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Nonlinear effects in the MICE Step IV lattice

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MICE CM41 meeting

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Goal of this study

- Analyze nonlinear effects in the MICE Step IV lattice and assess impact on the experimental program

Motivation

- The initial and final state (6-vector) of every particle will be measured
  - MICE is an extraordinarily unique experiment!
- Provides a means to
  - test our understanding of ionization energy loss dE/ds
    - infer properties that are not well known
  - observe muon cooling (Step IV)
  - observe sustained cooling with re-acceleration (final step)
  - demonstrate our ability to *predict in simulation* the performance of the key elements of a muon cooling channel
- Cooling can be quantified by the beam eigen-emittances
  - Two things in MICE affect eigen-emittances: (1) lattice nonlinearities, (2) beam-material interactions
  - Hence understanding nonlinearities is essential to disentangle these effects

For CM41 meeting skip ahead to slide 35

Approaches used in this initial study

- 1. Brute-force numerical integration of Lorentz force equation in z; convert to canonical momenta for diagnostics
  - easiest to implement; requires just B-field
- 2. Brute-force numerical integration of Hamilton's equations in z
  - requires EM potentials and first partial derivatives
- 3. Transfer map approach

These should give identical results. Doing both is just a consistency check

Results using (1) and (3) follow.
(2) was used just as a check.

Lorentz force w/ z as independent variable

- Consider quantities $(x, \gamma\beta_x, y, \gamma\beta_y, t, \gamma)$ (z)
- Equations of motion are:

$$\frac{dx}{dz} = \frac{\gamma\beta_x}{\gamma\beta_z}$$

$$\frac{d(\gamma\beta_x)}{dz} = \gamma \frac{q/mc^2}{\gamma\beta_z}\left[E_x + \frac{c}{\gamma}\left(\gamma\beta \times B\right)_x\right]$$

$$\frac{dy}{dz} = \frac{\gamma\beta_y}{\gamma\beta_z}$$

$$\frac{d(\gamma\beta_y)}{dz} = \gamma \frac{q/mc^2}{\gamma\beta_z}\left[E_y + \frac{c}{\gamma}\left(\gamma\beta \times B\right)_y\right]$$

$$\frac{dt}{dz} = \frac{1}{\beta_z c}$$

$$\frac{d\gamma}{dz} = \frac{q/mc^2}{\gamma\beta_z}\left[\gamma\beta \bullet E\right]$$

where $\gamma\beta_z = \sqrt{\gamma^2 - \left(\gamma\beta_x\right)^2 - \left(\gamma\beta_y\right)^2 - 1}$

Solenoid Channel Magnetic Field

For Lorentz force equation:

$$B_x=-(B'/2)x+(B'''/16)r^2x-(B^{(5)}/384)r^4x+(B^{(7)}/18432)r^6x$$
$$B_y=-(B'/2)y+(B'''/16)r^2y-(B^{(5)}/384)r^4y+(B^{(7)}/18432)r^6y$$
$$B_z=B-(B''/4)r^2+(B^{(4)}/64)r^4-(B^{(6)}/2304)r^6-\ldots$$

$$B_r=-(B'/2)r+(B'''/16)r^3-(B^{(5)}/384)r^4+(B^{(7)}/18432)r^6-\ldots$$

For diagnostics ($p^{canonical}=p^{mechanical}+qA$):

$$A_x=-(B/2)y + (B''/16)r^2y-(B^{(4)}/384)r^4y+(B^{(6)}/18432)r^6y-\ldots$$
$$A_y= (B/2)x - (B''/16)r^2x+(B^{(4)}/384)r^4x-(B^{(6)}/18432)r^6x-\ldots$$
$$A_z=0$$

Current Block Solenoid Model

- Superposition of 12 solenoids modeled via

$B(z)=\mu_0 J (r_2-r_1)[f(z-z_1)-f(z-z_2)]$

$$f(z)=\left[\frac{1}{2}+\frac{z}{2(r_2-r_1)}\log\frac{r_2+\sqrt{r_2^2+z^2}}{r_1+\sqrt{r_1^2+z^2}}\right]$$

Coil Parameters

$z_{center}(m)$	$z_{length}(m)$	$r_1(m)$	$r_2(m)$
-3200.d-3	110.6d-3	258.0d-3	325.8d-3
-2450.d-3	1314.3d-3	258.0d-3	280.1d-3
-1700.d-3	110.6d-3	258.0d-3	318.9d-3
-1300.d-3	199.5d-3	258.0d-3	288.9d-3
-861.d-3	201.3d-3	258.0d-3	304.2d-3
-202.75d-3	213.3d-3	267.0d-3	361.8d-3
202.75d-3	213.3d-3	267.0d-3	361.8d-3
861.d-3	201.3d-3	258.0d-3	304.2d-3
1300.d-3	199.5d-3	258.0d-3	288.9d-3
1700.d-3	110.6d-3	258.0d-3	318.9d-3
2450.d-3	1314.3d-3	258.0d-3	280.1d-3
3200.d-3	110.6d-3	258.0d-3	325.8d-3

Current density for 3 momenta

$140 MeV/c$	$200 MeV/c$	$240 MeV/c$
134.d6	134.d6	134.d6
147.d6	147.d6	147.d6
131.d6	131.d6	122.d6
104.d6	135.d6	148.d6
79.d6	113.d6	138.d6
71.d6	104.d6	120.d6
-71.d6	-104.d6	-120.d6
-75.d6	-112.d6	-129.d6
-106.d6	-140.d6	-154.d6
-131.d6	-131.d6	-122.d6
-147.d6	-131.d6	-147.d6
-134.d6	-134.d6	-134.d6

Beam initial conditions (per Chris Rogers)

Initial 2nd moment matrix was obtained based on Greg Penn analysis for a matched beam.
Computed at 140, 200, 240 MeV/c
Numerical distribution obtain from 6D uncorrelated Gaussian via Cholesky decompostion.
Energy spread and spread in time-of-flight were effectively zero.

Example:
Let u=(x, $m\gamma\beta_x$,y,$m\gamma\beta_y$).
4x4 beam initial 2nd moment matrix for 6mm transverse rms emittance, 200 MeV/c reference momentum, in a 4 T solenoid, is given by:

```
[ 1055.52342e-6   ,       0.        ,       0.        ,  -632.87592112e-3/m ]
[    0.        ,   760.21240555/m^2,    632.87592112e-3/m,       0.         ]
[    0.        ,   632.87592112e-3/m,   1055.52342e-6   ,       0.         ]
[ -632.87592112e-3/m,     0.        ,       0.        ,    760.21240555/m^2  ]
```

where m=105.6583715e6 eV/c, and the unit of length = meters

Hamilton's equations w/ z as independent variable

(in dimensionless units)

- Consider canonical quantities (x,p_x,y,p_y,t,p_t) (z)
- Define dimensionless variables
  - later we'll set $k=mc^2$, $l=1m$, $\omega=c$

$$\bar{x} = x/l \qquad \bar{y} = y/l \qquad \bar{t} = \omega t$$
$$\bar{p}_x = p_x/k \qquad \bar{p}_y = p_y/k \qquad \bar{p}_t = p_t/(\omega l k)$$

- The Hamiltonian is:

$$K(\bar{x},\bar{p}_x,\bar{y},\bar{p}_y,\bar{t},\bar{p}_t) = \frac{-1}{l}\sqrt{\left(\frac{\omega l}{c}\bar{p}_t + \frac{q}{kc}\psi\right)^2 - \left(\bar{p}_x - \frac{q}{k}A_x\right)^2 - \left(\bar{p}_y - \frac{q}{k}A_y\right)^2} - \frac{q}{lk}A_z$$

$$A = A(l\bar{x}, l\bar{y}, z, \bar{t}/\omega)$$

For this study, $\psi=0$

Only used as a consistency check in this study.

To integration Hamilton's equation, need the electromagnetic potentials and some derivatives

A_x=-(B/2)y + (B''/16)r^2y-($B^{(4)}$/384)r^4y+($B^{(6)}$/18432)r^6y-...
A_y= (B/2)x - (B''/16)r^2x+($B^{(4)}$/384)r^4x-($B^{(6)}$/18432)r^6x-...
A_z=0

$\partial A_x/\partial x=-\partial Ay/\partial y=(B''/8)xy-(B^{(4)}/96)r^2xy+(B^{(6)}/3072)r^4xy-...$

$\partial A_x/\partial y=-(B/2)+(r^2/16+y^2/8)B''-(r^2/384+y^2/96)r^2B^{(4)}+(r^2/18432+y^2/3072)r^4B^{(6)}$

$\partial A_y/\partial x=(B/2)-(r^2/16+x^2/8)B''+(r^2/384+x^2/96)r^2B^{(4)}-(r^2/18432+x^2/3072)r^4B^{(6)}$

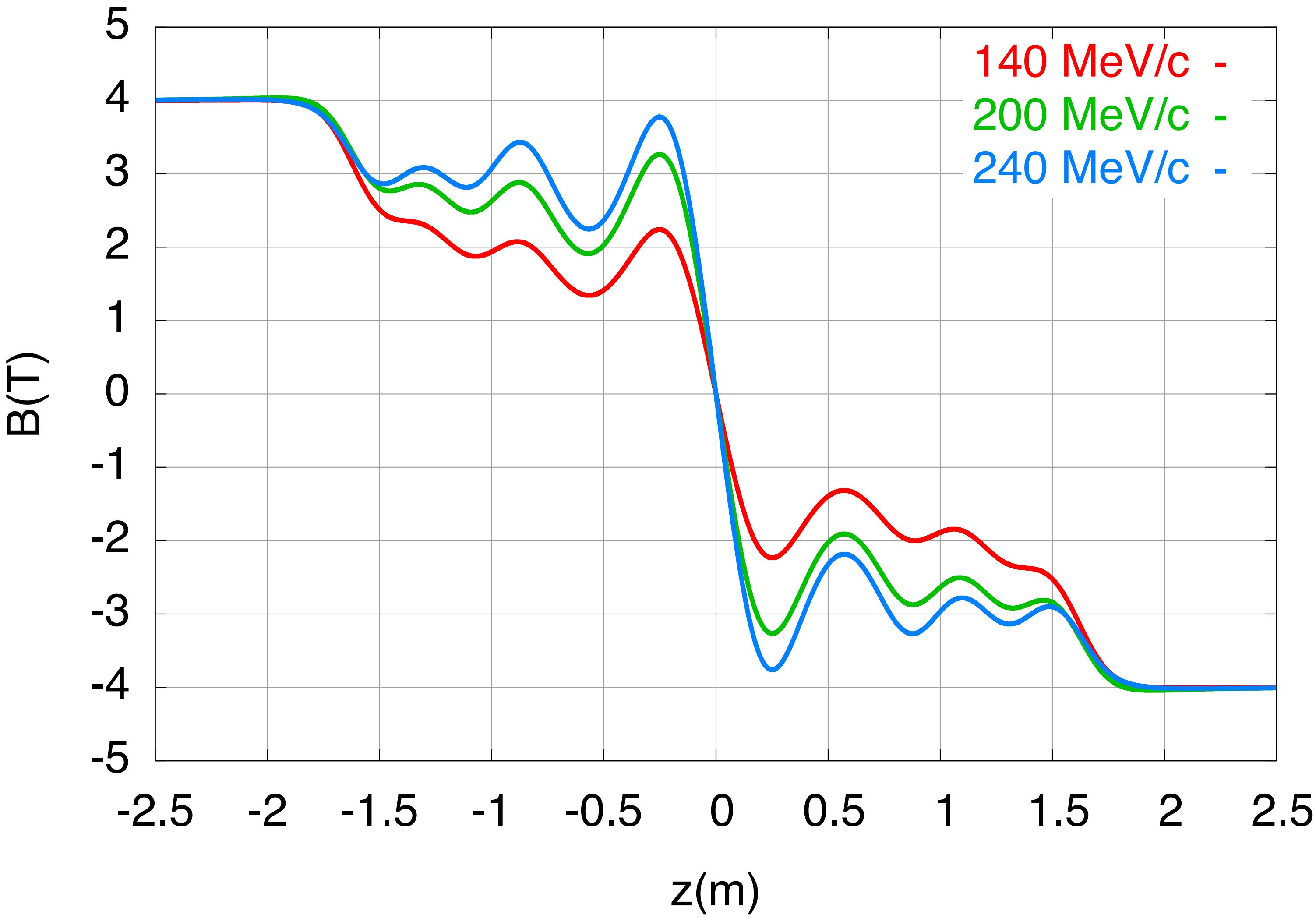
B_x=-(B'/2)x+(B'''/16)r^2x-($B^{(5)}$/384)r^4x+($B^{(7)}$/18432)r^6x
B_y=-(B'/2)y+(B'''/16)r^2y-($B^{(5)}$/384)r^4y+($B^{(7)}$/18432)r^6y
B_z=B-(B''/4)r^2+($B^{(4)}$/64)r^4-($B^{(6)}$/2304)r^6-...

B_r=-(B'/2)r+(B'''/16)r^3-($B^{(5)}$/384)r^4+($B^{(7)}$/18432)r^6-...

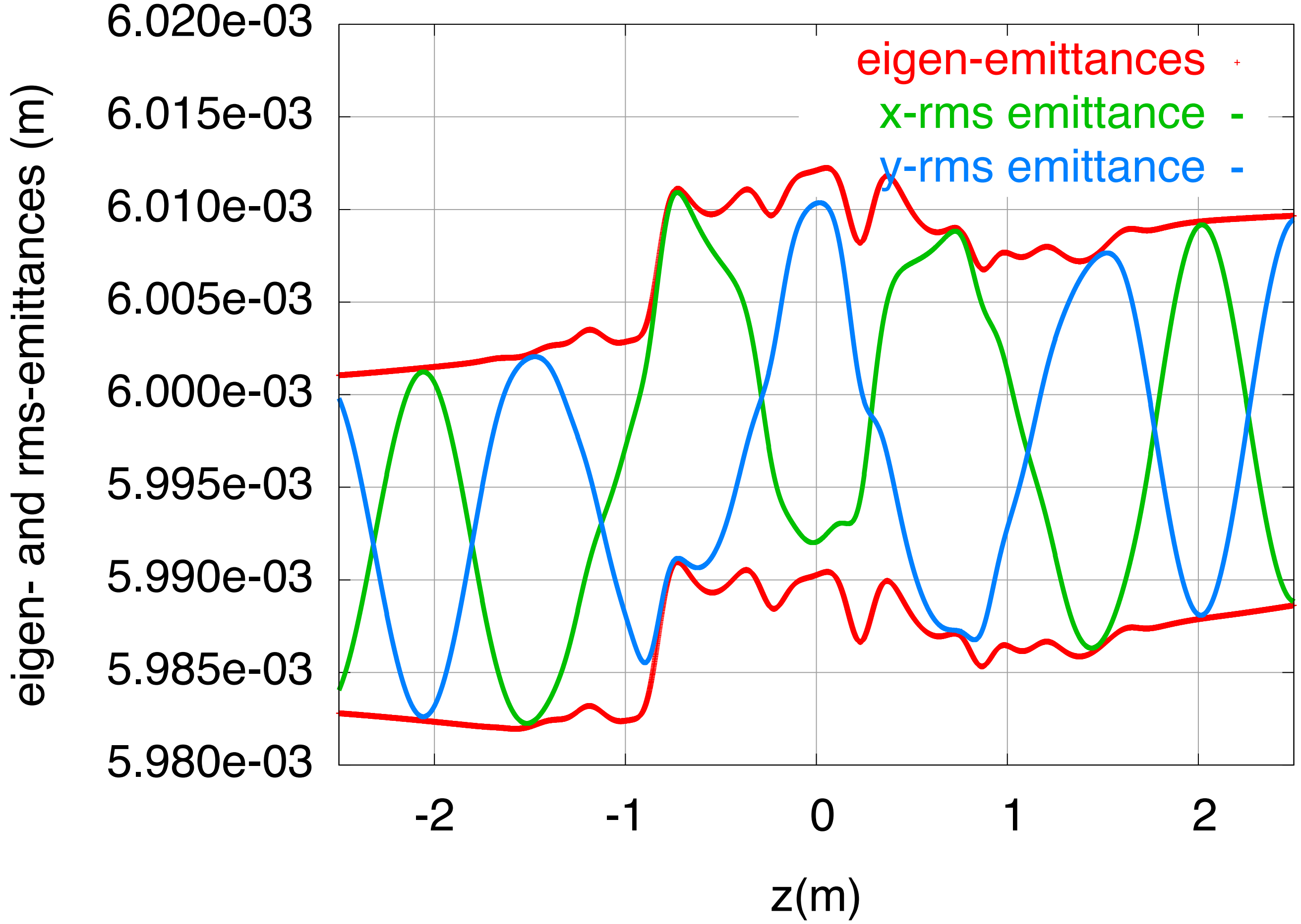
Simulation parameters

- ~1M particles (256 cores x 4K particles/core)
- 5000 z-steps (overkill)
  - Lorentz w/ RK4
  - Hamilton w/ Gauss4
- Simulation time: < 1 minute per run

On-axis B-field



Eigen-emittance growth for the case $\varepsilon_{x,n}=\varepsilon_{y,n}=6mm$, p=200 MeV/c



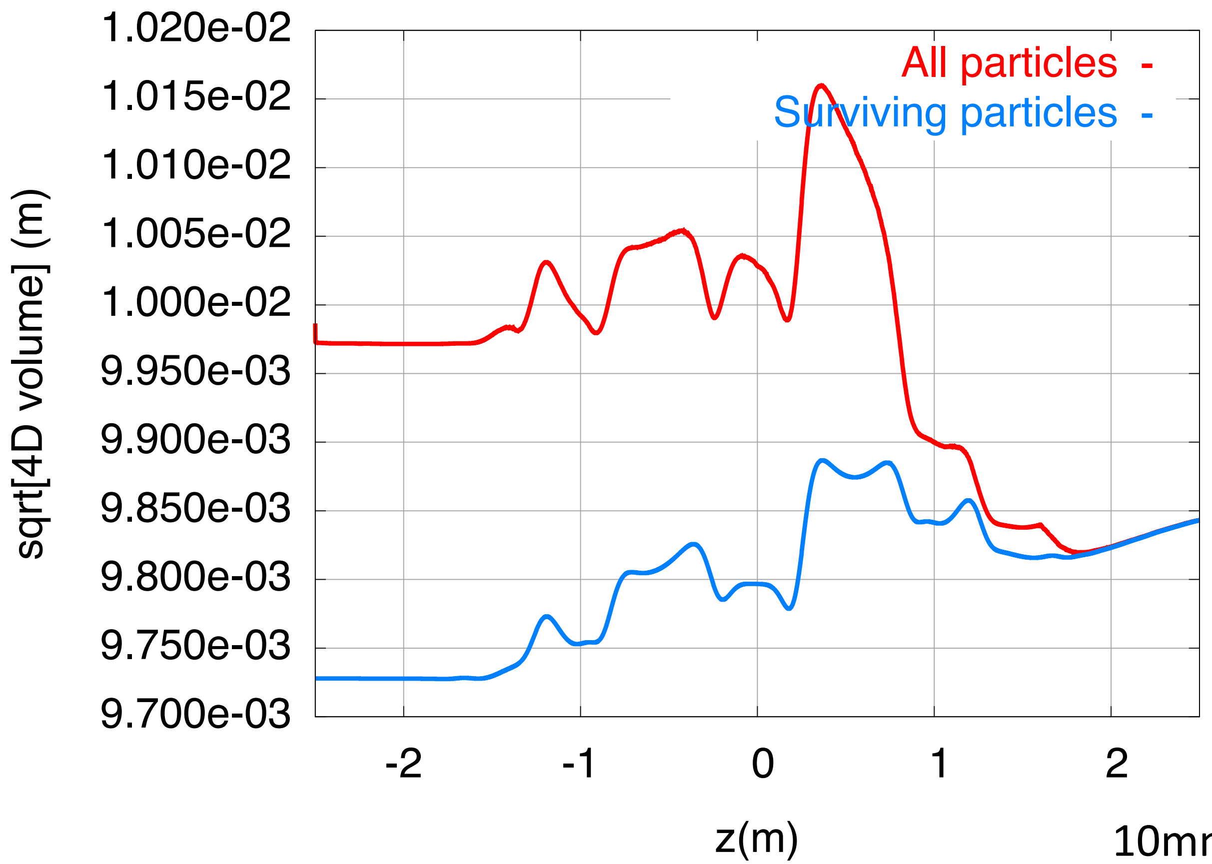
Choice of diagnostic quantity

- In the preceding plot, the horizontal and vertical should be identical but are not due to sampling w/ only ~ 1M particles
- Quantity of interest is overall emittance growth, not slight discrepancies in x and y due to sampling
- Instead will plot sqrt(eigen1*eigen2)
  - same in canonical and noncanonical variables
- From here on, "emittance" refers to sqrt(eigen1*eigen2) unless noted otherwise

Particle Loss:

If there is any particle loss, the simulation is run twice

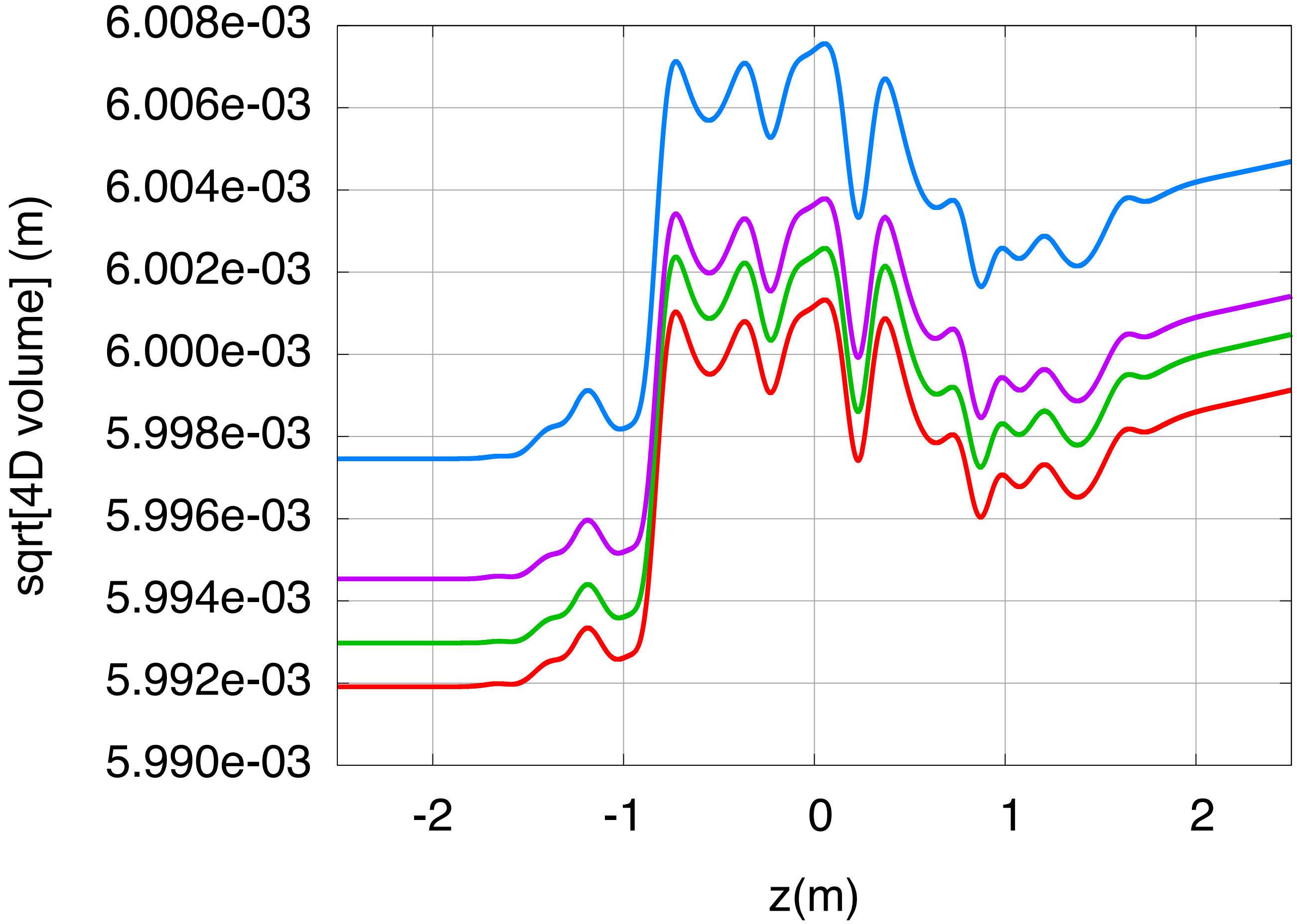
2^{nd} run computes diagnostics using just surviving particles



10mm, 140 MeV/c

Sampling:

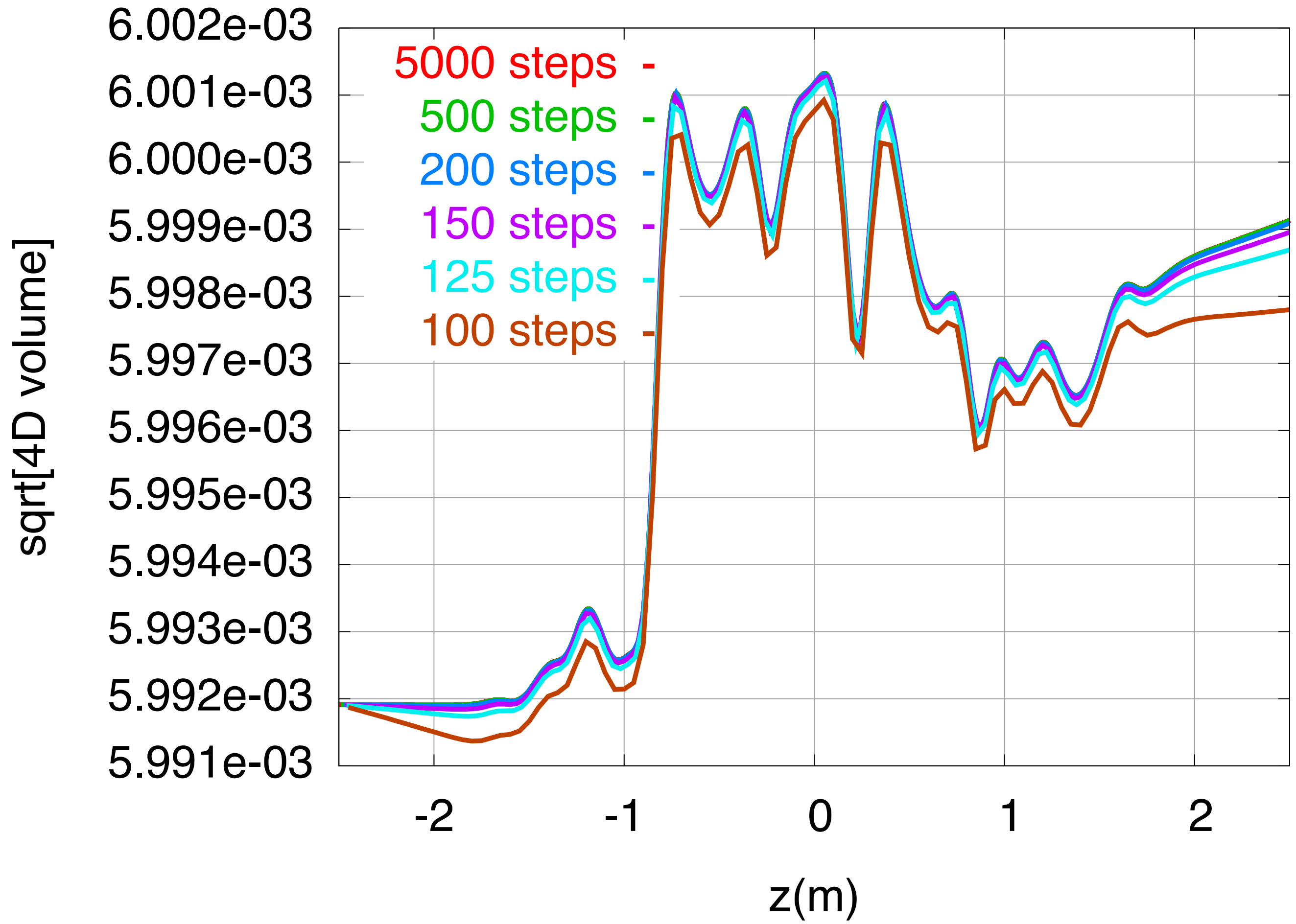
The emittance evolution is roughly the same for different random # seeds but shifted due to different initial value



Emittance evolution for 3 different random # seeds (6mm, 200 MeV/c)

Variation with integration step size:

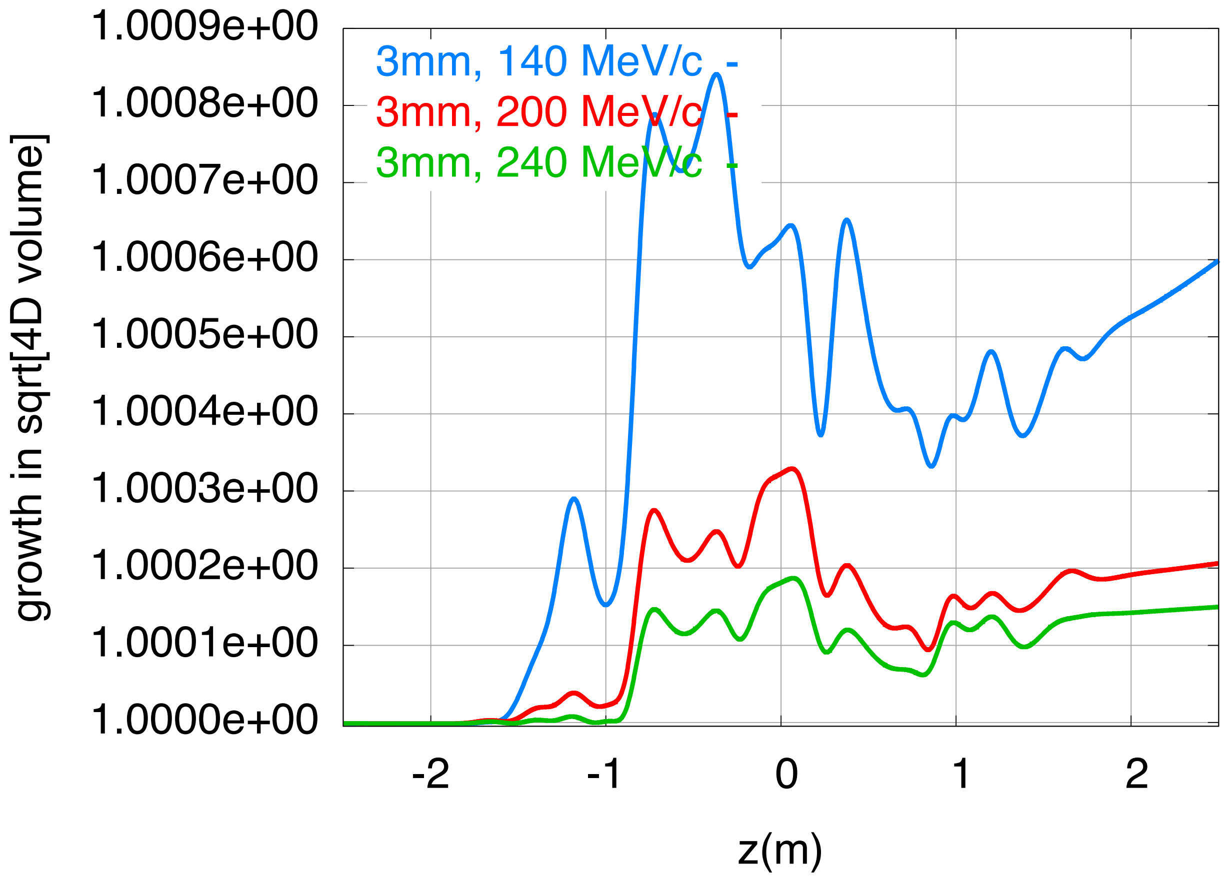
Good convergence when beamline is simulation w/ 200 steps;
This study used 5000 steps, which is more than sufficient



Emittance evolution for different step sizes (6mm, 200 MeV/c)

Simulation Results

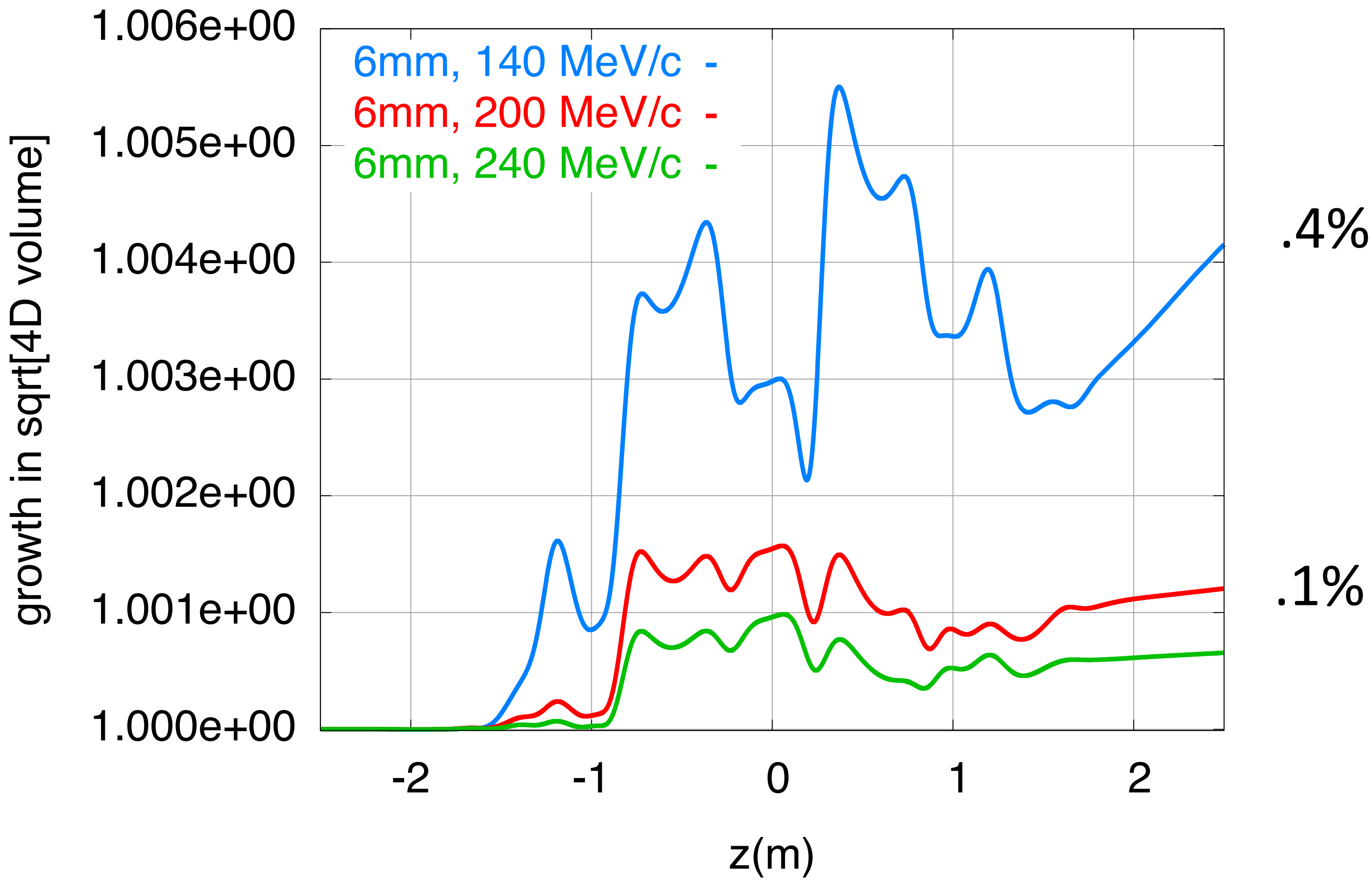
Emittance Growth for 3mm case is negligible



.06%

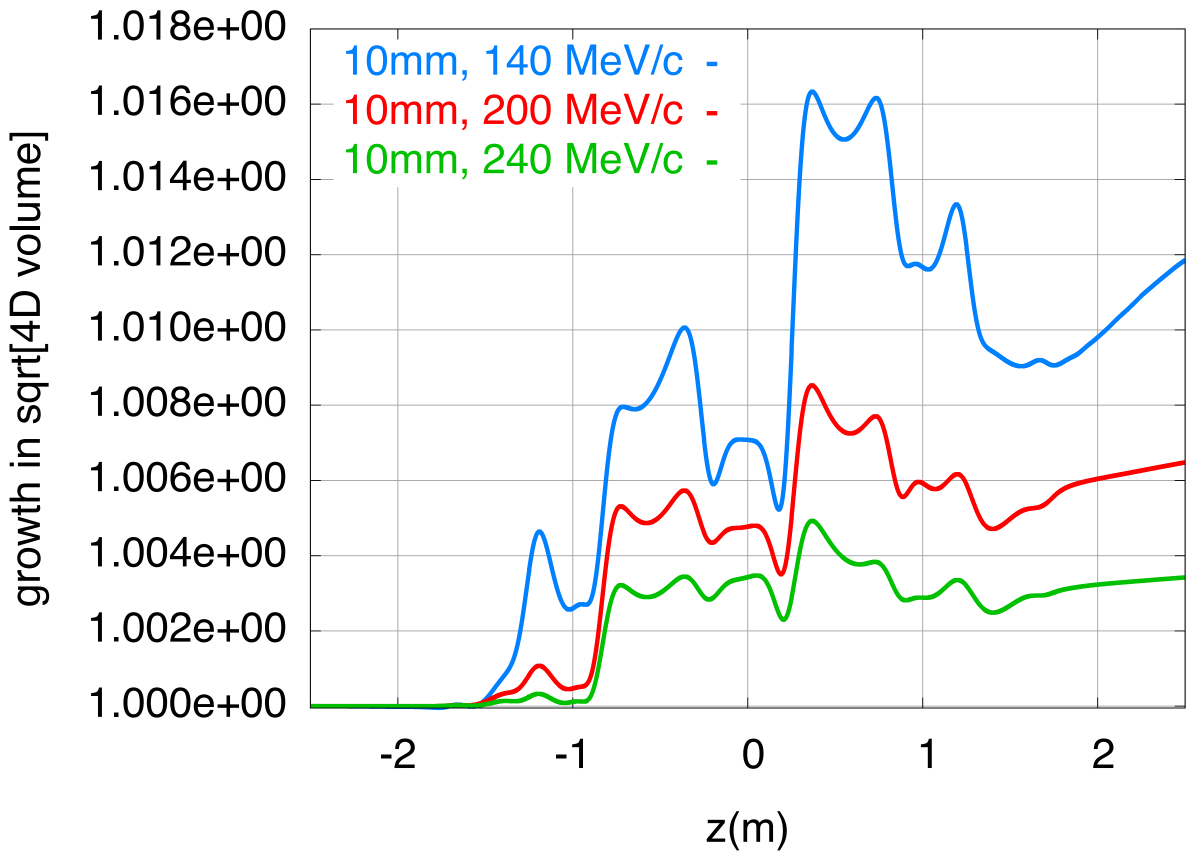
.02%

Emittance Growth for 6mm case:
~0.1% for 240,200 MeV/c; ~0.4% for 140 MeV/c



Emittance Growth for 10mm case

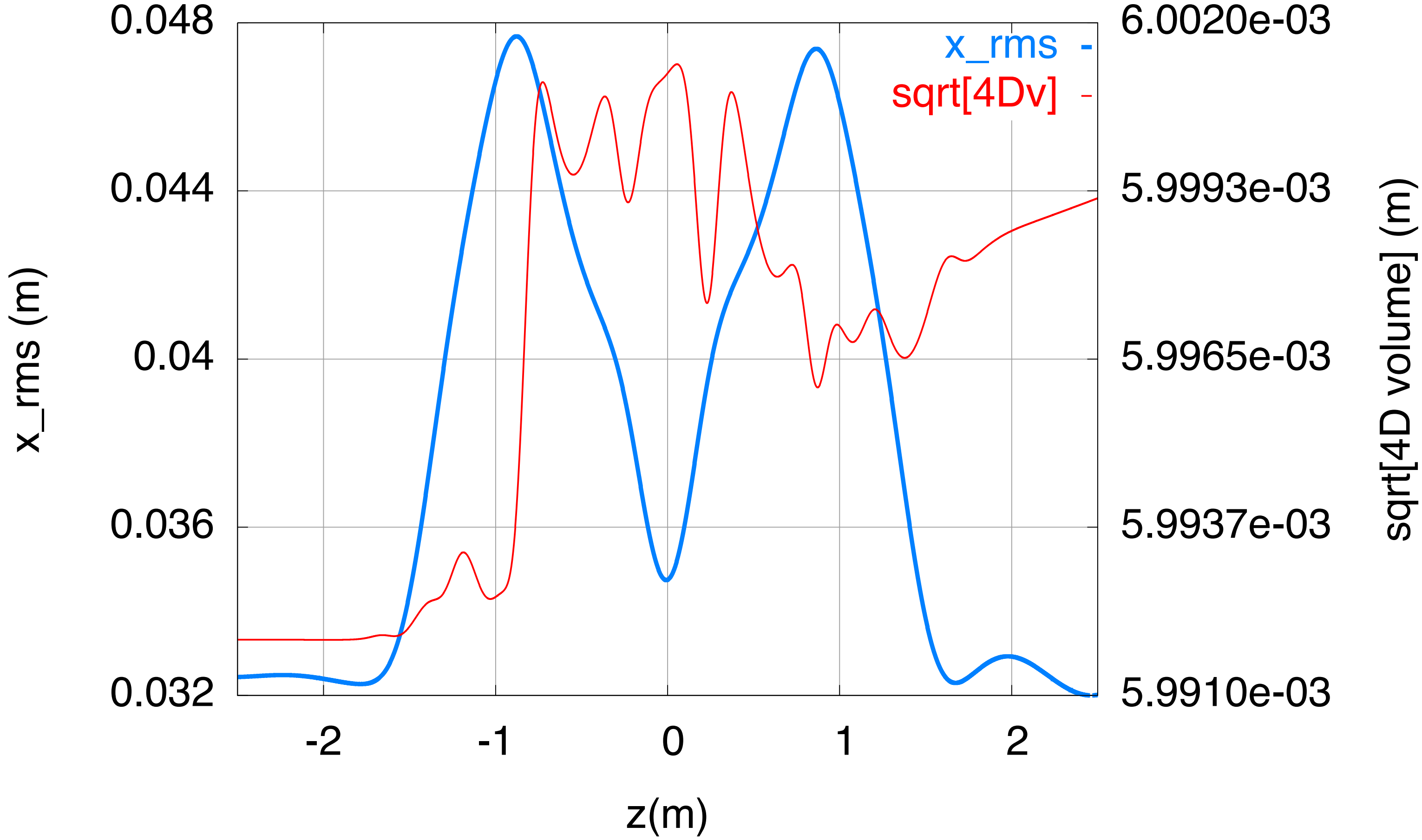
~0.3% for 240 MeV/c; ~0.6% for 200 MeV/c; ~1% for 140 MeV/c



1%

RMS size for 6mm, 200 MeV/c case

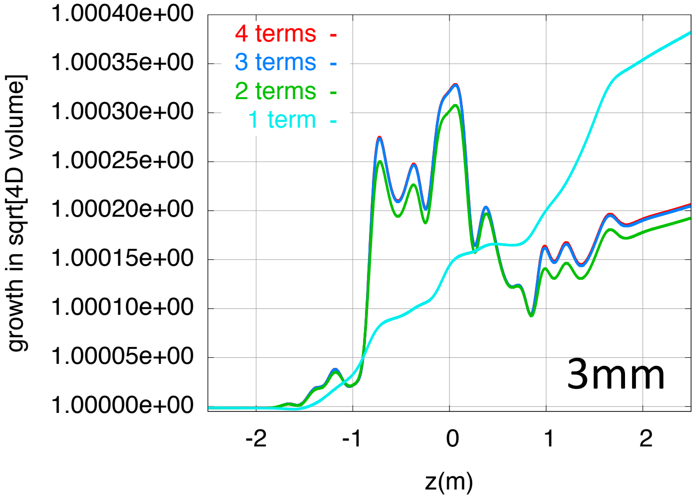
shown along with sqrt[4D volume]

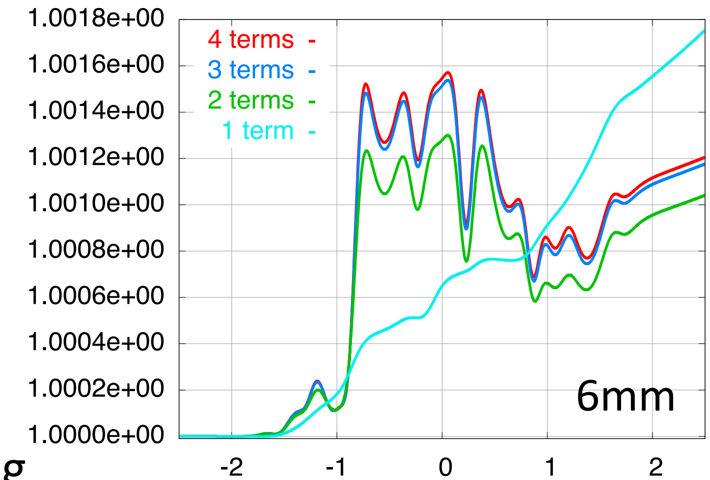


Initial increase in 4D volume is strongly correlated with initial increase in x_rms

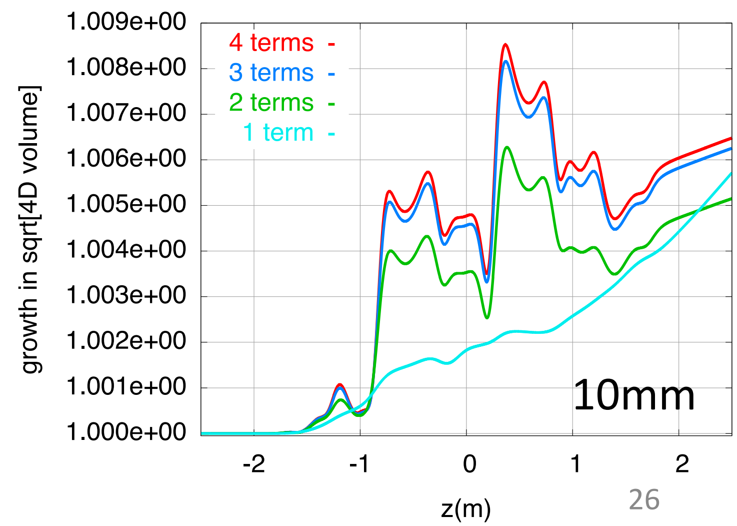
Importance of Nonlinearities

200 MeV/c; ε=3mm, 6mm, 10mm





- 1-term result is clearly wrong
- 2-term result is adequate
- 3- and 4-term results converging
  - "3-term" means keep through $B^{(4)}$ in B_z , in other words, 5^{th} order code in Hamiltonian formalism



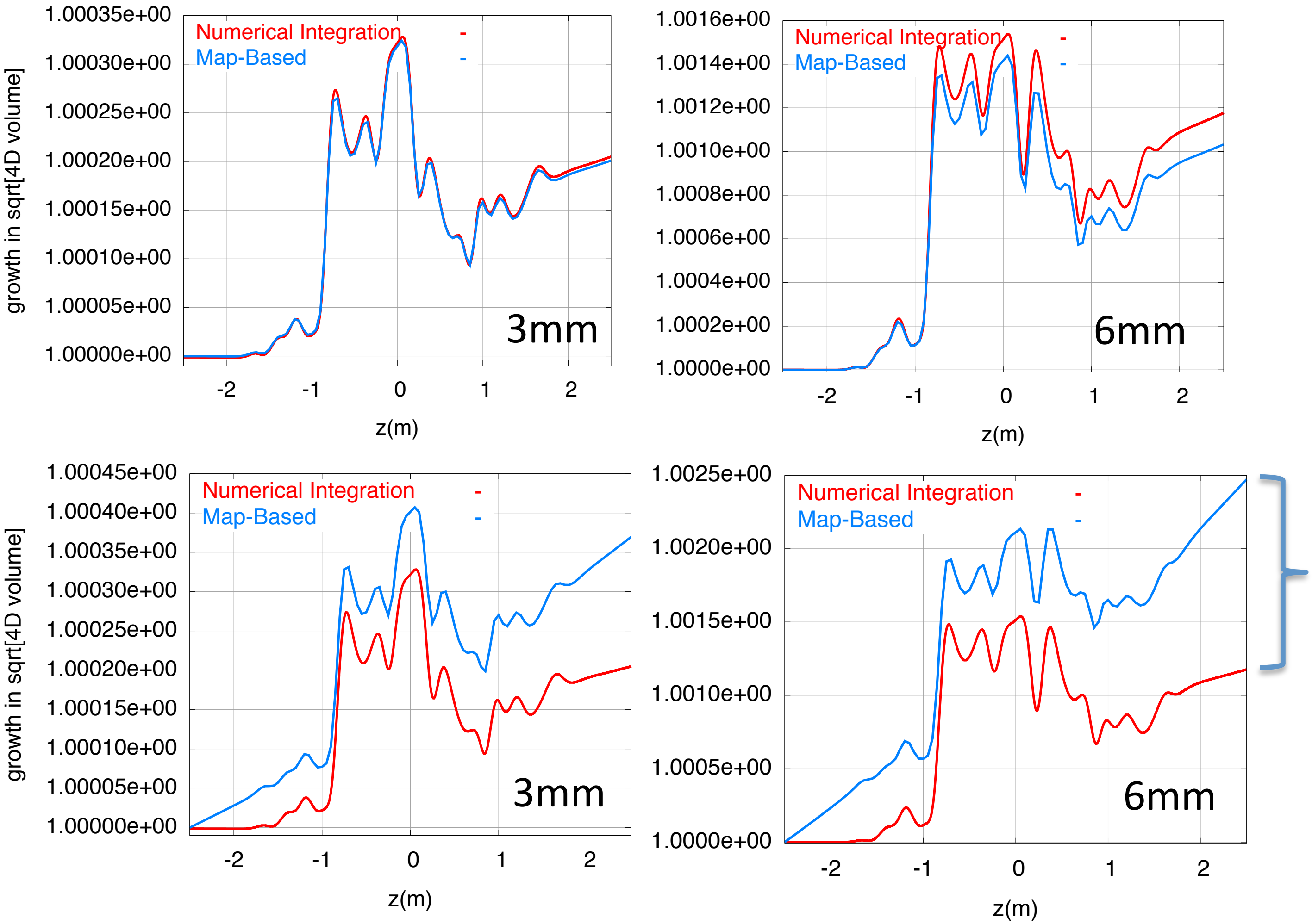
3 term

B_x=-(B'/2)x + (B'''/16)r^2x - ($B^{(5)}$/384)r^4x + ($B^{(7)}$/18432)r^6x
B_y=-(B'/2)y + (B'''/16)r^2y - ($B^{(5)}$/384)r^4y + ($B^{(7)}$/18432)r^6y
B_z= B - (B''/4)r^2 + ($B^{(4)}$/64)r^4 - ($B^{(6)}$/2304)r^6-...

Comparison w/ Map-based Code

- Preceding used high accuracy integration of exact equations of motion w/ approximate B field (expanded as a Taylor series in r)
- Map-based code expands w.r.t. reference particle
  - approximate B and approximate equations of motion
- The following uses modified version of MaryLie/IMPACT, which contains MaryLie 5th order routines
  - includes "genmap" routines for soft-edge magnets
  - the beamline was "sliced" and 5th order maps were generated for each slice and applied to particles

Map-based tracking results: 200 MeV/c; ε=3mm, 6mm



Symplectic tracking

Taylor map tracking

In this case symplectic tracking gives a more accurate prediction of emittance growth than Taylor map tracking. But it is probably academic because the difference is small.

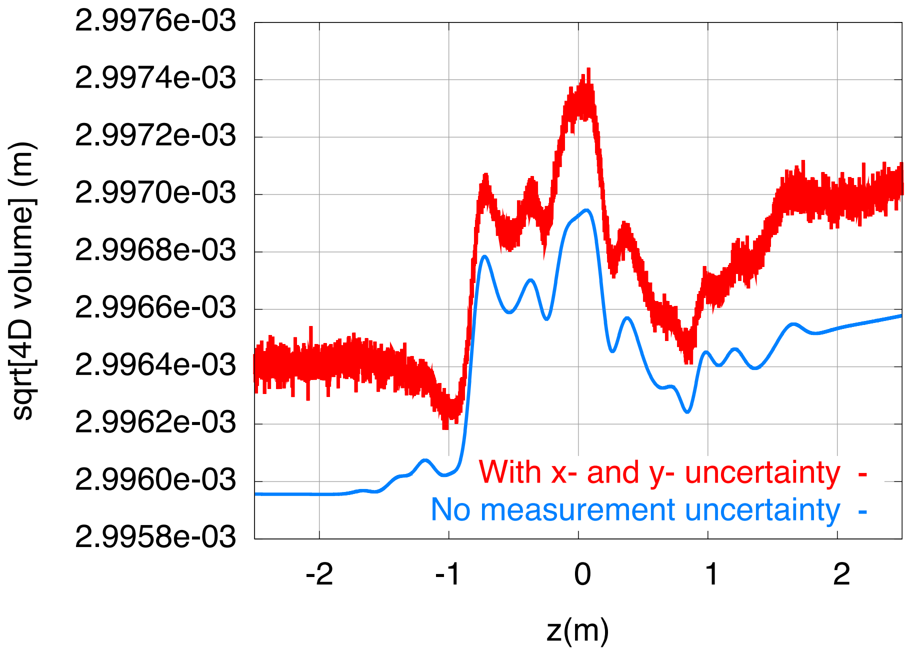
Impact of Measurement Uncertainty

- Tracker uncertainties in measured (x,px,y,py) are correlated w/ each other and w/ true values
- Here we use a simplified model:
  - $\sigma_x = \sigma_y = 400\mu m$, $\sigma_{px} = 2.36$ MeV/c/m_0, $\sigma_{py} = 1.8$ MeV/c/m_0
- Other effects not included in this study:
  - particle misidentification
  - windows
  - magnet misaligments
- In the following, we modified the simulation diagnostic to add random normal deviates at end of each z-step based on the above variances

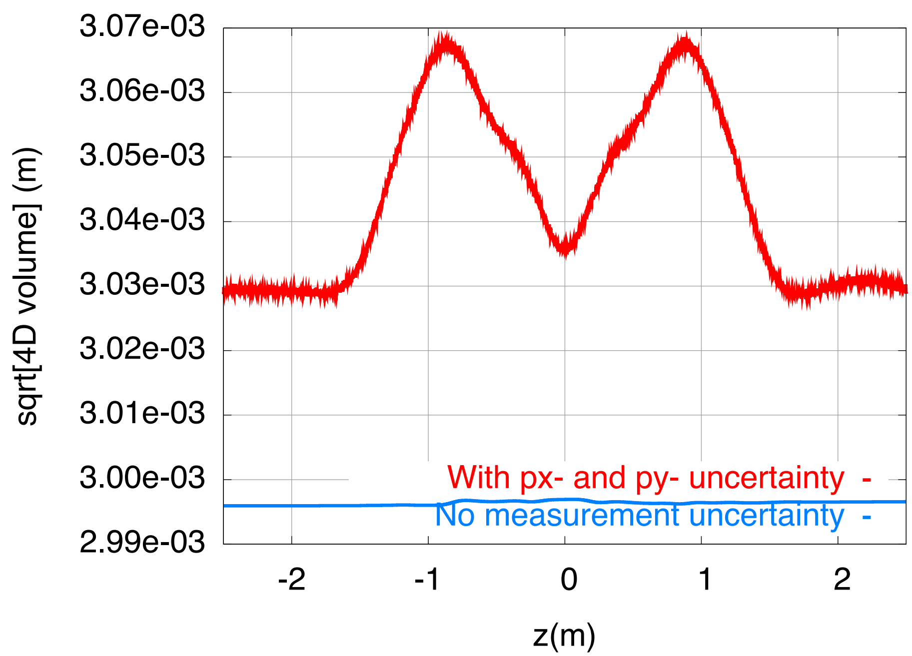
3mm, 200 MeV/c:

(x,y) measurement errors vs. (px,py) measurement errors

(x,y) errors



(px,py) errors



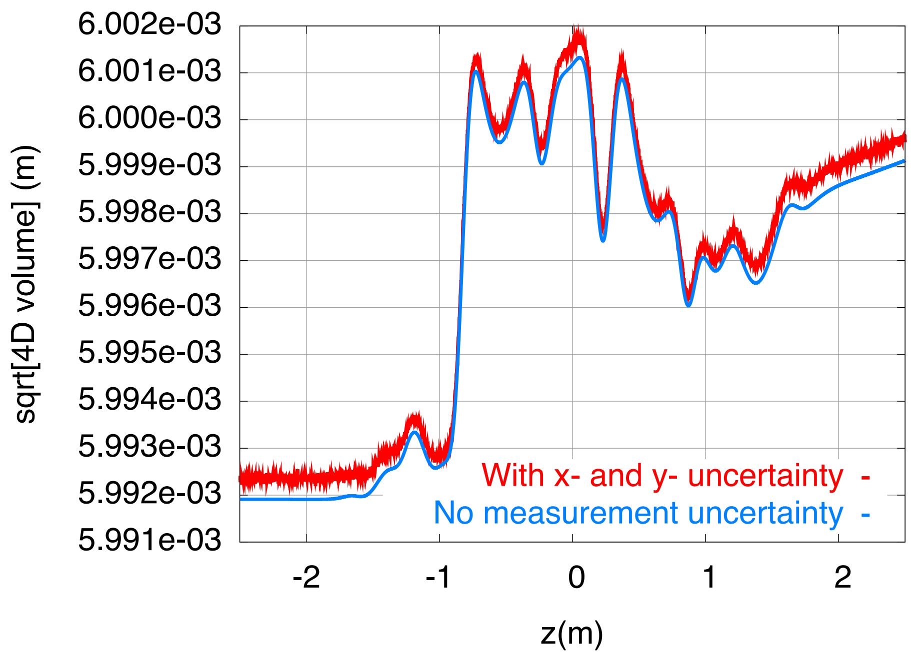
With (x,y) errors the emittance is slightly larger than ideal case

With (px,py) errors the true signal is not visible

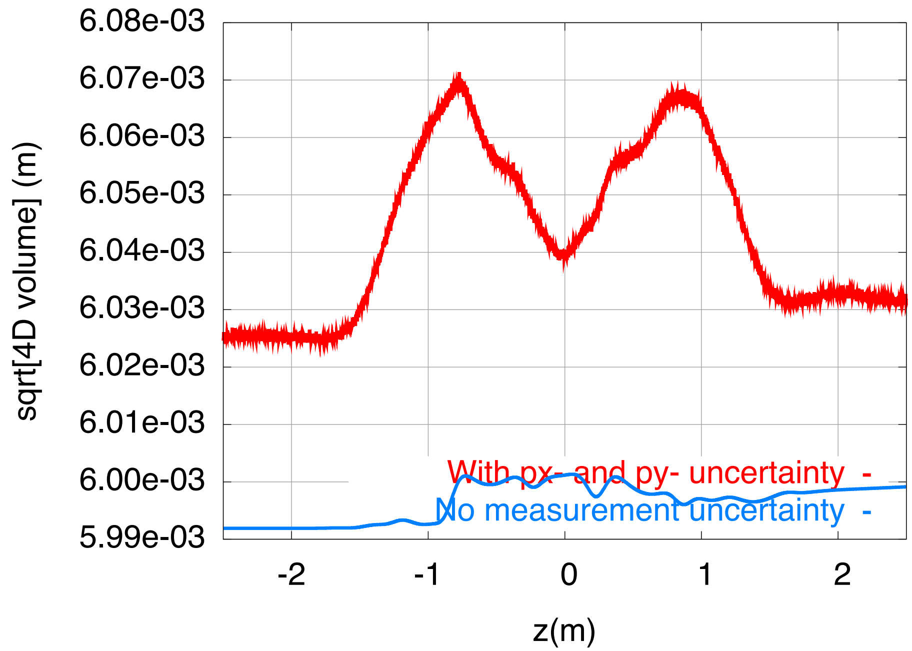
6mm, 200 MeV/c:

(x,y) measurement errors vs. (px,py) measurement errors

(x,y) errors



(px,py) errors

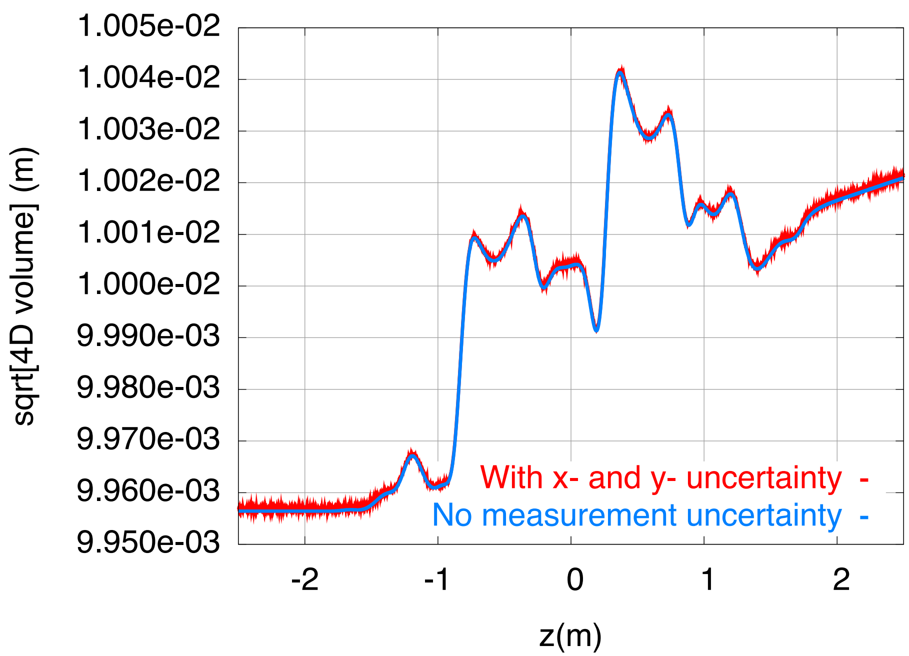


With (x,y) errors the emittance is very slightly larger than ideal case
With (px,py) errors the true signal is not visible

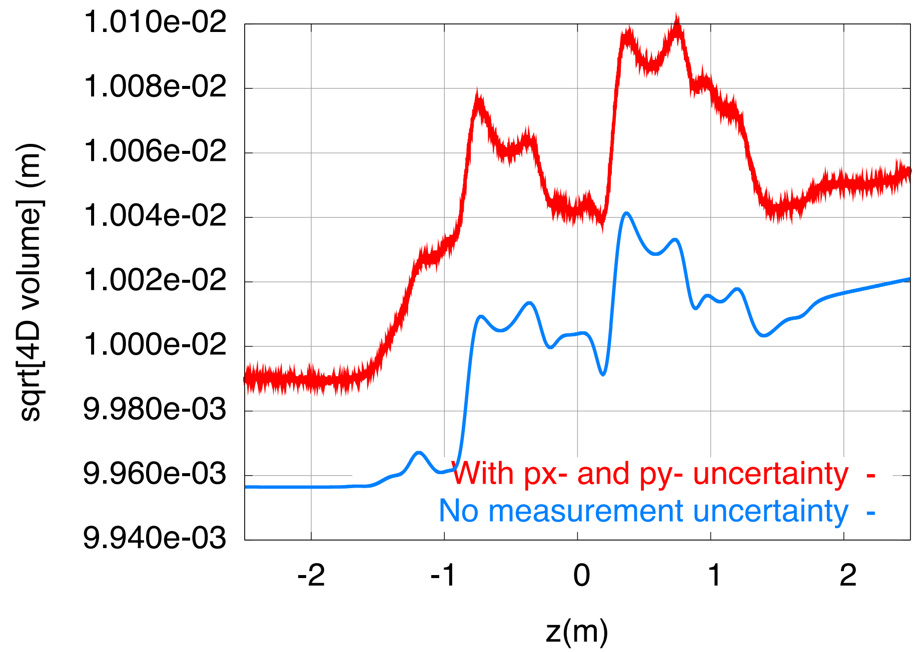
10mm, 200 MeV/c:

(x,y) measurement errors vs. (px,py) measurement errors

(x,y) errors



(px,py) errors



With (x,y) errors the emittance is just noisy compared with ideal case
With (px,py) errors the emittance is slightly larger than ideal case

Measurement uncertainty

- For these parameters, errors in momentum measurement are more important than errors in position measurement

basis of the following estimate. Consider that the 2D emittance, in the absence of correlations, is given by

$$\epsilon_{2D} = \sqrt{\sigma_x^2 \sigma_{px}^2} \tag{14}$$

Now consider a simple model in which the measured variance, $\sigma^2_{x,m}$ is related to the true variance, $\sigma^2_{x,t}$, and related to the variance associated with the measurement uncertainty, $\sigma^2_{x,u}$ by

$$\sigma^2_{x,m} = \sigma^2_{x,t} + \sigma^2_{x,u} \tag{15}$$

and similarly for $\sigma^2_{px,m}$. Then the measured 2D emitance satisifies,

$$\begin{aligned} \epsilon_{2D,m} &= \sqrt{\sigma^2_{x,m}\sigma^2_{px,m}} \\ &= \sqrt{(\sigma^2_{x,t}+\sigma^2_{x,u})(\sigma^2_{px,t}+\sigma^2_{px,u})} \\ &= \sqrt{\sigma^2_{x,t}\sigma^2_{px,t}} * \sqrt{1+\sigma^2_{x,u}/\sigma^2_{x,t}+\sigma^2_{px,u}/\sigma^2_{px,t}} \end{aligned} \tag{16}$$

where we have dropped the small term proportional to $\sigma^2_{x,u}\sigma^2_{px,u}$. Applying the binomial expansion,

$$\epsilon_{2D,m} = \epsilon_{2D,t} * \left[1 + \frac{\sigma^2_{x,u}}{2\sigma^2_{x,t}} + \frac{\sigma^2_{px,u}}{2\sigma^2_{px,t}} + ...\right] \tag{17}$$

Multiplying the 2D horizontal emittance and 2D vertical emittance, we have approximately (ignoring differences in the horizontal and vertical quantities), we find that the measured 4D emittance is related to the true 4D emittance by,

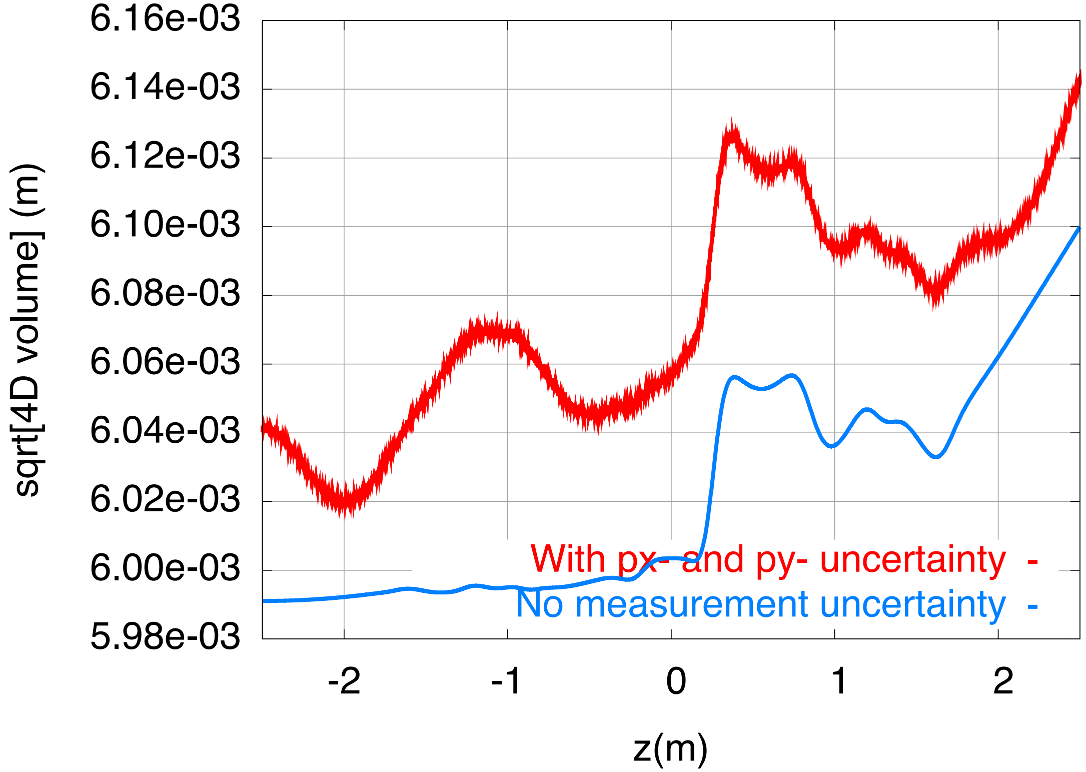
$$\epsilon_{4D,m} = \epsilon_{4D,t} * \left[1 + \frac{\sigma^2_{x,u}}{\sigma^2_{x,t}} + \frac{\sigma^2_{px,u}}{\sigma^2_{px,t}} + ...\right] \tag{18}$$

Now consider, for example, the 6mm - 200 MeV/c case. The measurement uncertainties and beam initial conditions are such that $\sigma^2_{x,u}/\sigma^2_{x,t} \sim 0.1\%$ and $\sigma^2_{px,u}/\sigma^2_{px,t} \sim 1\%$. Hence, it is not surprising that the previous plots show that momentum measurement errors are much more important than position measurement errors.

- But the magnitude of the measurement-induced errors is small

A Mismatched Case

6mm, 200 MeV/c case:
Mismatched induced by multiplying m_{11} and m_{33} by 1.5, dividing m_{22} and m_{44} by 1.5



- In this example, emittance growth due to mismatch is 2%, and the shift due to measurement uncertainty is an additional 1%

Conclusions (no absorber)

- Matched input beam emittance growth:
  - 3mm: negligible
  - 6mm: 0.1 - 0.4%
  - 10mm: 0.4 – 1.2%
- Effect of 5th order nonlinearities is visible
  - results are reasonably accurate keeping 3rd order but this is an idealized simulation
    - suggest keeping 5th order in simulations
- Map-based prediction is more accurate using symplectic tracking than Taylor maps, but the magnitude of the difference is small

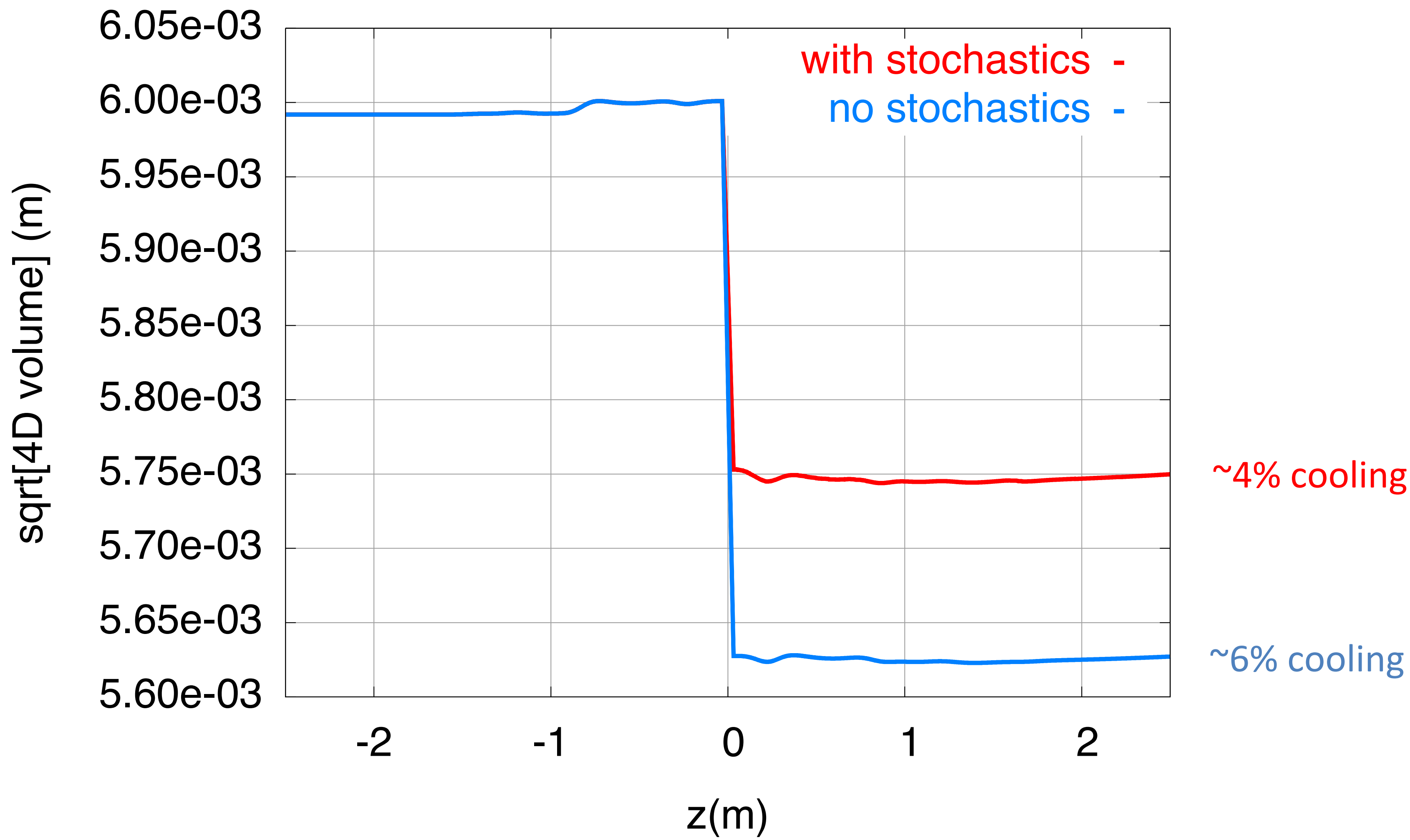
Conclusions (no absorber) cont.

- Based on this model of measurement uncertainty:
  - momentum errors more important than position errors
    - momentum errors swamp the signal in simulation, but probably irrelevant since we only measure initial & final state
    - magnitude of effect is small compared w/ expected cooling
      - 10mm, 200MeV/c case: leads to 0.3% effect
- Beam mismatch is a concern
  - In the example here, mismatch caused a 2% emittance growth
    - measurement uncertainty shifted the emittance by an additional ~1%, but not important since the shift is to both the initial and final state

Add LiH absorber

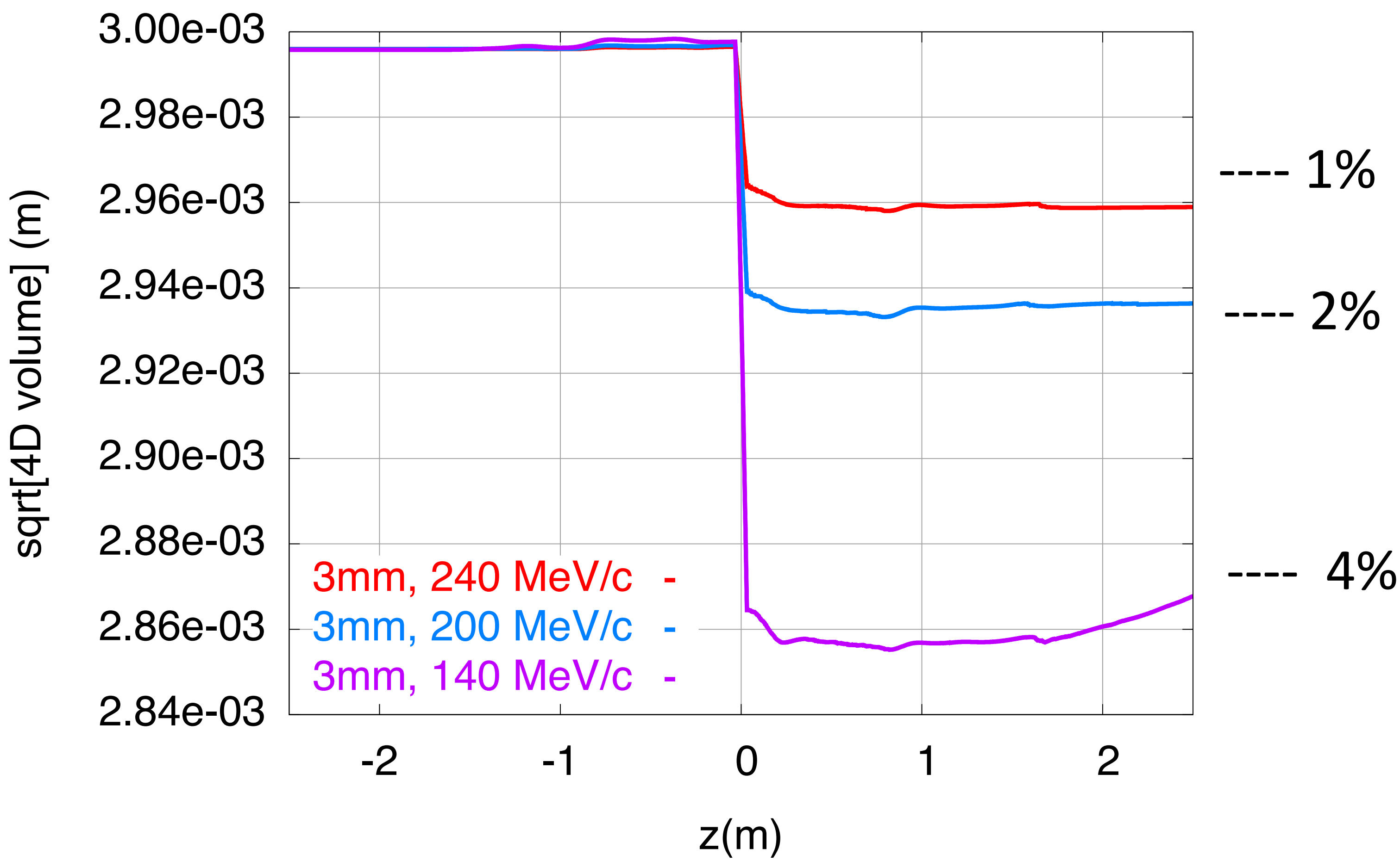
- Include 65mm thick LiH absorber in the simulations
- Beam-material interaction routines the same as in ICOOL

Emittance evolution for 6mm, 200 MeV/c case with LiH absorber : stochastics (multiple scattering & energy straggling) vs. no stochastics

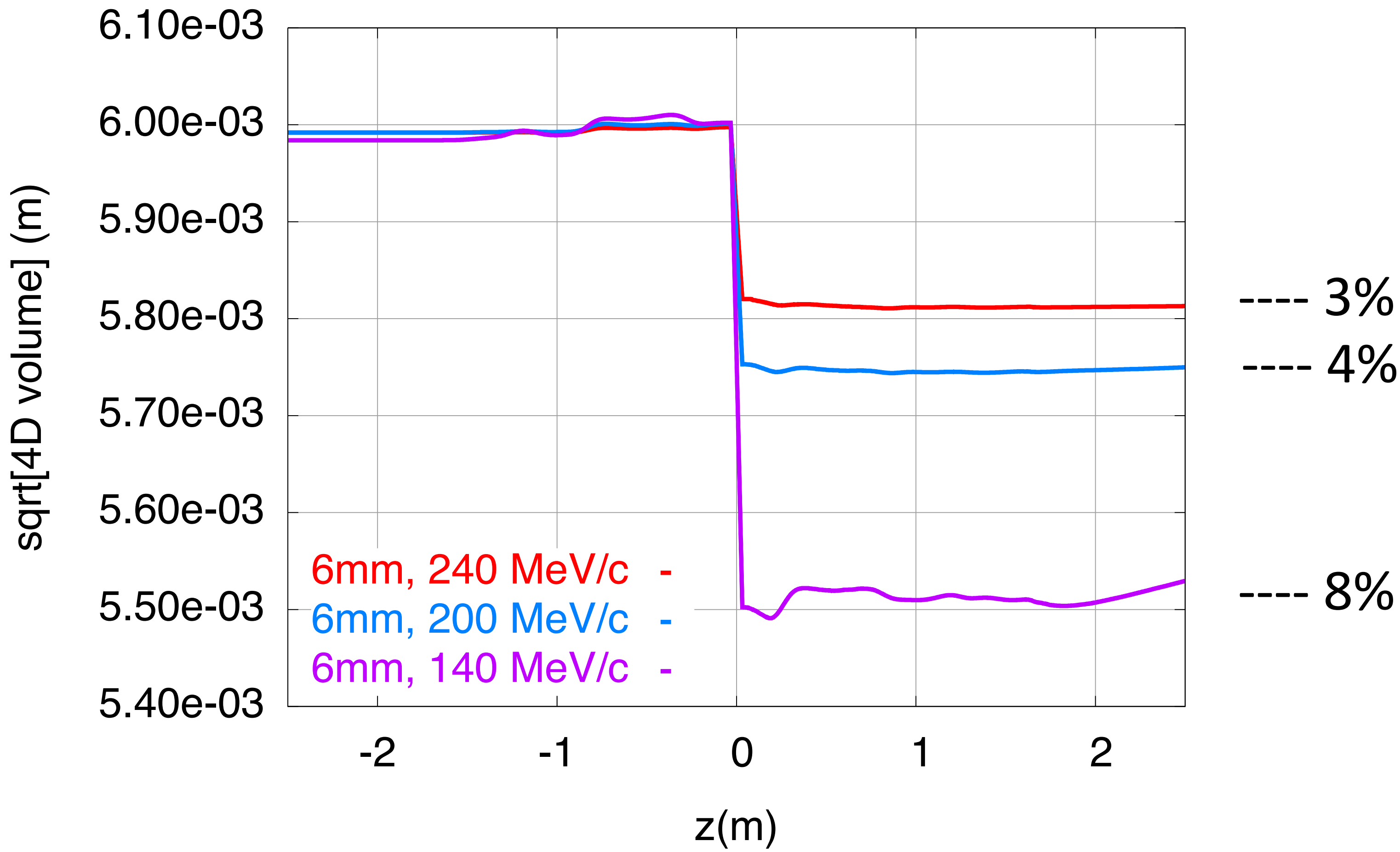


All results that follow include stochastics

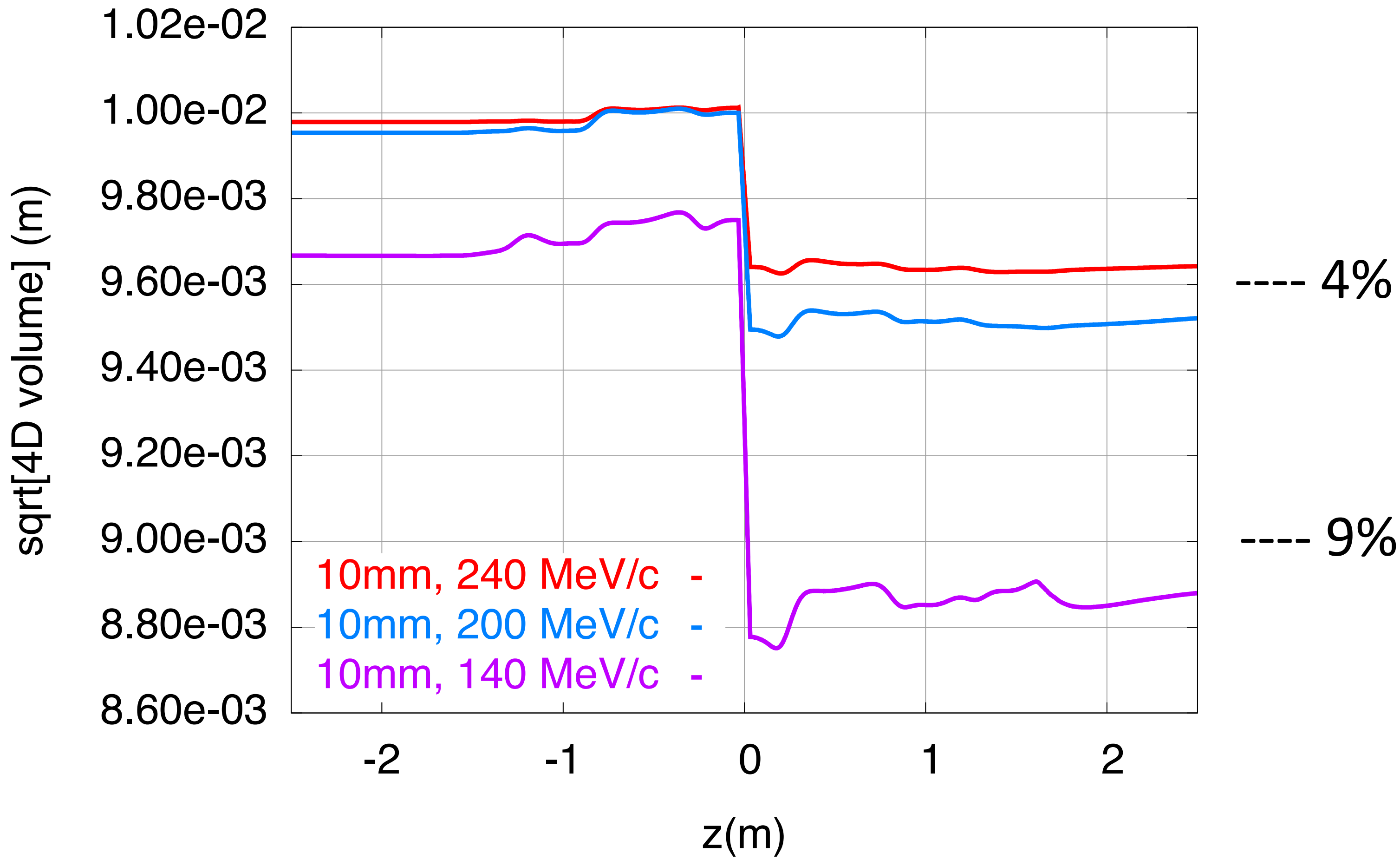
Emittance Evolution for 3mm cases with LiH absorber



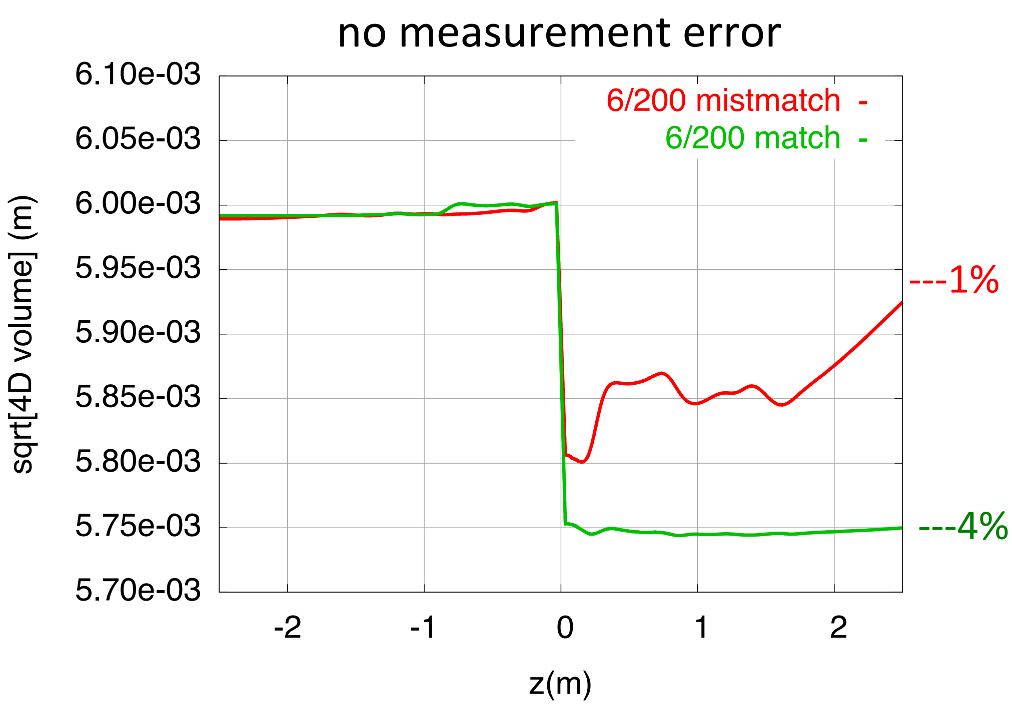
Emittance Evolution for 6mm cases with LiH absorber

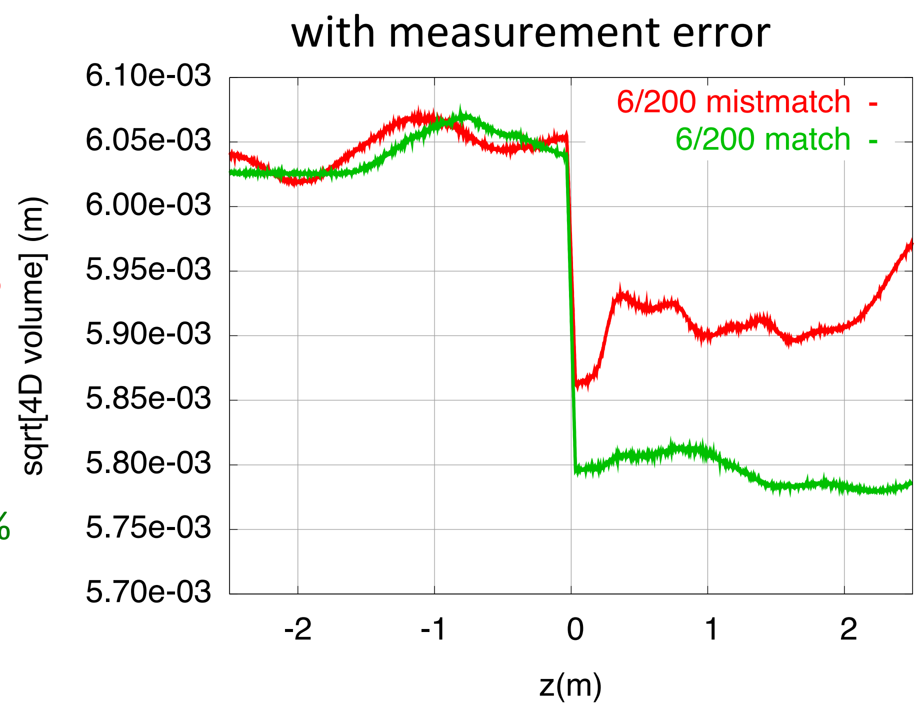


Emittance Evolution for 10mm cases with LiH absorber



Mismatched Case (multiply m_{11}, m_{33} by 1.5; divide m_{22} and m_{44} by 1.5): Emittance Evolution for 6mm, 200MeV/c case with LiH absorber

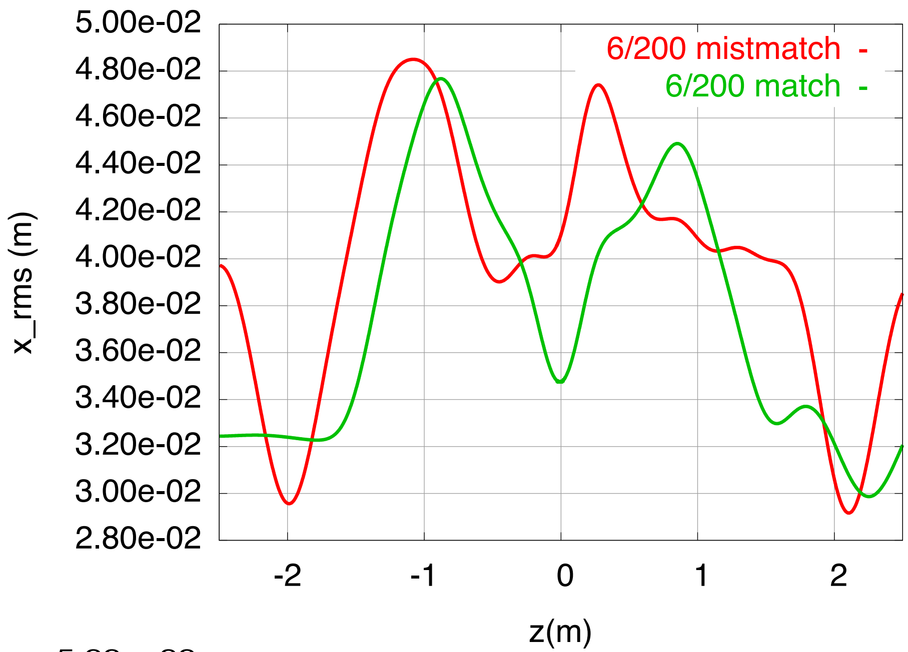


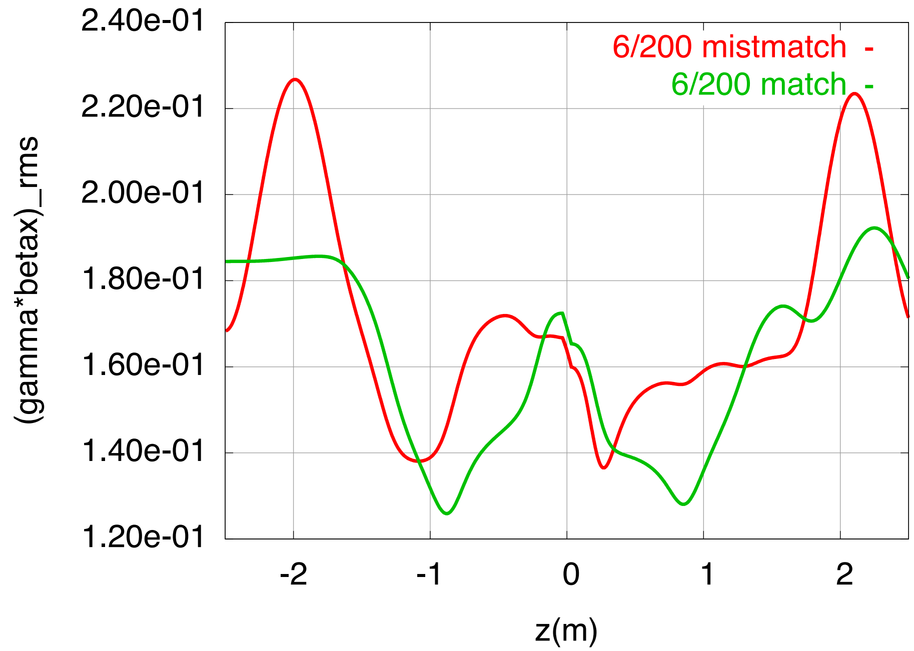


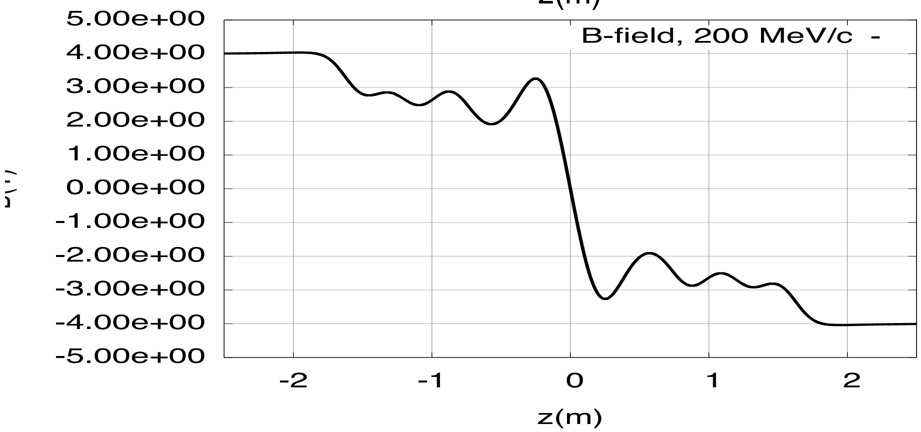
- Mismatch can cause significant emittance growth, obscure cooling;
- Measurement error (w/ the simple model used here) not significant.

- Need to discuss what amount of mismatch is reasonable to expect before drawing firm conclusions

Mismatched Case: x_rms and px_rms







- rms divergence at absorber is slightly less in mismatched case
- rms beam size at absorber is larger in mismatched case
- Larger excursions in rms divergence in mismatched case

Conclusions (with absorber)

- Mismatch can cause significant emittance growth, obscure the cooling
  - In the mismatched example studied here, a 4% cooling effect due to LiH absorber was reduced to a 1% effect
  - But this needs to be studied further using a degree of mismatch that is physically motivated
- Measurement error (based on the simple model used here) does not appear to be a significant problem

Future Plans

- Rerun some cases using MAUS to verify results
- Work w/ MICE personnel to understand the expected mismatch and perform simulations based on this
- Adapt statistical tools* to answer important questions:
  - What values of model parameters best describe the experimental observations (e.g., what can we infer about multiple scattering?)
  - If we turn knobs in the experiment and predict the outcome with simulation, what is the uncertainty in the predictions?

*D. Higdon, M. Kennedy, J. Cavendish, J. Cafeo, and R. Ryne, "Combining Field Data and Computer Simulations for Calibration and Prediction," SIAM J. Sci. Comput. 26,2 (2004).