Black holes in String Theory

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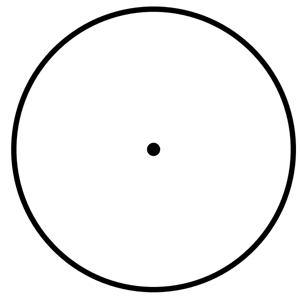
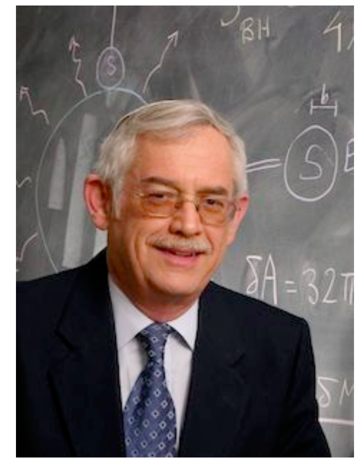
There are two main ideas in black hole physics …

The first is the notion that the entropy of a black hole is given by its surface area

$$S_{bek} = \frac{A}{4G}$$





(Bekenstein 72)

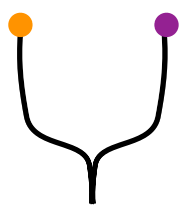
This is mysterious for two reasons:

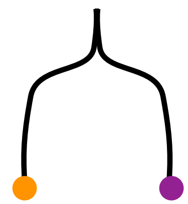
(a) The entropy is proportional to the area rather than the volume

(b) By the 'no-hair' theorem, there is nothing at the horizon, so what states are being counted by this entropy?

The other issue is the black hole information paradox …

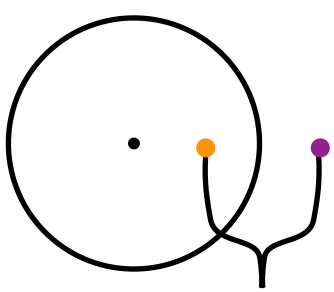
In quantum mechanics, the vacuum can have fluctuations which produce a particle-antiparticle pair





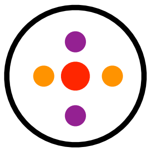
$$\Delta E \; \Delta t \sim \hbar$$

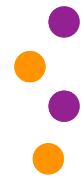
But if a fluctuation happens near the horizon, the particles do not have to re-annihilate : The negative gravitational potential gives the inner particle negative energy $\left(E = mc^2 - \frac{GMm}{r} \right)$



$$\Delta E = 0 \quad \rightarrow \quad \Delta t = \infty$$

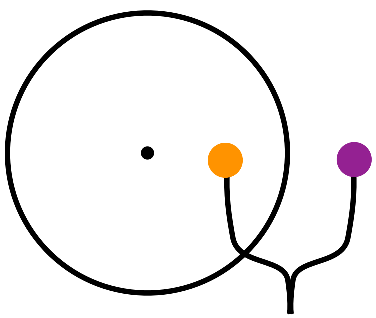
Thus real particle pairs are continuously created (Hawking 74)





Hole shrinks to
a small size

The essential issue: Vacuum fluctuations produce entangled states



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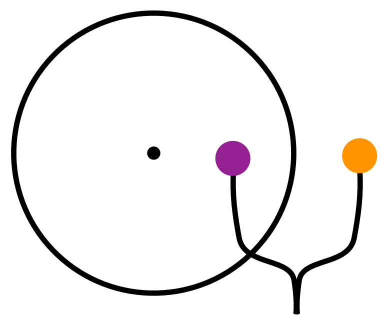


$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(e^+e^- + e^-e^+\right)$$

$$= \frac{1}{\sqrt{2}}\left(01 + 10\right)$$

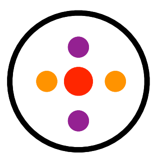
So the state of the radiation is entangled with the state of the remaining hole …

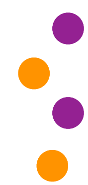
The radiation does not have a state by itself, the state can only be defined
when the radiation and interior are considered together

The amount of this entanglement is very large ...

If N particles are emitted, then there are 2^N possible arrangements

We can call an electron a 0 and a positron a 1





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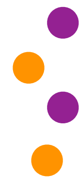
Possibility A: Information loss — The evaporation goes on till the remnant has zero mass. At this point the remnant simply vanishes

vacuum



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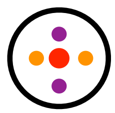
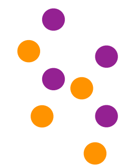
The radiation is entangled, but there is nothing that it is entangled WITH

The radiation cannot be assigned ANY quantum state ... it can only be described by a density matrix ... this is a violation of quantum mechanics (Hawking 1975)

Possibility B : Remnants: We assume the evaporation stops when we get to a planck sized remnant.

The remnant must have at least 2^N internal states

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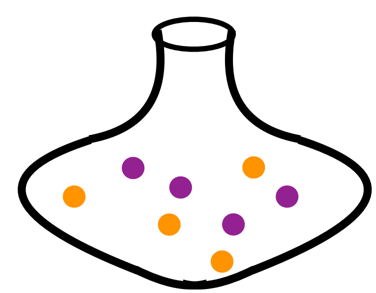
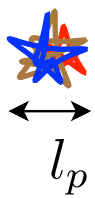
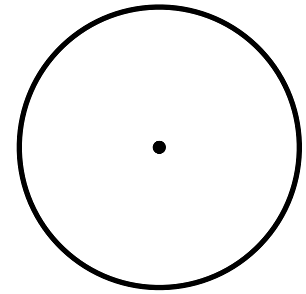
But how can we hold an unbounded number of states in planck volume with energy limited by planck mass? (Baby Universe ?)



The story of entropy

Consider a collection of strings and branes at weak coupling, and at strong coupling





weak coupling

strong coupling

If we are looking at supersymmetric states, the number of states should not change … (Vafa 94, Sen 95)

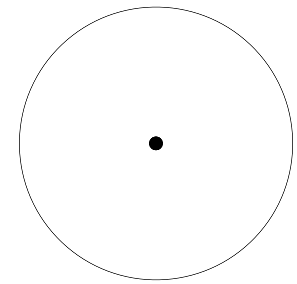
$$S_{micro} = S_{bek}$$

(Strominger+Vafa 96)

So it seems that the area entropy is indeed a count of states in some way. But the puzzle remains: what is at the horizon ?

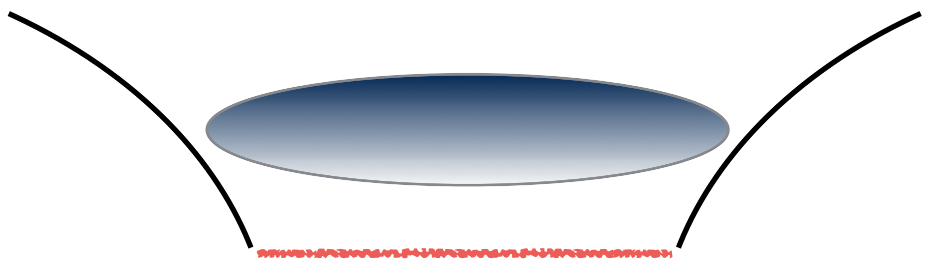
There was a general idea (Susskind …) that black holes have the maximal possible entropy allowed in a region …

Black holes



$$S = \frac{A}{4G}$$

But in Cosmology …



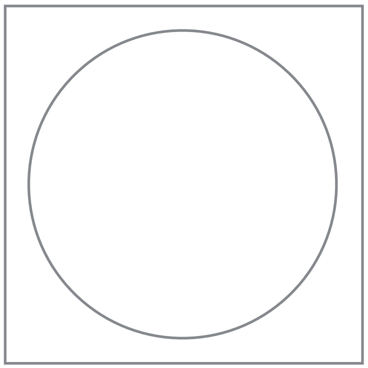
Homogenous cosmology

Entropy is proportional to volume …

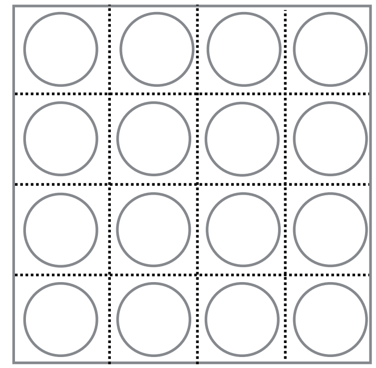
For a large enough volume, $S > \frac{A}{4G}$

In fact if we fix the volume, rather than the total energy, then we get much more entropy from a lattice of black holes …

E_1 S_1



E_2 S_2



much more entropy !

One then finds $$S \sim \sqrt{\frac{E\,V}{G}}$$

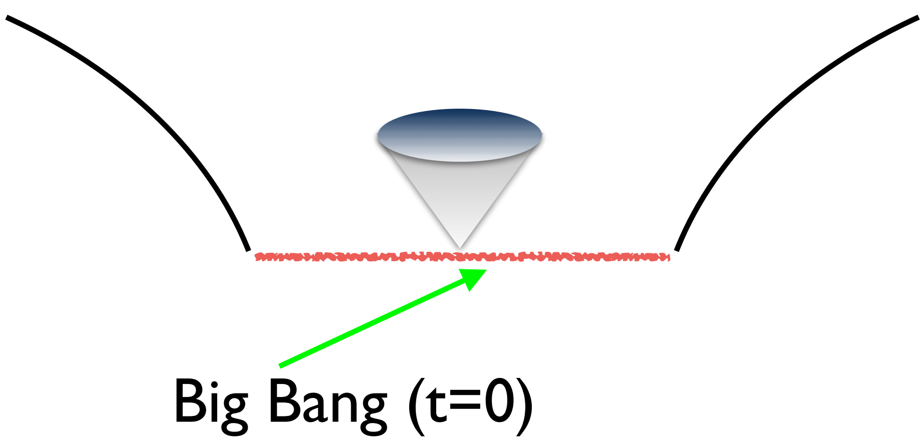
Writing $$\rho = \frac{E}{V}$$

we see that S is extensive

$$S \sim \sqrt{\frac{\rho}{G}}\, V$$

(Brustein+Veneziano, , Fischler+Susskind, Banks+Fischler, Masoumi+SDM)

So is there any role for the area formula $S = \frac{A}{4G}$?

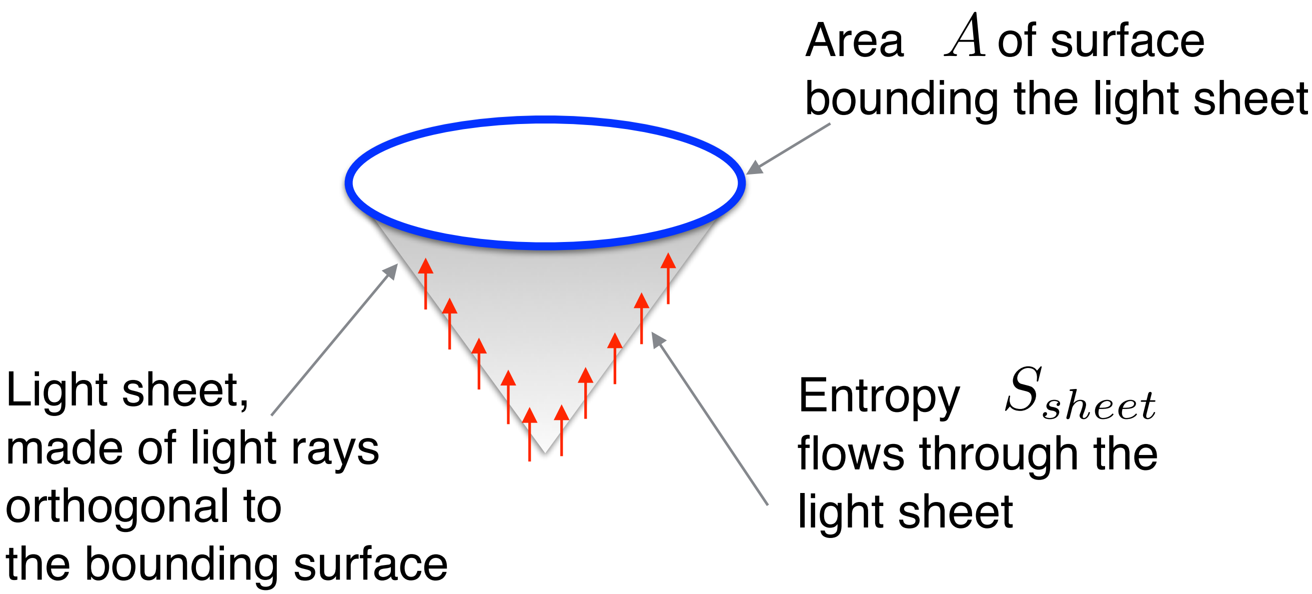


We should look only inside a region of size the cosmological horizon

Then we must have $S \leq \frac{A}{4G}$

(Fischler+Susskind 98)

This idea were made precise by Bousso, and encoded in a principle called the 'covariant entropy bound'



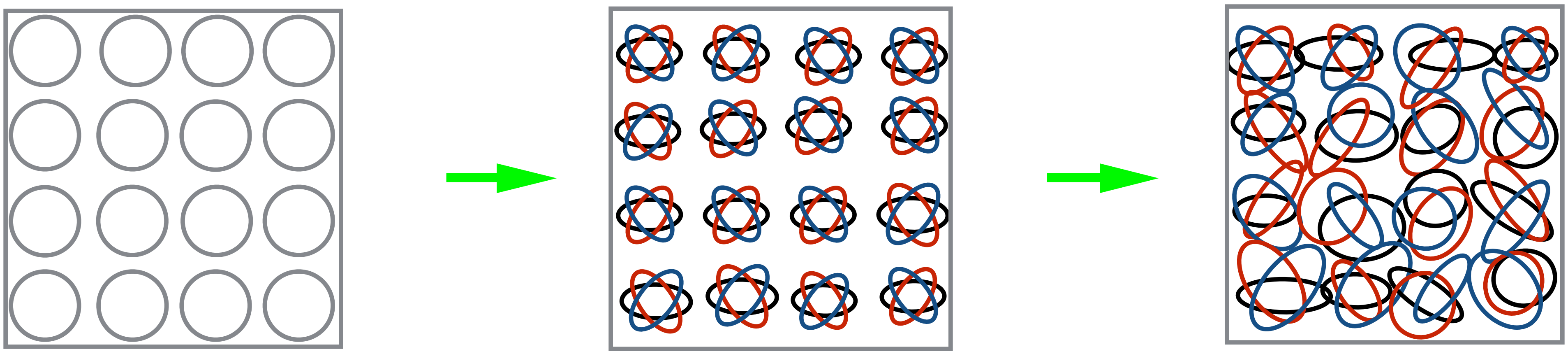
Covariant entropy bound
(Bousso 99)

$$S_{sheet} \leq \frac{A}{4G}$$

A 'proof' of this conjecture was given recently, both for free and interacting matter theories …

(Bousso, Cassini, Fisher, Maldacena)

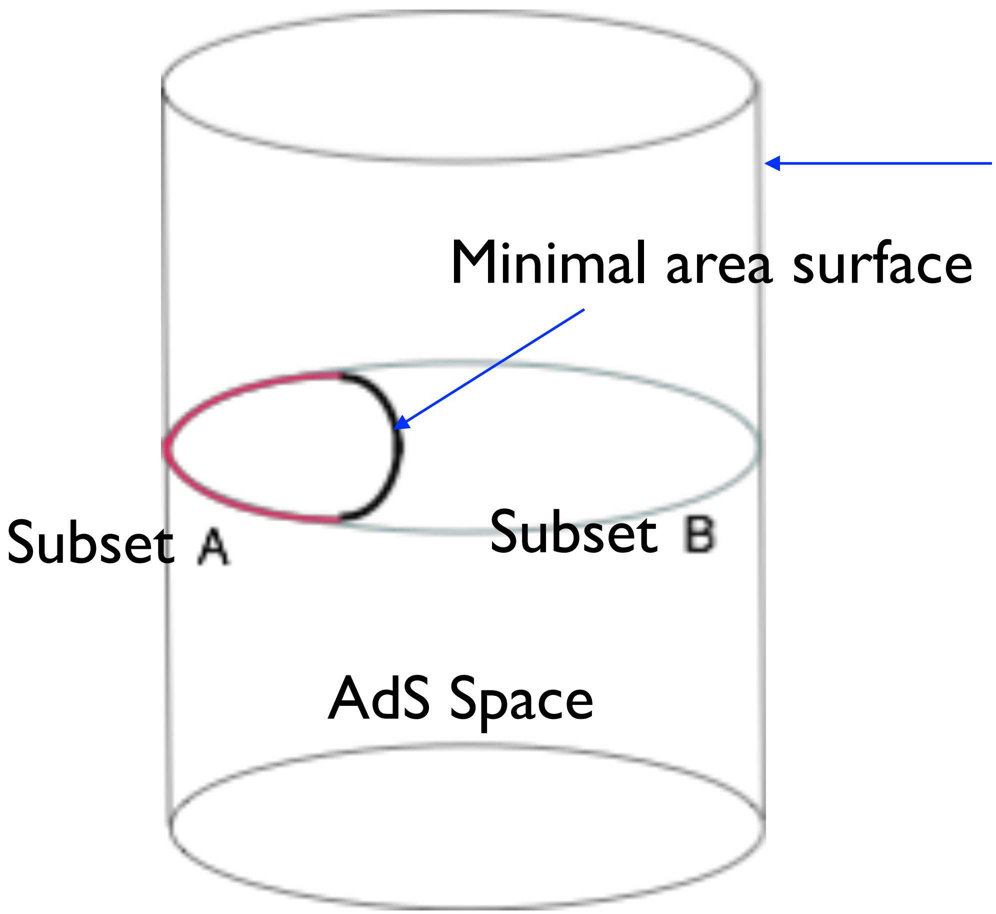
But in string theory it seems that we can realize the entropy $S \sim \sqrt{\frac{\rho}{G}}\, V$ as explicit string states ,…



It turns out that if we take an asymmetrically expansing Cosmology, this equation of state violates the Bousso bound … (Masoumi+SDM)

So, either the area bound is incorrect, or for some reason we cannot have asymmetric expansion with this equation of state (Banks)

A further puzzle is that we can get the area formula in situations where there is no black hole at all : The Ryu-Takayanagi conjecture:



The entanglement entropy of subset A with subset B is given by

$$\frac{A}{4G}$$

where A is the area of the minimal area surface in AdS that bounds the subset A

Here an entropy is given by an area, but there is no Black Hole anywhere !

We can create an artificial horizon by considering accelerated observers; such observers cannot see some part of spacetime

In this case the entropy describing the 'Information' which cannot be seen is related to the area of the horizon

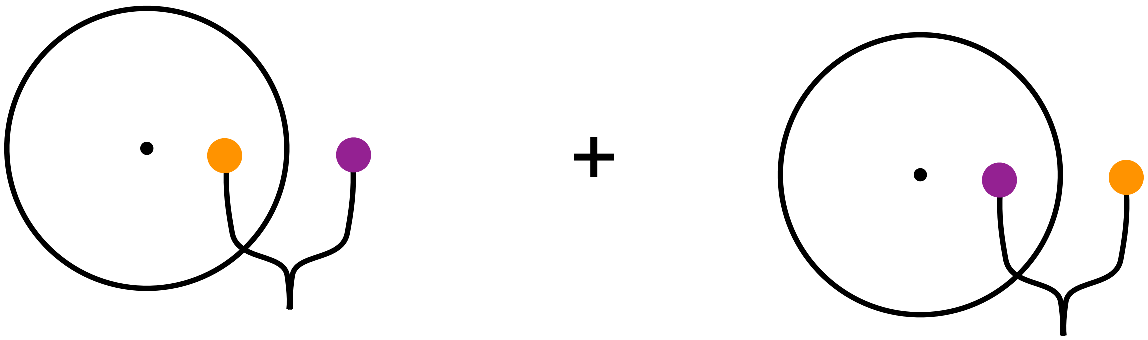
(Balasubramanian, Chowdhury, Czeck, de Boer)

Thus the relation of entropy to area remains mysterious ….

Resolving the information paradox in string theory: Fuzzballs

Avery, Balasubramanian, Bena, Carson, Chowdhury, de Boer, Gimon, Giusto, Halmagyi, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Niehoff, Park, Peet, Potvin, Puhm, Ross, Ruef, Saxena, Simon, Skenderis, Srivastava, Taylor, Turton, Vasilakis, Warner ...

Recall that the problem was created by the production of entangled pairs …



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(e^+e^- + e^-e^+\right)$$
$$= \frac{1}{\sqrt{2}}\left(01 + 10\right)$$

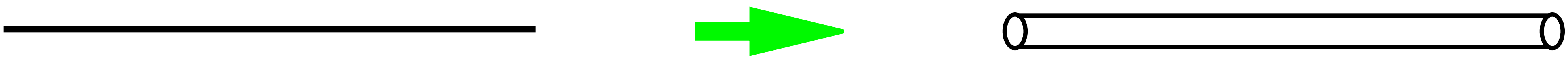
String theory cannot have information loss, since it is based on quantum mechanics

String theory also does not allow remnants: There are a finite number of states in a finite volume with a given energy, if AdS/CFT is to be correct

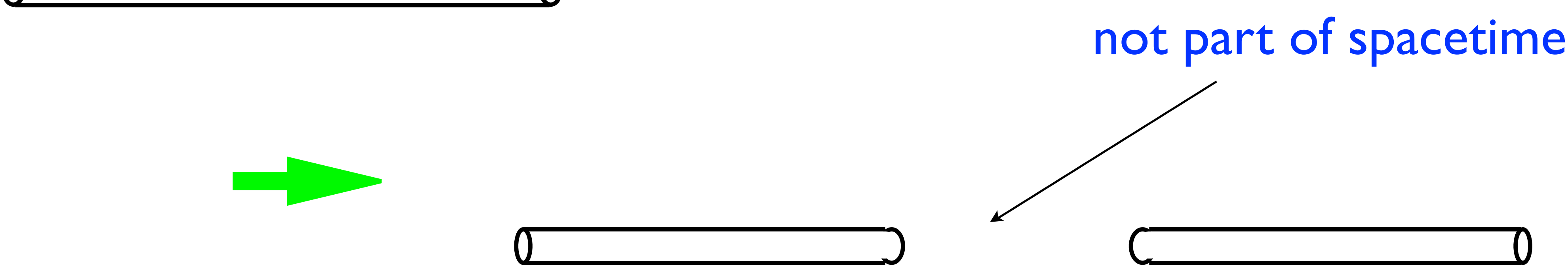
First consider a rough analogy …

Witten 1982: ‘Bubble of nothing’

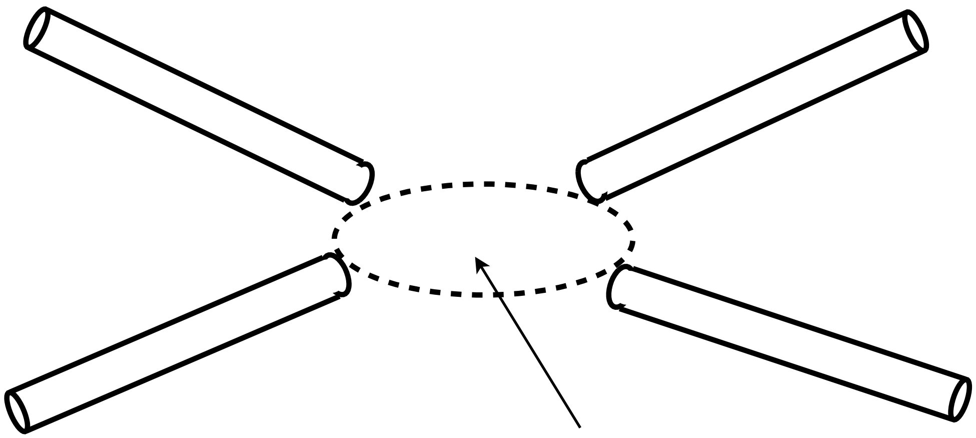
Consider Minkowski space with an extra compact circle



This space-time is unstable to tunneling into a ‘bubble of nothing’



In more dimensions :



not part of spacetime

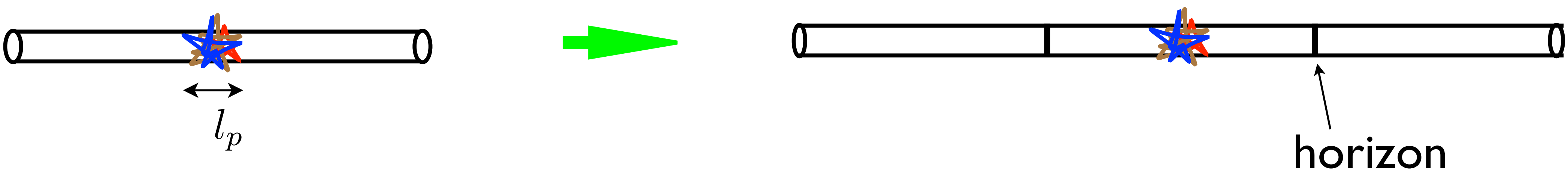
People did not worry about this instability too much, since it turns out that fermions cannot live on this new topology without having a singularity in their wave function …

But now consider the black hole …

Black holes:

The traditional expectation ...

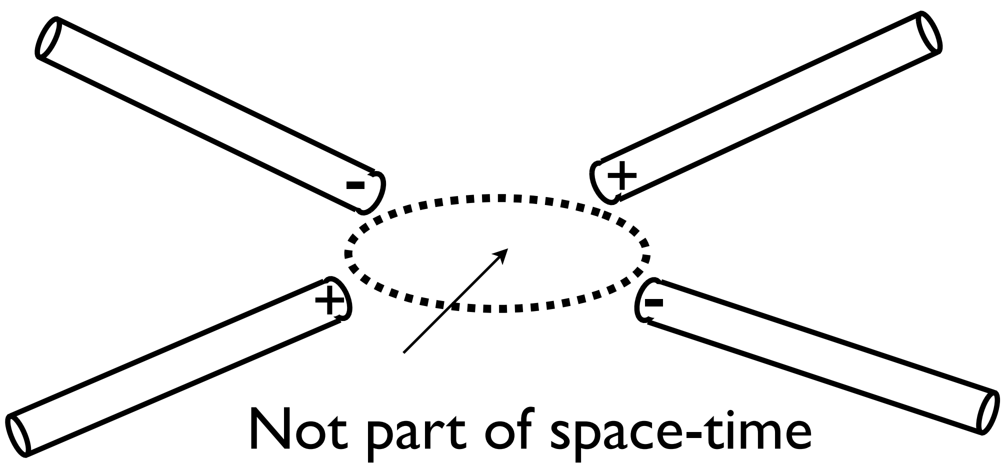
1-d spacetime + 1 compact direction



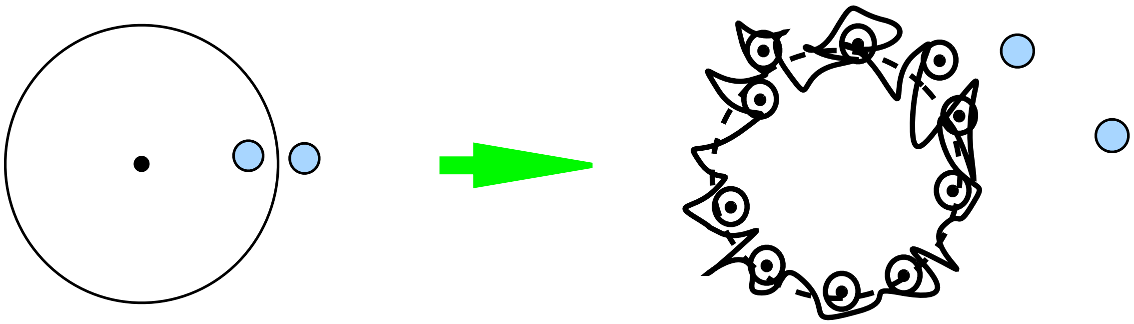
But one finds that something different happens ...

The geometry ‘caps off’ just outside the horizon (KK monopoles in simplest duality frame)

Mass comes from curvature, fluxes, strings, branes etc .. (spacetime ‘ends’ consistently in a set of valid sources in string theory)



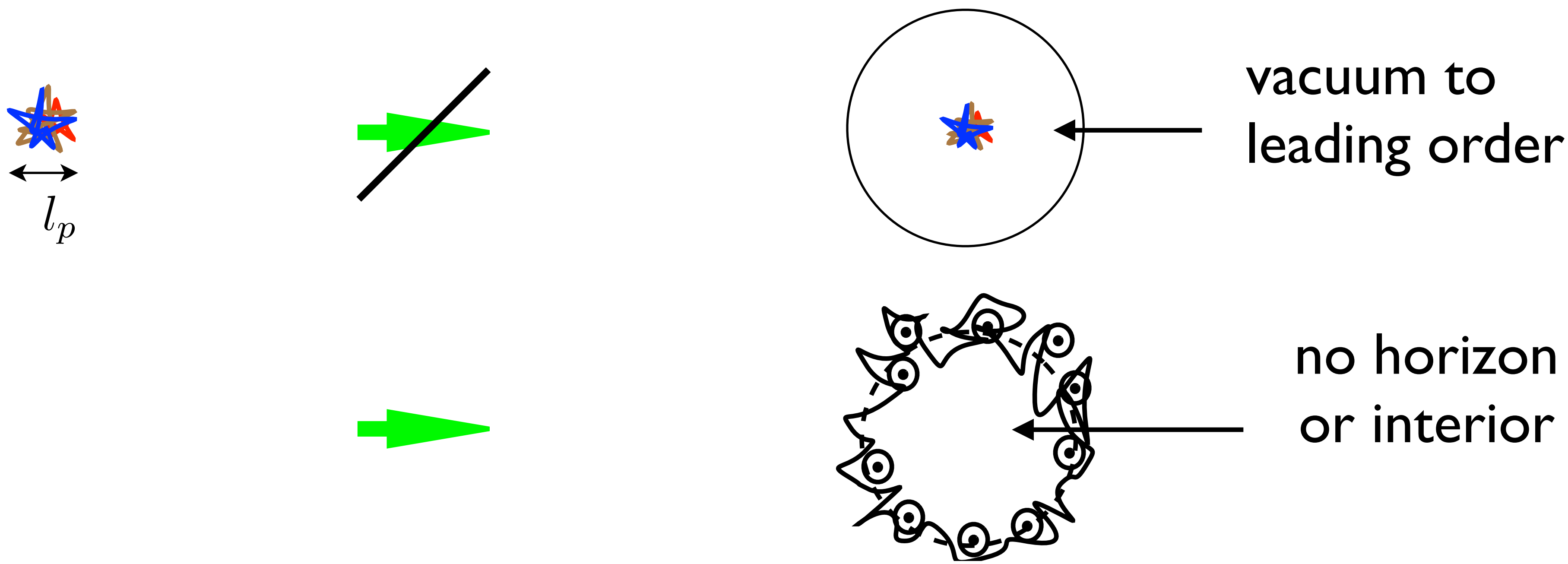
Fuzzball proposal:
All states of the hole are of this topology …
No state has a smooth horizon with an ‘interior’



The ‘fuzzball’ radiates from its surface just like a piece of coal, so there is no information paradox

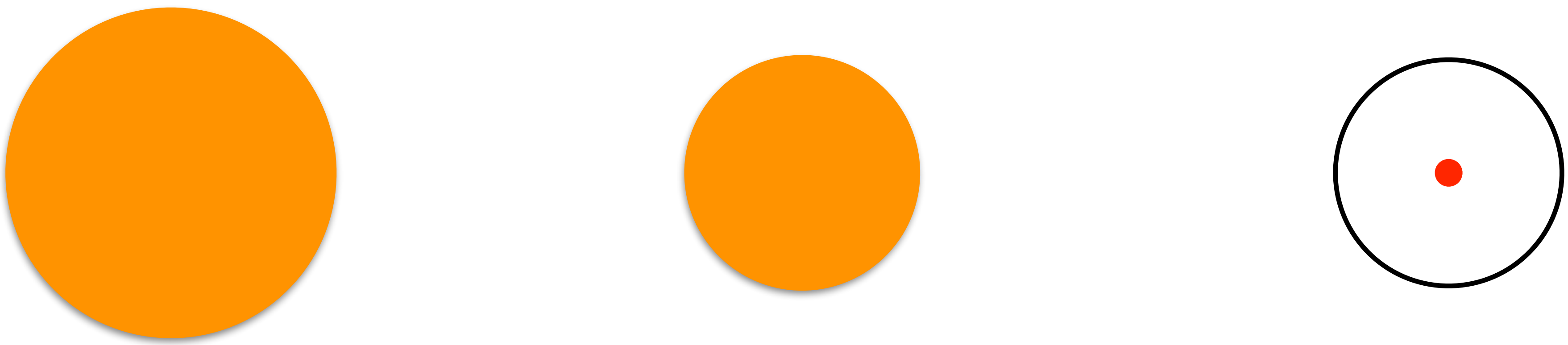
All states investigated so far have a fuzzball structure (extremal, near extremal, neutral with max rotation …)

Fuzzball conjecture: no state in string theory has a traditional horizon

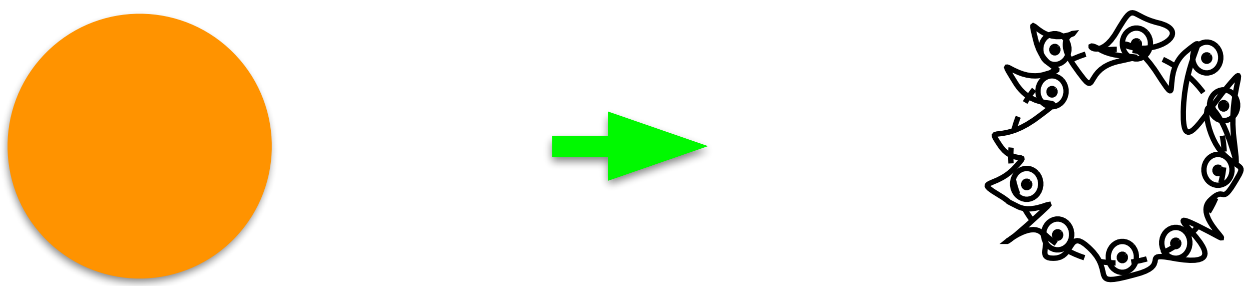


How could the black hole structure change in this radical way ?

Classically expected collapse of a star:



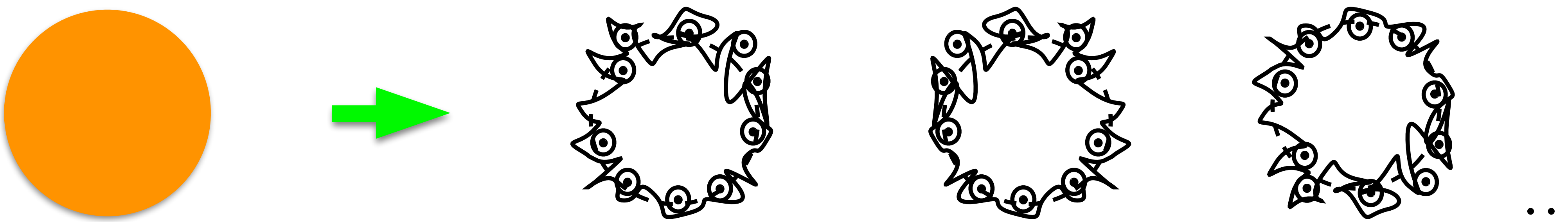
There is a small probability for the star to transition to a fuzzball state



$$\mathcal{P} \sim Exp[-S_{cl}] \sim Exp[-GM^2] \sim Exp[-S_{bek}]$$

(SDM 08, SDM 09, Kraus+SDM 15)

But we have to multiply this probability with a very large number of possible fuzzball states that the star can transition to …



Thus this very small number and this very large number *cancel* ...

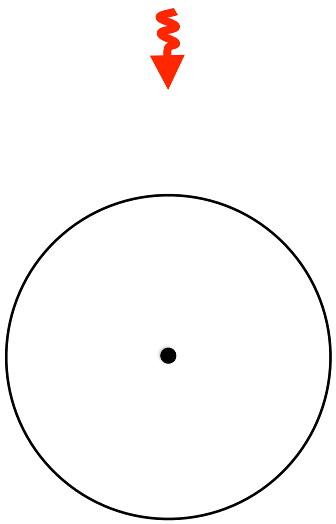
$$\mathcal{P}\, N \sim Exp[-S_{bek}] \times Exp[S_{bek}] \sim 1$$

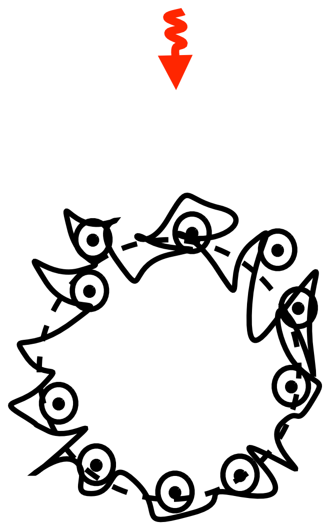
Thus the semiclassical approximation is broken because the measure competes with the classical action:

$$\mathcal{Z} = \int D[g]\, Exp[-S_{cl}(g)]$$

Fuzzball complementarity

What happens if an energetic photon falls towards the hole ?





In the old picture, it would fall in

In the fuzzball picture, there is no interior of the hole to fall into

One might think that the photon has hit a “brick wall” or a “firewall”

But there is a second, more interesting, possibility ….

→ ***The idea of fuzzball complementarity***

The dynamics of infall into a black hole are described by some frequencies

$$\nu_1^{bh},\ \nu_2^{bh},\ \nu_3^{bh},\ \ldots \nu_n^{bh}$$

Oscillations of the fuzzball are also described by some frequencies

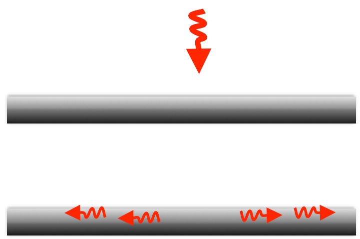
$$\nu_1^{fb},\ \nu_2^{fb},\ \nu_3^{fb},\ \ldots \nu_n^{fb}$$

What if

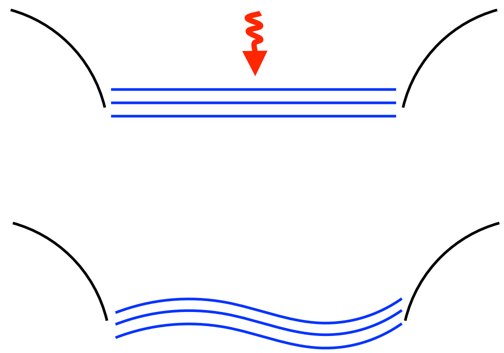
$$\nu_1^{bh},\ \nu_2^{bh},\ \nu_3^{bh},\ \ldots \nu_n^{bh} \quad \approx \quad \nu_1^{fb},\ \nu_2^{fb},\ \nu_3^{fb},\ \ldots \nu_n^{fb} \qquad ?$$

In that case falling onto the fuzzball will feel (approximately) like falling into a classical horizon …

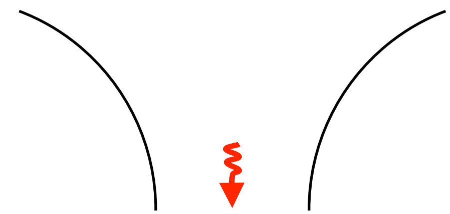
This may seem strange, but something like this happened with AdS/CFT duality …



Create random excitations



D-branes oscillate with some frequencies

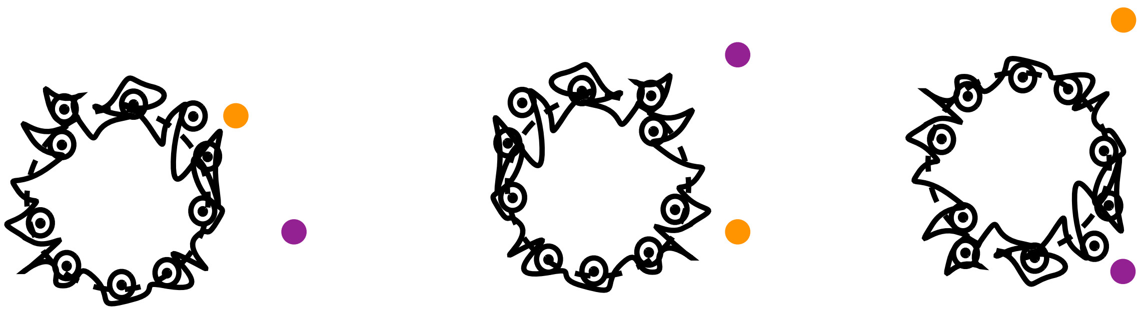


Gravitons in AdS space have the same frequency spectrum

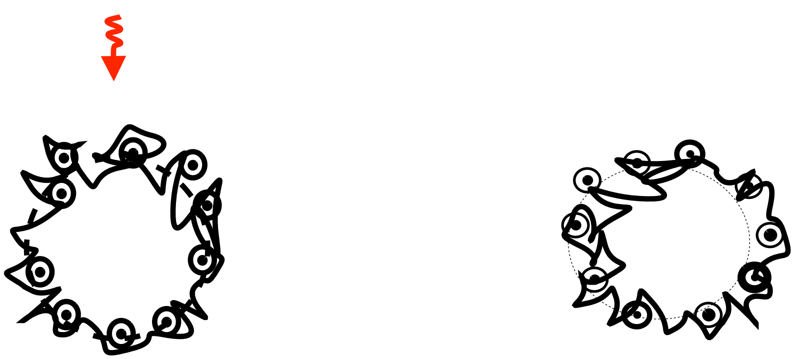
Maldacena 97

In our case, the frequencies of the traditional hole and of the fuzzball can be only approximately equal, since the fuzzballs are all a little different from each other …

This is crucial, since this is what allows information to escape !!

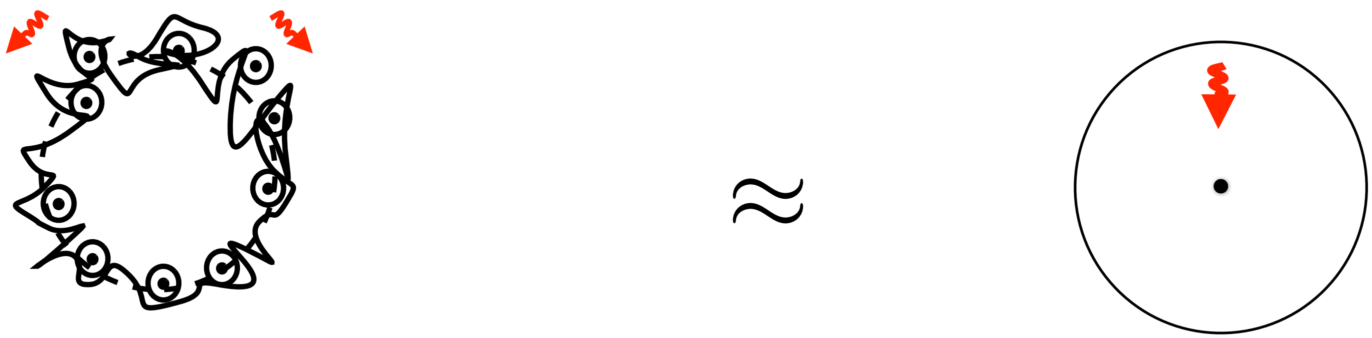


Low energy radiation
($E \sim T$)
is different between different fuzzballs, carries information



High energy impacts ($E \gg T$) give a near-universal set of frequencies, which reproduces the frequencies of classical infall

Thus we recover information, and also preserve, approximately, our classical intuition !!



The surface of the fuzzball behaves approximately like the membrane of the membrane paradigm, but this time with real degrees of freedom at the horizon, and spacetime does really end at this ‘membrane’

(SDM+PLumberg 2011)

Thus we get an approximate complementarity for freely falling observers with $E \gg T$, which is called fuzzball complementarity …

FIREWALLS :

Recently a group of people (Almheiri, Marolf, Polchiski, Sully: AMPS) argued that one CANNOT get complementarity ..

Thus the surface of any object that
solves the information paradox will behave like a 'firewall'

But there were two strange assumptions in their argument:

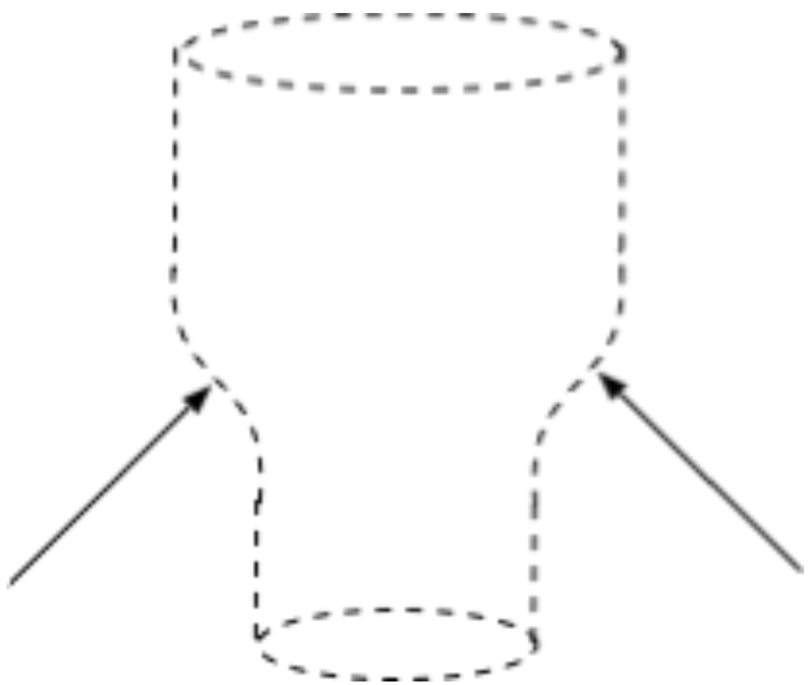
(a) AMPS focus on Hawking pairs, which are $E \sim T$, and do not take any limit $E \gg T$

We already know from fuzzballs that complementarity should be defined only in the $E \gg T$ limit (fuzzball complementarity)

(b) AMPS assume that an infalling observer feels nothing but semiclassical physics till he reaches within planck distance from the horizon

In particular, the surface of the black hole does not respond till it is hit by an infalling shell

But the horizon of a black hole is known to expand outwards BEFORE a shell falls onto it

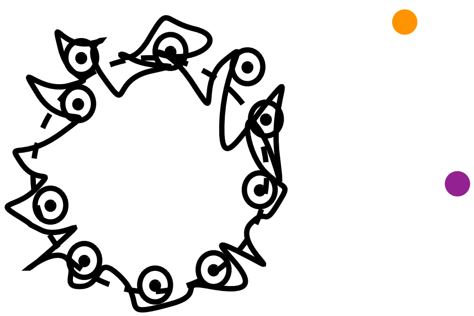


At 1 mm outside horizon, the temperature is lower than that of outer space …

⟶ No Firewall !!

Thus the firewall argument does not rule out the idea of fuzzball complementarity:

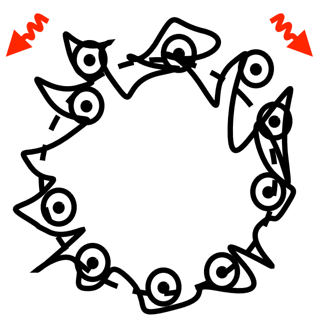
(A)



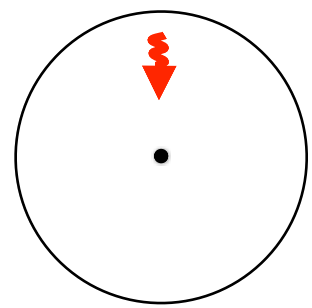
There is no black hole interior, so there is no information paradox

(The black hole radiates from the fuzzball surface like a normal body)

(B)



$\approx$



Oscillations of this surface mimic free infall to leading order under impact by freely falling objects with $E \gg T$

Summary

(A) The relation between area and entropy may be a special case of more general expression

$$S \sim \sqrt{\frac{\rho}{G}}\, V$$

(B) The information puzzle seems to be resolved in string theory through a radical change in the structure of the black hole (fuzzballs)