Cosmological (non)-Constant Problem:
*The case for TeV scale quantum gravity*

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Punchline!

• **Quantum Gravity** is *already* in the **Infrared**!

• Weakly coupled **QFT+GR** *must fail* at < 100 TeV!

→ **High-scale SUSY, GUT, (almost all) Inflation models**

→ **TeV-scale QG, Large Extra Dimensions**

→ **Strongly coupled UV completion (technicolor?, bootstrap?)**
Hierarchy problem(s)

- neutrino mass
- dark energy

$10^{-3}\text{eV}$  $10^{12}\text{eV}$  $10^{28}\text{eV}$

$\rho_{DE} \sim \Lambda^4$  $\delta m^2_H \sim \Lambda^2$  $\Lambda?$

Higgs, Electroweak  GUT...Planck  LHC
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GUT…Planck

old cosmological constant (CC) problem

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Does Quantum Gravity matter in the IR?

• Quantum Fluctuations do fluctuate!

\[ \langle T_{\mu\nu} \rangle = 0 \not\Rightarrow \langle T_{\mu\nu} T_{\alpha\beta} \rangle = 0 \]

• What is the analog of CC for the covariance of stress fluctuations?

• Can these fluctuations have an observable gravitational signature on large scales?

with Elliot Nelson (Penn-State → PI), 1504.00012
Vacuum Fluctuations in Linear Gravity

- Linearized Perturbations around FRW space-time

\[ ds^2 = a^2(\eta) \left[ -(1 + 2\phi)d\eta^2 + 2V_i dx_i d\eta + (1 - 2\psi) dx^2 \right] \]

- Einstein constraint sector: *scalars in longitudinal gauge and vectors*

\[ -k^2 \psi = 4\pi G \left( \delta T_{00} - \frac{3H}{k^2} i k^i \delta T_{i0} \right), \]

\[ -k^2 \phi = 4\pi G \left( \delta T_{00} - \frac{3H}{k^2} i k^i \delta T_{i0} + \left( \delta^{ij} - 3 \frac{k^i k^j}{k^2} \right) \delta T_{ij} \right), \]

\[ k^2 V_i = 16\pi G (\delta_{ij} - \hat{k}_i \hat{k}_j) \delta T_{j0}, \]
CnC: the upshot!
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- Random stress fluctuations at UV scale $\Lambda$

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\langle T_{ij}^{(V)}(x)T_{kl}^{(V)}(y) \rangle \sim \delta^3(x - y)\Lambda^5
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- Poisson eq. for anisotropic stress

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k^2 \Phi \sim M_p^{-2} A^{ij}T_{ij}
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- Variance of Metric perturbations grows as distance

\[ (\Delta^{(V)}_{\Phi})^2 \sim \frac{\Lambda^5}{M_p^4 k} \]
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\Lambda_{\text{IR}} = \frac{\Lambda_{\text{UV}}^5}{M_p^4}
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- Cosmology limits the UV scale

$$\langle T^{(V)}_{ij}(x)T^{(V)}_{kl}(y) \rangle \sim \delta^3(x - y)\Lambda^5$$

$$k^2 \Phi \sim M_p^{-2} A^{ij} T_{ij}$$

$$\left(\Delta^{(V)}_{\Phi}\right)^2 \sim \frac{\Lambda^5}{M_p^4 k}$$

$$\Lambda_{IR} = \frac{\Lambda_{UV}^5}{M_p^4}$$

$$\Lambda \lesssim (M_p^4 H_0)^{1/5} \approx 2 \text{ PeV}$$
Kallen-Lehmann spectral representation

- Most general expectation for stress correlators from
  \textit{Unitarity} + \textit{Lorentz symmetry}

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(y) \rangle = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot (x-y)} \int_0^\infty d\mu \left[ \rho_0(\mu) P_{\mu\nu} P_{\alpha\beta} + \rho_2(\mu) \left( \frac{1}{2} P_{\mu\alpha} P_{\nu\beta} + \frac{1}{2} P_{\mu\beta} P_{\nu\alpha} - \frac{1}{3} P_{\mu\nu} P_{\alpha\beta} \right) \right] \theta(k^0) 2\pi \delta(k^2 + \mu),$$

- \(\rho\)'s must positive.

- \textit{Cosmological} constraints will roughly translate to

$$\int \frac{d\mu}{\sqrt{\mu}} \rho_2(\mu) \lesssim (10 \text{ TeV} - 1 \text{ PeV})^5$$
E.g., a free scalar field

- For a weakly coupled scalar field

  \[ \rho_2(\mu) = \frac{\mu^2}{120\pi^2} \sqrt{\frac{1}{4} - \frac{m^2}{\mu}} \left[ \frac{1}{4} - \frac{m^2}{\mu} \right]^2 \Theta(\mu - 4m^2) \]

- For large scale, real-space correlations, one can deform the contour to get

  \[ \rho_{2,\text{eff}}(\mu) = \frac{m^5}{120\pi^2\sqrt{-\mu}} \Theta(-\mu) \]

- Poisson model …
CMB anisotropies

- Integrated Sachs-Wolfe (ISW) effect

\[
\frac{\delta T^{\text{ISW}}(\hat{r})}{T} = \int_{\eta_{\text{LSS}}}^{\eta_{\text{today}}} d\eta (\phi' + \psi' + V_i' \hat{r}^i),
\]

- ISW effect due to metric fluctuations, due to a scalar vacuum

\[
(\Delta^2_\ell)^{\text{ISW}} \equiv \frac{l(l + 1)C^\text{ISW}_l}{2\pi} \equiv \frac{49}{2880\pi^2} \frac{m^5 t_0}{M_p^4}
\]
power spectrum of CMB
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$m < 35 \text{ TeV}$!
CnC problem (take 2): A holographic entropy bound!

- Gravitational fine structure constant
  \[ \alpha_G \sim \frac{\tilde{E}^2}{M_p^2} \]

- Number of qubits in a Dirac field
  \[ \# = 2 \times 2 \times \text{Volume} \times \int \frac{d^3 k}{(2\pi)^3} = \frac{2\Lambda^3}{3\pi^2} \times \text{Volume}. \]

- Holographic Bound
  \[ S_{BH} = 2\pi M_p^2 \times \text{Area} \geq S = \# \times \alpha_G [1 - \ln(\alpha_G)] \sim \frac{2\Lambda^5 [1 + \ln(M_p^2/\Lambda^2)]}{3\pi^2 M_p^2} \times \text{Volume}. \]

- An IR cut-off for gravity
  \[ R \lesssim R_{\text{max}} \sim \frac{3\pi^3 M_p^4}{\Lambda^5 [1 + \ln(M_p^2/\Lambda^2)]}, \]

\[ \Lambda_{IR} \sim \frac{\pi}{R_{\text{max}}} \sim \frac{\Lambda^5 [1 + \ln(M_p^2/\Lambda^2)]}{3\pi^2 M_p^4}. \]

\[ \Lambda_{IR} < H_0 \simeq 9.5 \times 10^{-33} \text{eV} \Rightarrow \Lambda \lesssim 2.4 \text{ PeV}. \]
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- Vacuum energy-momentum \textit{fluctuations} can also source gravity
- They change the \textit{gravitational constraint} sector in the IR, thru \textit{equal-time correlators}
- Heisenberg Uncertainty principle for UV/IR observables
- \textit{CnC} problem is \textit{more severe} than the old CC problem, due to the positivity of the spectral functions or entropy, i.e. \textit{fine-tuning doesn’t work}
- What about Effective Field Theory?
On the folly of EFT
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• When it comes to gravity, EFT doesn’t have a good track record!
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• CC problem
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• When it comes to gravity, EFT doesn’t have a good track record!

• CC problem

• CnC problem

• and firewalls!
On the folly of EFT

• Gravitational Path Integral

\[ \int Dg D\varphi \times \text{Diff}^{-1}[g, \varphi] \times \exp \left( i \int d^4 x \sqrt{-g} \{ R[g] + \mathcal{L}_m[\varphi, g] \} \right). \]

• Naïve Effective Action

\[ \exp(iS_{\text{eff, naive}}[g]) = \exp(iS_{\text{GR}}[g]) \times \int D\varphi \exp \left( i \int d^4 x \sqrt{-g} \mathcal{L}_m[\varphi, g] \right) \]

• Ignores GR Constraints :-(

\[ \text{Diff}^{-1}[g, 0] \exp(iS_{\text{eff, naive}}[g]) \neq \exp(iS_{\text{GR}}[g]) \times \int D\varphi \times \text{Diff}^{-1}[g, \varphi] \times \exp \left( i \int d^4 x \sqrt{-g} \mathcal{L}_m[\varphi, g] \right) \]
Open Questions

• Seriously?!  
• What about the early universe/inflation?  
• Is there a gauge-invariant description of this effect?  
• What happens beyond linear order?  
• Nature of IR cut-off? massive gravity, Dark Energy?  
• What will a 100 TeV collider see?
Final Thoughts
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- **CnC problem:** Quantum Gravity effects must show up around 20 TeV-PeV scale
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- **EFT:** Just think outside the box!
BONUS SLIDES
Interaction entropy

- Imagine a system with Hamiltonian $H_0$, in its ground state $|0\rangle_0$, and zero energy.

- Now, turn on $H_{\text{int}}$; To 1st order, new eigenstates are

- Time-Averaged density matrix:

$$
\rho_{\text{int}} = \sum_n |\langle n|0\rangle_0|^2 |n\rangle \langle n| = \sum_n \frac{\langle n|H_{\text{int}}|0\rangle|^2}{E_n^2} |n\rangle \langle n|,
$$

- Entropy of a 2-state system

$$
S_{\text{qubit}} = -tr(\rho_{\text{int}} \ln \rho_{\text{int}}) \simeq \alpha [1 - \ln(\alpha)] + \mathcal{O}(\alpha^2),
$$

- Fine structure constant

$$
\alpha \equiv \frac{|\langle 1|H_{\text{int}}|0\rangle|^2}{E_1^2},
$$
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- Imagine particles of mass $m$, uniformly sprinkled in the phase space with density $\langle f_0 \rangle$

\[
\langle T_{\mu\nu}(y, t') T^{\alpha\beta}(y + x, t' + t) \rangle = m^5 \langle f_0 \rangle \frac{x^\mu x^\nu x^\alpha x^\beta}{(-x_\gamma x_\gamma)^{7/2}} \Theta(-x_\gamma x_\gamma)
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- We shall see that this structure occurs in generic quantum field theories.
Gravity of Poisson vacuum

- Solving Einstein equations, we find the spectrum of metric perturbations: \( \Delta^2_\phi \sim \frac{m^5 \langle f_0 \rangle}{M_p^4 k} \)

- or \( \Delta^2_\phi \sim 4 \times 10^{-9} \left( \frac{m}{50 \text{ TeV}} \right)^5 \left( \frac{\langle f_0 \rangle}{1/2} \right) \left( \frac{k/a}{2 \times 10^{-4} \text{Mpc}^{-1}} \right)^{-1} \)

- spectrum of CMB anisotropies (Integrated Sachs-Wolfe, or ISW effect):

\[
(\Delta^2_l)_{\text{ISW}} \equiv \frac{l(l + 1)C^\text{ISW}_l}{2\pi} = \frac{49\pi}{720} \frac{m^5 t_0}{M_p^4} \langle f_0 \rangle \\
(t_0 = 13.7 \text{ billion years})
\]
I: An offset in Hubble law

- Particle action
  \[ S_p = -m \int dt \sqrt{1 + 2\phi(x, t)} - |\dot{x}|^2, \]

- To 2nd order in \( \phi \)
  \[ S_p \approx m \int d\tau \left[ -1 + \frac{1}{2}|\dot{x}|^2 + \phi(0, t) - \phi(x, t) + \frac{1}{2}\phi(x, t)^2 - \frac{3}{2}\phi(0, t)^2 + \phi(x, t)\phi(0, t) \right]. \]

- Effective Newtonian potential
  \[ \Phi_N(x, t) \simeq -\langle \phi(x, t)\phi(0, t) \rangle \]

- An offset in the Hubble law
  \[ v \simeq Hr - \frac{1}{32\pi HM_p^4} \int \frac{d\mu}{\sqrt{\mu}} \rho_2(\mu) \]

- Planck cluster kSZ monopole
  \[ \langle v_r \rangle = 72 \pm 60 \text{ km/s}, \]
  \[ \left[ \frac{1}{2} \int \frac{d\mu}{\sqrt{\mu}} \rho_2(\mu) \right]^{1/5} < 1.1 \text{ PeV}, \]
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