Towards precise predictions for $pp \to 4l$ processes at the LHC

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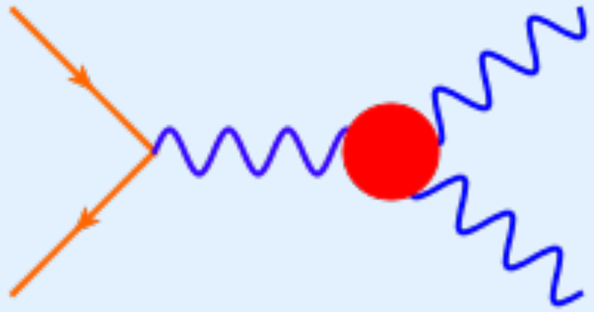
based on work with K. Melnikov, J.M. Henn, A.V. Smirnov, V.A. Smirnov

CERN, Feb. 20th 2015

Di-boson processes: very interesting physics…

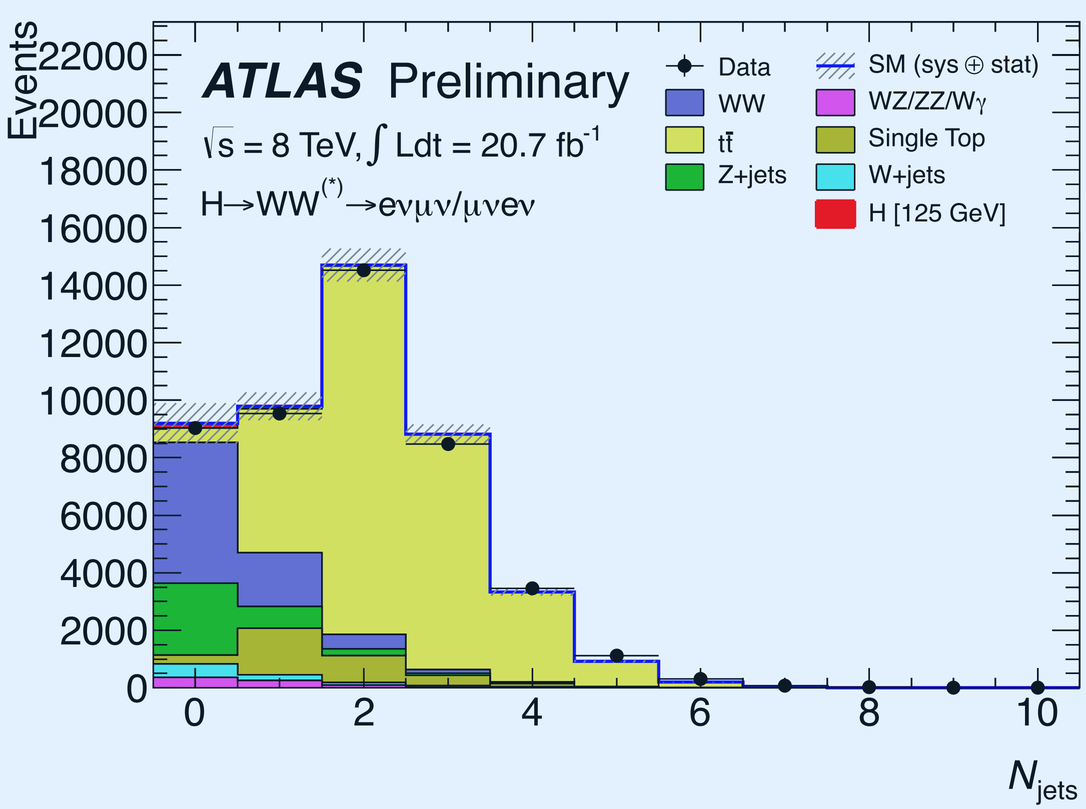
On-shell VV production

- test gauge structure of the SM
- background for BSM searches

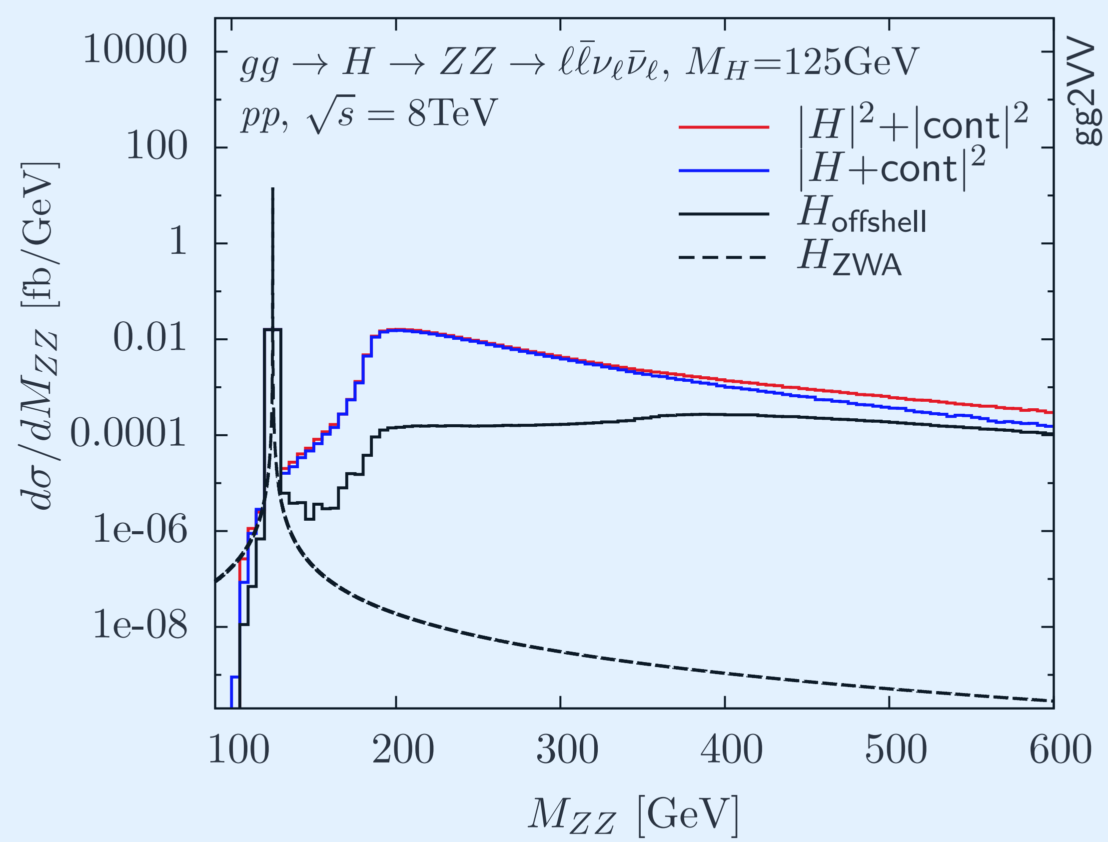


VV production and Higgs physics

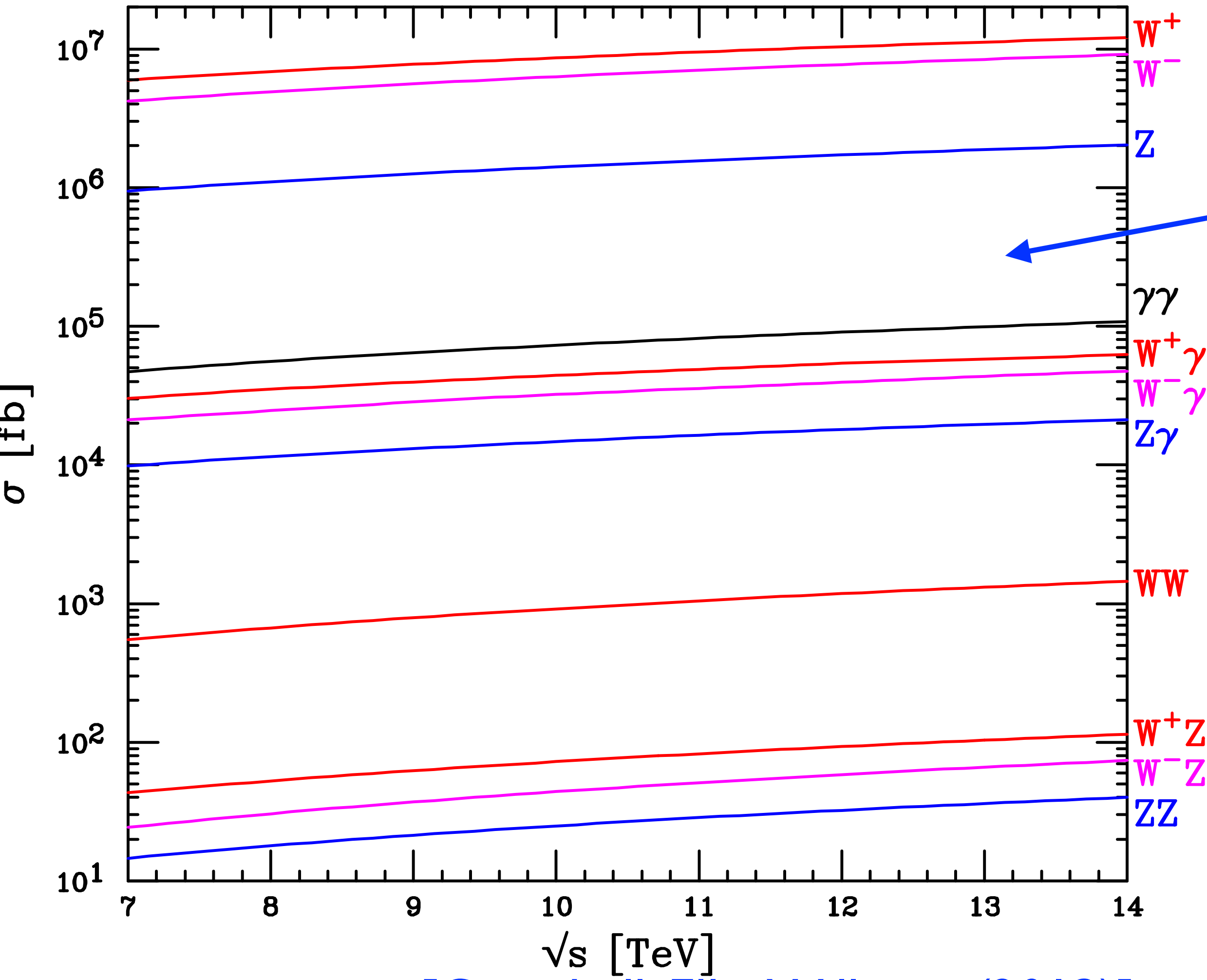
As a background: pp->(H)->WW*



The off-shell pp->(H)->4l tail and Higgs properties



On-shell production: a lot is known



[Campbell, Ellis, Williams (2013)]

Very efficient NLO implementations

Process		Syntax	Cross section (pb)			
Vector-boson pair +jets			LO 13 TeV		NLO 13 TeV	
b.1	$pp \to W^+W^-$ (4f)	`p p > w+ w-`	$7.355 \pm 0.005 \cdot 10^1$	${}^{+5.0\%}_{-6.1\%}$ ${}^{+2.0\%}_{-1.5\%}$	$1.028 \pm 0.003 \cdot 10^2$	${}^{+4.0\%}_{-4.5\%}$ ${}^{+1.9\%}_{-1.4\%}$
b.2	$pp \to ZZ$	`p p > z z`	$1.097 \pm 0.002 \cdot 10^1$	${}^{+4.5\%}_{-5.6\%}$ ${}^{+1.9\%}_{-1.5\%}$	$1.415 \pm 0.005 \cdot 10^1$	${}^{+3.1\%}_{-3.7\%}$ ${}^{+1.8\%}_{-1.4\%}$
b.3	$pp \to ZW^\pm$	`p p > z wpm`	$2.777 \pm 0.003 \cdot 10^1$	${}^{+3.6\%}_{-4.7\%}$ ${}^{+2.0\%}_{-1.5\%}$	$4.487 \pm 0.013 \cdot 10^1$	${}^{+4.4\%}_{-4.4\%}$ ${}^{+1.7\%}_{-1.3\%}$
b.4	$pp \to \gamma\gamma$	`p p > a a`	$2.510 \pm 0.002 \cdot 10^1$	${}^{+22.1\%}_{-22.4\%}$ ${}^{+2.4\%}_{-2.1\%}$	$6.593 \pm 0.021 \cdot 10^1$	${}^{+17.6\%}_{-18.8\%}$ ${}^{+2.0\%}_{-1.9\%}$
b.5	$pp \to \gamma Z$	`p p > a z`	$2.523 \pm 0.004 \cdot 10^1$	${}^{+9.9\%}_{-11.2\%}$ ${}^{+2.0\%}_{-1.6\%}$	$3.695 \pm 0.013 \cdot 10^1$	${}^{+5.4\%}_{-7.1\%}$ ${}^{+1.8\%}_{-1.4\%}$
b.6	$pp \to \gamma W^\pm$	`p p > a wpm`	$2.954 \pm 0.005 \cdot 10^1$	${}^{+9.5\%}_{-11.0\%}$ ${}^{+2.0\%}_{-1.7\%}$	$7.124 \pm 0.026 \cdot 10^1$	${}^{+9.7\%}_{-9.9\%}$ ${}^{+1.5\%}_{-1.3\%}$
b.7	$pp \to W^+W^-j$ (4f)	`p p > w+ w- j`	$2.865 \pm 0.003 \cdot 10^1$	${}^{+11.6\%}_{-10.0\%}$ ${}^{+1.0\%}_{-0.8\%}$	$3.730 \pm 0.013 \cdot 10^1$	${}^{+4.9\%}_{-4.9\%}$ ${}^{+1.1\%}_{-0.8\%}$
b.8	$pp \to ZZj$	`p p > z z j`	$3.662 \pm 0.003 \cdot 10^0$	${}^{+10.9\%}_{-9.3\%}$ ${}^{+1.0\%}_{-0.8\%}$	$4.830 \pm 0.016 \cdot 10^0$	${}^{+5.0\%}_{-4.8\%}$ ${}^{+1.1\%}_{-0.9\%}$
b.9	$pp \to ZW^\pm j$	`p p > z wpm j`	$1.605 \pm 0.005 \cdot 10^1$	${}^{+11.6\%}_{-10.0\%}$ ${}^{+0.9\%}_{-0.7\%}$	$2.086 \pm 0.007 \cdot 10^1$	${}^{+4.9\%}_{-4.8\%}$ ${}^{+0.9\%}_{-0.7\%}$
b.10	$pp \to \gamma\gamma j$	`p p > a a j`	$1.022 \pm 0.001 \cdot 10^1$	${}^{+20.3\%}_{-17.7\%}$ ${}^{+1.2\%}_{-1.5\%}$	$2.292 \pm 0.010 \cdot 10^1$	${}^{+17.2\%}_{-15.1\%}$ ${}^{+1.0\%}_{-1.4\%}$
b.11*	$pp \to \gamma Zj$	`p p > a z j`	$8.310 \pm 0.017 \cdot 10^0$	${}^{+14.5\%}_{-12.8\%}$ ${}^{+1.0\%}_{-1.0\%}$	$1.220 \pm 0.005 \cdot 10^1$	${}^{+7.3\%}_{-7.4\%}$ ${}^{+0.9\%}_{-0.9\%}$
b.12*	$pp \to \gamma W^\pm j$	`p p > a wpm j`	$2.546 \pm 0.010 \cdot 10^1$	${}^{+13.7\%}_{-12.1\%}$ ${}^{+0.9\%}_{-1.0\%}$	$3.713 \pm 0.015 \cdot 10^1$	${}^{+7.2\%}_{-7.1\%}$ ${}^{+0.9\%}_{-1.0\%}$
b.13	$pp \to W^+W^+jj$	`p p > w+ w+ j j`	$1.484 \pm 0.006 \cdot 10^{-1}$	${}^{+25.4\%}_{-18.9\%}$ ${}^{+2.1\%}_{-1.5\%}$	$2.251 \pm 0.011 \cdot 10^{-1}$	${}^{+10.5\%}_{-10.6\%}$ ${}^{+2.2\%}_{-1.6\%}$
b.14	$pp \to W^-W^-jj$	`p p > w- w- j j`	$6.752 \pm 0.007 \cdot 10^{-2}$	${}^{+25.4\%}_{-18.9\%}$ ${}^{+2.4\%}_{-1.7\%}$	$1.003 \pm 0.003 \cdot 10^{-1}$	${}^{+10.1\%}_{-10.4\%}$ ${}^{+2.5\%}_{-1.8\%}$
b.15	$pp \to W^+W^-jj$ (4f)	`p p > w+ w- j j`	$1.144 \pm 0.002 \cdot 10^1$	${}^{+27.2\%}_{-19.9\%}$ ${}^{+0.7\%}_{-0.5\%}$	$1.396 \pm 0.005 \cdot 10^1$	${}^{+5.0\%}_{-6.8\%}$ ${}^{+0.7\%}_{-0.6\%}$
b.16	$pp \to ZZjj$	`p p > z z j j`	$1.344 \pm 0.002 \cdot 10^0$	${}^{+26.6\%}_{-19.6\%}$ ${}^{+0.7\%}_{-0.6\%}$	$1.706 \pm 0.011 \cdot 10^0$	${}^{+5.8\%}_{-7.2\%}$ ${}^{+0.8\%}_{-0.6\%}$
b.17	$pp \to ZW^\pm jj$	`p p > z wpm j j`	$8.038 \pm 0.009 \cdot 10^0$	${}^{+26.7\%}_{-19.7\%}$ ${}^{+0.7\%}_{-0.5\%}$	$9.139 \pm 0.031 \cdot 10^0$	${}^{+3.1\%}_{-5.1\%}$ ${}^{+0.7\%}_{-0.5\%}$
b.18	$pp \to \gamma\gamma jj$	`p p > a a j j`	$5.377 \pm 0.029 \cdot 10^0$	${}^{+26.2\%}_{-19.8\%}$ ${}^{+0.6\%}_{-1.0\%}$	$7.501 \pm 0.032 \cdot 10^0$	${}^{+8.8\%}_{-10.1\%}$ ${}^{+0.6\%}_{-1.0\%}$
b.19*	$pp \to \gamma Zjj$	`p p > a z j j`	$3.260 \pm 0.009 \cdot 10^0$	${}^{+24.3\%}_{-18.4\%}$ ${}^{+0.6\%}_{-0.6\%}$	$4.242 \pm 0.016 \cdot 10^0$	${}^{+6.5\%}_{-7.3\%}$ ${}^{+0.6\%}_{-0.6\%}$
b.20*	$pp \to \gamma W^\pm jj$	`p p > a wpm j j`	$1.233 \pm 0.002 \cdot 10^1$	${}^{+24.7\%}_{-18.6\%}$ ${}^{+0.6\%}_{-0.6\%}$	$1.448 \pm 0.005 \cdot 10^1$	${}^{+3.6\%}_{-5.4\%}$ ${}^{+0.6\%}_{-0.7\%}$

Automatic NLOPS + merging

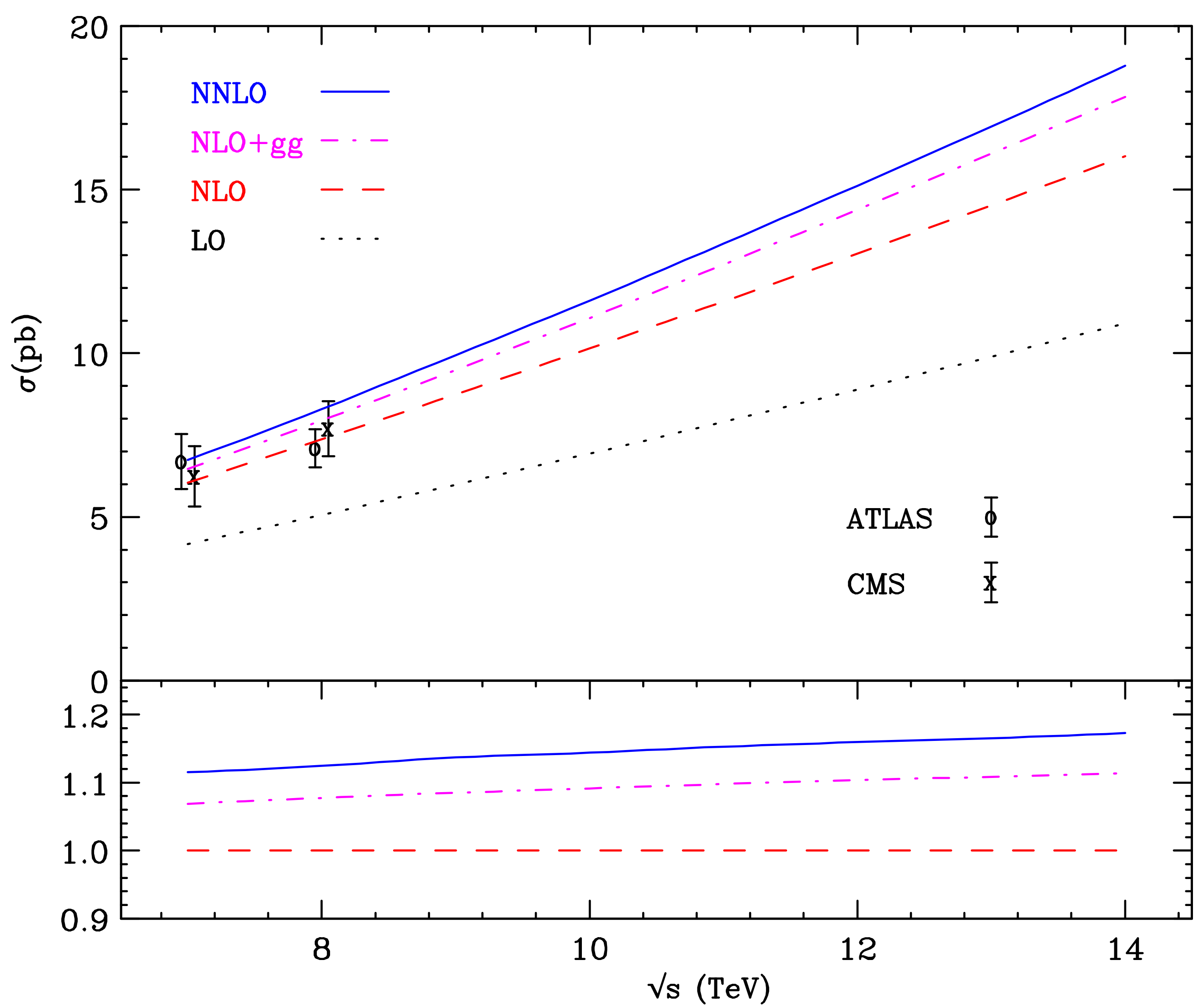
[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro; MadGraph5_aMC@NLO (2014)]

Typical size of NLO corrections: ~ 50%

On-shell production: a lot is known

~50% NLO corrections -> NNLO is desirable

First results for fully inclusive VV started to appear: ZZ



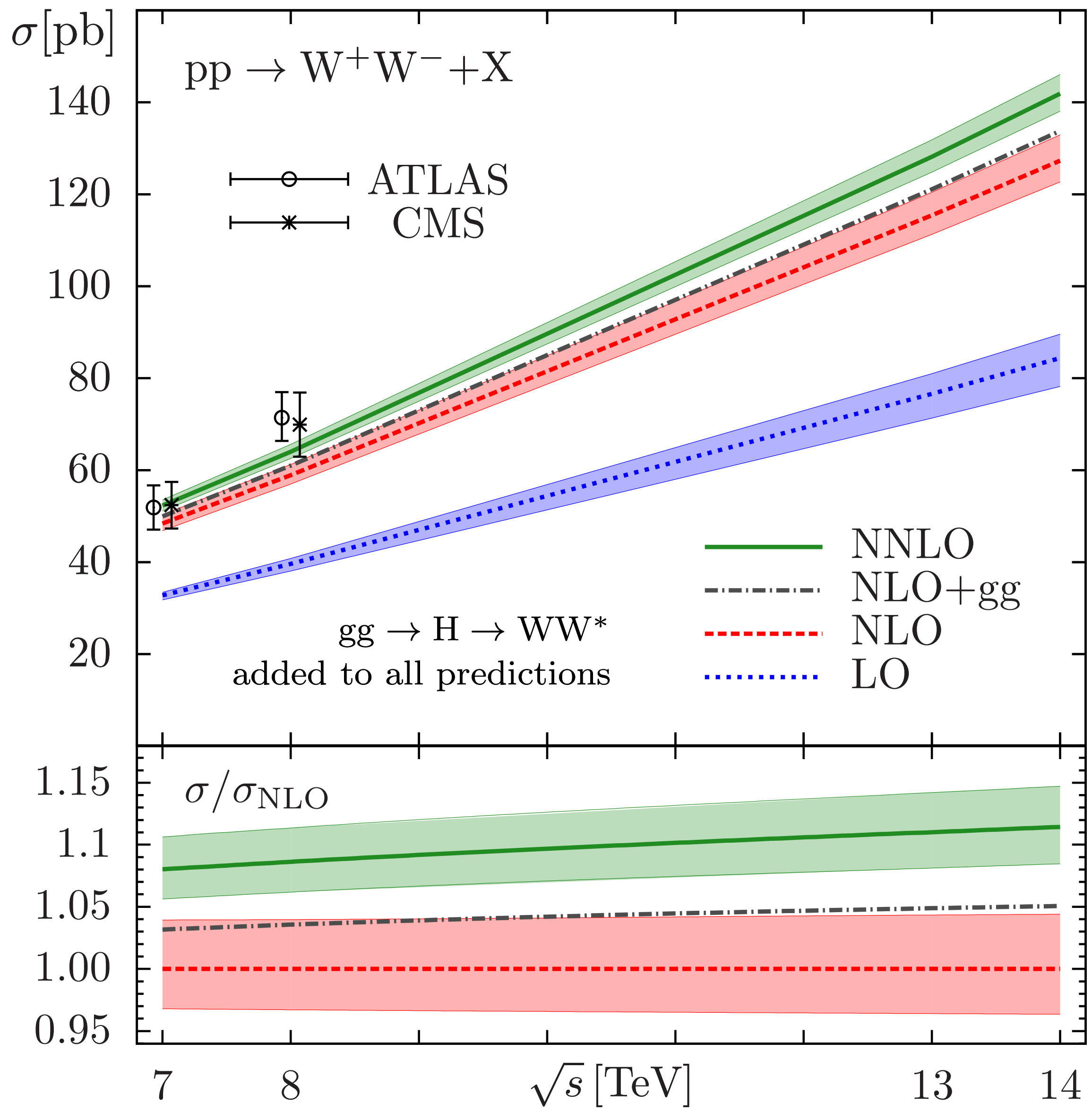
[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhoefer, v. Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)]

- Good agreement with exp. measurements
- Non-negligible NNLO corrections (~10%)
- gg->VV accounts for ~60% of the full NNLO
- NLO scale variation underestimates error
- ~ 3% th uncertainty

On-shell production: a lot is known

~50% NLO corrections -> NNLO is desirable

First results for fully inclusive VV started to appear: WW



[Cascioli, Gehrmann, Grazzini, Kallweit, Maierhoefer, v. Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)]

- Non-negligible NNLO corrections (~10%)
- gg->VV: ~35% of full NNLO
- NLO scale variation underestimates error
- ~ 3% th uncertainty
- Tension with exp. understood [Monni, Zanderighi (2014)]
- Fiducial NNLO predictions highly desirable

pp->4l and Higgs: less satisfactory situation

For Higgs analysis, fully inclusive predictions for stable pp->VV are clearly not adequate

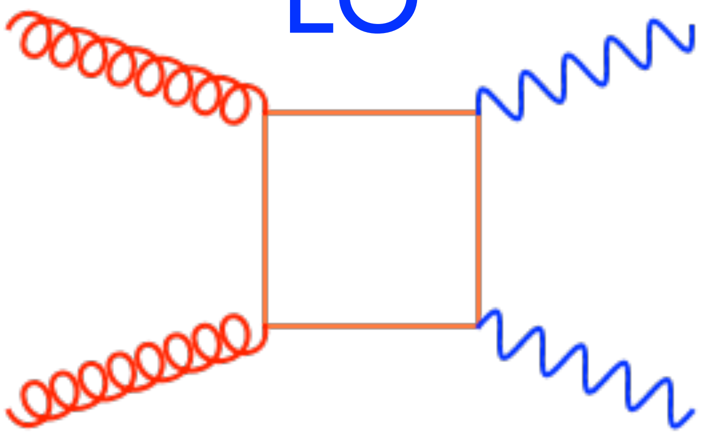
- standard 'on-peak' H->WW* analysis:
  off-shell Ws, good control on lepton distribution shapes
- 'off-shell' analysis of the H->ZZ tail:
  lepton correlations to reduce qq->VV background,
  signal/background gg->(H)->ZZ interference effects

NNLO predictions for pp->(H)->4l more complicated than inclusive pp->VV -> only NLO is known

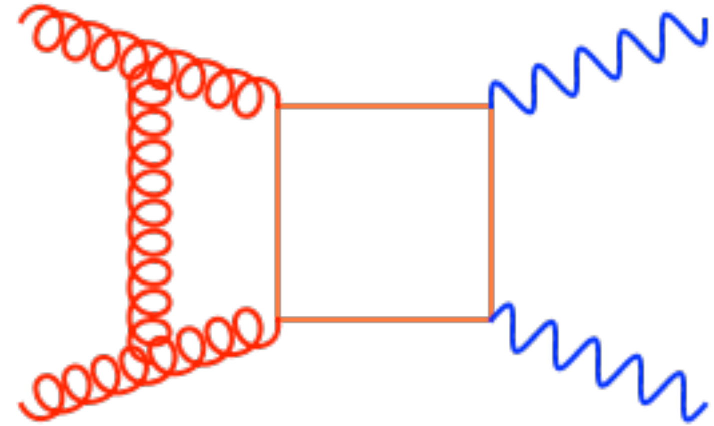
Given the accuracy goals of the Higgs program,
FULL (NNLL+)NNLO PREDICTIONS HIGHLY DESIRABLE

pp->4l and Higgs: the case of gg->VV

LO



NLO, not known

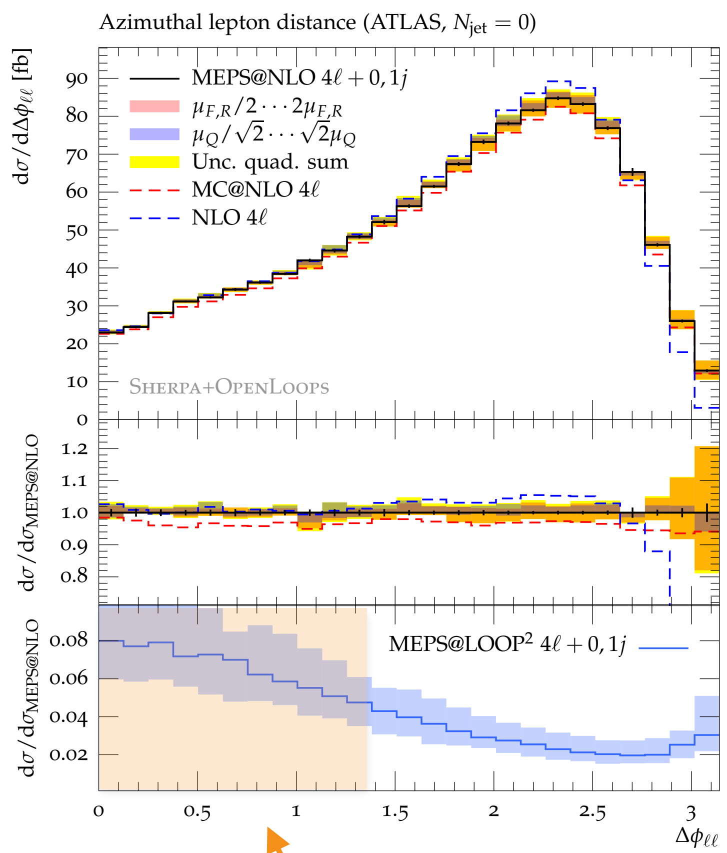


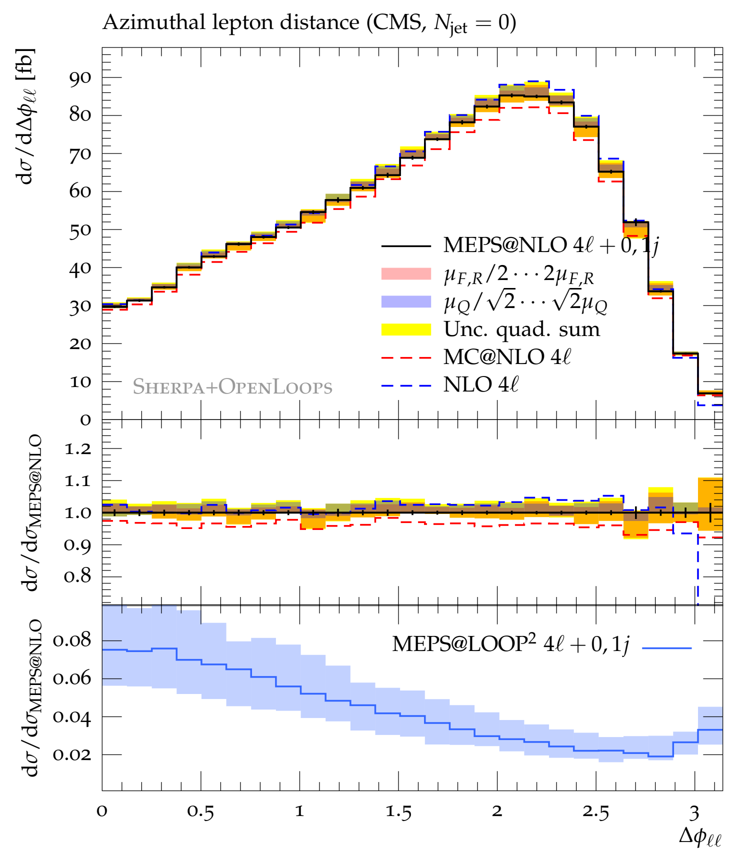
- NNLO QCD, but enhanced by gluon flux
- In general ~ 3% of NLO result, but for Higgs analysis can be as large as ~10-30%
- Non-trivial modifications of lepton shapes
- Can interfere with the Higgs signal

- Gluon initiated-process -> expect large radiative corrections (~ to gg->H [Bonvini, FC, Forte, Melnikov, Ridolfi (2013)])
- First corrections already involve complicated 2-loop amps.

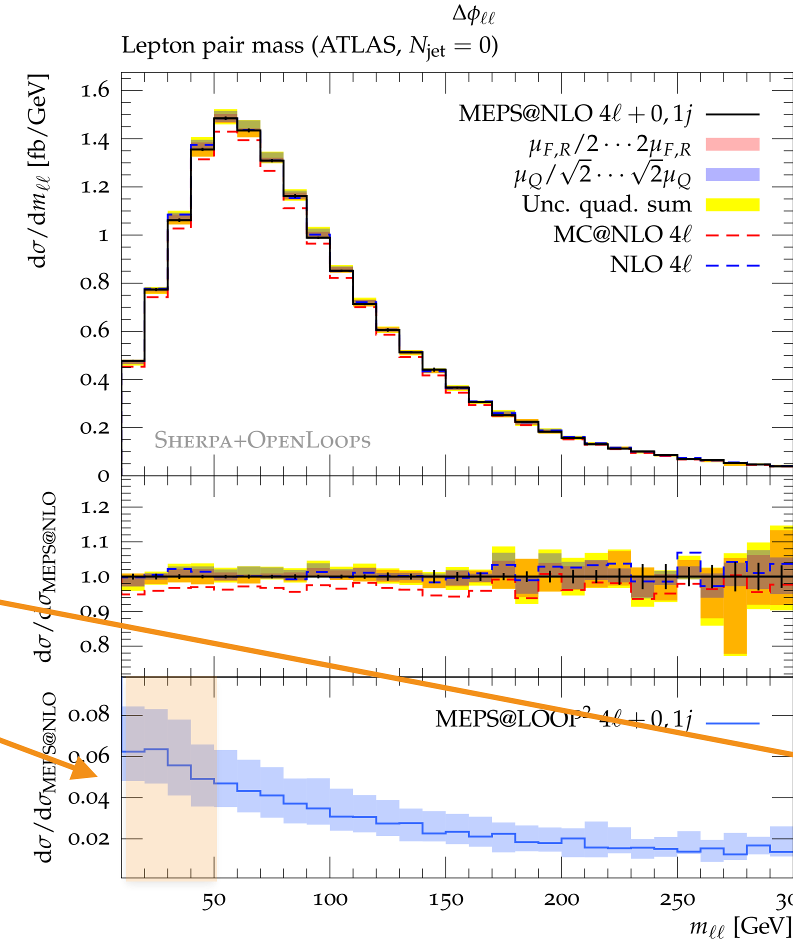
Example I: pp->4l and H->WW*

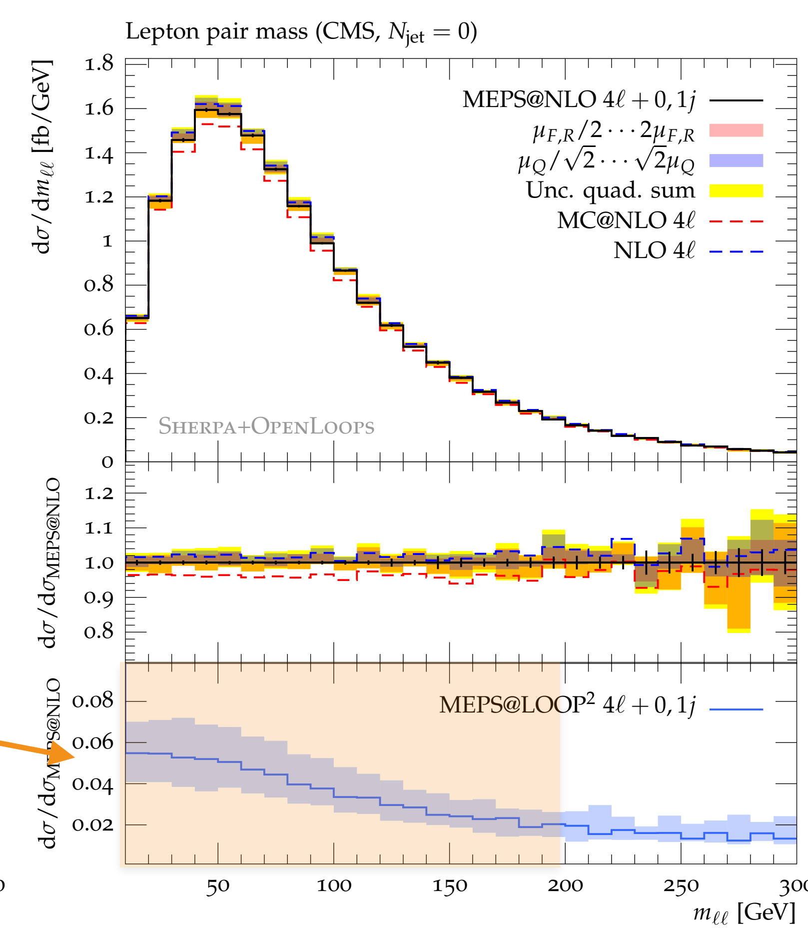
[Cascioli, Hoeche, Krauss, Maierhoefer, Pozzorini, Siegert (2013)]





$\Delta\Phi_{ll}$ and m_{ll} after pre-selection cuts

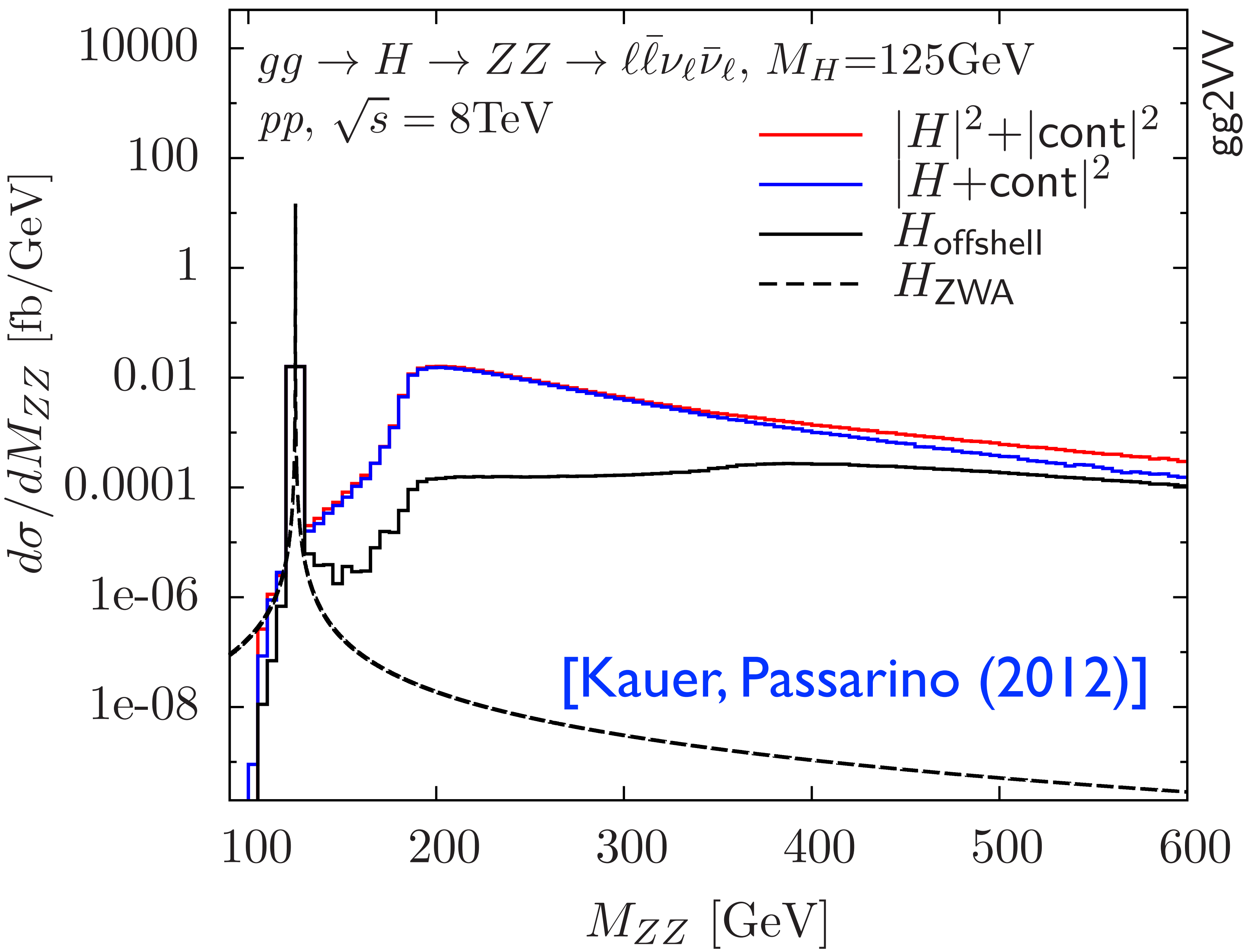




Bulk of effects in the Higgs signal region

Non trivial shapes

Example II: the off-shell H -> ZZ tail



- Past the VV threshold, enhanced decay into longitudinal vector bosons compensate the rapidly falling Higgs propagator
- Small but persisting off-shell tail, O(10%) of the peak cross-section
- Irrelevant for standard analysis if proper selection cuts are applied
- If looked for, can give complementary information w.r.t traditional searches
- Example: bounds on the Higgs total width [FC, Melnikov (2013); CMS/ATLAS measurements (2014)]

Example II: the off shell H->ZZ tail

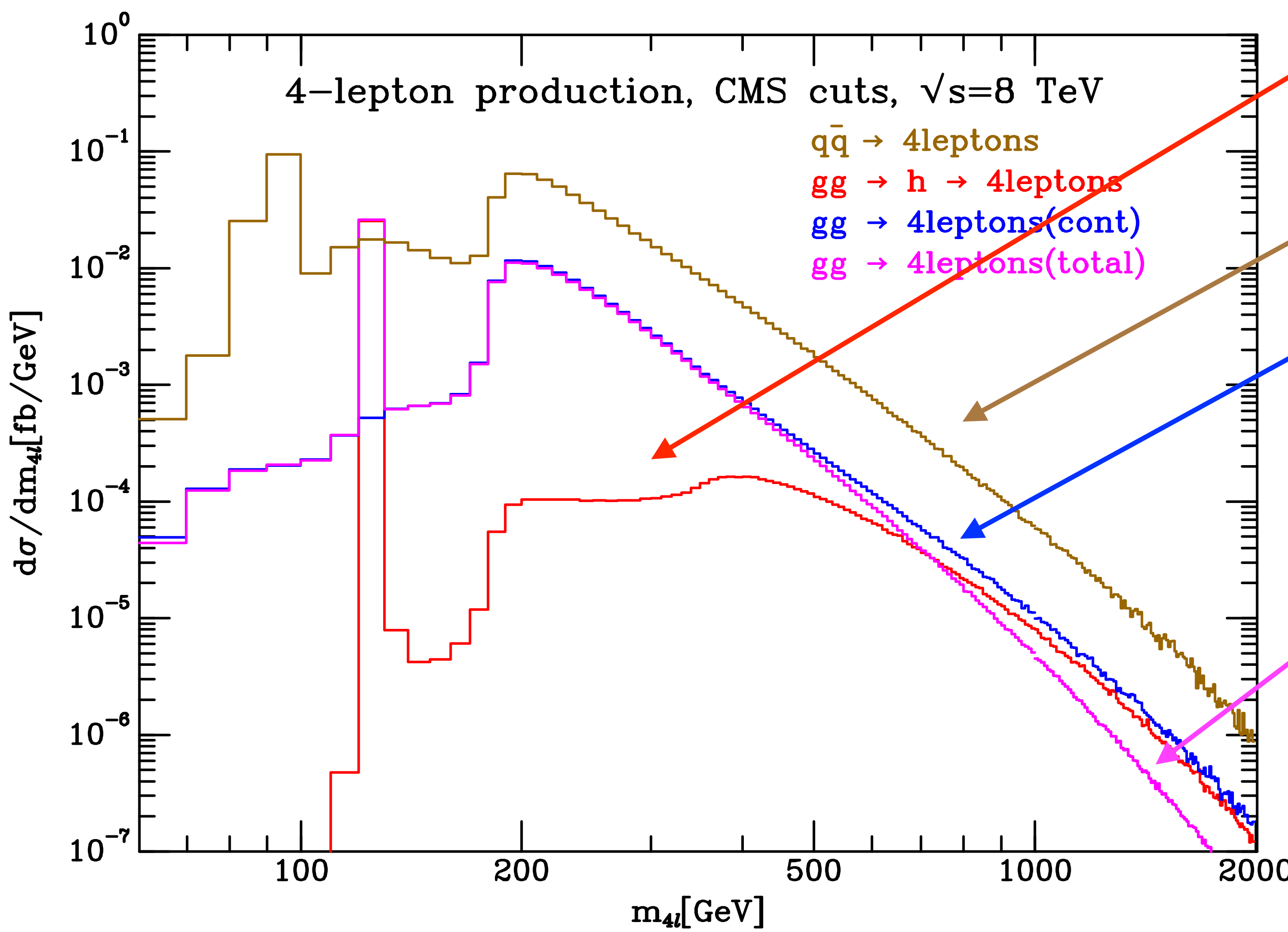
Off-shell analysis based on (more or less refined) counting of events in the Higgs tail -> good control of pp->4l mandatory

$N_H \sim 5 \times 10^{-2} \times N_{\text{tot}}$

$N_{qq \to ZZ} \approx N_{\text{tot}}$

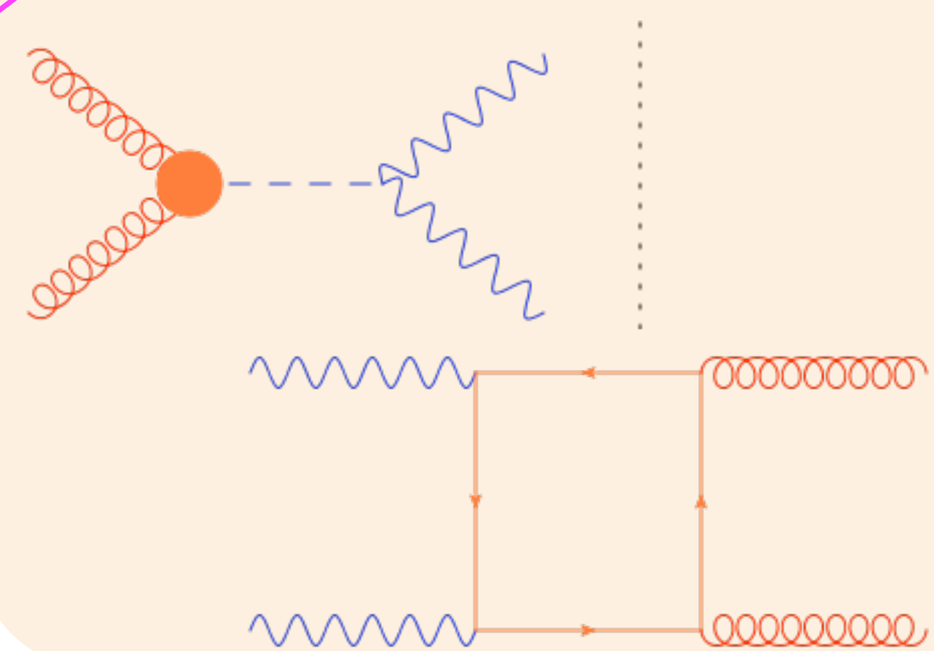
$N_{gg} \sim 10^{-1} \times N_{\text{tot}}$

$N_{\text{off}} \sim 10^{-2} N_{\text{tot}}$



[Campbell, Ellis, Williams (2013)]

$N_{\text{int}} \sim -2 \times 10^{-2} N_{\text{tot}}$



pp->4l and Higgs analysis: what is needed

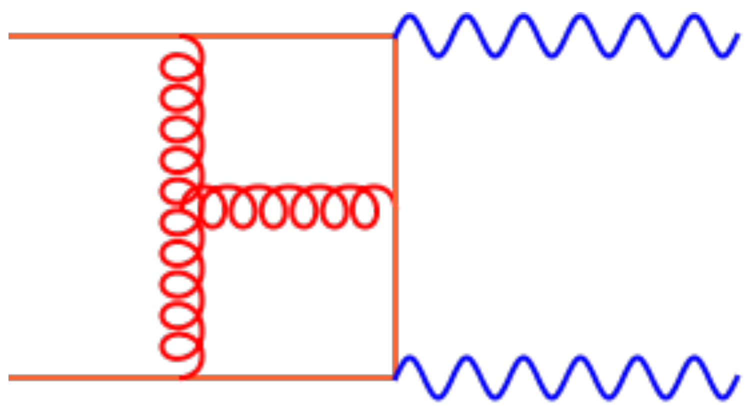
Precise Higgs analysis in the di-boson channels require sophisticated QCD predictions.

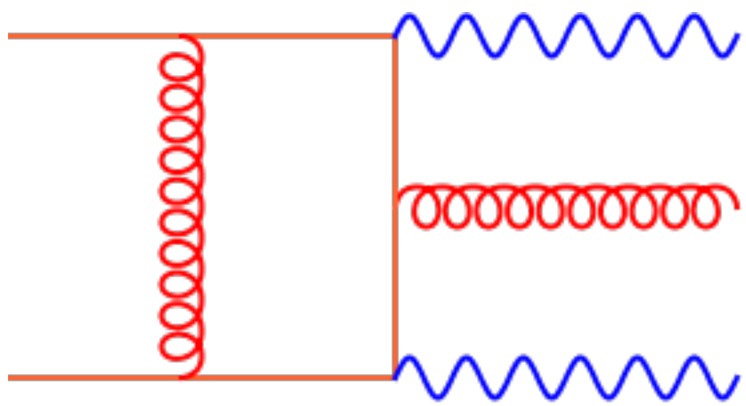
In particular, the following f.o. results are highly desirable:

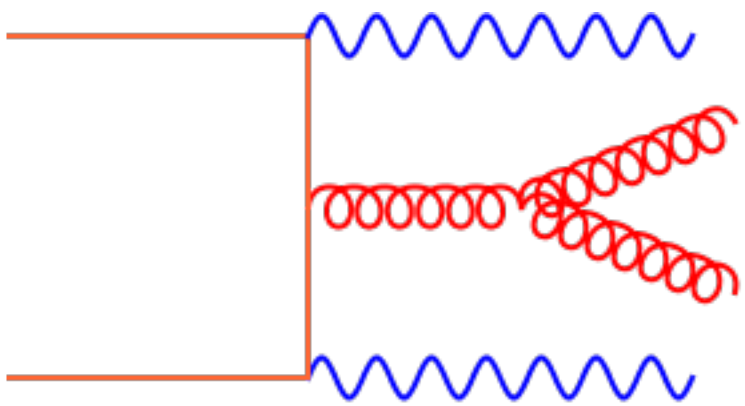
- WW^* background in Higgs searches:
  - fully exclusive pp -> 4l @ NNLO QCD
  - gg -> 4l @ NLO QCD from off-shell vector-bosons
- Off-shell tail in the 4l invariant mass distribution:
  - fully exclusive pp -> VV -> 4l @ NNLO QCD
  - gg -> 4l @ NLO QCD
  - signal / background gg -> (H) -> 4l interferences

(+NNLL, PS…)

Anatomy of a NNLO computation…

VV 

RV 

RR 

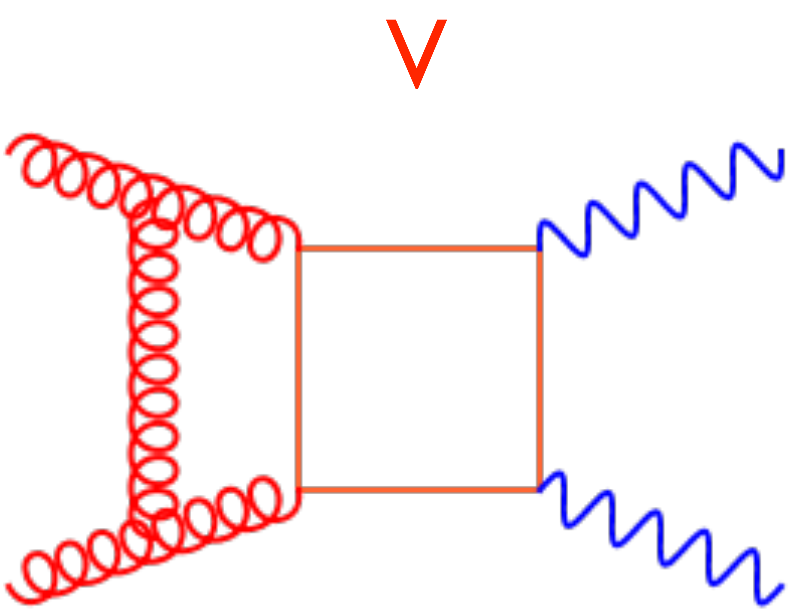
$$\int \left[\frac{\mathrm{vv}_4}{\epsilon^4} + \frac{\mathrm{vv}_3}{\epsilon^3} + \frac{\mathrm{vv}_2}{\epsilon^2} + \frac{\mathrm{vv}_1}{\epsilon} + \mathrm{vv}_0\right] d\phi_2$$

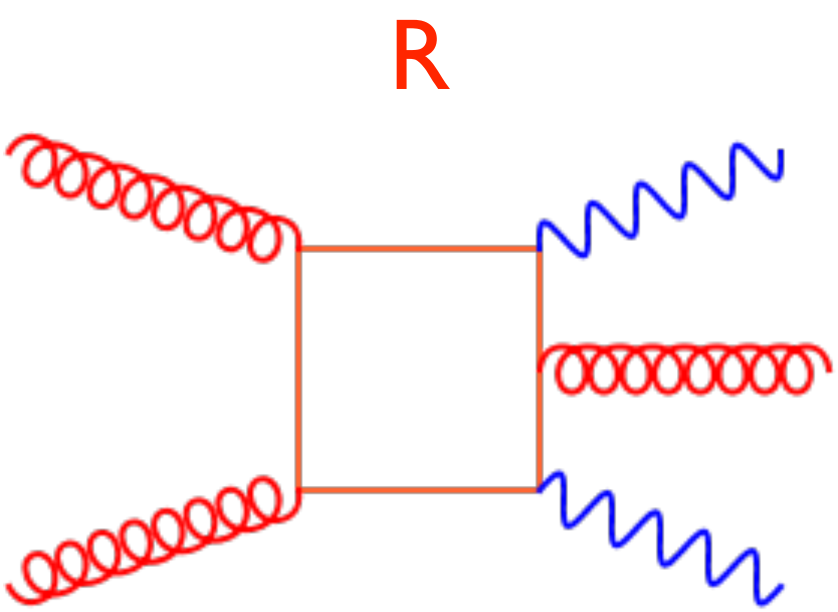
$$\int \left[\frac{\mathrm{rv}_2}{\epsilon^2} + \frac{\mathrm{rv}_1}{\epsilon} + \mathrm{rv}_0\right] d\phi_3$$

$$\int [\mathrm{rr}_0]\, d\phi_4$$

- In general, the complicated part of fully differential NNLO computations is to consistently extract IR singularity from double-real emission/real-virtual emission
- In this case however, the singularity structure is very simple (colorless final state) -> this problem is well understood
- The missing piece are COMPLICATED TWO-LOOP AMPLITUDES

…and of a NLO one



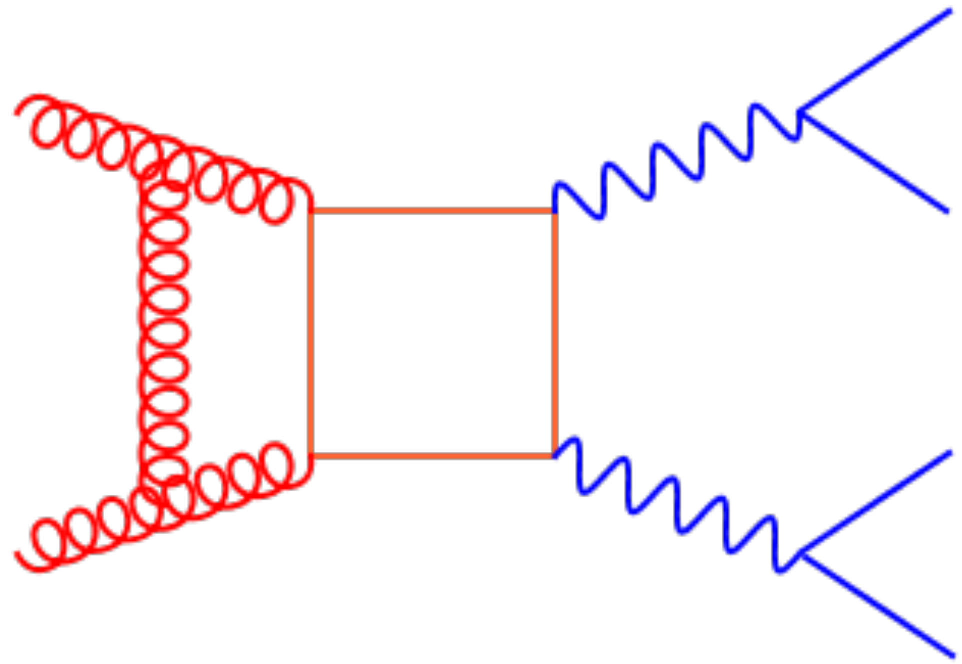


- By today's standards, gg->VVj @ 1-loop is well within reach
- On the other hand, gg->VV* @ 2-loop is a 2->2 process involving many different scales (s, t , $M_{V,1}^2$, $M_{V,2}^2$) and as such it is very challenging
- Nevertheless, lot of recent developments and new ideas in the multi-loop community made this computation possible

  (Same story for qq->VV @ 2-loop
  [FC, Henn, Melnikov, A.V. Smirnov, V.A. Smirnov (2014)])

Computing 2-loop gg->VV amplitudes

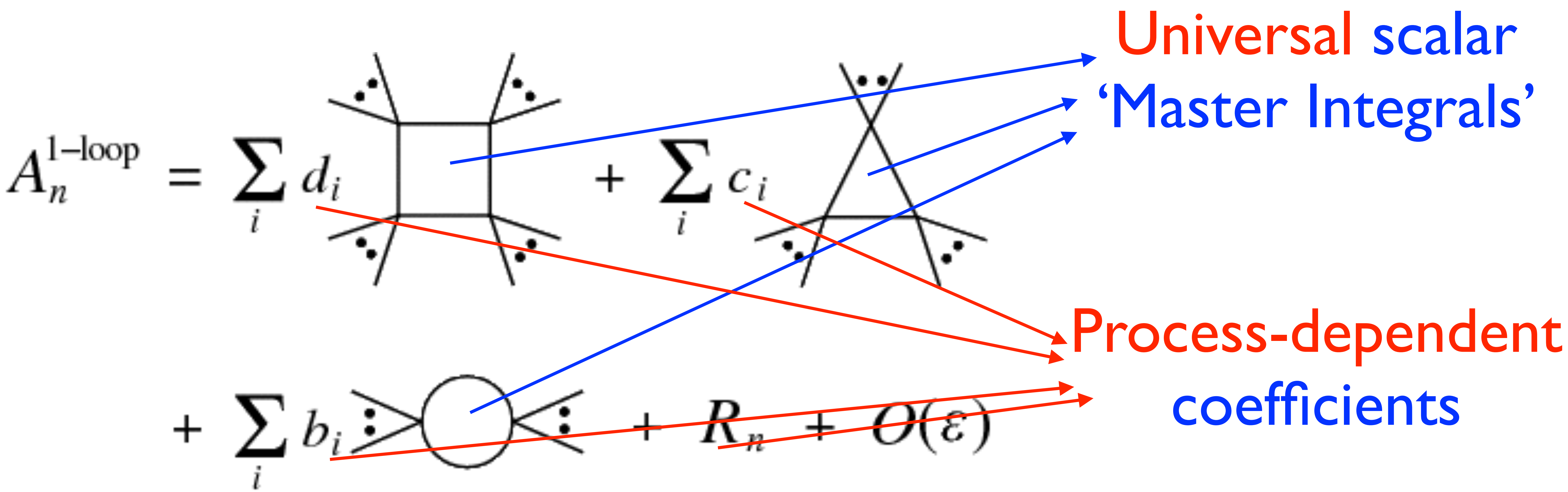
gg->VV: what we computed



- Generic amplitude for gg -> VV^* -> 4l, for $V=\gamma^*$,Z,W and $l=l^{\pm},\nu$
- Computation at the amplitude-level -> possible to interfere with (well-known) Higgs amplitudes
- For now, we neglected however contributions from the third generation (=massive quarks in the loop). Good approximation at low invariant masses, but can be problematic beyond the ttbar threshold (in the H->ZZ off-shell tail…)

Computing amplitudes: the 1-loop paradigm

No matter how complicated it is, near D=4 any 1-loop amplitude can be written as



- The goal of the game is then 'just' to isolate the coefficients d,c,b,R (PV, generalized unitarity…)
- The full procedure is algebraic
- MI are computed once and for all

The problem at 2-loop

Despite a lot of interesting recent progress, we do not have a similar picture at 2-loop

- No process-independent basis of master integrals
- No algebraic way to reduce the full amplitude to a sum of (a minimal set of) master integrals times process-dependent coefficients
- Fortunately, non-trivial identities between the (many and complicated) tensor integrals contributing to a 2-loop amplitude can still be found, thanks to their symmetry properties
- Main tool: IBP identities
  [Tkachov (1981), Chetyrkin and Tkachov (1981)]

IBP in a nut-shell

- In dimensional regularization, multi-loop integrals are invariant under shift of the loop-momentum $k \to k + \alpha q$

$$\int d^d k F(k; \{p_j\}) = \int d^d(k + \alpha q) F(k + \alpha q; \{p_j\})$$

- The above (trivial) condition can give interesting information if considered for infinitesimal α

$$\alpha \int d^d k \frac{\partial}{\partial k} \cdot [q F(k, \{p_j\})] = 0$$

- When acting on the Feynman integrand F, ∂_k changes the numerator / propagator structure -> relations between different integrals

IBP in a nut-shell

- The systematic application of IBPs for all possible $\partial_i\,[q_j\ldots]$ leads to many relations between different Feynman integrals
- Through these relations, all the relevant integrals of a multi-loop amplitude can be related to a minimal set of process-dependent basic 'master integral'
- Although this step is highly non-trivial, it can be done in an algorithmic way [Laporta (2001)]
- Nowadays, many public computer implementation of the Laporta algorithm are available (Air, FIRE, Reduze…)
- Reducing all the relevant integrals to a minimal set is a solved problem, for a generic process in principle and for not too complicated kinematics in practice

IBPs for gg->VV: from amplitude to form factors

- In order for IBP to work effectively, the numerator of the amplitude integrand must be of the form $f(p_i \cdot k_j)$
- In general, this is not the case for amplitudes, as k_j can be contracted with polarization vectors $k_j \cdot \epsilon_V \sim \langle l k_j \bar{l}]$
- Typically, this is solved by considering $\sum_{pol} 2\mathrm{Re}\left(\mathcal{A}^*_{tree}\mathcal{A}_{loop}\right)$
- However, we are interested in the amplitude (lepton decay, interferences…)
- To solve this issue: project the amplitude onto helicity-stripped form-factors.
- These should be independent, to avoid the appearance of spurious singularities

IBPs for gg->VV: from the amplitude to form factors

$$g(p_1) + g(p_2) \to \left[V_3(p_3) \to l(p_5)\bar{l}(p_6)\right] + \left[V_4(p_4) \to l(p_7)\bar{l}(p_8)\right]$$

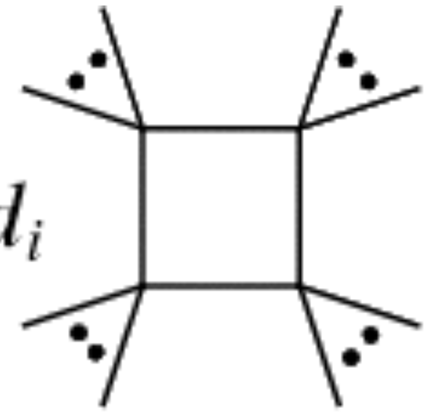
- In our case, this can be achieved with 9 helicity-dependent form factors

$$\begin{aligned}
\mathcal{A}_{3L4L}^{\lambda_1\lambda_2} =& \mathcal{N}_{\lambda_1\lambda_2}\Big\{\Big(F_1^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_2^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\langle 17\rangle[81] \\
&+ \Big(F_3^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_4^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\langle 27\rangle[82] + 2F_5^{\lambda_1\lambda_2}\langle 57\rangle[86] \\
&+ \frac{1}{2}\Big(F_6^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_7^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\Big(\langle 12\rangle\langle 78\rangle[81][82] + \langle 17\rangle\langle 27\rangle[21][87]\Big) \\
&- \frac{1}{2}\Big(F_8^{\lambda_1\lambda_2}\langle 17\rangle[81] + F_9^{\lambda_1\lambda_2}\langle 27\rangle[82]\Big)\Big(\langle 12\rangle\langle 56\rangle[61][62] + \langle 15\rangle\langle 25\rangle[21][65]\Big)\Big\}.
\end{aligned}$$

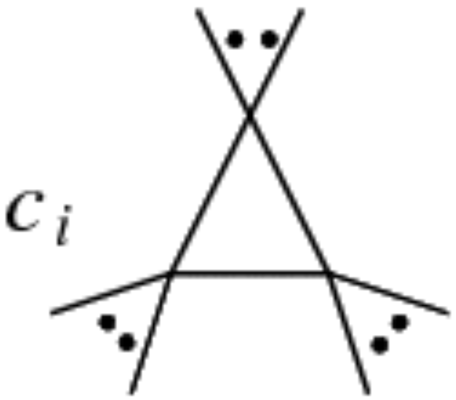
- At the integrand level, $F_i = F_i\,(s,t,M_3^2,M_4^2;\{k_i \cdot p_{1,2,3}, k_1 \cdot k_2\})$
- As such, each integral in F_i can be reduced to a minimal set of master integrals via IBP relations

Remember the 1-loop story

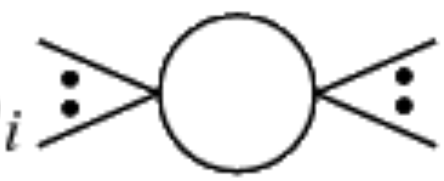
$$A_n^{1\text{-loop}} = \sum_i d_i$$



$$+ \sum_i c_i$$



$$+ \sum_i b_i$$



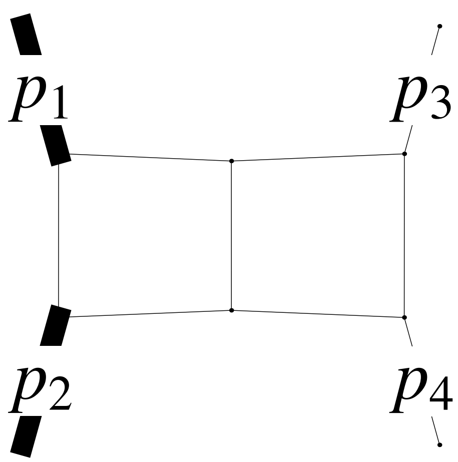
$$+ R_n + O(\varepsilon)$$

- After IBP reduction, we effectively computed the coefficient in front of a minimal set of integrals, i.e. the 2-loop equivalent of d,c,b,R
- At 1-loop, this would be the end of the story
- At 2-loop however the (many) basis integrals are not known, and must be computed
- THIS IS THE MOST CHALLENGING TASK TO PERFORM

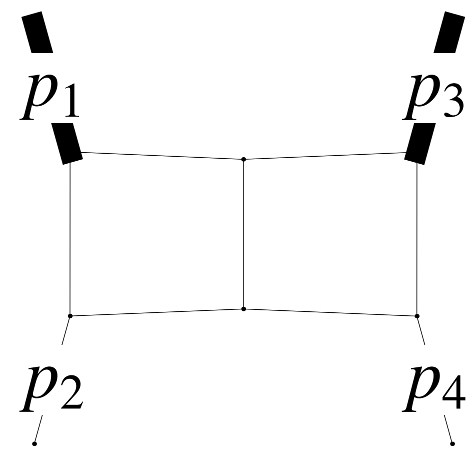
Master integrals for gg->VV

At 2-loop, 6 distinct families of master integrals

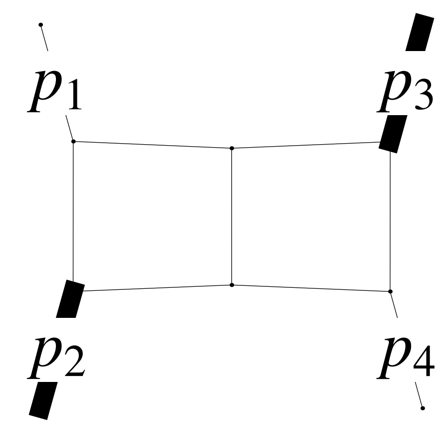
Each contains several independent (scalar and tensor) master integrals



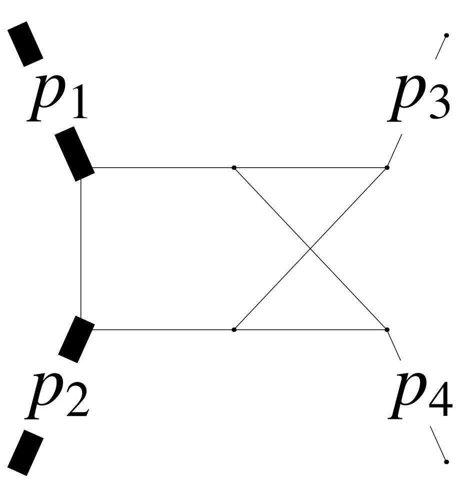
P_{12}, 31 MI



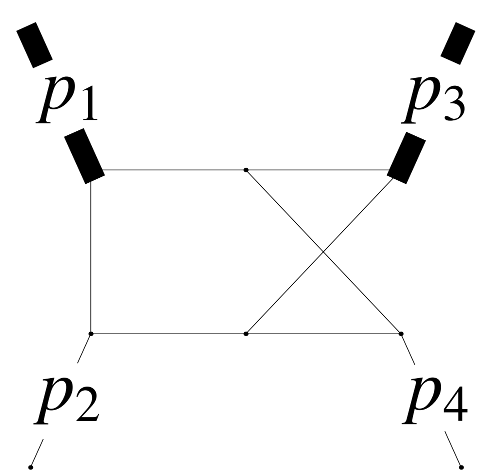
P_{13}, 29 MI



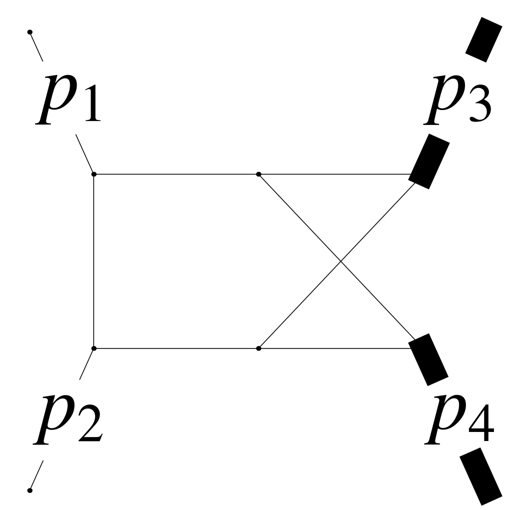
P_{13}, 28 MI



N_{12}, 35 MI



N_{13}, 43 MI



N_{34}, 51 MI

Evaluating MI

- A lot of different two-loop integrals to evaluate
- Although there are several overlaps between different families (bubbles, triangles…), still ~ hundred integrals to compute
- Some are easy, but many are very complicated

- Ideally, we would like to avoid a brute force computation of individual integrals
- Can we group 'nicely' the integrals?
- Can we integrated many of them at once?

Preliminary: loop integrals via differential equations

[Kotikov (1991), Remiddi (1997)]

- Loop integrals in generic kinematics: very hard
- In general however, they simplify for specific kinematics configurations (threshold, high-energy…)
- If derivatives of MI are known, one can use them to transport simple kinematics to generic kinematics
- Taking derivatives of MI: ~ IBP procedure -> DERIVATIVES OF MI ARE LINEAR COMBINATIONS OF MI

$$\partial_x \vec{f}(x) = A(\epsilon, x) \cdot \vec{f}(x)$$

Preliminary: loop integrals via differential equations

$$\partial_x \vec{f}(x) = A(\epsilon, x) \cdot \vec{f}(x)$$

- Hard problem split into two somewhat simpler
  1. solving the differential equation
  2. evaluating boundary values
- Group together several master integrals
- If we would be able to integrate the system in its matrix form, problem solved at once
- However, for generic A, this is obviously impossible: highly coupled differential equations in many variables…

Differential equations made simple

[Henn (2013)]

$$\partial_x \vec{f}(x) = A(\epsilon, x) \cdot \vec{f}(x)$$

- We are dealing with a physical problem -> *A* IS NOT A GENERIC FUNCTION, constrains from singularity structure of Feynman integrals
- Near singular points (threshold,…): $f(x) \sim (x - x_0)^{a(\epsilon)}$
- System can be put in a Fuchsian form (i.e. singularity structure can be made manifest)

$$\partial_x \vec{g}(x) = \sum_i \frac{A_i(\epsilon)}{(x - x_i)} \vec{g}(x) \qquad \big(g(x) = T(x, \epsilon) f(x)\big)$$

Differential equations made simple

$$\partial_x \vec{f}(x) = A(\epsilon, x) \cdot \vec{f}(x) \longrightarrow \partial_x \vec{g}(x) = \sum_i \frac{A_i(\epsilon)}{(x - x_i)} \vec{g}(x)$$

- Near singular points (threshold,…): $f(x) \sim (x - x_0)^{a(\epsilon)}$ with *a* linear in ε (theory of asymptotic expansions)

- Simplest possible case:

$$\partial_x \vec{h}(x) = \epsilon \sum_i \frac{A_i}{(x - x_i)} \vec{h}(x) \text{ or } d\vec{h}(x) = \epsilon \sum_i A_i d \ln(x - x_i) \vec{h}(x)$$

- While this last step not possible in general, it is POSSIBLE FOR OUR MASTER INTEGRALS (as well as for many other examples involving massless propagators)

- (Algorithmic to tell whether $A_i(\epsilon)$ -> ε A_i: [R. Lee (2014)])

Integrating the differential equations:

$$\partial_x \vec{f}(x) = A(\epsilon, x) \cdot \vec{f}(x) \longrightarrow \boxed{\partial_x \vec{h}(x) = \epsilon \sum_i \frac{A_i}{(x - x_i)} \vec{h}(x)}$$

- While in general integrating the system can still be hard, it is trivial to get an expansion around ε = 0 in terms of iterated integrals.

- All the system can be integrated at once and expressed in terms of Goncharov poly-logarithms

$$G(a_1, ..., a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, ..., a_n; t) \qquad G(a_1; z) = \int_0^z \frac{dt}{t - a_1}$$

- If coefficients of MI in the amplitude do not have spurious 1/ε singularities: results only up to weight 4 are needed

Differential equations made simple: recap

$$\partial_x \vec{f}(x) = A(\epsilon, x) \cdot \vec{f}(x) \longrightarrow \boxed{\partial_x \vec{h}(x) = \epsilon \sum_i \frac{A_i}{(x - x_i)} \vec{h}(x)}$$

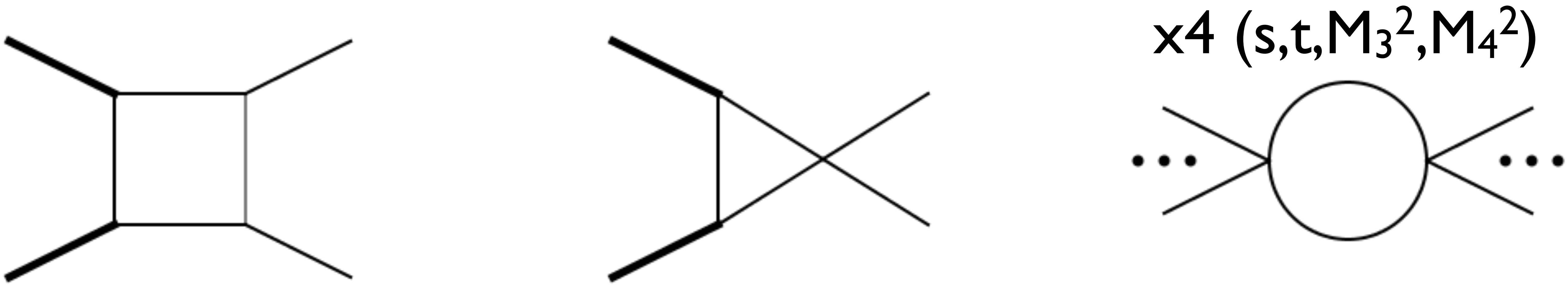
If a change of basis can put the MI in a 'canonical form':

- all integrals can be straightforwardly evaluated at once, as a series expansion in ε. THE SIZE OF THE SYSTEM IS IRRELEVANT
- results are expressible in terms of Goncharov poly-logarithms (numerical implementations available, GiNaC)
- at a given order in ε, the solution is a pure-function (no rational pre-factors) of uniform weight

HOW CAN WE FIND A CANONICAL FORM?

The canonical form for gg->VV: 1-loop

P12 family at 1-loop: 6 independent master integrals



As it is, not in a canonical form. To get there: solution in the canonical form must be pure function of uniform weight

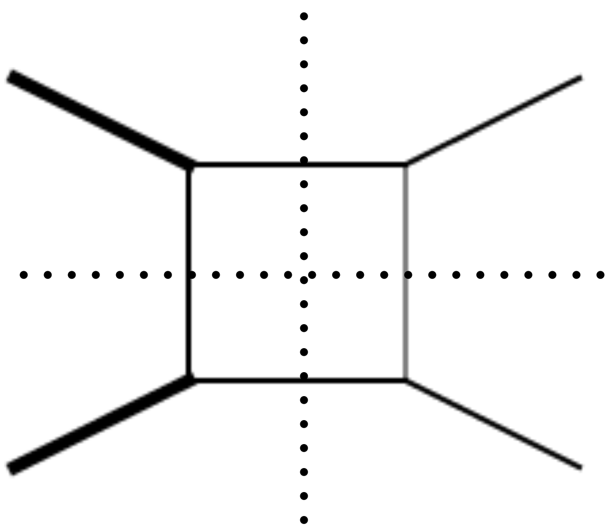
Simple integrals: fix by hand

$$B \sim \frac{(p^2)^{-\epsilon}}{\epsilon(1-2\epsilon)} = \frac{1}{\epsilon}\left[1+\epsilon(-\ln p^2 + 2) + \ldots\right] \longrightarrow B \to (1-2\epsilon)B$$

The canonical form for gg->VV: 1-loop

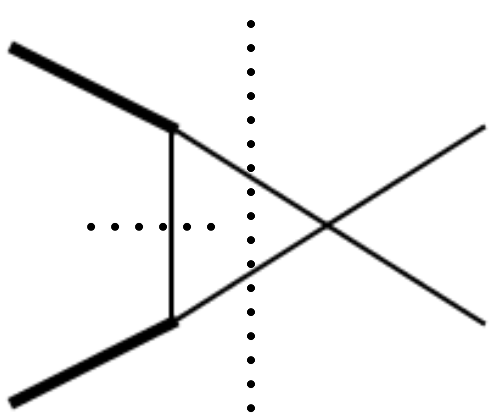
Canonical form: pure function of uniform weight

This property must reflect in the cut-structure of the integral



$$J = \frac{1}{st}$$

Good candidate:
$D \to stD$



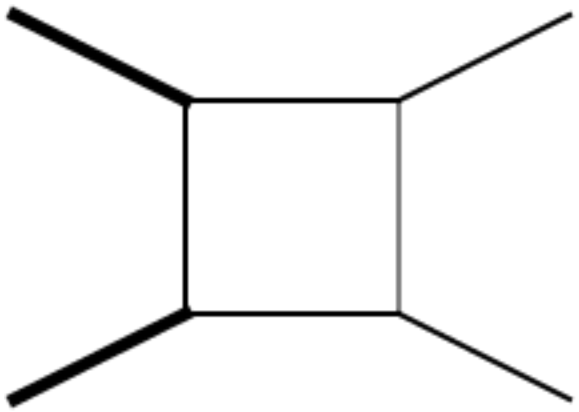
$$\int \frac{dt}{t} \frac{1}{\sqrt{\Delta}}$$

Good candidate:
$C \to \sqrt{\Delta} C$

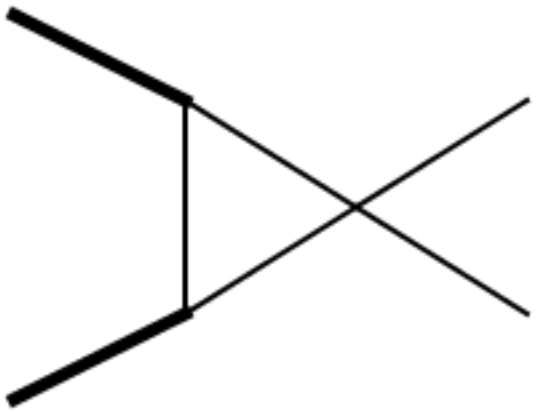
$$\Delta = s^2 + m_3^4 + m_4^4 - 2(sm_3^2 + sm_4^2 + m_3^2 m_4^2)$$

The canonical form for gg->VV: 1-loop

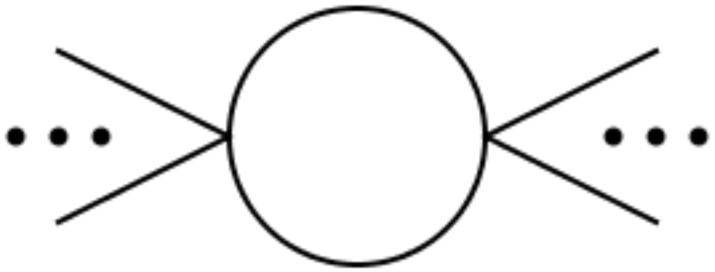
This guess turns out to be right -> CANONICAL FORM



$D \to stD$



$C \to \sqrt{\Delta} C$



$B \to (1-2\epsilon) B$

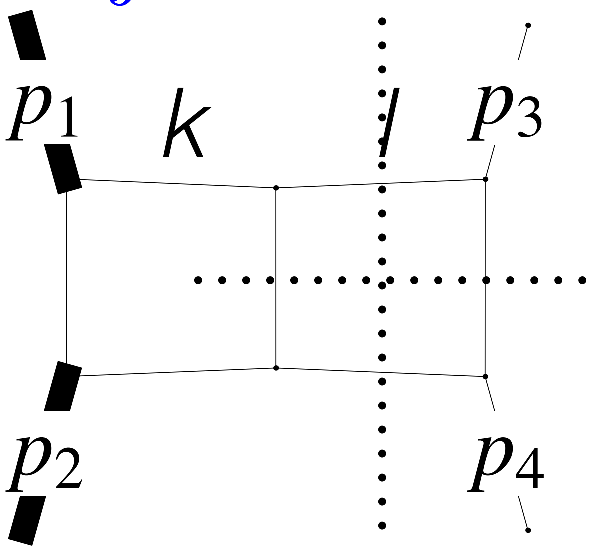
$$d\vec{f}(s,t,m_i) = \epsilon \sum_k A_i d\ln(\alpha_k) \vec{f}(s,t,m_i)$$

$$\alpha = \Big\{ s, t, m_i^2, m_i^2 - t, p_\perp^2, \Delta, \sqrt{\Delta} - m_3^2 - m_4^2 + s,$$
$$(m_3^2 + m_4^2)s - (m_3^2 - m_4^2)(\sqrt{\Delta} + m_3^2 - m_4^2),$$
$$m_3^2 m_4^2 (\sqrt{\Delta} - m_3^2 - m_4^2 + s) + 4m_3^2 m_4^2 t - t^2(\sqrt{\Delta} - s + m_3^2 + m_4^2) \Big\}$$

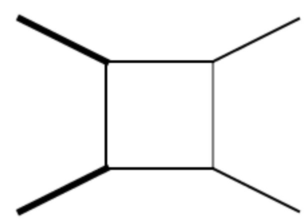
The canonical form for gg->VV: 2-loop

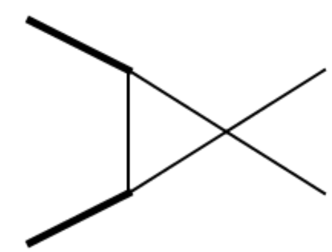
Example: complicated P12 structures

$$I = \int d^d k d^d l \frac{N(k)}{k^2(k+p_1)^2(k+p_{12})^2 l^2 (l+p_1)^2 (l-p_3)^2 (k-l)^2}$$



$$\longrightarrow \int d^d k \left[\frac{1}{s(k-p_3)^2}\right] \frac{N(k)}{k^2(k+p_1)^2(k+p_{12})^2}$$





Good candidates: $N(k) = s^2 t,$ $N(k) = s(k-p_3)^2\sqrt{\Delta}$

As in the 1-loop cases, these ansatz proves correct

USING THESE IDEAS, FULL CANONICAL BASIS CAN BE FOUND

Integrating the canonical form: square roots

$$d\vec{f}(s,t,m_i) = \epsilon \sum_k A_i d\ln(\alpha_k)\vec{f}(s,t,m_i)$$

$$G(a_1,\ldots,a_n;z) = \int_0^z \frac{dt}{t-a_1} G(a_2,\ldots,a_n;t)$$

3-mass triangle: square-root singularity in the diff. eq.

Evaluation in terms of Goncharov poly-logarithms problematic

To solve this problem:

rational re-mapping to eliminate all square-roots

$$s \to m_3^2(1+x)(1+xy), \quad t \to -m_3^2 xz, \quad m_4^2 \to m_3^2 x^2 y$$

$$\alpha = \{x, y, z, 1+x, 1-y, 1-z, 1+xy, -y+z, 1+y+xy-z, xy+z,$$
$$1+x(1+y-z), 1+xz, 1+y-z, xyz+z+x(-y+z), xyz-y+yz+z\}$$

Physical region x>0, 0<y<z<1: alphabet is sign-definite

The last step: fixing the boundary condition

So far, only solved half of the problem. Diff. Eq. must be supplemented by appropriate boundary conditions

Although much simpler than the full integral, computing results in specific kinematics configuration can still be challenging

Thanks to its manifest singularity structure, the differential equation in the canonical form + physics intuition can help to reduce these computations to a minimum

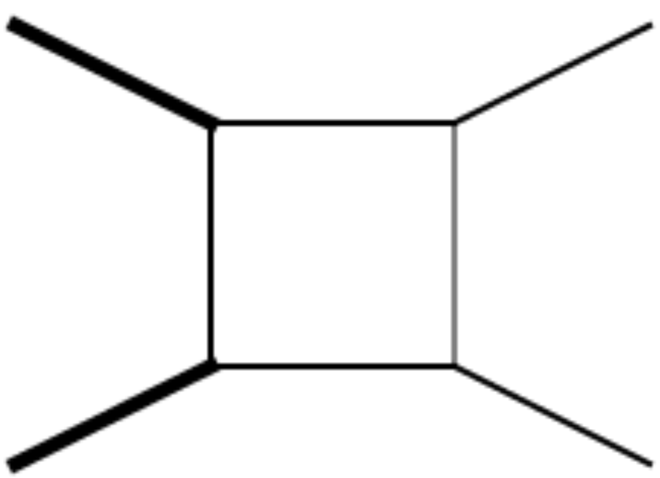
The last step: fixing the boundary condition

Example: boundary condition for 1-loop box

As a boundary condition, we considered forward scattering at threshold, with $M_4^2/M_3^2 \to 0$

In this limit, the diff. eq. develops a spurious DPS $p_T \to 0$ singularity

$$\partial_{p_\perp} D = \frac{\epsilon}{p_\perp}(B_{M_3} + B_{M_4} + B_s - 2B_t - C - D)$$



This box cannot have such singularity ->

$$D = B_{M_3} + B_{M_4} + B_s - 2B_t - C - D$$

at the boundary, to all orders in ε

The last step: fixing the boundary condition

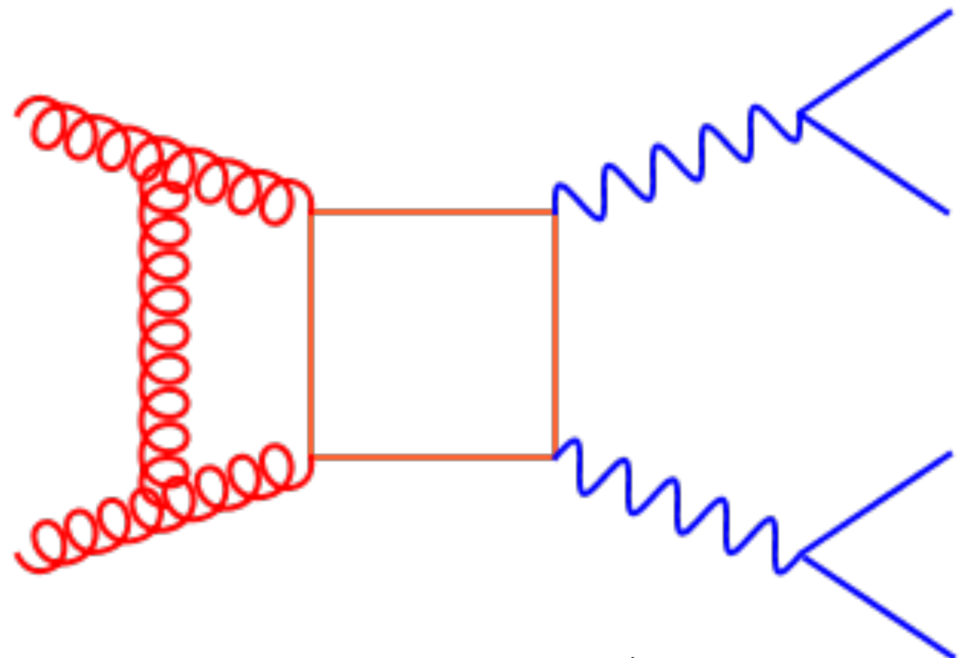
- Similar arguments can be used to obtain other non-trivial boundary conditions
- THESE ARGUMENTS APPLY VERBATIM AT TWO-LOOP
- As a consequence, VIRTUALLY ALL BC FOR THE GG->VV AMPLITUDE CAN BE FIXED BY CONSISTENCY RELATIONS, without doing an actual computation
- As a check, we recomputed brute-force the (very hard!) boundary conditions. Full agreement is observed

MI for gg->VV: final remarks

- Thanks to new ideas for multi-loop computations, this very hard problem can be made relatively simple
- all MI have been computed, in terms of Goncharov Poly-Logs (numerical evaluation: GiNaC [Vollinga, Weinzierl (2005)])
- In principle possible to remap GNs in terms of classical poly-logarithms plus one extra function, i.e. Li22 (in-house numerical implementation -> improved speed-stability)
- Checks on the result:
  - against results in special cases ([Gehrmann et al, (2014)])
  - numerically for one phase-space point (FIESTA)
  - after our result was published, independently reproduced by two groups ([Papadopoulos, Tommasini, Wever, (2014); v. Manteuffel, Tancredi (to appear)])

The gg->VV 2-loop amplitude

$$g(p_1) + g(p_2) \to \left[V_3(p_3) \to l(p_5)\bar{l}(p_6)\right] + \left[V_4(p_4) \to l(p_7)\bar{l}(p_8)\right]$$

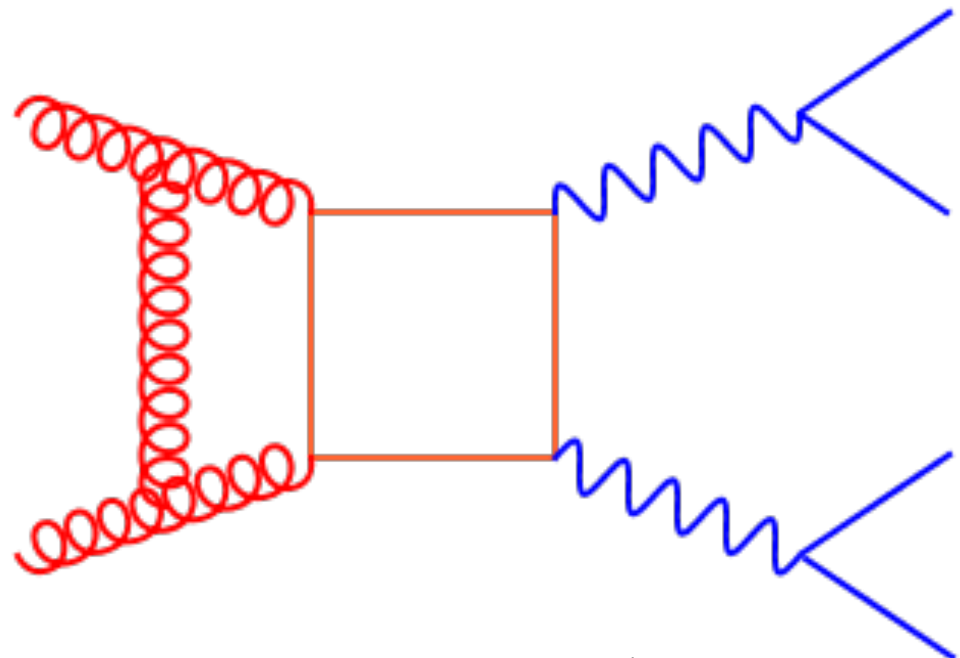


$$\begin{aligned}\mathcal{A}_{3L4L}^{\lambda_1\lambda_2} =& \mathcal{N}_{\lambda_1\lambda_2}\Big\{\Big(F_1^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_2^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\langle 17\rangle[81] \\ &+ \Big(F_3^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_4^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\langle 27\rangle[82] + 2F_5^{\lambda_1\lambda_2}\langle 57\rangle[86] \\ &+ \frac{1}{2}\Big(F_6^{\lambda_1\lambda_2}\langle 15\rangle[61] + F_7^{\lambda_1\lambda_2}\langle 25\rangle[62]\Big)\Big(\langle 12\rangle\langle 78\rangle[81][82] + \langle 17\rangle\langle 27\rangle[21][87]\Big) \\ &- \frac{1}{2}\Big(F_8^{\lambda_1\lambda_2}\langle 17\rangle[81] + F_9^{\lambda_1\lambda_2}\langle 27\rangle[82]\Big)\Big(\langle 12\rangle\langle 56\rangle[61][62] + \langle 15\rangle\langle 25\rangle[21][65]\Big)\Big\}.\end{aligned}$$

Combining IBP reduction + MI in the form factors: FULL ANALYTIC EXPRESSIONS FOR THE 2-LOOP AMPLITUDE.

The gg->VV 2-loop amplitude

$$g(p_1)+g(p_2)\to\left[V_3(p_3)\to l(p_5)\bar{l}(p_6)\right]+\left[V_4(p_4)\to l(p_7)\bar{l}(p_8)\right]$$



$$\begin{aligned}\mathcal{A}^{\lambda_1\lambda_2}_{3L4L}=&\mathcal{N}_{\lambda_1\lambda_2}\Big\{\Big(F_1^{\lambda_1\lambda_2}\langle15\rangle[61]+F_2^{\lambda_1\lambda_2}\langle25\rangle[62]\Big)\langle17\rangle[81]\\&+\Big(F_3^{\lambda_1\lambda_2}\langle15\rangle[61]+F_4^{\lambda_1\lambda_2}\langle25\rangle[62]\Big)\langle27\rangle[82]+2F_5^{\lambda_1\lambda_2}\langle57\rangle[86]\\&+\frac{1}{2}\Big(F_6^{\lambda_1\lambda_2}\langle15\rangle[61]+F_7^{\lambda_1\lambda_2}\langle25\rangle[62]\Big)\Big(\langle12\rangle\langle78\rangle[81][82]+\langle17\rangle\langle27\rangle[21][87]\Big)\\&-\frac{1}{2}\Big(F_8^{\lambda_1\lambda_2}\langle17\rangle[81]+F_9^{\lambda_1\lambda_2}\langle27\rangle[82]\Big)\Big(\langle12\rangle\langle56\rangle[61][62]+\langle15\rangle\langle25\rangle[21][65]\Big)\Big\}.\end{aligned}$$

Combining IBP reduction + MI in the form factors: FULL ANALYTIC EXPRESSIONS FOR THE 2-LOOP AMPLITUDE.

The gg->VV 2-loop amplitude: checks

- The infra-red singularity structure of the amplitude is known from first principles. In this case, it reads

$$\mathcal{A}_2 = \frac{0}{\epsilon^4} + \frac{0}{\epsilon^3} - \frac{C_A^2}{\epsilon^2} s^{-\epsilon} e^{i\pi\epsilon} \mathcal{A}_1 + \mathcal{O}(\epsilon^0) \qquad \text{(unrenorm.)}$$

- Highly non-trivial check on the computation, computation does not separate convergent and divergent part until the very end.
- The above structure is not automatically manifest in the computation (nor in the actual result)
- CHECK ESTABLISHED NUMERICALLY, TO BETTER THAN 16 DIGITS ACCURACY

The gg->VV 2-loop amplitude: stability issues

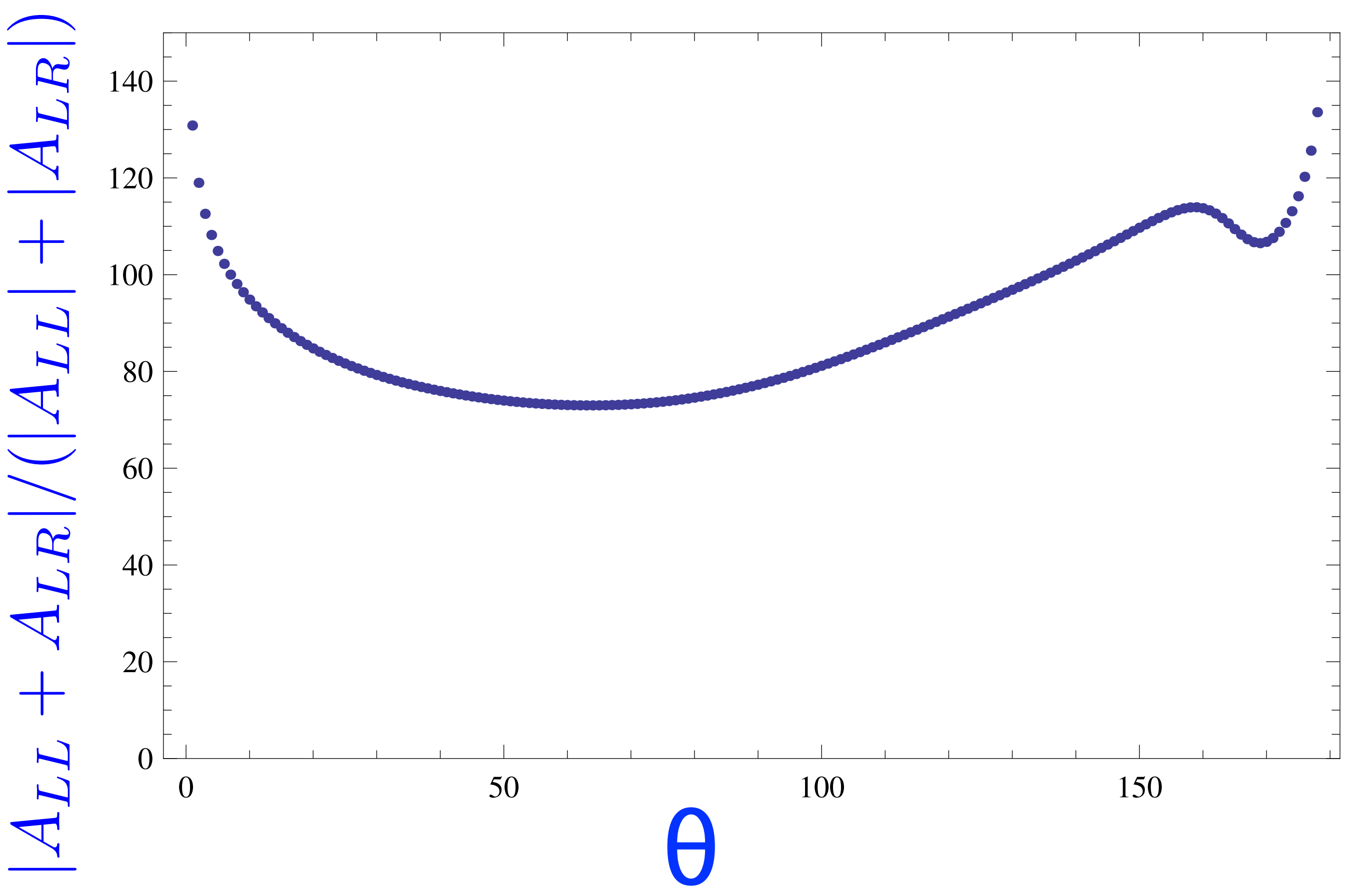
- Already at one-loop, gg->VV amplitudes suffer from numerical instabilities created by spurious gram singularities.
- Typical manifestation: (spurious) poles in $p_{T,V}$. This region is not removed by experimental cuts (cut on leptons)
- At two-loop, the situation can only be worse. Potential to make the computation useless in practice
- To investigate this issue: compare final Fortran implementation with (basically) infinite precision in Mathematica

The gg->VV 2-loop amplitude: stability issues

$$g(p_1) + g(p_2) \to [V_3(p_3) \to l(p_5)\bar{l}(p_6)] + [V_4(p_4) \to l(p_7)\bar{l}(p_8)]$$

The set-up:

- $\sqrt{s_{part}}$ = 125 GeV, M_{V1} = 80.419 GeV, M_{V2} = 25 GeV
- scattering angles for leptons in the C.o.M. of the parent V: $\{\theta_{56}=\pi/4, \varphi_{56}=\pi/2\}$; $\{\theta_{78}=\pi/6, \varphi_{78}=\pi\}$
- Scan in the scattering angle of the VV system



- Result is stable down to θ~1//179° -> p_T ~ 0.5 GeV
- COMPARABLE TO 1-LOOP STABILITY
- COMPUTATION IS RELIABLE

Conclusions

- 4l final states very interesting processes at the LHC, both per se and in Higgs-related analysis
- NLO corrections are sizable ~ 50% -> need for NNLO
- Situation even worse for gg->VV, only known at LO

- Major bottleneck for such predictions: complicated 2-loop amplitudes (many different scales, full 2->2 topologies)
- Thanks to interesting new idea, these problems are manageable
- THE FULL 2-LOOP AMPLITUDE FOR GG->4L IS NOW AVAILABLE
- Along with our previous result for the qqb -> 4l amplitude, this REMOVES THE LAST OBSTACLE FOR PRECISE (F.O.) PREDICTIONS FOR (OFF-SHELL) PP->4L (leptons fiducial volume, signal/background interferences…)

Outlook

The technical part is gone ->

NOW IT'S TIME FOR PHENOMENOLOGY

- Fully exclusive NNLO, matched with jet veto
- Phenonemological studies for H->WW* background
- gg->ZZ @ NLO and the Higgs off-shell cross-section
- PS @ NNLO in a more complicated environment?

Thank you

for your attention

Backup

On-shell production: the WW puzzle

Naively, there seemed to be a slight tension between measurements / predictions for the total WW cross section

$$\sigma_{ATLAS} = 71.4 \pm 5.6, \quad \sigma_{CMS} = 69.9 \pm 7.0$$
$$\sigma_{NLO} = 54.77 \pm 1.6, \quad \sigma_{NNLO} = 59.84 \pm 1.3 \text{ pb}$$

- Although not very significant, this drew a lot of attention as it has explanations in terms of natural SUSY [Meade et al. (2013-2014), Rolbiecki and Sakurai (2013)]
- tension reduced at NNLO
- Measurements involve veto on jet activity -> theory used to extrapolate from fiducial to total cross-section may not be adequate [Meade, Ramani, Zeng (2014); Monni, Zanderighi (2014)]
- should compare FIDUCIAL CROSS-SECTIONS

On-shell production: the WW puzzle

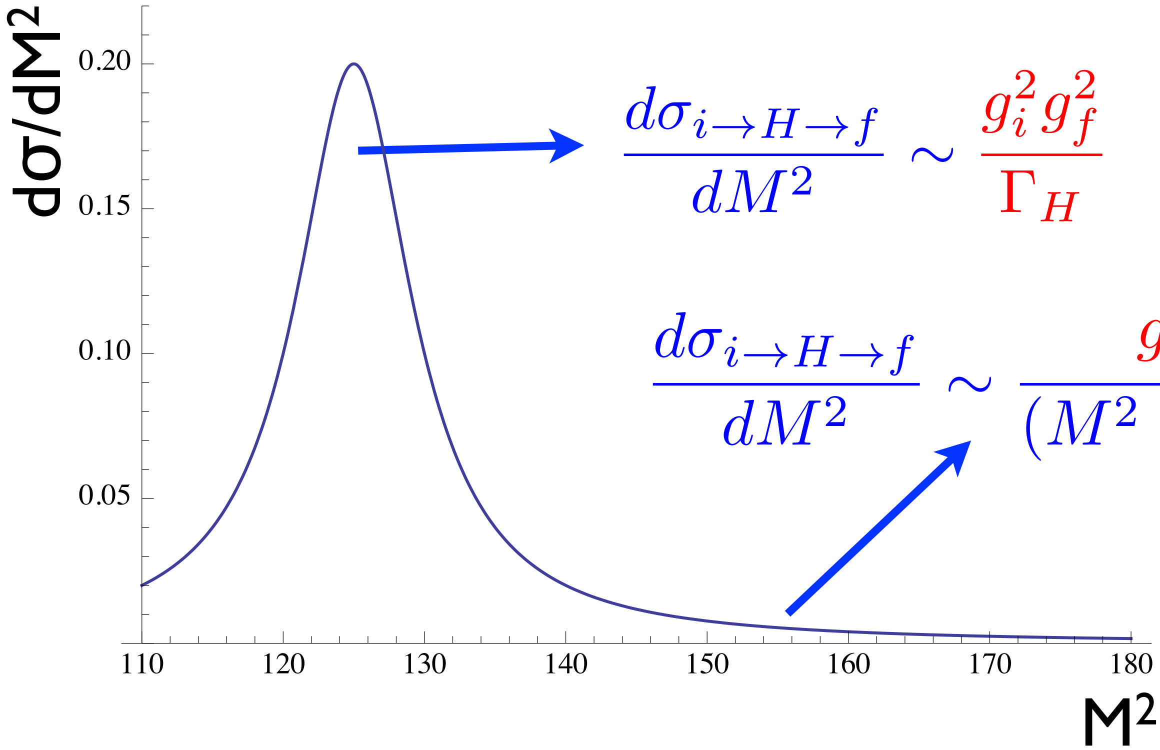
Estimated cross section for the fiducial region
from extrapolating NNLO+NNLL

$$\sigma_{ATLAS}(e\mu) = 377.8 \pm 27.3, \quad \sigma_{ext,th} = 357.9 \pm 14.4 \text{ fb}$$
$$\sigma_{ATLAS}(ee) = 68.5 \pm 8.5, \quad \sigma_{ext,th} = 69.0 \pm 2.7 \text{ fb}$$
$$\sigma_{ATLAS}(\mu\mu) = 74.4 \pm 7.6, \quad \sigma_{ext,th} = 75.1 \pm 3.0 \text{ fb}$$

- VERY GOOD AGREEMENT BETWEEN THEORY AND EXPERIMENT
- tension in the total cross section seems due to powheg overestimating Sudakov suppression and distortion of leptons p_T, EXTRAPOLATION ISSUE [Monni, Zanderighi (2014)]
- Corrections to gg->4l can play a relevant role (~10% of NLO in the fiducial region, large corrections expected)
- To perform a full study: DIFFERENTIAL NNLO FOR PP -> 4L AND NLO FOR GG -> 4L HIGHLY DESIRABLE

Bounds on the Higgs width

[FC, Melnikov (2013)]



$$\frac{d\sigma_{i\to H\to f}}{dM^2} \sim \frac{g_i^2 g_f^2}{\Gamma_H}$$

$$\frac{d\sigma_{i\to H\to f}}{dM^2} \sim \frac{g_i^2 g_f^2}{(M^2 - m_H^2)^2}$$

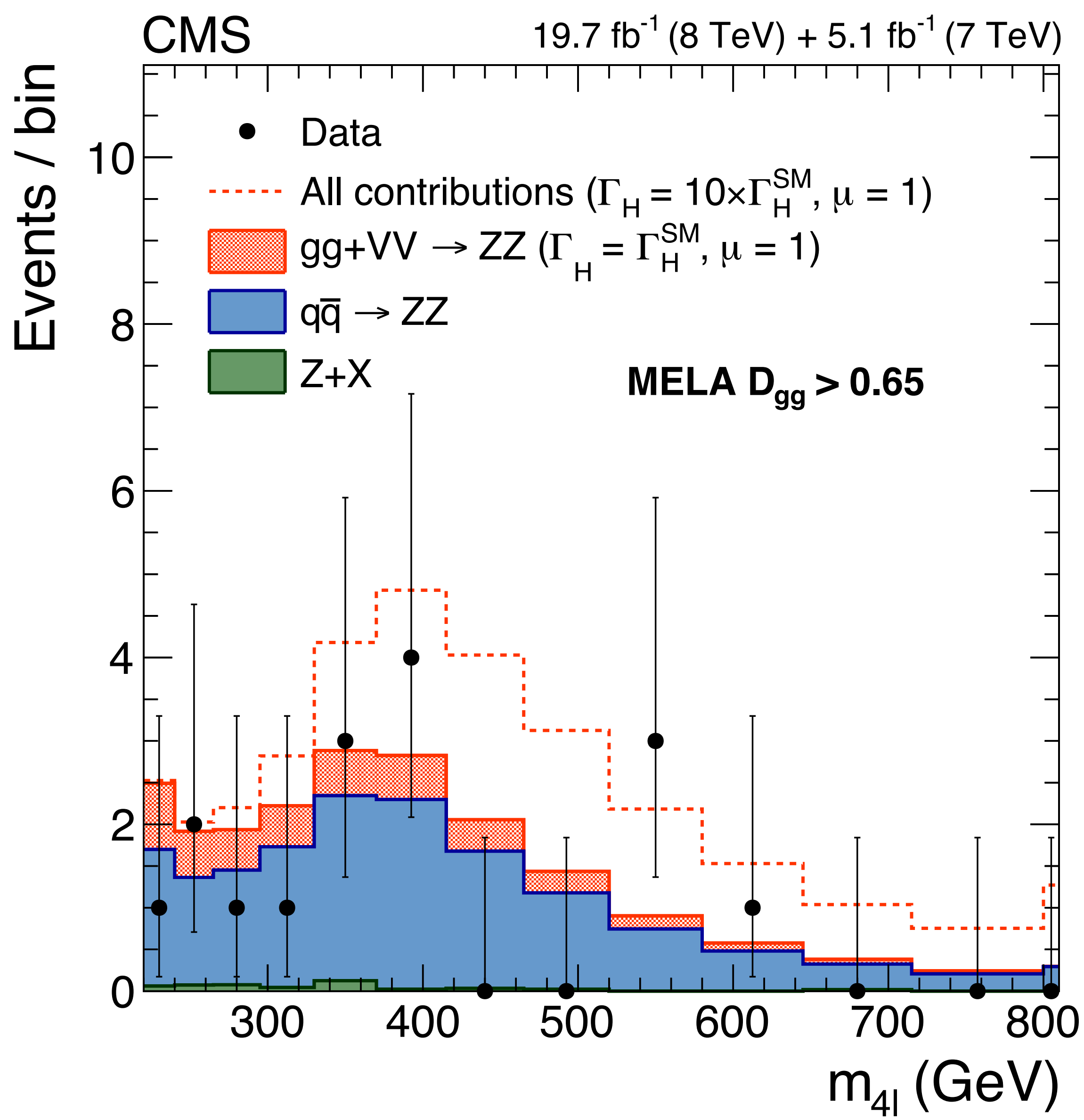
- On the peak, only access to coupling x BR
- Off the peak, Γ_H independent
- Because of this, constraints on Γ_H

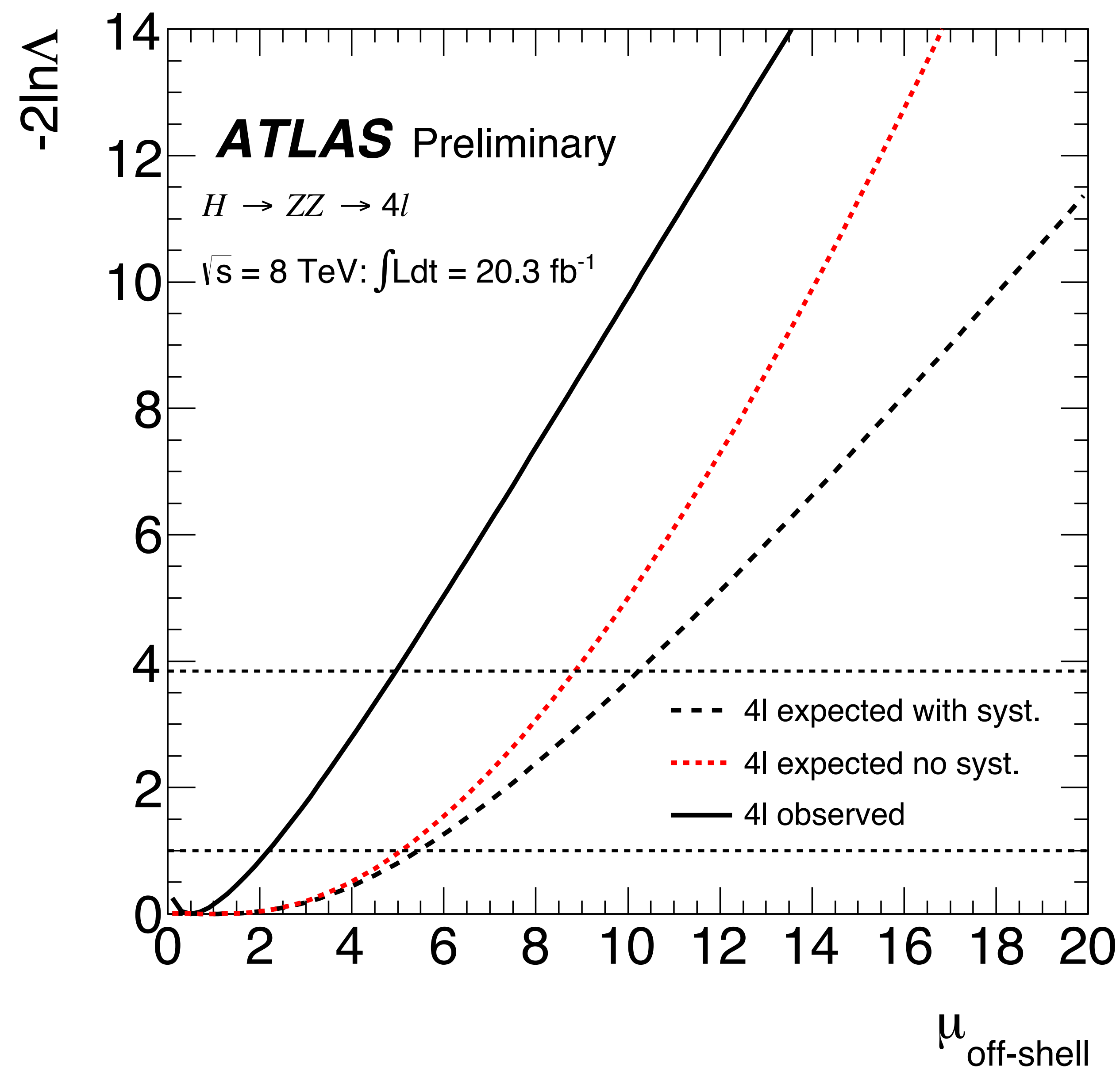
- Peak looks SM-like -> $\frac{g_i^2 g_f^2}{\Gamma_H} = \frac{g_{i,\rm SM}^2 g_{f,\rm SM}^2}{\Gamma_{H,\rm SM}} \longrightarrow g = \xi g_{\rm SM},\ \Gamma_H = \xi^4 \Gamma_{H,\rm SM}$
- Off the peak -> $N_{obs}^{off} \propto g_i^2 g_f^2 = \xi^4 g_{i,\rm SM}^2 g_{f,\rm SM}^2 \propto \xi^4 N_{\rm SM}^{off} = \frac{\Gamma_H}{\Gamma_{H,\rm SM}} N_{\rm SM}^{off}$

Bounds of the order ~ 10-20 $\Gamma_{H,SM}$ can be achieved

Refined tools available [Kauer (2008, 2012); Campbell, Ellis, Williams (2013)]
Thorough phenomenological studies [Campbell, et al (2013-2014)]

Analysis is doable (and actually done)





CMS: $\Gamma_H < 5.4\ \Gamma_{H,SM}$ = 22 MeV @ 95CL

ATLAS: $\Gamma_H < 4.8\text{-}7.7\ \Gamma_{H,SM}$ = 20-32 MeV @ 95CL

Assuming correlation of on/off-shell couplings