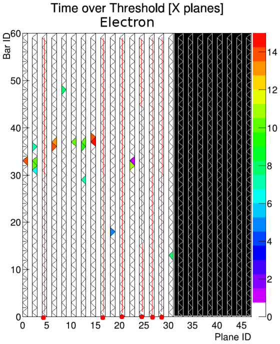
The use of these variables as a combined test statistic will prove to be a strong tool to reject electrons and tag real muons in the detector as we will see in the following sections

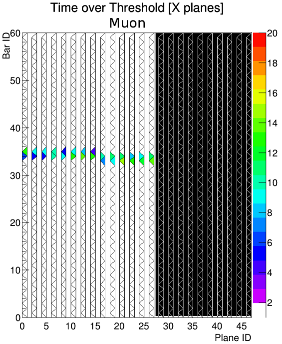
Plane Density ρ_p

The plane density is defined as the percentage of the planes that record a signal on the path of the particle or its shower, i.e.

$$\rho_p = \frac{number\, of\, planes\, hit}{\Delta z} = \frac{N_X + N_Y}{z_X + z_Y} \tag{2}$$

with Δz the depth of the particle expressed in number of planes.

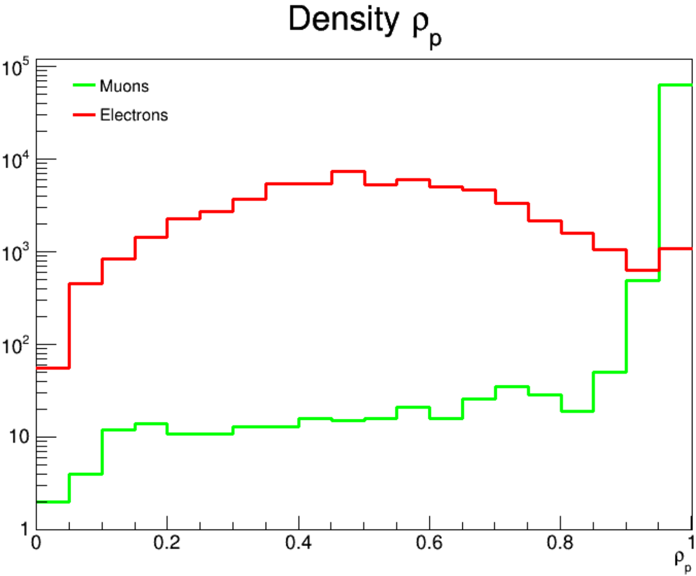




Electrons: 9 planes hit over a span of 15, $\rho_p = 60\%$;

Muons: 14 planes hit over a span of 14, $\rho_p = 100\%$.

Muon vs electron: Density (normalized)



Spread in terms of χ^2/N in the two projections (+ App. C)

One way to express that angular spread of an electromagnetic shower is to fit it with a line a evaluate its χ^2 normalized to the amount of hits N:

$$\chi^2/N = \frac{1}{N}\sum_i \frac{(y_i - (ax_i + b))^2}{\sigma_i^2} \tag{3}$$

For a given array of hits (x_i, y_i), the exact value of this parameter is expressed in term of the spread $\sigma_y^2 = E\left[(y - \overline{y})^2\right]$ as:

$$\chi^2/N = \sigma_y^2(1 - \rho^2) \tag{4}$$

with $\rho = \texttt{Cov}(x, y)/\sigma_x\sigma_y$. This is exactly what we want as:

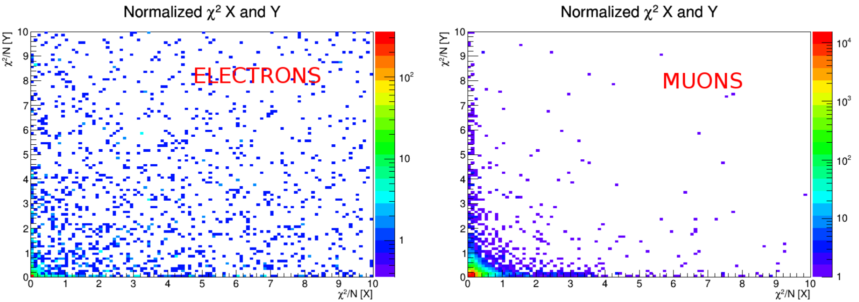
- Electrons have a significant spread σ_y and the hits they produce are weakly correlated ($(1 - \rho^2)\rightarrow 1$), so that $\chi^2/N \rightarrow \sigma_y^2 \gg 1$
- Muons have a small spread σ_y (centre of the detector) and are strongly correlated (line, $(1 - \rho^2) \rightarrow 0$), so that $\chi^2/N \ll 1$

χ^2/N in the X and Y projections

Distribution of the electron and the muon χ^2/N in the two projections:

- The electrons don't exhibit an obvious pattern in their distribution
- The muons are concatenated around $(0,0)$ as we would expect

$\rightarrow$ A natural choice of variable to test when it comes to a combination of two similar statistic is their product $\chi^2_X \times \chi^2_Y$



3.4 First selection attempt

At first glance, the most efficient variable to reject the electrons at all momenta is the plane density ρ_p. Even if it performs well on its own, adding a cut on $\chi^2_X \times \chi^2_Y$ improves the rejection without reducing the acceptance.

Hypothesis testing :

- H_0 is the null hypothesis, the particle X is a muon. H_1 is the alternative, i.e. X is an electron.
- $\alpha = p(X \in w|H_0)$ is the **loss**, the probability that X is tagged as an electron, given that X is a muon (w the critical region)
- $\beta = p(X \in (W - w)|H_1)$ is the **contamination**, the probability that X is tagged as a muon, given that X is an electron (W the space)

$\rightarrow$ We want to define w such that the power $1 - \beta$ is maximized without losing too much of the initial sample (given α)
$\rightarrow$ The real contamination is in fact $R_e\beta$ with R_e the abundance of electrons in the beam, i.e. $R_e = N_e/N_\mu$ (= 11.7% in the test beam)

Chi squared vs Density

We want to select a critical region w_α of the space $W(\rho_P\,, \chi^2_X \chi^2_Y)$ for a given loss α. We follow the curve $1-\alpha$ and minimize for β, we get, for $\alpha = 1\%$:

- $\rho_p > .9$
- $\chi^2_X \chi^2_Y < 0.75$

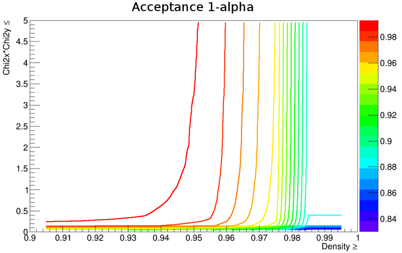
This yields a contamination of:

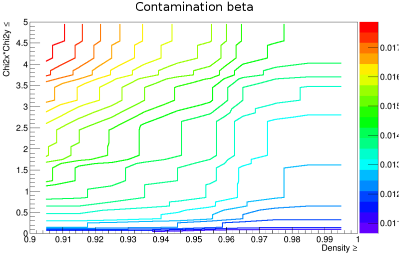
→ $\boxed{\beta \simeq 1.4\%}$

→ This value is independent of the beam setting as it was obtained for the whole electron sample.

→ It is achieved without the input of any other detector

→ In the case of this test beam, the purity reaches $1 - R_e\beta = 99.84\%$.





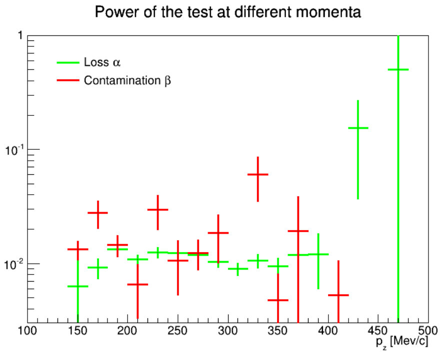
Rejection power at different momenta

Is this method efficient for every momenta?

→ The contamination fluctuates with the setting but there is no strong correlation between the setting and β. The worst is reached for $p_z =$ 322 MeV/c @Q9 for which $\beta = 6\%$.

→ The loss of muons is kept around 1% for every setting.

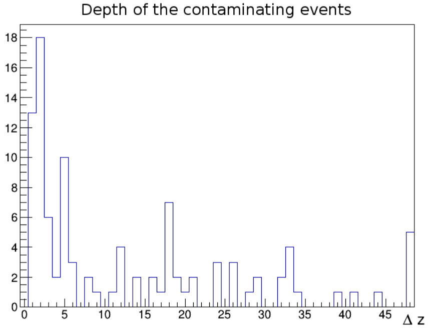
→ The highest momenta experience huge losses because the electron peak and muon peak merge and the muons are incorrectly tagged.



Problematic events

Electron events with high density ρ_P and low spread $\chi^2_X\chi^2_Y$:

- Out of the 7608 electrons tested, only 101 of them make the cut
- 50% are very low range electrons $\rightarrow \rho_P \sim 1,\ \chi^2_i \ll 1$
- The events with $\Delta_z = 48$ could be mistagged very high p_z muons
- The rest are electrons that didn't shower in KL or for which the photons of the shower didn't produce secondary particles (random)



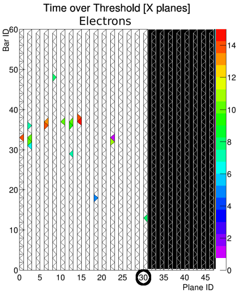
$\rightarrow$ An appropriate cut on the range could get rid of the last few electrons. Unfortunately this cut would need to be a function of momentum, $R_C(p_z)$, as we certainly don't want to reject additional muons.

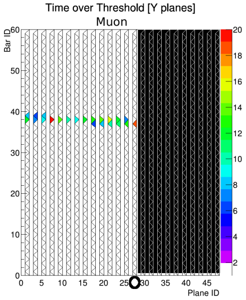
3.5 Range R

The range $R = \Delta_Z / (\cos\theta_X \cos\theta_Y)$ is defined as the distance the particles travels through the EMR before it stops. It can be expressed in number of planes or in mm (1 plane = 17mm).

$\rightarrow$ For μ and π it corresponds beautifully to the range as their path is more or less straight forward along the BL;

$\rightarrow$ for e it gives us an idea of the range of the electromagnetic shower but is much less precise as the angle of the linear fit is not obvious





Electrons: the last hit is in plane 30, $R \simeq 30$;

Muons: the muon stopped in plane 27, $R \simeq 29$.

Electromagnetic shower range

We can't infer the electron momentum from the TOF information as they are all ultra-relativistic. Even if we could, the showering in KL is such that there is no strong correlation between initial momentum and shower depth.

→ The **whole** electron sample must be rejected to prove efficiency

→ The range **is not** a strong way to reject electrons

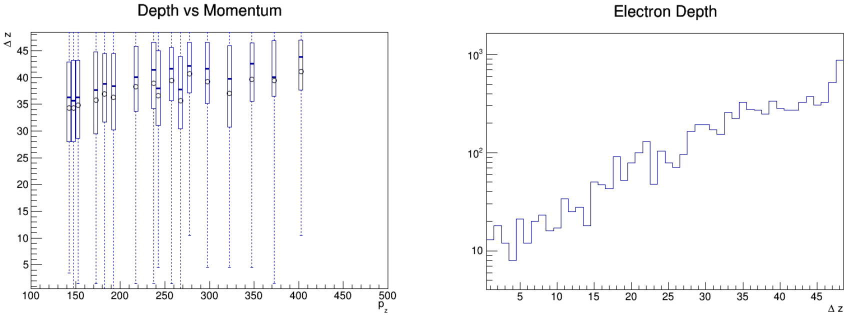


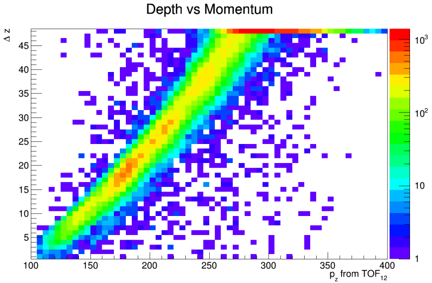
Figure: The momentum is inferred at Q9 from the beam setting

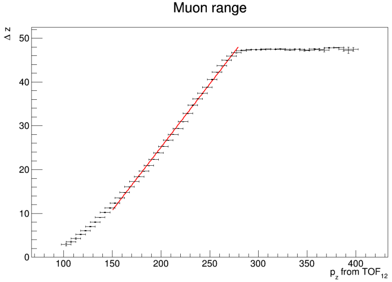
Muon range in the EMR

Unlike the electrons, the muons don't shower in KL. They loose energy in the EMR until they stop or cross the whole detector before stopping.
A simple linear fit yields the formulas, for $150 MeV/c < p_z < 280 MeV/c$:

$$R_\mu(p_z) \simeq (0.29 \times p_z - 32.13)\texttt{planes}\,, \quad (5)$$

$$R_\mu(p_z) \simeq (0.49 \times p_z - 54.62)\texttt{cm}\,. \quad (6)$$



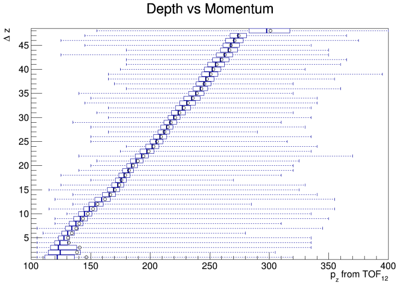


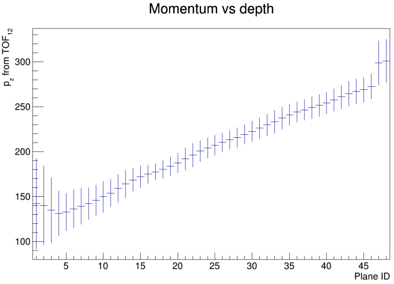
Uncertainty on the mean used here

Resolution on the muon momentum

We can also inverse this relation to get an estimation of the momentum of the muon as a function of the range measured in the EMR. The **uncertainty** on the reconstructed momentum is about **10 MeV/c** for every range between 5 and 45 planes (NB: uncertainty on p_z from TOF)
Inverting the relation yields, for $5 < R_\mu < 45$:

$$p_z(R_\mu) \simeq ([3.5 \times R_\mu + 110.8] \pm 10) MeV/c\,. \tag{7}$$





4. Conclusions

What is completed and will stay **unchanged** in the final EMR Step I paper:

- Introduction, presentation of MICE and the role of the EMR
- Technical description of the detector and its features
- TOF analysis to extract the momentum between TOF1 and TOF2

→ Final version for this at `http://micewww.pp.rl.ac.uk/issues/1472`
What will be **added** or **replaced** in the paper:

- Energy loss correction of the muon and pion impinging momentum after going through **TOF2** and **KL**
- New variables to tag muons, reject electrons (density, spread, range) → **New PID efficiency analysis**
- Contamination constrained down to 1.4% for 1% loss.

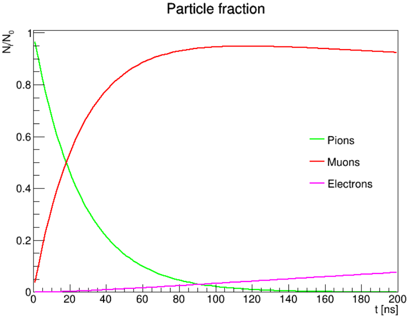
Appendix A: Contamination in the beam

First of all, what is the composition of the pion beam as a function of its position z along the beamline. If we assume the same acceptance for all particles in the test beam, then we have:

$$\frac{dN_\pi}{dt} = -\lambda_\pi N_\pi,\ \frac{dN_\mu}{dt} = +\lambda_\pi N_\pi - \lambda_\mu N_\mu,\ \frac{dN_e}{dt} = +\lambda_\mu N_\mu \quad (8)$$

which solves for $N_e(t)$ in terms of its fraction of the whole sample N_0:

$$\frac{N_e(t)}{N_0} = 1 - \frac{\lambda_\mu}{\lambda_\mu - \lambda_\pi} e^{-\lambda_\pi t} + \frac{\lambda_\pi}{\lambda_\mu - \lambda_\pi} e^{-\lambda_\mu t} \quad (9)$$

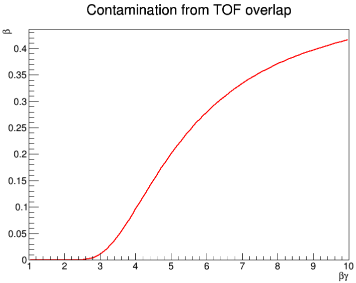


What are the electrons that contaminate the muons sample? As the particles are tagged by the TOFs, it comes from an overlap between the muon and electron peaks. The normalized peaks overlap at:

$$t_C(p_z) = \frac{\mu_\mu(p_z) - \mu_e}{\sigma_e + \sigma_\mu} \tag{10}$$

with μ_μ the position of the muon peak (function of muon momentum), $\mu_e \simeq 31.84$ ns, $\sigma_e \simeq 0.25$ ns and $\sigma_\mu \simeq 0.5$ ns. The electrons beyond this point constitute contamination and the muons below, loss:

$$\begin{aligned} \alpha(p_z) &= \beta(p_z) = P(t_e > t_C) \\ &= \frac{1}{\sqrt{(2\pi)}} \int_{t_C(p_z)}^{+\infty} e^{-x^2/2} dx \\ &= \frac{1}{2} \text{Erfc}\left(\frac{t_C(p_z)}{\sqrt{2}}\right) \end{aligned} \tag{11}$$



In our analysis, they are 3 main sources of contamination linked to **TOF**:

1. The electrons that come from TOF overlap (negligible at low p_z)

$\rightarrow$ $N_e^C(p_z) = \beta(p_z) \int_0^{z_{TOF1}} \frac{dN_e(z,p_z)}{dz} dz \simeq 0.04$ % for 500MeV/c @target

2. The muons the decay between TOF1 and TOF2 (still tagged as μ?)

$\rightarrow$ The muon peak moves towards the electron peak as function of its decay position z, a z close to TOF2 gives electrons in the muon peak.

$\rightarrow$ $N_e^C(p_z) = \int_{z_{TOF1}}^{z_{TOF2}} \frac{dN_e(z,p_z)}{dz} \beta(z,p_z) dz \simeq 0.075$ % for 500MeV/c @target

3. The muons that decay after TOF2 (short distance $\sim$ 0.37 m)

$\rightarrow$ All the decays beyond this point contaminate, i.e $\beta(p_z) = 1$:

$\rightarrow$ $N_e^C(p_z) = \int_{z_{TOF2}}^{z_{EMR}} \frac{dN_e(z,p_z)}{dz} dz \simeq 0.02$ % for 280 Mev/c @target

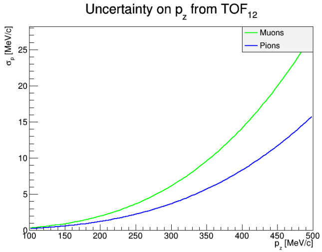
Appendix B: Uncertainty on the momentum measurement

The source of the momentum measurements in the test beam is reconstructed (provided TOF PID) from the time of flight $t \equiv TOF_{12}$:

$$|\overrightarrow{p}| \equiv p = \frac{m_i c}{\sqrt{(\frac{ct}{D})^2 - 1}} \tag{12}$$

With uncertainty:

$$\begin{aligned} \sigma_p &= \left.\frac{\partial p}{\partial t}\right|_{\bar{t},\overline{D}} \sigma_t \oplus \left.\frac{\partial p}{\partial D}\right|_{\bar{t},\overline{D}} \sigma_D \\ &= \frac{m_i c^3 t}{D^2}\left[(\frac{ct}{D})^2 - 1\right]^{-3/2} \sigma_t \\ &\oplus \frac{m_i c^3 t^2}{D^3}\left[(\frac{ct}{D})^2 - 1\right]^{-3/2} \sigma_D \end{aligned} \tag{13}$$



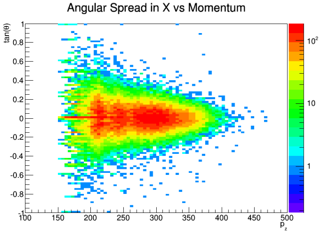
with $\sigma_t \sim 70$ ps and $\sigma_D \sim 8$ mm.

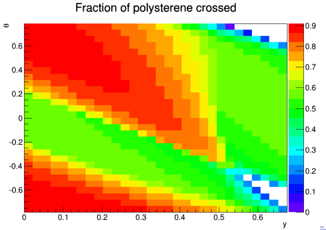
The other main source of error on the reconstructed momentum at the entrance of the EMR is the correction for the energy loss in KL. A simplified model has been developed to simulate the spread in EL:

- KL is made out of PS fibres threaded through sheets of Pb ($V_{PS}/V_{Pb} \sim 2$)
- We reduce to the smallest repeatable lattice element and define the function:

$$\Delta x(PS)/\Delta x \equiv f(y_0, \theta) \quad (14)$$

→ The proportion of PS the track goes through in this element.
→ Small angles are favoured (see data).



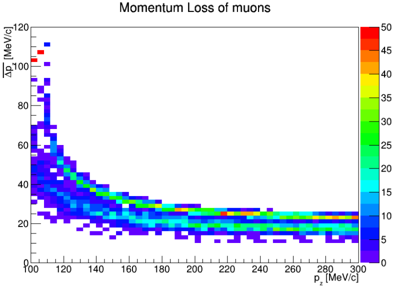


Given the angular distribution (0 ± 0.13 radians) and a uniform distribution in y_0 (no favored initial position), we can simulate the EL.

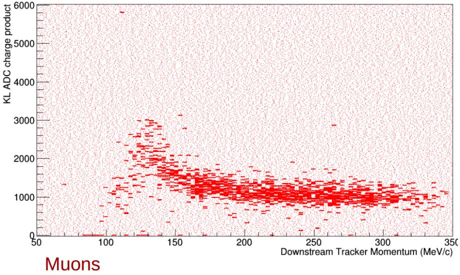
→ Beautiful agreement between simulation and data

→ Offset of 15 MeV/c of the peak, energy loss in TOF2 !

→ For MIP particles, $\overline{\Delta p_z} \simeq (20 \pm 5)$ Mev/c is a good approximation



Simulated p loss



Real KL ADC product

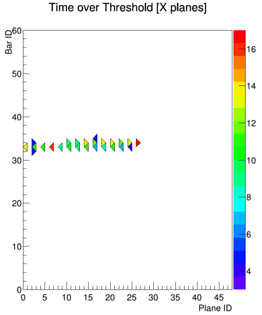
In fine, for muons @200Mev/c (MIP particles), uncertainty of **2MeV/c** from 200Mev/c and **5MeV/c** from 200Mev/c. It increases at low p_z because of KL EL and at high p_z because of TOF.

Appendix C: Linear fits of events in the EMR

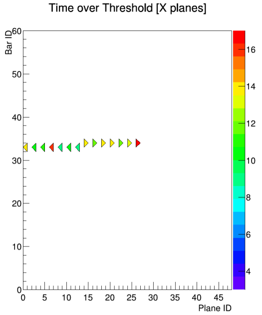
The first step before fitting is to only keep the highest energy hits

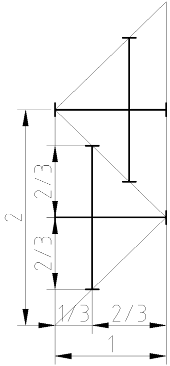
- $\rightarrow$ Only perfectly efficient way of getting way of getting rid of crosstalk
- $\rightarrow$ The coordinates of the channel with the highest charge are saved into the arrays $(\overrightarrow{x}, \overrightarrow{y})$, x along the beamline and y perpendicular to it
- $\rightarrow$ For given bar, its coordinates are given by its COM

BEFORE



AFTER





From $\chi^2/N = \frac{1}{N}\sum_{i=1}^{N}(y_i - (ax_i + b))^2/\sigma_i^2$, provided that $\sigma_i = \sigma_0$, $\forall i$:

$$\chi^2/N = \frac{1}{N\sigma_0^2}\sum_{i=1}^{N}[y_i^2 - 2y_i(ax_i + b) + (a^2x_i^2 + 2abx_i + b^2)] \quad (15)$$

To minimize it for a and b, we derive about these two parameters:

$$\frac{\partial(\chi^2/N)}{\partial a} = 0 \iff \frac{1}{N}\sum_{i=1}^{N}[-2x_iy_i + 2ax_i^2 + 2bx_i] = 0$$
$$\iff - < xy > +a < x^2 > +b < x >= 0 \quad (16)$$

$$\frac{\partial(\chi^2/N)}{\partial b} = 0 \iff \frac{1}{N}\sum_{i=1}^{N}[-2y_i + 2ax_i^2 + 2b] = 0$$
$$\iff - < y > +a < x > +b = 0 \quad (17)$$

Introducing $b =< y > -a < x >$ into Eq. 16, we get:

$$- < xy > +a < x^2 > + < x >< y > -a < x >^2 = 0 \tag{18}$$

We know that the variance of x is $\sigma_x^2 = E[(x - \overline{x})^2] =< x^2 > - < x >^2$ and the covariance of x and y is $\sigma_{xy}^2 =< xy > - < x >< y >$:

$$\boxed{a = \sigma_{xy}^2/\sigma_x^2} \text{ and } \boxed{b =< y > -a < x >} \tag{19}$$

Finally, imputing these values into Eq. 15 with arbitrary $\sigma_0 = 1$:

$$\begin{aligned}
\chi^2/N &= \sigma_y^2 - 2a\sigma_{xy}^2 + a^2\sigma_x^2 \\
&= \sigma_y^2 - 2\sigma_{xy}^4/\sigma_x^2 + \sigma_{xy}^4/\sigma_x^2 \\
&= \sigma_y^2(1 - \frac{\sigma_{xy}^4}{\sigma_x^2\sigma_y^2}) \\
&= \boxed{\sigma_y^2(1 - \rho^2)} \qquad (20)
\end{aligned}$$