

# Mathematica Toolbox for PDF Uncertainties and Applications to New Physics Searches



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# Outline

Parton Distribution Functions

ManeParse

- Purpose
- Construction
- Speed
- Validation
- Examples

Applications to New Physics Searches

- Intrinsic Bottom Quark
- Ultrahigh Energy Neutrinos

Future Improvements and Applications

# Parton Distribution Functions (PDFs)

- Parameterized fits to experimental data
  - Different collaborations have different fits
- Describe the momentum fraction carried by each component of hadron
- Broken down into grids by:
  - All quark flavors + gluon
  - Hard scattering energy  $Q$
  - Bjorken  $x$
- Essential for relating theoretical predictions to experiment

# Purpose

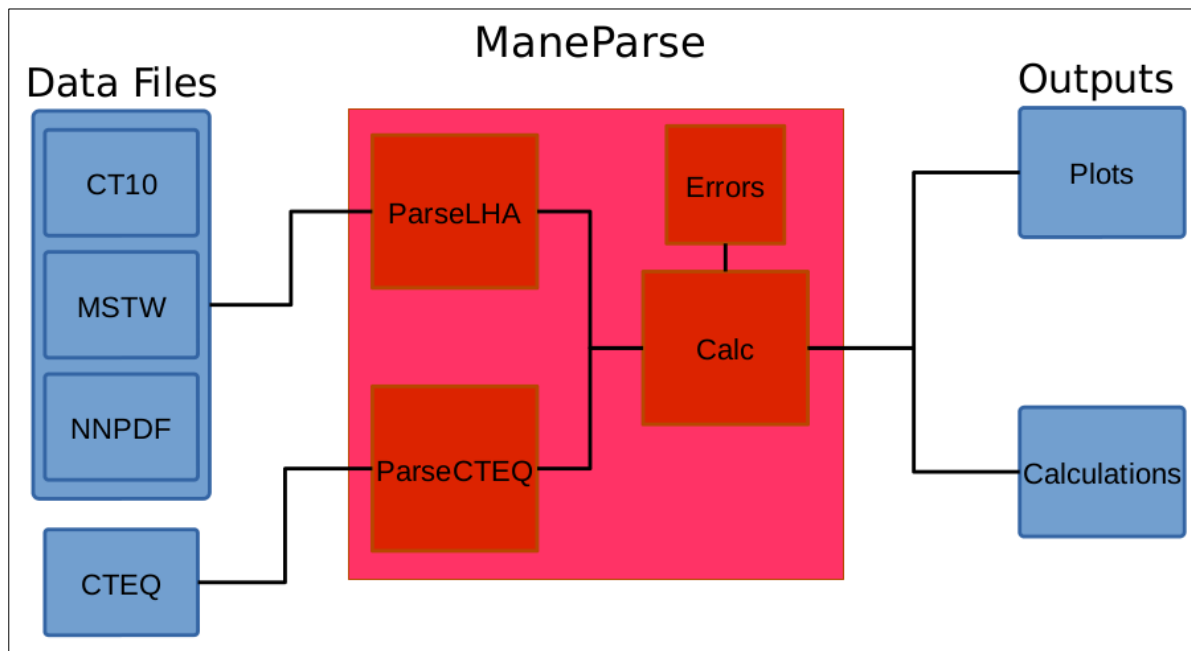
- Develop Lightweight PDF Reader for multiple collaboration's PDFs
  - LHAPDF6
  - CTEQ PDS
- Custom 4-point Lagrange Interpolation routine
  - Fast, reliable, transparent
  - Gives grids functional dependence
- Mathematica environment
  - User-Friendly
  - Built-in Plotting and calculation functions
- Implement multiple Error propagation techniques
  - Hessian
  - Monte Carlo
- Able to do calculations of observables
  - Cross Sections
  - Luminosities

# ManeParse

- pdfParseLHA
- pdfParseCTEQ
- pdfCalc
- pdfErrors

ManeParse  
Schematic

▼ pdfCalc`			
pdfAlphaS	pdfGetInfo	pdfLowFunction	pdfSetList
pdfFlavor	pdfGetQlist	pdfNumQpartition	pdfSetListDisplay
pdfFunction	pdfGetXlist	pdfReset	pdfXmin
▼ pdfErrors`			
pdfHessianCorrelation	pdfLuminosity	pdfMCCorrelation	
pdfHessianError	pdfMCCentral	pdfMCError	
▼ pdfParse`			
pdfFamilyParseCTEQ		pdfParseCTEQ	
▼ pdfParseLHA`			
pdfFamilyParseLHA		pdfParseLHA	



## Available User Functions

Able to load unlimited  
data sets at once  
CTEQ, NNPDF, MSTW,  
HERA... and more

# Speed

PDF Set	Number of Files	Read Time	Read Time per File	Time 1000 x pdfFunction Call
CT10	53	1.852258	0.0349483	0.635051
MSTW	41	1.193042	0.0290986	0.592335
NNPDF	101	4.109711	0.04069021	0.606711
CTEQ66	45	0.914369	0.0203193	0.624650

Times in Seconds

- Speeds comparable to LHAPDF6 and CTEQ code (C++ and Fortran respectively)
- Hundredths of a second to read each file
- Approximately 2000 pdfFunction calls per second

# Validation

## Sum Rules for multiple PDF Sets

- Sum Rules provide important “sanity check”
- Momentum fraction of all partons must add to 1
- Any errors in parsing or interpolating would disrupt the sum

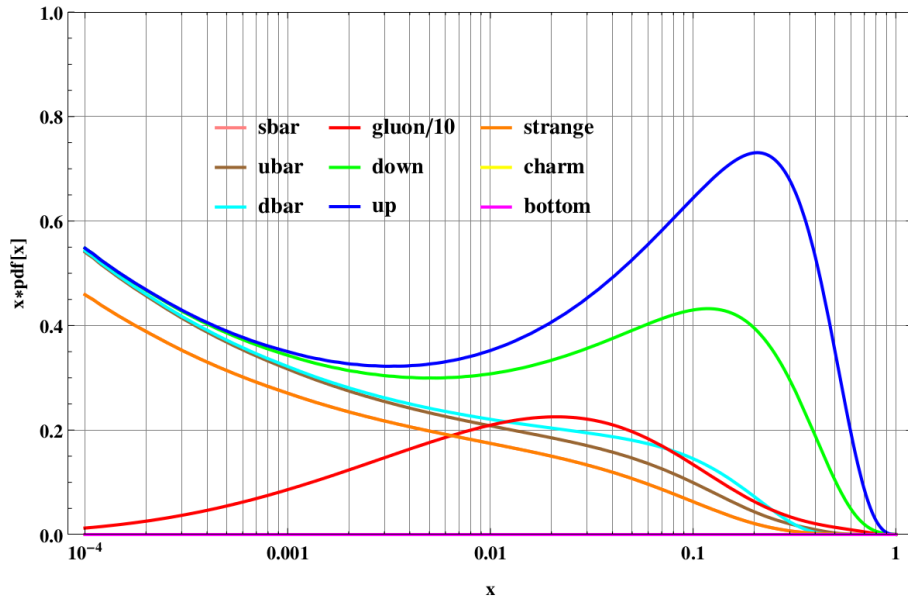
Flavor	CT10	NNPDF	MSTW	CTEQ66
bbar	0.	0.	0.	0.
cbar	0.01	0.01	0.01	0.01
sbar	0.02	0.01	0.02	0.02
ubar	0.03	0.03	0.03	0.03
dbar	0.04	0.03	0.04	0.04
gluon	0.45	0.5	0.47	0.45
down	0.14	0.13	0.13	0.14
up	0.28	0.26	0.27	0.28
strange	0.02	0.01	0.02	0.02
charm	0.01	0.01	0.01	0.01
bottom	0.	0.	0.	0.
Total	0.999677	0.999563	1.00467	0.99969



# Examples: I

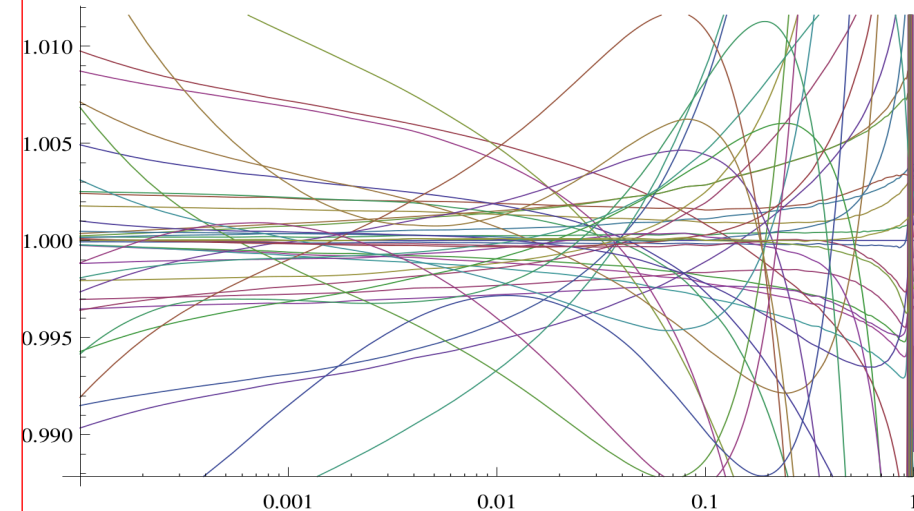
## Plot of Multiple Partons for a Single PDF

CT10 Central Value @  $Q=1.3$



Ratio of Error Sets to Central Value

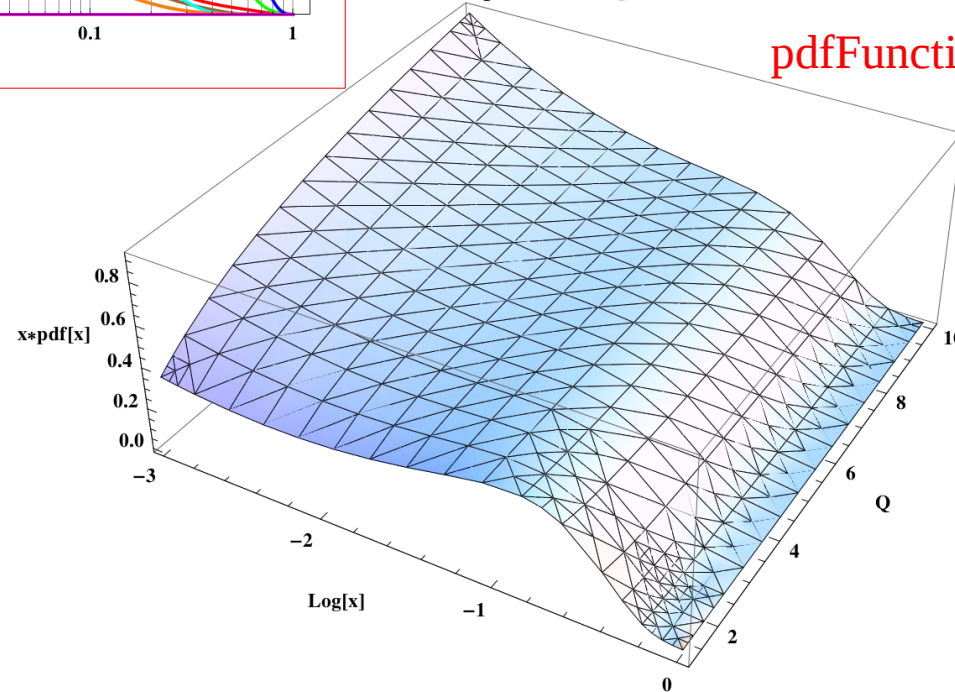
MSTW



PhaseSpace: Down Quark

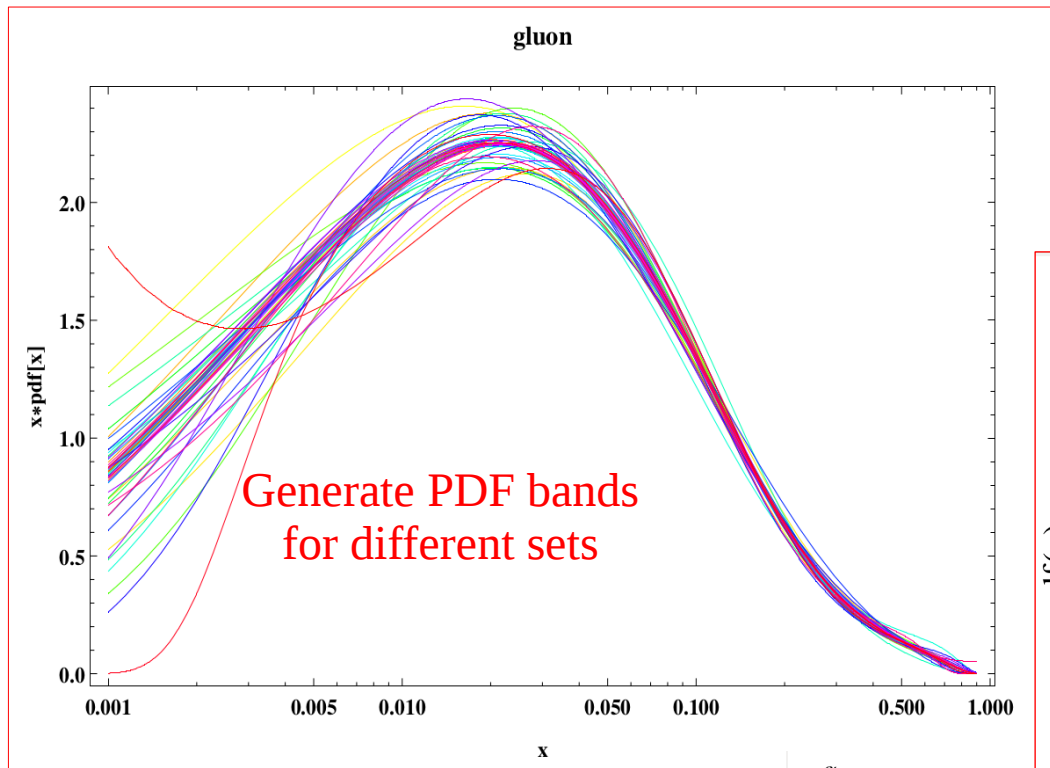
Change PDF Sets easily  
`pdfFunction[setNumber,flavor,x,Q]`

Visualize Phase  
Space with 3D  
plotting



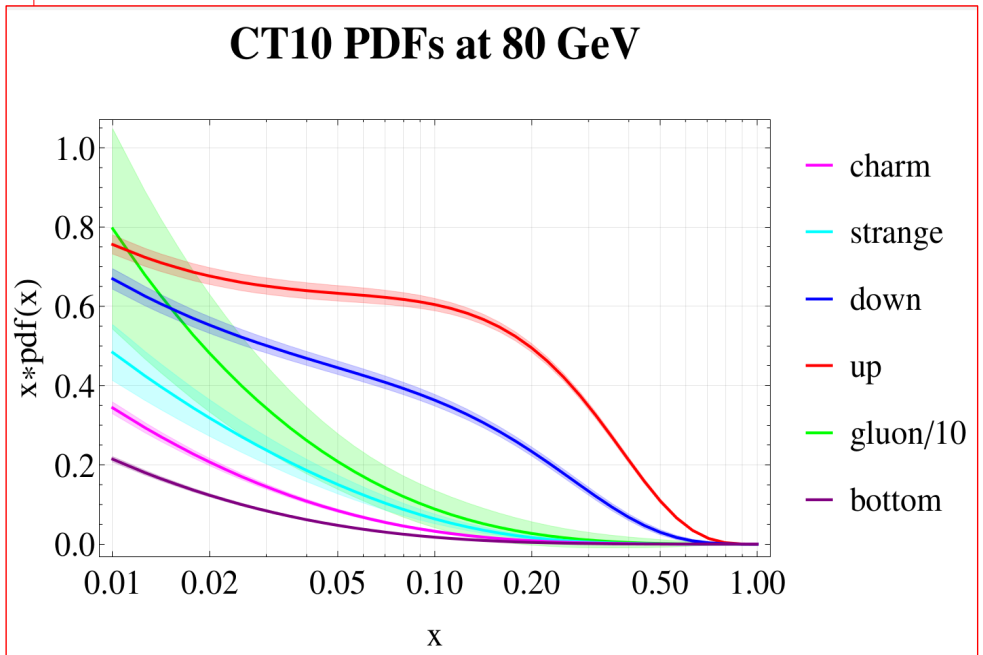


# Examples: II



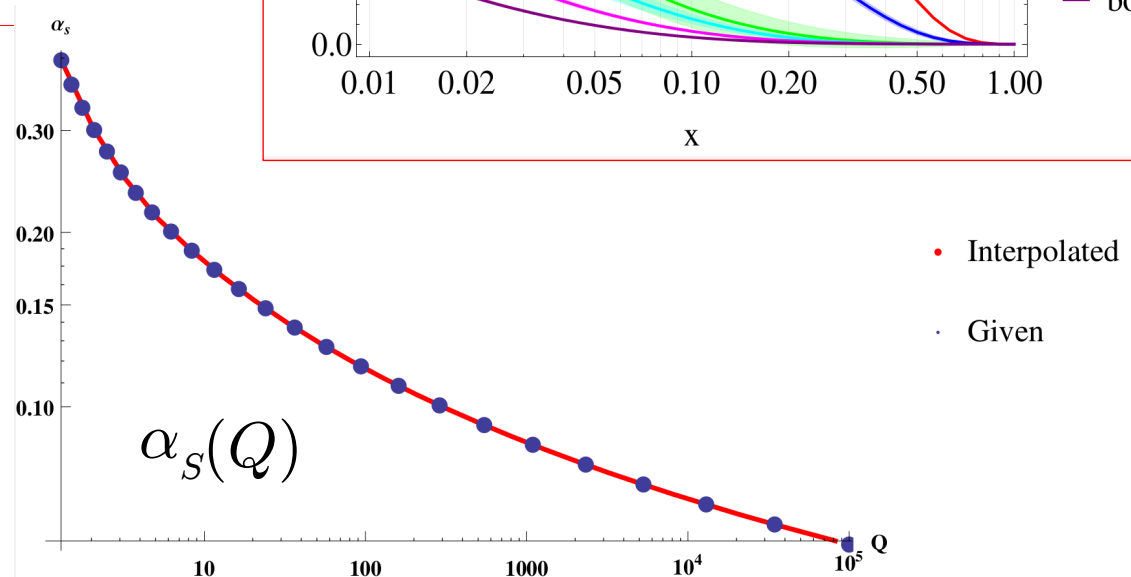
Full sets of PDFs inside Mathematica

Easy to manipulate



$\alpha_s(Q)$  is available  
Interpolated values match grid values

Proper  $\alpha_s$  is essential for  
NLO+ calculations



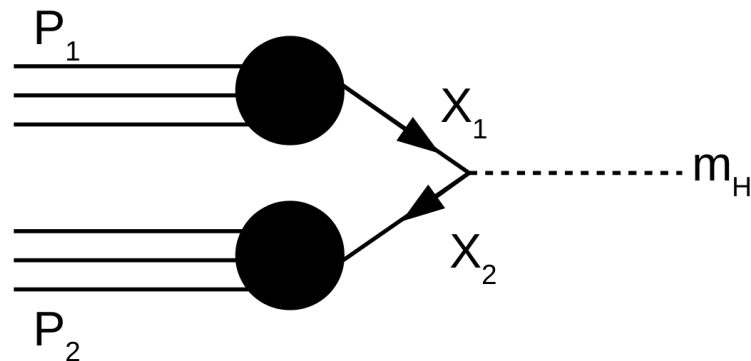
# Luminosities at LHC

Luminosities connect experimental and theoretical cross sections

$$\sigma_{pp \rightarrow H+X} = \sum_{ij} \int_{\tau}^1 d\tau \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}_{ij}(\hat{S})$$

$$\frac{d\mathcal{L}_{ij}}{d\tau}(\tau, \mu) = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{1}{x} \left[ f_i(x, \mu) f_j(\tau/x, \mu) + f_j(x, \mu) f_i(\tau/x, \mu) \right] dx$$

Production of heavy scalar

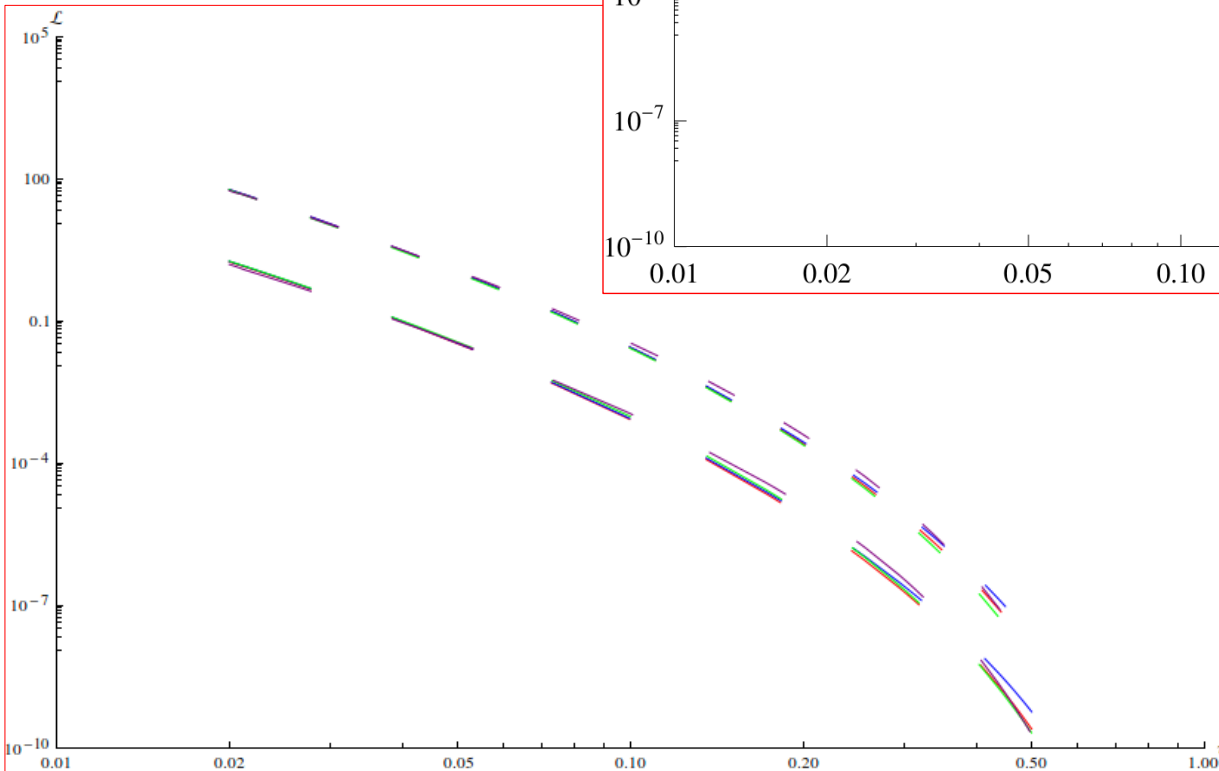
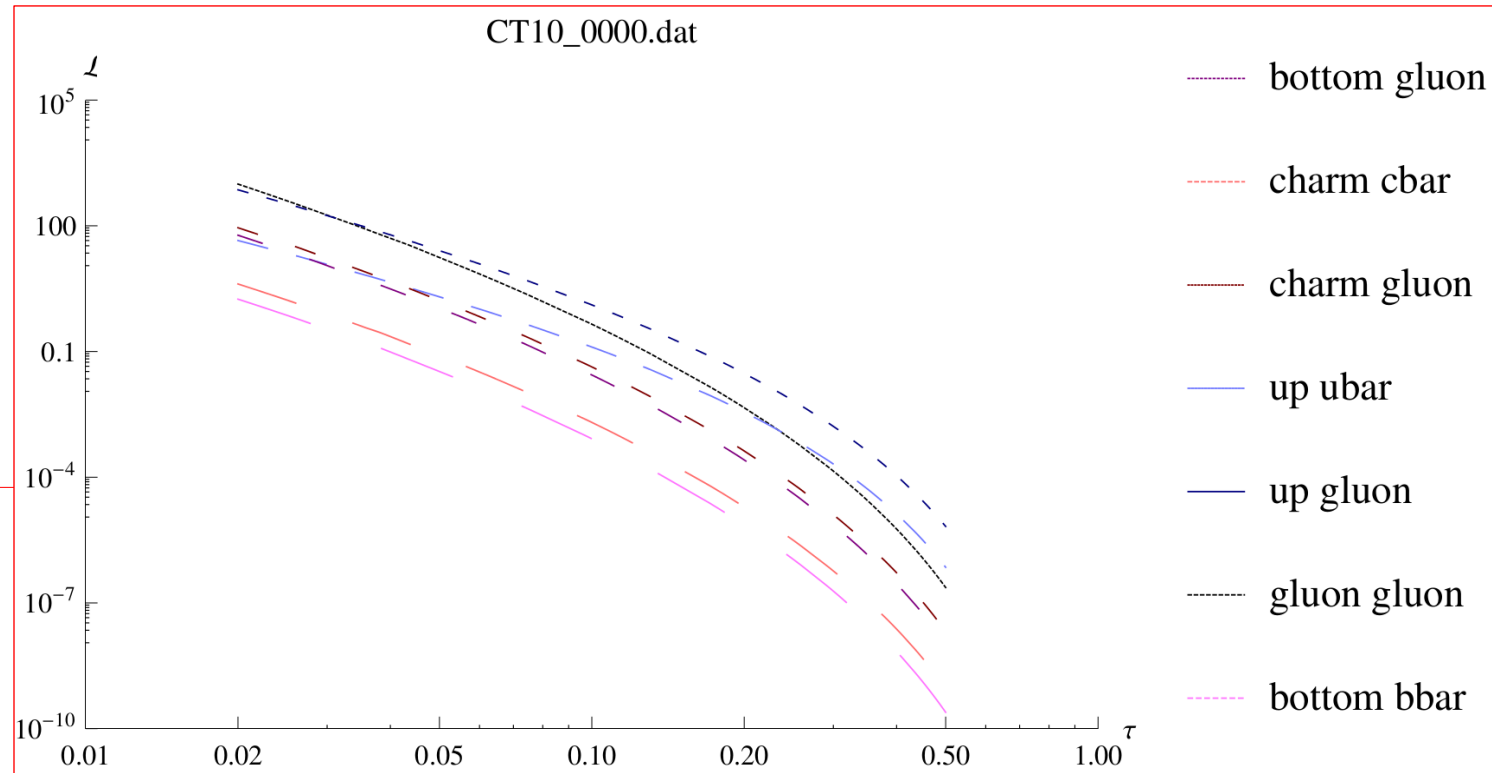


Kinematics:

$$m_H^2 = x_1 x_2 S = \tau S \therefore \tau = \frac{m_H^2}{S}$$

# Luminosities in ManeParse

Luminosities  
for different  
parton  
interactions

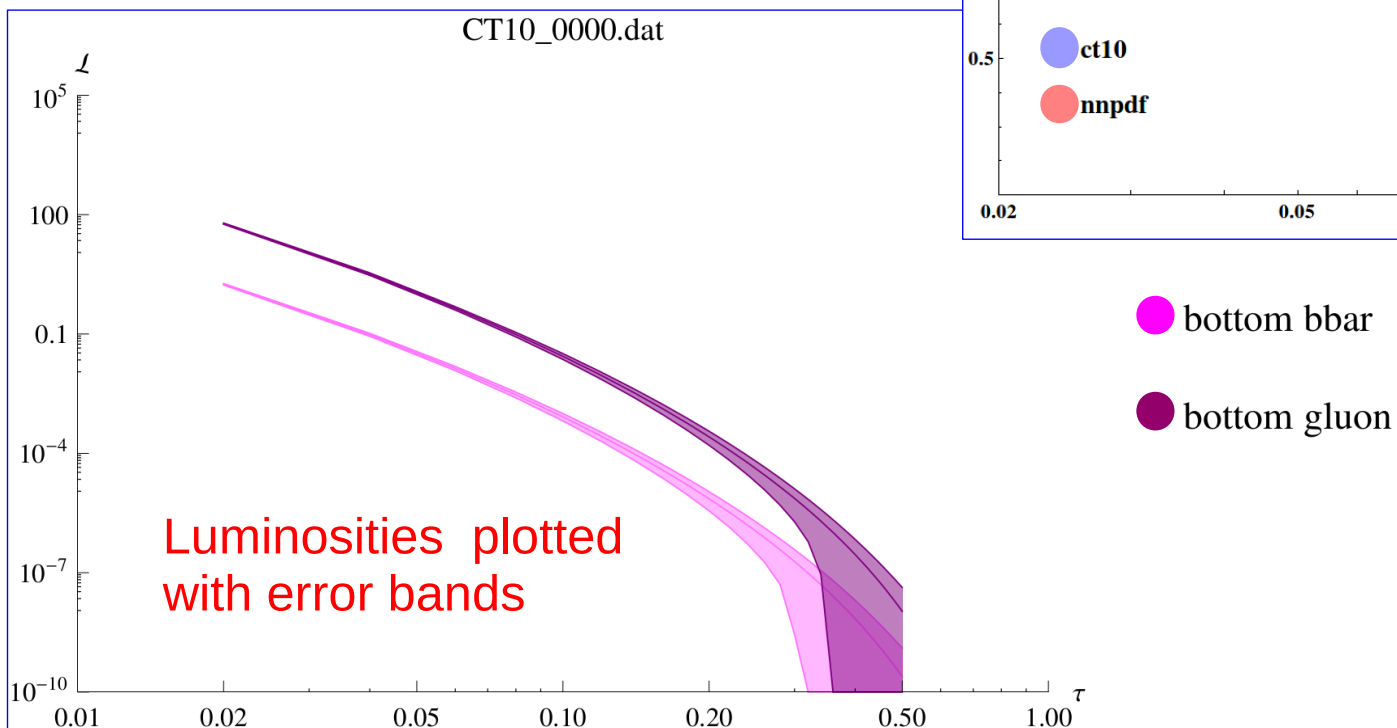
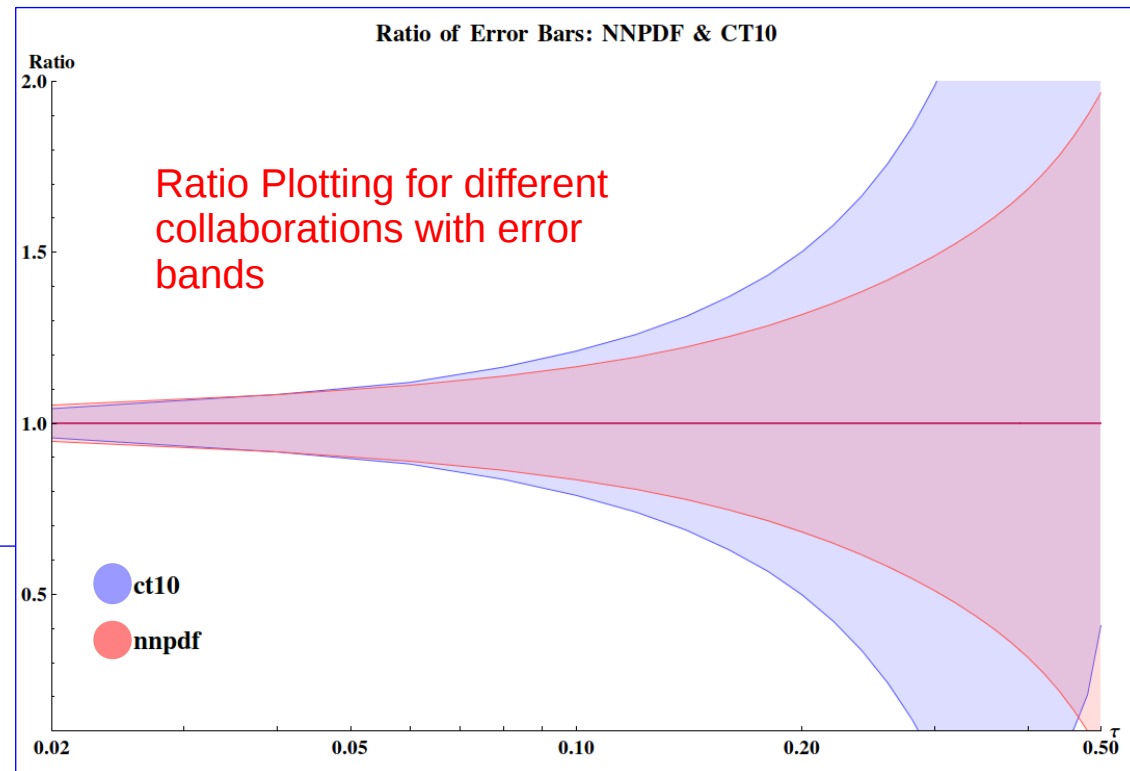


Bottom glue (top) and  
Bottom Bbar luminosities  
for multiple collaborations

# Luminosities in ManeParse

Error bands can be added using the pdfError package

- Hessian Fit for CT10
- Monte Carlo Fit for NNPDF
- Errors for functions can be applied directly

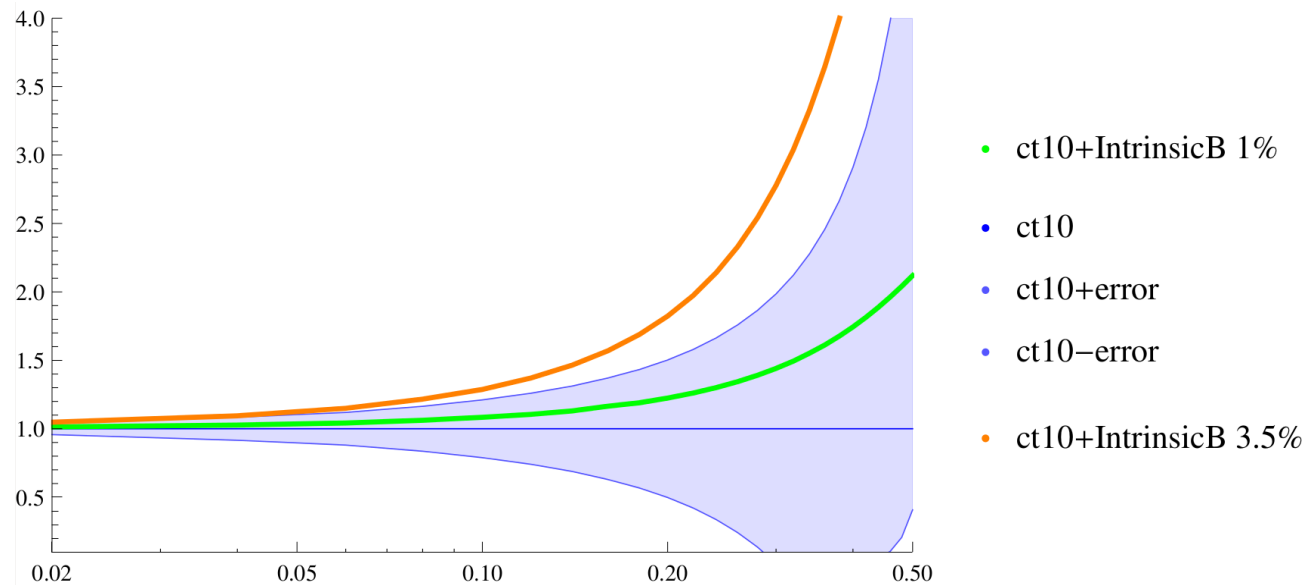


# Intrinsic Bottom PDF

- Custom PDF
- Intrinsic PDF can be added to an existing extrinsic PDF directly due to decoupling of DGLAP evolution equations\*

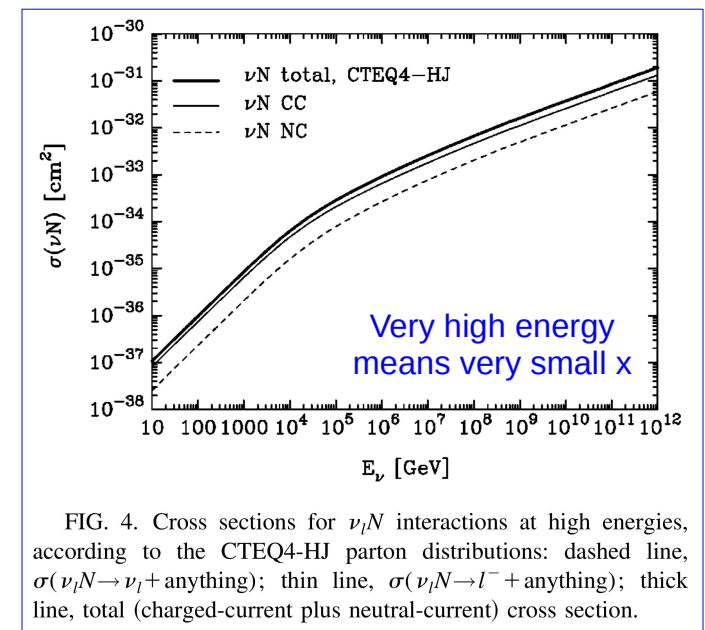
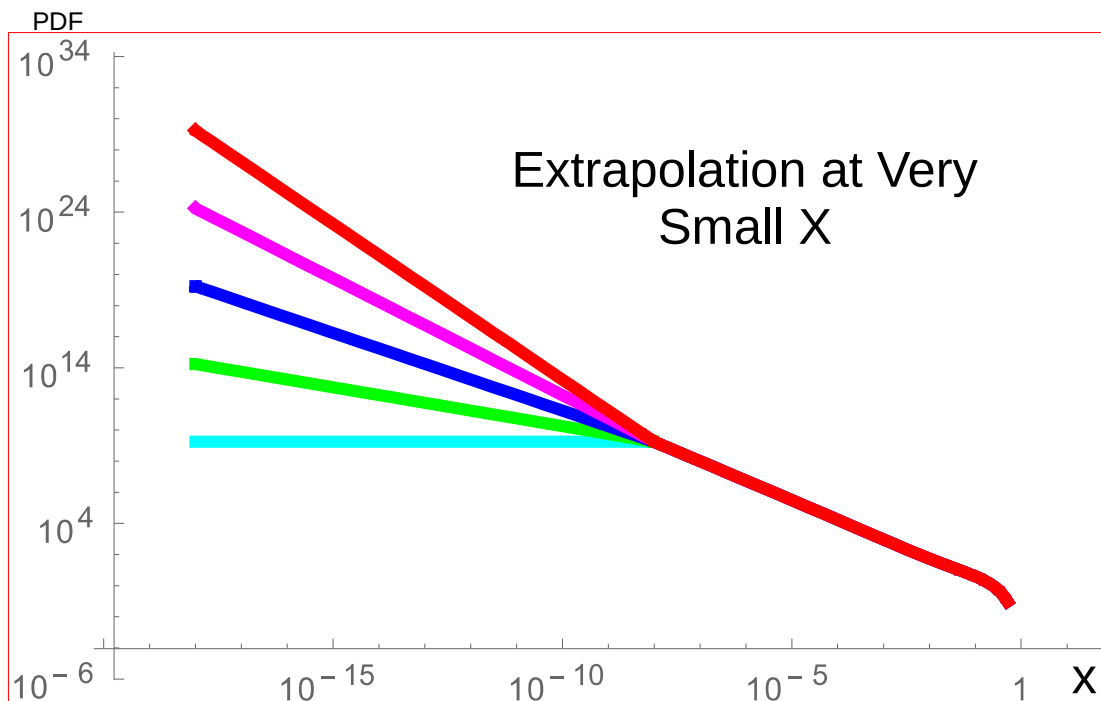
$$\text{PDF} = \text{PDF}_{\text{extrinsic}} + \text{PDF}_{\text{intrinsic}}$$

Ratio Plot for  
CT10 with  
Intrinsic Bottom  
component



# Ultrahigh Energy Neutrinos

- Calculations at very low  $x$  values
- Package allows for interpolation to these very low values with ability to tune to fit data



Reference: arXiv:hep-ph/9807264 R.  
Gandhi et al.



# Conclusions & Future Work

ManeParse is ready for release.

Complete Demo notebook available.

To Download visit:

[ncteq.org](http://ncteq.org)

[ncteq.hepforge.org/code/pdf.html](http://ncteq.hepforge.org/code/pdf.html)



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Florian Lyonnet

Aleksander Kusina

Questions?

Please Contact:

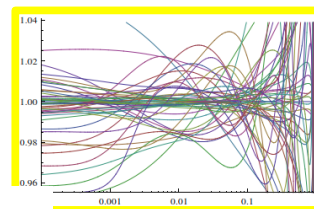
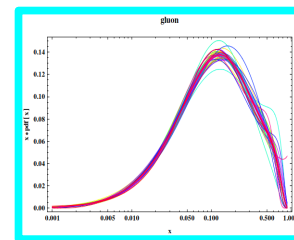
Eric Godat [egodat@smu.edu](mailto:egodat@smu.edu)

# Backup Slides

# Introduction: Mathematica Interface for LHAPDF6

- Package designed to allow for user interaction

- Check values in PDFs
- Make plots
- Check ratios between PDF sets
- Integrate and check sum rules



-5	bbar	0
-4	cbar	0
-3	sbar	2
-2	ubar	3
-1	dbar	4
0	gluon	42
1	down	15
2	up	32
3	strange	2
4	charm	0
5	bottom	0

- User accessible functions

## ▼ pdfCalc`

pdfAlphaS	pdfGetInfo	pdfLowFunction	pdfSetList
pdfFlavor	pdfGetQlist	pdfNumQpartition	pdfSetListDisplay
pdfFunction	pdfGetXlist	pdfReset	pdfXmin

## ▼ pdfErrors`

pdfHessianCorrelation	pdfLuminosity	pdfMCCorrelation
pdfHessianError	pdfMCCentral	pdfMCErrors

## ▼ pdfParse`

pdfFamilyParseCTEQ	pdfParseCTEQ
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## ▼ pdfParseLHA`

pdfFamilyParseLHA	pdfParseLHA
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fo stored in the PDF sets

- pdfFuctionLHA uses encapsulated interpolation

# LHAPDF<sup>6</sup> Features

LHAPDF the Les Houches Accord PDF Interface

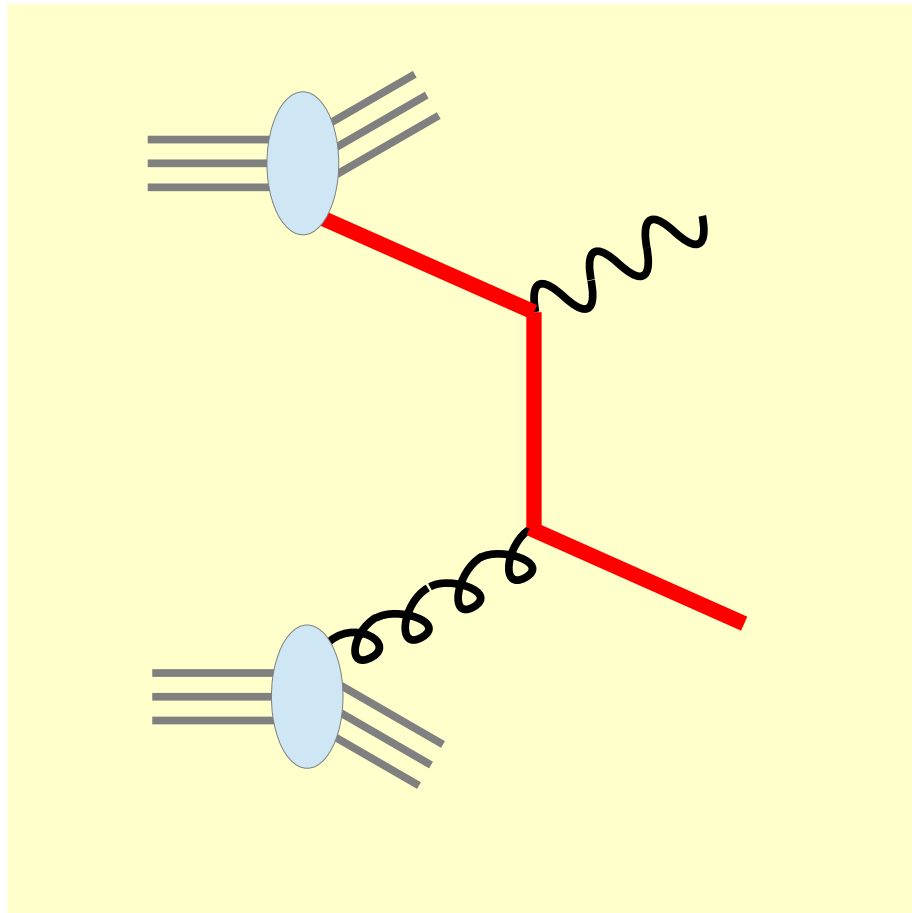
- All PDF sets defined with a unified grid structure
  - .. can access grids with a single interface
- New “*dat*” files available for:
  - CTEQ
  - MSTW
  - NNPDF
- ... *and under construction*:
  - ABKM, JR, CJ, nCTEQ, ATLAS, HERA

*... what about the*

# Heavy Quarks

*c & b: Extrinsic & Intrinsic*

*Historically, these have been a challenge because  $Q \sim m_{c,b}$*



$$c \, g \rightarrow c \, \gamma$$

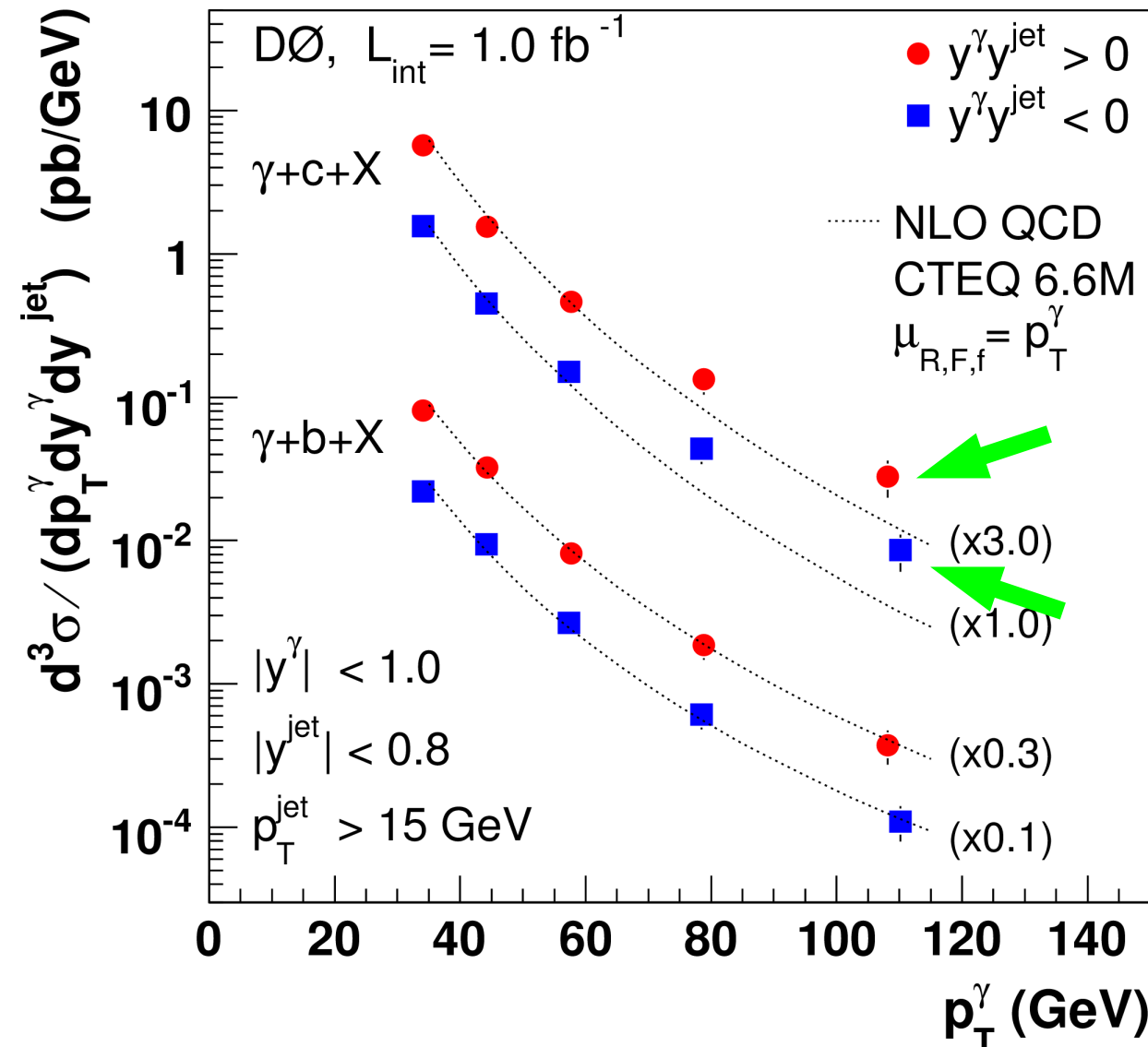
$$b \, g \rightarrow b \, \gamma$$

$$s \, g \rightarrow c$$

$$W$$

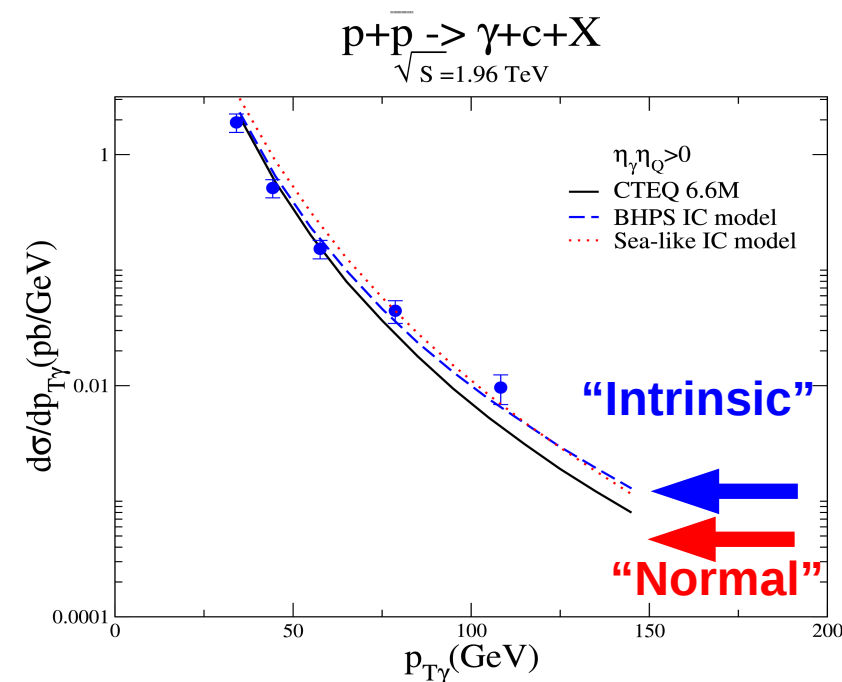
$$c \, q \rightarrow b$$





Excess in Charm,  
NOT Bottom

only at high PT



DGLAP Evolution equations ...

including **ordinary**  $Q_0$  and **intrinsic**  $Q_1$  heavy quark

$$\begin{aligned}\dot{g} &= P_{gg} \otimes g + P_{gq} \otimes q + P_{gQ} \otimes Q_0 + \cancel{P_{gQ} \otimes Q_1}, \\ \dot{q} &= P_{qg} \otimes g + P_{qq} \otimes q + P_{qQ} \otimes Q_0 + \cancel{P_{qQ} \otimes Q_1}, \\ \dot{Q}_0 + \dot{Q}_1 &= P_{Qg} \otimes g + P_{Qq} \otimes q + P_{QQ} \otimes Q_0 + P_{QQ} \otimes Q_1.\end{aligned}$$

neglect

**Equations decouple:**

Intrinsic component evolves independently

Scale set by  $m_Q$

Adjust normalization by simple rescaling

$$\dot{Q}_1 = P_{QQ} \otimes Q_1.$$

$$c_1(x) = \bar{c}_1(x) \propto x^2 [6x(1+x) \ln x + (1-x)(1+10x+x^2)]$$

