Cosmological (non)-Constant Problem:
The case for TeV scale quantum gravity

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Punchline!

- **Quantum Gravity** is ***already*** in the **Infrared!**
- *Weakly coupled* **QFT+GR *must fail* at < 100 TeV!**

→ ~~*High scale SUSY, GUT, (almost all) Inflation models*~~

→ *TeV-scale QG, Large Extra Dimensions*

→ *Strongly coupled UV completion (technicolor?, bootstrap?)*

Hierarchy problem(s)

- neutrino mass
- dark energy

Higgs, Electroweak

LHC

GUT…Planck

$10^{-3}\mathrm{eV}$ $\qquad$ $10^{12}\mathrm{eV}$ $\qquad$ $10^{28}\mathrm{eV}$

$\rho_{\mathrm{DE}} \sim \Lambda^4$ $\qquad$ $\delta m_H^2 \sim \Lambda^2$ $\qquad$ $\Lambda?$

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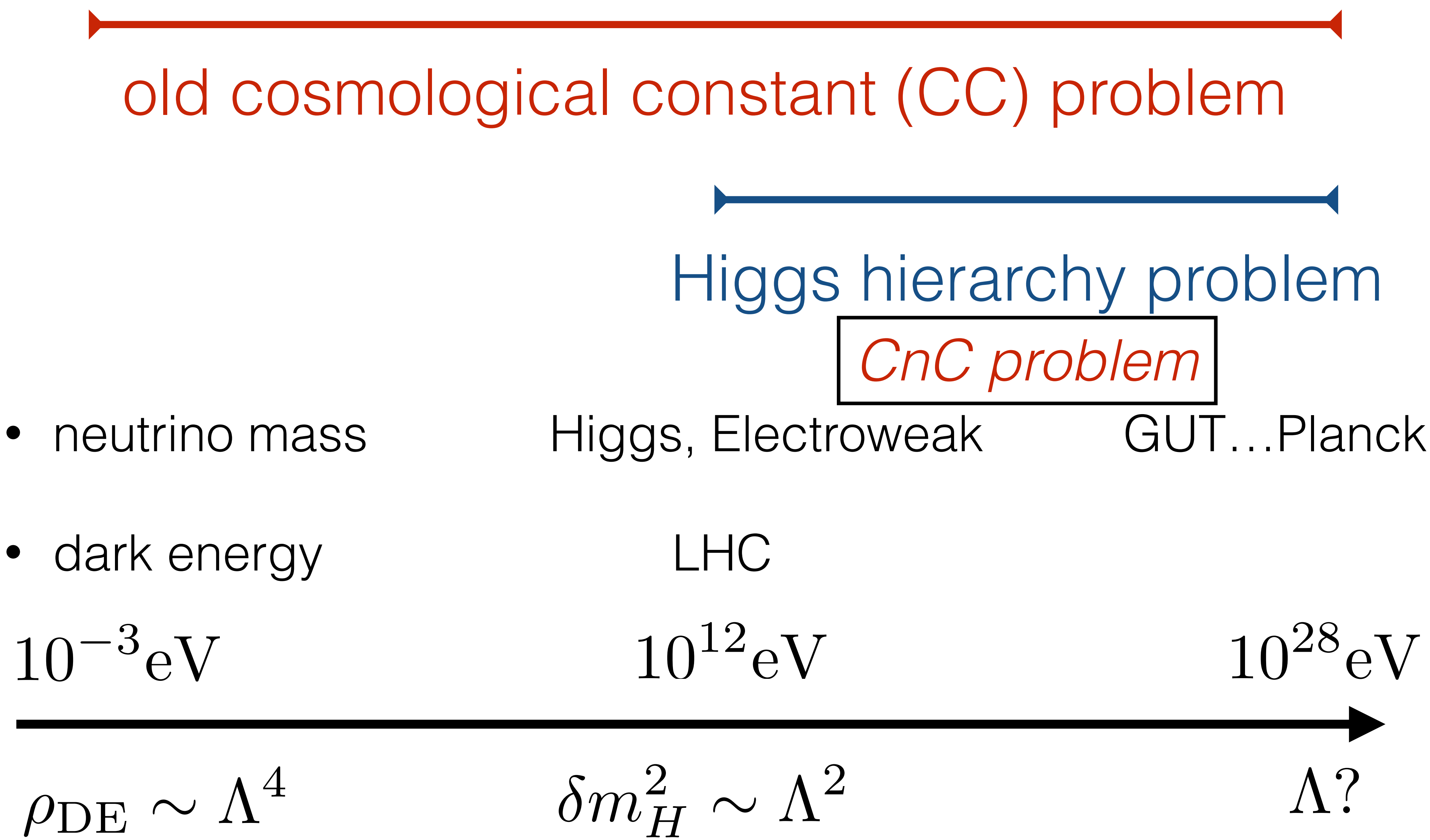

Does Quantum Gravity matter in the IR?

- Quantum Fluctuations do fluctuate!

$$\langle T_{\mu\nu} \rangle = 0 \not\Rightarrow \langle T_{\mu\nu} T_{\alpha\beta} \rangle = 0$$

- What is the analog of CC for the covariance of stress fluctuations?

- Can these fluctuations have an observable gravitational signature on large scales?

with Elliot Nelson (Penn-State → PI), 1504.00012

Vacuum Fluctuations in Linear Gravity

- Linearized Perturbations around FRW space-time

$$ds^2 = a^2(\eta)\left[-(1+2\phi)d\eta^2 + 2V_i dx_i d\eta + (1-2\psi)d\mathbf{x}^2\right]$$

- Einstein constraint sector: *scalars in longitudinal gauge and vectors*

$$-k^2\psi = 4\pi G\left(\delta T_{00} - \frac{3\mathcal{H}}{k^2} i k^i \delta T_{i0}\right),$$

$$-k^2\phi = 4\pi G\left(\delta T_{00} - \frac{3\mathcal{H}}{k^2} i k^i \delta T_{i0} + \left(\delta^{ij} - 3\frac{k^i k^j}{k^2}\right)\delta T_{ij}\right),$$

$$k^2 V_i = 16\pi G(\delta_{ij} - \hat{k}_i \hat{k}_j)\delta T_{j0},$$

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- Random stress fluctuations at UV scale Λ

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- Cosmology limits the UV scale

$$\Lambda \lesssim (M_{\rm p}^4 H_0)^{1/5} \approx 2\ \mathrm{PeV}$$

Kallen-Lehmann spectral representation

- Most general expectation for stress correlators from *Unitarity*+*Lorentz symmetry*

$$\langle T_{\mu\nu}(x)T_{\alpha\beta}(y)\rangle = \int \frac{d^4k}{(2\pi)^4}e^{ik\cdot(x-y)}\int_0^\infty d\mu\left[\rho_0(\mu)P_{\mu\nu}P_{\alpha\beta}+\rho_2(\mu)\left(\frac{1}{2}P_{\mu\alpha}P_{\nu\beta}+\frac{1}{2}P_{\mu\beta}P_{\nu\alpha}-\frac{1}{3}P_{\mu\nu}P_{\alpha\beta}\right)\right]\theta(k^0)2\pi\delta(k^2+\mu),$$

- ρ's must positive.

$$P_{\mu\nu} \equiv \eta_{\mu\nu} - k_\mu k_\nu/k^2$$

- *Cosmological* constraints will roughly translate to

$$\int \frac{d\mu}{\sqrt{\mu}}\rho_2(\mu) \lesssim (10\ \mathrm{TeV} - 1\ \mathrm{PeV})^5$$

E.g., a free scalar field

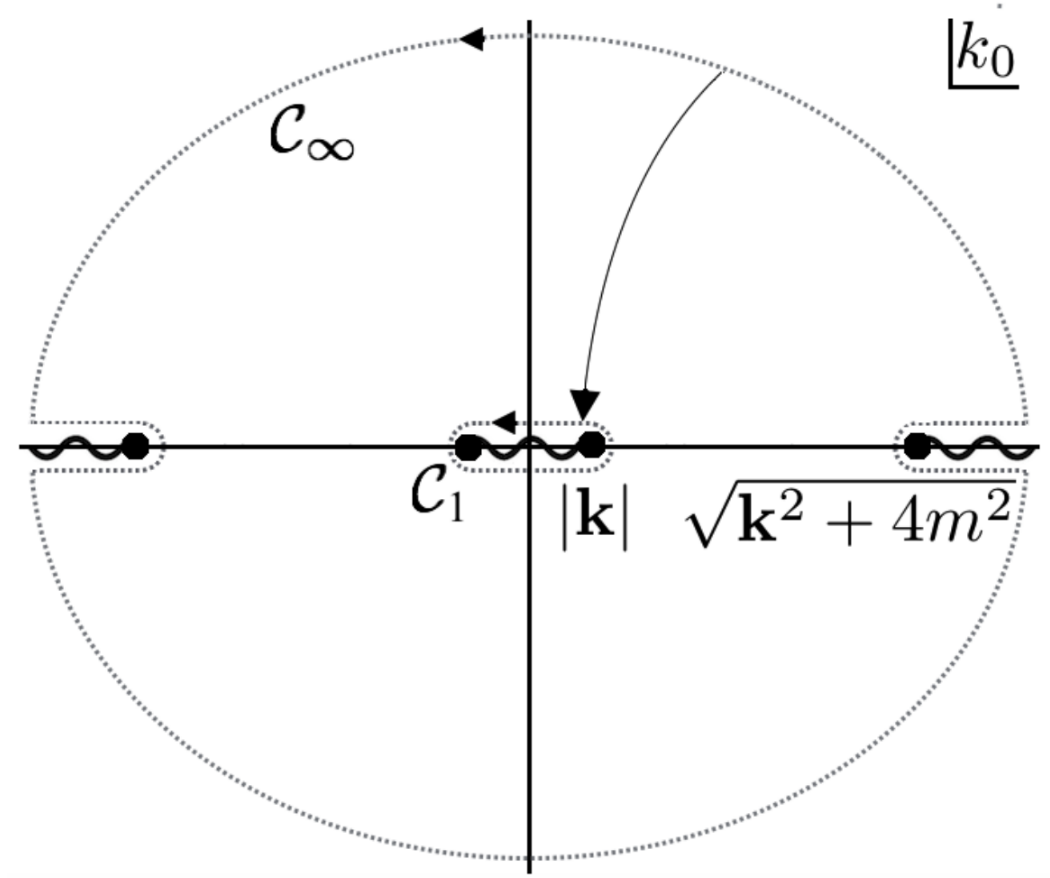
- For a weakly coupled scalar field

$$\rho_2(\mu) = \frac{\mu^2}{120\pi^2}\sqrt{\frac{1}{4}-\frac{m^2}{\mu}}\left[\frac{1}{4}-\frac{m^2}{\mu}\right]^2\Theta(\mu-4m^2)$$

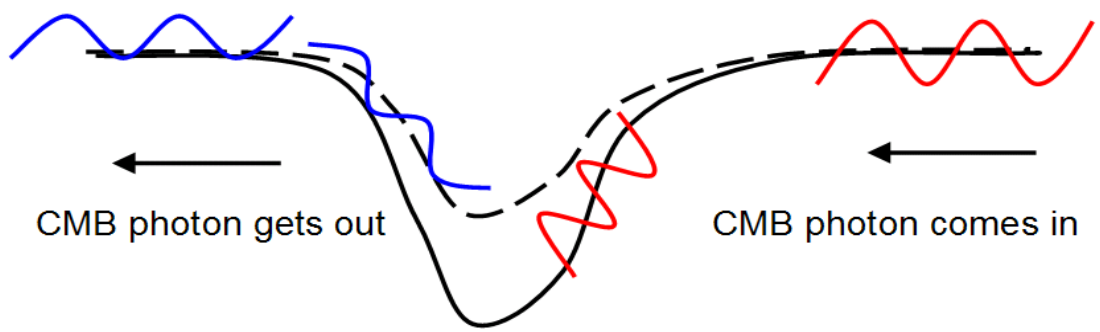
- For large scale, real-space correlations, one can deform the contour to get

$$\rho_{2,\mathrm{eff}}(\mu) = \frac{m^5}{120\pi^2\sqrt{-\mu}}\Theta(-\mu)$$

- *Poisson model …*



CMB anisotropies



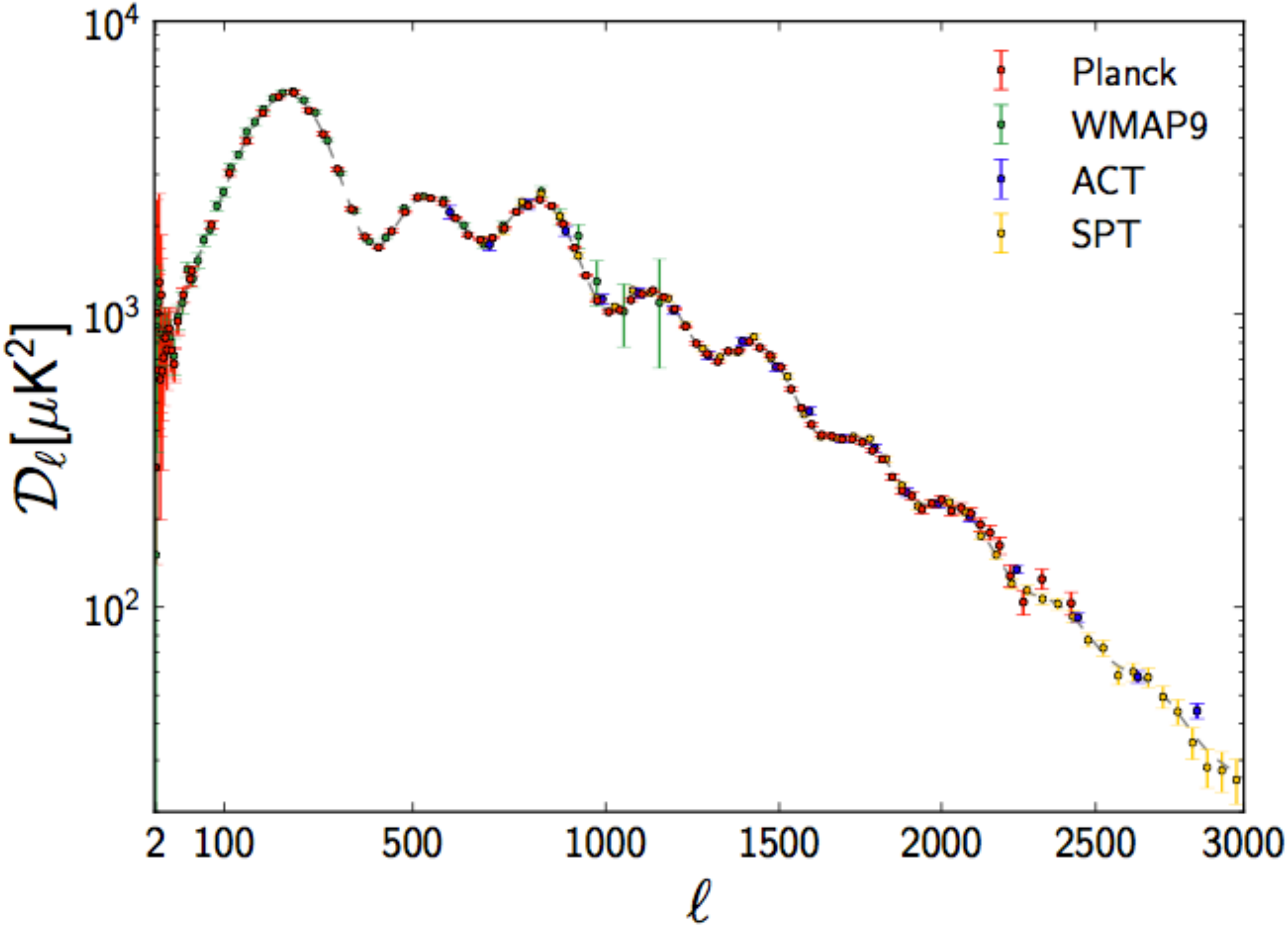
- Integrated Sachs-Wolfe (ISW) effect

$$\frac{\delta T^{\mathrm{ISW}}(\hat{\mathbf{r}})}{T} = \int_{\eta_{LSS}}^{\eta_{\mathrm{today}}} d\eta(\phi' + \psi' + V_i'\hat{r}^i),$$

- ISW effect due to metric fluctuations, due to a scalar vacuum

$$(\Delta_l^2)^{\mathrm{ISW}} \equiv \frac{l(l+1)C_l^{\mathrm{ISW}}}{2\pi} = \frac{49}{2880\pi^2}\frac{m^5 t_0}{M_p^4}$$

power spectrum of CMB



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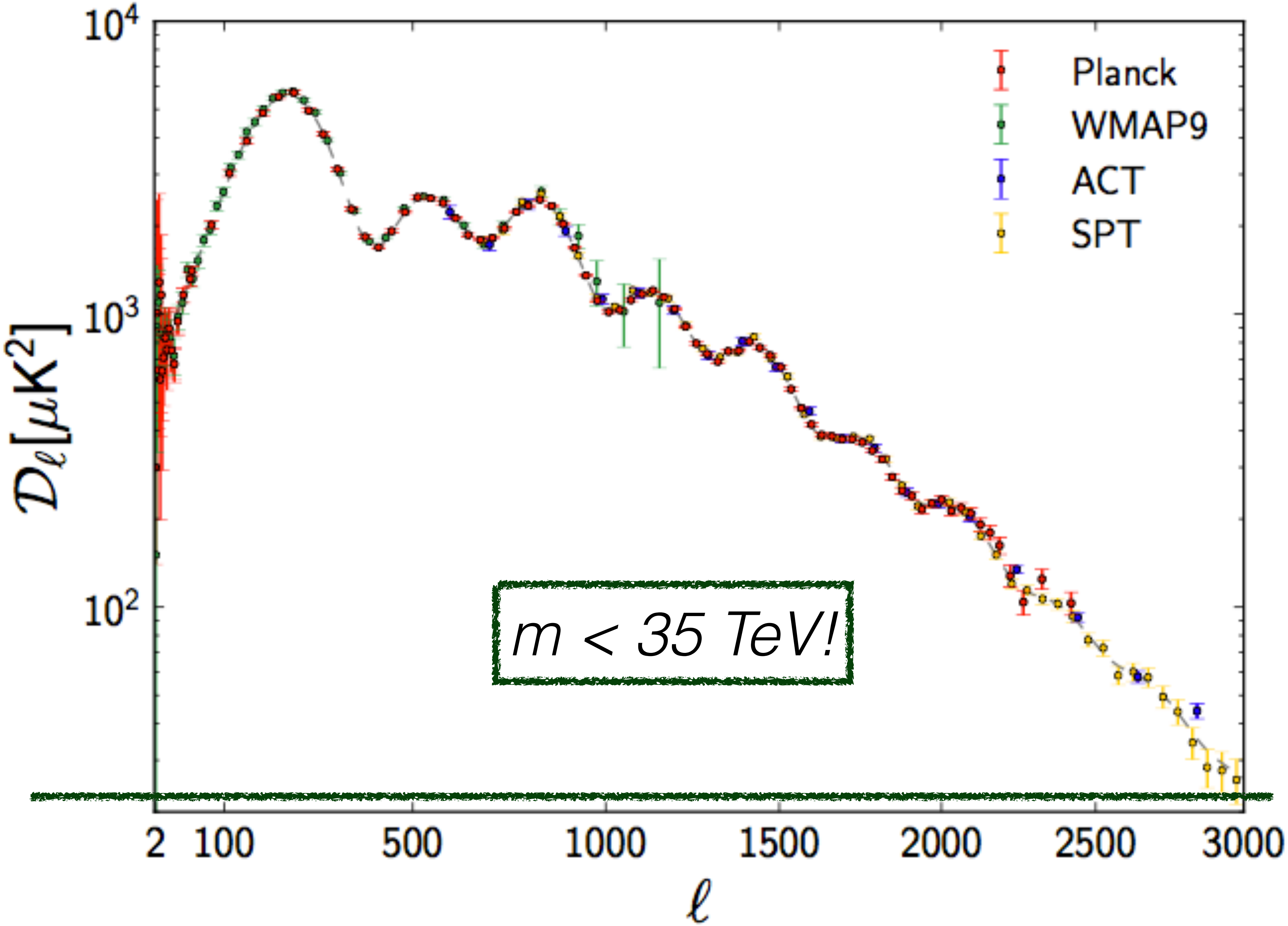

CnC problem (take 2):

A holographic entropy bound!

- Gravitational fine structure constant $\alpha_G \sim E^2/M_p^2$

- Number of qubits in a Dirac field

$$\# = 2\times 2\times Volume \times \int^{\Lambda} \frac{d^3k}{(2\pi)^3} = \frac{2\Lambda^3}{3\pi^2}\times Volume,$$

- Holographic Bound

$$S_{BH} = 2\pi M_p^2 \times Area > S = \# \times \alpha_G \left[1 - \ln(\alpha_G)\right] \sim \frac{2\Lambda^5\left[1+\ln(M_p^2/\Lambda^2)\right]}{3\pi^2 M_p^2}\times Volume.$$

- An IR cut-off for gravity

$$R \lesssim R_{\max} \sim \frac{3\pi^3 M_p^4}{\Lambda^5\left[1+\ln(M_p^2/\Lambda^2)\right]},$$

$$\Lambda_{IR} \sim \frac{\pi}{R_{\max}} \sim \frac{\Lambda^5\left[1+\ln(M_p^2/\Lambda^2)\right]}{3\pi^2 M_p^4}.$$

$$\boxed{\begin{aligned}&\Lambda_{IR} < H_0 \simeq 9.5\times 10^{-33}\mathrm{eV}\\ &\Rightarrow \boxed{\Lambda \lesssim 2.4\ \mathrm{PeV}.}\end{aligned}}$$

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- What about Effective Field Theory?

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- and firewalls!

On the folly of EFT

- Gravitational Path Integral

$$\int DgD\varphi \times \mathrm{Diff}^{-1}[g,\varphi] \times \exp\left(i\int d^4x\sqrt{-g}\left\{R[g]+\mathcal{L}_m[\varphi,g]\right\}\right).$$

- Naïve Effective Action

$$\exp(iS_{\mathrm{eff,naive}}[g]) \equiv \exp(iS_{\mathrm{GR}}[g]) \times \int D\varphi \exp\left(i\int d^4x\sqrt{-g}\mathcal{L}_m[\varphi,g]\right)$$

- Ignores GR Constraints :-(

$$\mathrm{Diff}^{-1}[g,0]\exp(iS_{\mathrm{eff,naive}}[g]) \neq \exp(iS_{\mathrm{GR}}[g])\times\int D\varphi\times\mathrm{Diff}^{-1}[g,\varphi]\times\exp\left(i\int d^4x\sqrt{-g}\mathcal{L}_m[\varphi,g]\right)$$

Open Questions

- Seriously?!
- What about the early universe/inflation?
- Is there a gauge-invariant description of this effect?
- What happens beyond linear order?
- Nature of IR cut-off? massive gravity, Dark Energy?
- What will a 100 TeV collider see?

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  - ➡ Also motivated by solving the *Higgs hierarchy problem, e.g. Large Extra Dimensions*
  - ➡ *A definite target for particle colliders (e.g. 100 TeV collider)*
- **EFT:** Just think outside the box!

BONUS SLIDES

Interaction entropy

- Imagine a system with Hamiltonian H_o, in its ground state $|0\rangle_\circ$, and zero energy

- Now, turn on H_{int};To 1st order, new eigenstates are

$$\langle n|0\rangle_\circ \simeq -\frac{\langle n|H_{\rm int}|0\rangle}{E_n},$$

- Time-Averaged density matrix:

$$\rho_{\rm int} = \sum_n |\langle n|0\rangle_\circ|^2 |n\rangle\langle n| = \sum_n \frac{|\langle n|H_{\rm int}|0\rangle|^2}{E_n^2}|n\rangle\langle n|,$$

- Entropy of a 2-state system

$$S_{qubit} = -tr(\rho_{\rm int}\ln\rho_{\rm int}) \simeq \alpha\,[1-\ln(\alpha)] + \mathcal{O}(\alpha^2),$$

- *Fine structure constant*

$$\alpha \equiv \frac{|\langle 1|H_{\rm int}|0\rangle|^2}{E_1^2},$$

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- Imagine particles of mass *m,* uniformly sprinkled in the phase space with density $\langle f_0 \rangle$

$$\langle T^{\mu\nu}(\mathbf{y},t')T^{\alpha\beta}(\mathbf{y}+\mathbf{x},t'+t)\rangle = m^5\langle f_0\rangle \frac{x^\mu x^\nu x^\alpha x^\beta}{(-x_\gamma x^\gamma)^{7/2}}\Theta(-x_\gamma x^\gamma)$$

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- We shall see that this structure occurs in generic quantum field theories.

Gravity of Poisson vacuum

- Solving Einstein equations, we find the spectrum of metric perturbations: $\Delta_\phi^2 \simeq \frac{m^5 \langle f_0 \rangle}{M_{\rm p}^4 k}$
- or $\Delta_\phi^2 \simeq 4 \times 10^{-9} \left(\frac{m}{50\ \mathrm{TeV}}\right)^5 \left(\frac{\langle f_0 \rangle}{1/2}\right) \left(\frac{k/a}{2 \times\ 10^{-4}\mathrm{Mpc}^{-1}}\right)^{-1}$
- spectrum of CMB anisotropies (Integrated Sachs-Wolfe, or ISW effect):

$$(\Delta_l^2)^{\rm ISW} \equiv \frac{l(l+1)C_l^{\rm ISW}}{2\pi} = \frac{49\pi}{720} \frac{m^5 t_0}{M_{\rm p}^4} \langle f_0 \rangle$$

(t_0 = *13.7 billion years*)

I: An offset in Hubble law

- Particle action $$S_p = -m \int dt \sqrt{1 + 2\phi(\mathbf{x}, t) - |\dot{\mathbf{x}}|^2},$$
- To 2nd order in ϕ

$$S_p \simeq m \int d\tau \left[-1 + \frac{1}{2}|\dot{\mathbf{x}}|^2 + \phi(0,t) - \phi(\mathbf{x},t) + \frac{1}{2}\phi(\mathbf{x},t)^2 - \frac{3}{2}\phi(0,t)^2 + \phi(\mathbf{x},t)\phi(0,t) \right].$$

- Effective Newtonian potenial $$\Phi_N(\mathbf{x}, t) \simeq -\langle \phi(\mathbf{x}, t)\phi(0, t) \rangle$$
- An offset in the Hubble law $$v \simeq Hr - \frac{1}{32\pi H M_p^4} \int \frac{d\mu}{\sqrt{\mu}} \rho_2(\mu)$$
- Planck cluster kSZ monopole $$\boxed{\left[\frac{1}{2} \int \frac{d\mu}{\sqrt{\mu}} \rho_2(\mu) \right]^{1/5} < 1.1 \text{ PeV},}$$

$$\langle v_r \rangle = 72 \pm 60 \text{ km/s}.$$

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