Geometric constraints on rank-1 $\mathcal{N} = 2$ superconformal field theories

Matteo Lotito

in collaboration with

P. Argyres, Y. Lü, M. Martone (arXiv:1505.04814)

Y. Lü talk: Fri. @ 4:00 PM (parallel session)

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Study of $\mathcal{N} = 2$ SCFTs

Our interest is to study and classify CFTs. Some amount of supersymmetry should help us in the classification, so in particular we are focusing on $\mathcal{N} = 2$ SCFTs.

The Problem

- Understand structure of $\mathcal{N} = 2$ SCFTs by studying their moduli spaces of vacua;
- Understand the connection between moduli space geometries and microscopic CFT data.

We apply Seiberg-Witten techniques to carry out the analysis:

Our Approach

- Classify all possible scale-invariant Coulomb branches (CBs);
- Classify the possible deformations.

Review of SW

An $\mathcal{N} = 2$ vector multiplet has the form:
$$\begin{array}{ccccccc} & \swarrow\!\!\nearrow & \lambda_\alpha & \leftrightarrow & A_\mu \\ \phi & \leftrightarrow & \tilde{\lambda}_\alpha & \swarrow\!\!\nearrow & \end{array}$$

The scalar component ϕ can acquire a vev, giving rise to a moduli space of vacua (Coulomb branch):

$$\langle \phi \rangle = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_n \end{pmatrix}, \quad \sum_i^n a_i = 0$$

For the case of interest, *Rank-1 Coulomb branches*, we start with an $SU(2)$ gauge theory, so the vev of ϕ takes the form: $\langle \phi \rangle = \mathrm{diag}(a, -a)$.

On the CB, the low energy theory has a residual $U(1)$ gauge symmetry, governed by the gauge coupling τ.

Holomorphic gauge coupling

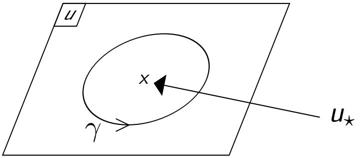
From the structure of $\mathcal{N} = 2$ supersymmetry, the gauge coupling:

$$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

is a holomorphic function of the coordinate u (gauge-invariant function of the vev a: $u = {}^1\!/\!{}_2\langle \mathrm{Tr}\phi^2 \rangle = a^2$).

Moreover, on the CB, it can undergo a monodromy (Electric-Magnetic duality transformation):

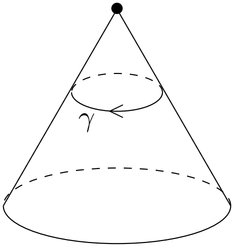
$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \qquad M_\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

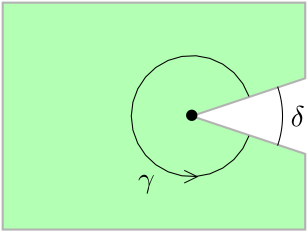


The monodromy γ uniquely characterizes the singularity at $u_\star$.
At a singularity additional massless states appear.

Scale Invariant Moduli Spaces

Scale invariant geometries must have at most one singular point (from scale invariance), this implies that the geometry of the Coulomb branch is a complex cone, with arbitrary deficit angle δ.





This is further restricted by $\mathcal{N} = 2$ supersymmetry and electric-magnetic duality of the low energy theory, giving a discrete set of allowed deficit angles.

Simplifying assumptions

- *Rank-1 Coulomb branches:*
  Rank of the gauge group of the microscopic theory, or, for non lagrangian theories, we call rank the (complex) dimensionality of the Coulomb branch.
  Implies that monodromies are $\in SL(2,\mathbb{Z})$, in the general case they are $\in Sp(2r,\mathbb{Z})$, where $r = \dim_{\mathbb{C}}(CB)$.
- *Regularity of the 2-form:*
  There is a holomorphic 2-form defined on the total space of the CB (CB with complex tori fibered over each point),
  another simplifying condition we use is that it is regular (non vanishing) at singular points on the CB.

Scale invariant CBs

Allowed geometries correspond to Kodaira classification of elliptic curves:

Singularity	Curve $y^2 =$	$D(u)$	$\text{ord}_0(\Delta)$	M_0	δ
II^*	x^3+u^5	6	10	ST	$\pi/3$
III^*	x^3+u^3x	4	9	S	$\pi/2$
IV^*	x^3+u^4	3	8	$-(ST)^{-1}$	$2\pi/3$
I_0^*	$x^3+\tau u^2 x+u^3$	2	6	$-I$	π
IV	x^3+u^2	3/2	4	$-ST$	$4\pi/3$
III	x^3+ux	4/3	3	S^{-1}	$3\pi/2$
II	x^3+u	6/5	2	$(ST)^{-1}$	$5\pi/3$
$I_{n\geq 1}$	$(x-1)(x^2+(u/\Lambda)^n)$	1	n	T^{-n}	2π (cusp)
$I^*_{n>1}$	$x^3+ux^2+u^{n+3}\Lambda^{-2n}$	2	$n+6$	$-T^{-n}$	2π (cusp)

Table: Col 1: Kodaira type of singularity. Col 2: Scale invariant SW curve. Col. 3: Mass dimension of u. Col.4: Order of the discriminant. Col.5: Monodromy around singularity. Col.6: Deficit angle.

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IV	$x^3 + u^2$	3/2	4	$-ST$	$4\pi/3$
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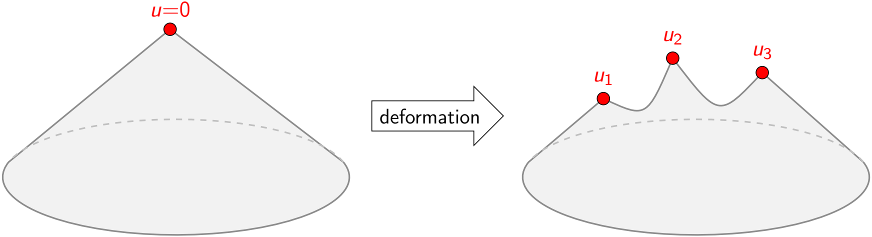
However, it can't be the end of the story, there are multiple examples of **SCFTs with the same Kodaira singularity**. → We need something more.

Deformations

$\mathcal{N} = 2$ preserving relevant deformations of CFTs can be introduced.



A singular point can split into multiple singularities, each of which will again be of one of the Kodaira types described earlier.

E.g.:

$$II^{\star} \to \{I_1^{10}\}$$

Maximal deformation of the $II^{\star}$ singularity splits up into ten I_1 sings.

Safely irrelevant conjecture

- There is another physical requirement we have to use, to be able to write the possible deformations of scale invariant geometries:

Safely irrelevant conjecture: There are no dangerously irrelevant operators in $\mathcal{N} = 2$ SCFTs.

What this is saying, basically, is that a deformation with generic coefficients always splits into a set of singularities, which can't be split any further (I_n and $I_n^\star$ type, with the exception of possible "frozen" singularities).

Dirac quantization condition

For deformations involving I_n and I_n^* singularities, we can set additional constraints by the fact that these correspond to specific IR-free theories having states with fixed E-M charges, the condition on the possible deformations is that the charges have to be commensurate.

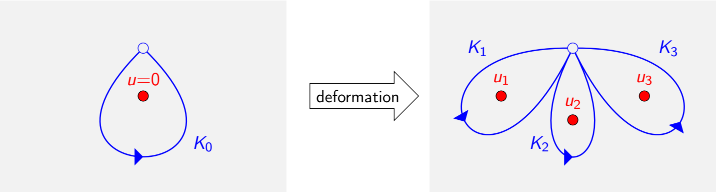
sing.	$\text{ord}_0(D_x)$	deformation pattern
II^*	10	$\{{I_1}^{10}\}$, $\{{I_1}^6, I_4\}$, $\{{I_1}^2, I_4^2\}$, $\{{I_1}^3, I_1^*\}$, $\{I_1, III^*\}$, $\{{I_1}^2, IV^*\}$, $\{I_2, IV^*\}$, $\{I_1, I_9\}$, $\{I_2^5\}$, $\{I_2, I_8\}$, $\{I_5^2\}$, $\{I_2, I_2^*\}$
III^*	9	$\{{I_1}^9\}$, $\{{I_1}^5, I_4\}$, $\{{I_1}^2, I_1^*\}$, $\{I_1, IV^*\}$, $\{I_1, {I_4}^2\}$, $\{{I_3}^3\}$
IV^*	8	$\{{I_1}^8\}$, $\{{I_1}^4, I_4\}$, $\{I_1, I_1^*\}$, $\{{I_4}^2\}$, $\{{I_2}^4\}$
I_0^*	6	$\{{I_1}^6\}$, $\{{I_1}^2, I_4\}$, $\{{I_2}^3\}$, $\{{I_3}^2\}$
IV	4	$\{{I_1}^4\}$, $\{{I_2}^2\}$
III	3	$\{{I_1}^3\}$
II	2	$\{{I_1}^2\}$

Table: List of deformation patterns of Kodaira singularities allowed by the Dirac quantization condition. The ones in grey are incompatible with the $\mathrm{SL}(2, \mathbb{Z})$ monodromies of the Kodaira singularities (next slide).

- Order of the discriminant has to be invariant under deformations.

Special Kähler conditions

From the geometry/monodromy point of view,



Deformations should also satisfy conditions:

- K_0, monodromy at infinity has to be of Kodaira type;
- $K_0 = K_1 K_2 K_3 \in SL(2, \mathbb{Z})$, product of undeformable sings in the same conjugacy class as the original monodromy.

We further restrict number of consistent deformations.
Maximal deformations of Kodaira singularities are always allowed.
By allowing sub-maximal deformations, we obtain more interesting results.

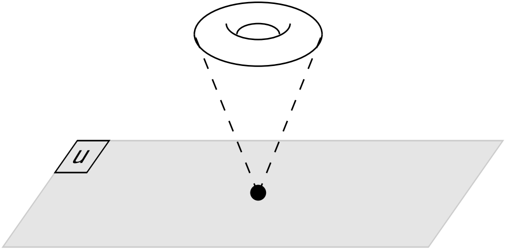
Deformations allowing frozen sings.

Safe deformations of regular, rank 1, scale-invariant CBs							
Kodaira singularity	deformation pattern	flavor symmetry	central charges			Higgs branches	
			k_F	$12 \cdot c$	$24 \cdot a$	h_1	h_0
II^*	$\{{I_1}^{10}\}$	E_8	12	62	95	0	29
	$\{{I_1}^6, I_4\}$	$\mathrm{sp}(10)$	7	49	82	5	16
III^*	$\{{I_1}^9\}$	E_7	8	38	59	0	17
	$\{{I_1}^5, I_4\}$	$\mathrm{sp}(6) \oplus \mathrm{sp}(2)$	$5 \oplus 8$	29	50	3	8
IV^*	$\{{I_1}^8\}$	E_6	6	26	41	0	11
	$\{{I_1}^4, I_4\}$	$\mathrm{sp}(4) \oplus \mathrm{u}(1)$	$4 \oplus ?$	19	34	2	4
	$\{I_1, I_1^*\}$	$\mathrm{u}(1)$	?	$14+h$	$29+h$	h	?
I_0^*	$\{{I_1}^6\}$	$\mathrm{so}(8)$	4	14	23	0	5
	$\{{I_1}^2, I_4\} \simeq \{{I_2}^3\}$	$\mathrm{sp}(2)$	3	9	18	1	1
IV	$\{{I_1}^4\}$	$\mathrm{su}(3)$	3	8	14	0	2
III	$\{{I_1}^3\}$	$\mathrm{su}(2)$	8/3	6	11	0	1
II	$\{{I_1}^2\}$	–	–	22/5	43/5	0	0
with the assumption of a frozen IV* SCFT with central charge c'							
II^*	$\{I_2, IV^*\}$	$\mathrm{su}(2)$	?	$24c'+h+4$	$24c'+h+37$	h	?
or							
II^*	$\{{I_1}^2, IV^*\}$	G_2	?	$24c'+h+10$	$24c'+h+43$	h	?
III^*	$\{I_1, IV^*\}$	$\mathrm{su}(2)$	?	$16c'+h+\frac{10}{3}$	$16c'+h+\frac{73}{3}$	h	?
with the assumption of a frozen III* SCFT with central charge c''							
II^*	$\{I_1, III^*\}$	$\mathrm{su}(2)$	?	$18c''+h+5$	$18c''+h+38$	h	?

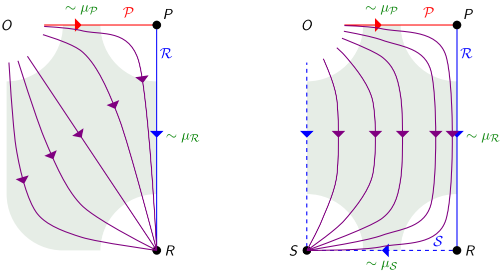
Backup

Regularity assumption

- *Regularity of the 2-form*
  Total space of the CB is $\mathcal{X} = \mathcal{X}_u \tilde{\times} \mathcal{M}_V$, $\mathcal{M}_V$ is the CB and the $\mathcal{X}_u$ are complex tori fibered over each point u.



There exists a holomorphic 2-form: $\Omega = dz \wedge du$ uniquely defined on the total space $\mathcal{X}$. The regularity condtion is the requirement that $\Omega(u_\star) \neq 0$ for any singular point $u_\star \in \mathcal{M}_V$.



(a) **(b)**

Fig. a: $\mathcal{P}$ is a safely irrelevant operator, turning it on doesn't modify the behaviour of $\mathcal{R}$ flow to R in the IR.

Fig. b: upon turning on $\mathcal{P}$, we move from the R to the the S fixed point, therefore, despite being irrelevant at $\mathcal{P}$, it changes the IR behaviour of the flow, producing a relevant deformation ($\mathcal{S}$) from R to S.

From the cases constructed explicitly we find that it is always possible to construct a relevant deformation from a scale invariant CB to all the allowed physical deformations, therefore we introduce the *safely irrelevant conjecture*: in $\mathcal{N} = 2$ SCFTs there are no dangerously irrelevant operators.