## Lattice QCD at the particle frontier

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I. Flavour physics
II.(g-2)
III. Future prospects: dark matter, neutrinos and ILC

## [Many topics not covered]

- Provide accurate, predictive calculational capability at hadronic scales enabling tests of the SM and searches for physics beyond it
- Understand the emergence of hadrons, nuclei and extreme forms of matter (RHIC, n-stars)
- Useful toolkit to study strongly interacting field theories beyond the SM
- Lattice QCD: tool to deal with quarks and gluons
- Correlation functions as functional integral over quark and gluon d.o.f. on R4

$$
\langle\mathcal{O}\rangle=\int d A_{\mu} d q d \bar{q} \mathcal{O}[q, \bar{q}, A] e^{-S_{Q C D}[q, \bar{q}, A]}
$$

perform quark integrals exactly

- Discretise and compactify system
- Finite but large number of d.o.f $\left(10^{10}\right)$
- Numerically integrate via importance sampling
 (average over important configurations)
- Undo the harm done in previous steps
- Lattice $\mathrm{QCD} \Rightarrow \mathrm{QCD}$
- Lattice QCD has advanced markedly in the last 5 years
- Faster computers and better algorithms
- New approaches to physics challenges
- State-of-the-art calculations address all systematics (discretisation, finite size, ...)
- Sub percent accuracy on some quantities
- Necessitates consideration of small effects: QED, isospin breaking (significant intellectual challenges to overcome)
- Scope increasing from meson physics to baryons and to light nuclei
- Different groups around the world using many technically different approaches to LQCD (c.f. 200 I state-of the-art)


- Flavour physics has been a mainstay of LQCD
- Many calculations of what are now "simple" quantities
- FLAG (PDG for LQCD)
- Members from most of the major collaborations
- Evaluates and grades different aspects of each calculation
- Provides averages as the "LQCD community consensus" value for a given quantity
- Summary report every couple of years I I 05.3453, I 304.5422
- New version early 2016: expanded scope and coverage
- Quark masses, decay constants, form factors, kaon mixing, LECs...
- Colour coded for quality of calculation (\# lattice spacings, volumes,...)


- Long running tension between $\mathrm{V}_{\mathrm{ub}}$ ( $\mathrm{and} \mathrm{V}_{\mathrm{cb}}$ ) extractions from inclusive $B \rightarrow X_{u}\left(B \rightarrow X_{c}\right)$ and exclusive decays $B \rightarrow \pi(B \rightarrow D)$


$$
\mathcal{H}_{\mathrm{eff}}=\frac{G_{F}}{\sqrt{2}} V_{u b} \underbrace{\bar{u} \gamma^{\mu}\left(1-\gamma_{5}\right) b}_{\equiv J^{\mu}} \bar{\ell} \gamma_{\mu}\left(1-\gamma_{5}\right) \nu
$$

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Inclusive


$$
\begin{aligned}
& \frac{\mathrm{d} \Gamma}{\mathrm{~d} q^{2} \mathrm{~d} E_{\ell}} \propto\left|V_{u b}\right|^{2}(\ldots)_{\mu \nu} \\
& \times \operatorname{Im}(\underbrace{-i \int \mathrm{~d}^{4} x e^{-i q \cdot x}\langle B| \mathbf{T} J^{\mu \dagger}(x) J^{\nu}(0)|B\rangle}_{\text {OPE, HQET }})
\end{aligned}
$$

Exclusive


$$
\frac{\mathrm{d} \Gamma}{\mathrm{~d} q^{2}} \propto\left|V_{u b}\right|^{2}|(\ldots)_{\mu} \underbrace{\langle\pi| J^{\mu}|B\rangle}_{\text {lattice } Q C D}|^{2}
$$

## Inclusive vs exclusive $\mathrm{V}_{\mathrm{ub}}$ \& $\mathrm{V}_{\mathrm{cb}}$

- Long running tension between $\mathrm{V}_{\mathrm{ub}}$ ( $\mathrm{and} \mathrm{V}_{\mathrm{cb}}$ ) extractions from inclusive $B \rightarrow X_{u}\left(B \rightarrow X_{c}\right)$ and exclusive decays $B \rightarrow \pi(B \rightarrow D)$

- Possible to reconcile through BSM scenarios that produce RH currents at low energy

$$
\mathcal{H}_{\text {eff }}=\frac{G_{F}}{\sqrt{2}} V_{u b}^{L}\left[\left(1+\epsilon_{R}\right) \bar{u} \gamma^{\mu} b-\left(1-\epsilon_{R}\right) \bar{u} \gamma^{\mu} \gamma_{5} b\right] \bar{\ell} \gamma_{\mu}\left(1-\gamma_{5}\right) \nu
$$


figure modified from LHCb I504.01568

- Bottom baryons provide another exclusive decay channel: $\Lambda_{b} \rightarrow \mathrm{plv}$
- LHCb: branching fraction ratio measured


$$
\text { [1504.0|568=Nature Phys. } 11 \text { (20|5)] }
$$

- Extraction of $\left|V_{\mathrm{ub}} / \mathrm{V}_{\mathrm{cb}}\right|$ requires hadronic matrix elements

$$
\begin{aligned}
& \qquad \begin{array}{ll}
\langle p| \bar{u} \gamma^{\mu} b\left|\Lambda_{b}\right\rangle, & \langle p| \bar{u} \gamma^{\mu} \gamma_{5} b\left|\Lambda_{b}\right\rangle, \\
\left\langle\Lambda_{c}\right| \bar{c} \gamma^{\mu} b\left|\Lambda_{b}\right\rangle, & \left\langle\Lambda_{c}\right| \bar{c} \gamma^{\mu} \gamma_{5} b\left|\Lambda_{b}\right\rangle \\
\text { from LQCD }
\end{array}
\end{aligned}
$$



- 12 form factors needed
- Careful consideration of systematic uncertainties
- Precise at large $q^{2}$
- Coordinated with LHCb (ffs needed during analysis)
- Compare partial integrals

$$
\left|\frac{V_{u b}}{V_{c b}}\right|=0.083(4)_{\operatorname{expt}}(4)_{\mathrm{latt}}
$$

- Combine with exclusive $\mathrm{V}_{\mathrm{cb}}$ to get |Vub|

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## Inclusive vs exclusive $V_{\text {ub }}$

- Consistent with mesonic exclusive measurement

$$
\left|V_{u b}\right|=3.27(0.15)_{\operatorname{expt}}(0.16)_{\mathrm{latt}}(0.06)_{V_{c b}} \times 10^{-3}
$$



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$$
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$$



$$
\begin{aligned}
& \text { Inclusive [PDG 2014] } \\
& \mathrm{B} \rightarrow \pi / v[\mathrm{PDG} 2014] \\
& \Lambda_{b} \rightarrow \mathrm{plv}[\mathrm{DLM} / \mathrm{LHCb} 2015] \\
& \mathrm{B} \rightarrow \pi / v[\mathrm{RBC/UKQCD} 2015] \\
& \mathrm{B} \rightarrow \pi / v[\text { FNALMILC 2015] }
\end{aligned}
$$

- New LQCD calculations for $B \rightarrow \pi$ decays too!


## Inclusive vs exclusive $V_{u b}$

- Different dependence of baryon decay disfavours RH currents as a solution to inclusive/exclusive tension

figure modified from
LHCb I504.01568
- Exclusive extractions:
- very different experimental and theoretical systematics
- Mutual consistency ( $p=0.26$ )
- Inclusive extractions creates significant tension
- Solution from RH currents disfavoured by baryonic extraction
- ?????

- Neutral kaon systems of intense interest for 50+ years
- Physical states are combinations of CP eigenstates

$$
\begin{aligned}
& \left|K_{L}\right\rangle \sim\left|K_{-}^{0}\right\rangle+\bar{\epsilon}\left|K_{+}^{0}\right\rangle \\
& \left|K_{S}\right\rangle \sim\left|K_{+}^{0}\right\rangle+\bar{\epsilon}\left|K_{-}^{0}\right\rangle
\end{aligned} \quad\left|K_{ \pm}^{0}\right\rangle=\frac{1}{\sqrt{2}}\left[\left|K^{0}\right\rangle \mp\left|\bar{K}^{0}\right\rangle\right] \quad \begin{aligned}
& K^{0}=\bar{s} \gamma_{5} d \\
& \bar{K}^{0}=\bar{d} \gamma_{5} s
\end{aligned}
$$

- First observation of CP violation [Christenson, Cronin, Fitch \& Turlay 1964]
- Both direct and indirect CP violation

- PDG values
$\operatorname{Re} \epsilon=(1.657 \pm 0.021) \times 10^{-3} \quad \operatorname{Re}\left(\frac{\epsilon^{\prime}}{\epsilon}\right)=(1.67 \pm 0.26) \times 10^{-3}$
$\operatorname{Im} \epsilon=(1.572 \pm 0.022) \times 10^{-3}$
- Indirect CP violation arises from mixing

$$
i \frac{d}{d t}\left(\left\lvert\, \begin{array}{l}
\left.K^{0}\right\rangle \\
\left.K^{0}\right\rangle
\end{array}\right.\right)=\left[\begin{array}{ll}
H_{00} & H_{0 \overline{0}} \\
H_{\overline{0} 0} & H_{\overline{0} \overline{0}}
\end{array}\right]\binom{\left|K^{0}\right\rangle}{\left.K^{0}\right\rangle}
$$

- Predominantly from box diagram ( $\Delta \mathrm{S}=24 \mathrm{q}$ operator)


$$
|\epsilon|=\frac{\operatorname{Im}\left\langle\bar{K}^{0}\right| \mathcal{O}_{L L}\left|K^{0}\right\rangle}{\Delta M_{K}}+\frac{\operatorname{Im} A_{0}}{\operatorname{Re} A_{0}}
$$

- Conventional approach: take expt for Im $A_{0} / \operatorname{Re} A_{0}, \Delta M_{k}$, "bag parameter" from LQCD

$$
\left\langle\bar{K}^{0}\right| \mathcal{O}_{L L}^{\Delta S=2}\left|K^{0}\right\rangle=\frac{8}{3} F_{K}^{2} M_{K}^{2} B_{K}(\mu)
$$

- $B_{k}$ under precise control

$$
\hat{B}_{K}=\left(\frac{\bar{g}(\mu)^{2}}{4 \pi}\right)^{-\gamma_{0} / 2 \beta_{0}} \exp \left\{\int_{0}^{\bar{g}(\mu)} d g\left(\frac{\gamma(g)}{\beta(g)}+\frac{\gamma_{0}}{\beta_{0} g}\right)\right\} B_{K}(\mu)
$$

- Decays through either $\Delta \mathrm{I}=1 / 2,3 / 2(\Delta \mathrm{~S}=\mathrm{I})$ with amplitudes ( $\delta_{\mathrm{I}}=$ strong scattering phases)

$$
A\left(K \rightarrow \pi \pi_{I}\right) \equiv A_{I} e^{i \delta_{I}} \quad I=0,2
$$

- Long standing puzzle ( $\Delta \mathrm{I}=\mathrm{l} / 2$ rule)

$$
\omega=\frac{\operatorname{Re} A_{2}}{\operatorname{Re} A_{0}} \sim \frac{1}{22}
$$

- Amplitudes relate to CP violating parameters as

$$
\begin{aligned}
\epsilon^{\prime} & =\frac{i \omega}{\sqrt{2}} e^{i\left(\delta_{2}-\delta_{0}\right)}\left[\frac{\operatorname{Im} A_{2}}{\operatorname{Re} A_{2}}-\frac{\operatorname{Im} A_{0}}{\operatorname{Re} A_{0}}\right] \\
|\epsilon| & =\frac{\operatorname{Im}\left\langle\bar{K}^{0}\right| \mathcal{O}_{L L}\left|K^{0}\right\rangle}{\Delta M_{K}}+\frac{\operatorname{Im} A_{0}}{\operatorname{Re} A_{0}}
\end{aligned}
$$

- Many new developments by the RBC/UKQCD collaboration
- Quantitative understanding of $\Delta \mathrm{I}=1 / 2$ rule [P Boyle et al. (RBC/UKQCD) Phys.Rev.Lett. I 10 (2013) I 5, I 5200 I; T Blum et al. (RBC/UKQCD) Phys.Rev. D9I (2015) 7, 074502 ]
- Important progress in calculating all pieces of $\varepsilon$ and $\varepsilon^{\prime}$ directly from SM
- Full calculations of decay amplitudes [Z Bai, et al. (RBC/UKQCD), I 505.07863]
- Kaon mass difference: long distance contributions [Z Bai, et al. (RBC/UKQCD) Phys.Rev.Lett. I I 3 (20|4) II 2003]
- Additional progress on rare kaon decays [ N Christ et al (RBC/UKQCD) I507.03094]
- Decays through weak $\Delta \mathrm{I}=\mathrm{I} / 2,3 / 2(\Delta \mathrm{~S}=\mathrm{I})$ process

- Ten relevant operators in effective Hamiltonian (weak decays, QCD and EW penguin operators)

$$
\mathcal{H}_{\Delta S=1}=\frac{G_{F}}{\sqrt{2}} \sum_{i=1}^{10}\left[V_{u d} V_{u s}^{*} z_{i}(\mu)-V_{t d} V_{t s}^{*} y_{i}(\mu)\right] \mathcal{Q}_{i}^{\overline{M S}}(\mu)
$$

- Short distance physics in Wilson coefficients $z_{i}(\boldsymbol{\mu})$ and $y_{i}(\boldsymbol{\mu})$
- LQCD to determine matrix elements

$$
\langle\pi \pi| \mathcal{Q}_{j}^{\text {latt }}\left|\bar{K}^{0}\right\rangle \quad \mathcal{Q}_{i}^{\overline{M S}}(\mu)=Z_{i j}^{\text {latt } \rightarrow \overline{M S}}(\mu, a) \mathcal{Q}_{j}^{\text {latt }}
$$

(need to convert lattice operators to MSbar scheme)

- Consider operator $Q_{(27,1)}^{3 / 2}=\left(\bar{s}^{i} d^{i}\right)_{L}\left\{\left(\bar{u}^{j} u^{j}\right)_{L}-\left(\bar{d}^{j} d^{j}\right)_{L}\right\}+\left(\bar{s}^{i} u^{i}\right)_{L}\left(\bar{u}^{j} d^{j}\right)_{L}$
- Matrix element evaluation can be Fierzed into two "colour contractions"

- Vacuum saturation would give (2) $\sim 1 / 3$ (1) Numerical evaluation gives (2) $\sim-0.7$ (1)

- For $\Delta \mathrm{I}=3 / 2$ contributions add (1) + (2) so significant cancelation
- More complex for $\Delta \mathrm{I}=\mathrm{I} / 2$, but roughly enhanced rather than suppressed

$$
\frac{\operatorname{Re} A_{2}}{\operatorname{Re} A_{0}} \sim \frac{(1)+(2)}{2(2)-(1)} \sim 10-15
$$



## Muon g-2

- Long standing discrepancy between measured value and SM estimate for muon anomalous magnetic moment ( $\sim 3 \sigma$ )

- Sign of new physics or problem with theory?
- New experiments aiming at 4-fold uncertainty reduction (E989 @ Fermilab, E34 @ JPARC)
- Requires commensurate control of theory


## Standard Model (g-2) $\mu$

Snowmass:The Muon (g-2) Theory Value: Present and Future [T Blum et al. 13। I.2198]

QED (5 loop) [Aoyama et al. 2012]

## - Measured value

$$
a_{\mu}^{\mathrm{E} 821}=(116592089 \pm 63) \times 10^{-11} \quad(0.54 \mathrm{ppm})
$$

- Breakdown of contributions (2 evaluations of HVP)

|  | VALUE $\left(\times 10^{-11}\right)$ UNITS |
| :--- | ---: |
| QED $(\gamma+\ell)$ | $116584718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_{\alpha}$ |
| HVP(lo) $[20]$ | $6923 \pm 42$ |
| HVP(lo) $[21]$ | $6949 \pm 43$ |
| HVP(ho) $[21]$ | $-98.4 \pm 0.7$ |
| HLbL | $105 \pm 26$ |
| EW | $154 \pm 1$ |
| Total SM [20] | $116591802 \pm 42_{\text {H-LO }} \pm 26_{\text {H-HO }} \pm 2_{\text {other }}\left( \pm 49_{\text {tot }}\right)$ |
| Total SM [21] | $116591828 \pm 43_{\text {H-LO }} \pm 26_{\text {H-HO }} \pm 2_{\text {other }}\left( \pm 50_{\text {tot }}\right)$ |

## - Deviation

$$
\begin{aligned}
\Delta a_{\mu}(\mathrm{E} 821-\mathrm{SM}) & =(287 \pm 80) \times 10^{-11}[20] \\
& =(261 \pm 78) \times 10^{-11}[21]
\end{aligned}
$$



- Hadronic vacuum polarisation


Hadronic light-by-light


Electroweak (2 loop) [Czarnecki et al. 2006]


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- LQCD requirements

| Lattice |  | precision | timescale |
| :--- | :---: | :--- | :--- |
| benchmark |  |  |  |
| HVP | $1-2 \%$ | Few years | $\tau--e^{-} e^{-}$discrepancy |
| HVP | sub-\% | This decade | Competitive w/ e+e- |
| hLbL | any | soon | Course Verification of models |
| hLbL | $\sim 30 \%$ | $3-5$ years | Competitive with models |
| hLbL | $\sim 10 \%$ | Ultimate goal | Replace models |

[B Casey Lattice 2014 projections]

- Hugely active area of LQCD
- Efforts to increase precision on HVP
- Exploration of techniques to address HLbL

- Current theoretical estimate from dispersive treatment

- Use data on $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow\right.$ hadrons $)$ combining many data sets

$$
\left(q^{2} g_{\mu \nu}-q_{\mu} q_{\nu}\right) \Pi_{V}\left(q^{2}\right)
$$

$$
s=q^{2}
$$



$$
\begin{gathered}
\Pi_{V}\left(k^{2}\right)-\Pi_{V}(0)=\frac{k^{2}}{\pi} \int_{4 m_{\pi}^{2}}^{\infty} d s \frac{\operatorname{Im} \Pi_{V}(s)}{s\left(s-k^{2}-i \epsilon\right)} \\
\operatorname{Im} \Pi_{V}(s)=\frac{s}{4 \pi \alpha} \sigma_{\text {tot }}\left(e^{+} e^{-} \rightarrow X\right) \\
a_{\mu}^{\text {had }}=\int d s
\end{gathered}
$$

$$
a_{\mu}^{\mathrm{HVP}}=\frac{1}{4 \pi^{2}} \int_{4 m_{\pi}^{2}}^{\infty} d s K(s) \sigma_{\text {total }}(s)
$$

- Can be computed from SM directly [Blum PRL9I (2003) 05200I]
- Analytically continue to Euclidean space [T Blum PRL9I (2003) 05200I]

$$
K^{2}=-q^{2}>0
$$

Use modified kernel

$$
a_{\mu}=\frac{g-2}{2}=\left(\frac{\alpha}{\pi}\right)^{2} \int_{0}^{\infty} d K^{2} f\left(K^{2}\right) \hat{\Pi}\left(K^{2}\right)
$$

- Precision goal is challenging, but calculations rapidly improving
- Major technical improvements at low $K^{2}$ where $f\left(K^{2}\right)$ is peaked
- Ready for large scale calculations
$\mathrm{e}^{+} \mathrm{e}^{-}$(Davier 2011)


T Izubuchi Lattice 2015 (already out of date)

- HLbL smaller but hard to determine
- Currently guesstimated from models (see Colangelo et al. for dispersive analysis of some pieces)
- Accessible in LQCD from 4-pt correlator


$$
\begin{aligned}
\Gamma_{\mu}^{(\mathrm{Hlbl})}\left(p_{2}, p_{1}\right)= & i e^{6} \int \frac{d^{4} k_{1}}{(2 \pi)^{4}} \frac{d^{4} k_{2}}{(2 \pi)^{4}} \frac{\Pi_{\mu \nu \rho \sigma}^{(4)}\left(q, k_{1}, k_{3}, k_{2}\right)}{k_{1}^{2} k_{2}^{2} k_{3}^{2}} \\
& \times \gamma_{\nu} S^{(\mu)}\left(p_{2}+k_{2}\right) \gamma_{\rho} S^{(\mu)}\left(p_{1}+k_{1}\right) \gamma_{\sigma}
\end{aligned}
$$

$$
\begin{gathered}
\Pi_{\mu \nu \rho \sigma}^{(4)}\left(q, k_{1}, k_{3}, k_{2}\right)=\int d^{4} x_{1} d^{4} x_{2} d^{4} x_{3} \exp \left[-i\left(k_{1} \cdot x_{1}+k_{2} \cdot x_{2}+k_{3} \cdot x_{3}\right)\right] \\
\times\langle 0| T\left[j_{\mu}(0) j_{\nu}\left(x_{1}\right) j_{\rho}\left(x_{2}\right) j_{\sigma}\left(x_{3}\right)\right]|0\rangle
\end{gathered}
$$

- 32 relevant tensor structures!
- Required for all $k_{1}, k_{2}$ !
- Recent calculation at fixed kinematics [Green et al., 1507.01577$]$
- Comparison to model derived from dispersive analysis of $\gamma^{*} \gamma^{*} \rightarrow$ hadrons
- Not viable for all kinematics but may help constrain models



## Hadronic light-by-light



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$q=2 \pi / L N_{\text {prop }}=81000 \longmapsto$ $q=0 N_{\text {prop }}=26568 \longmapsto \bigcirc$

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- Still requires many further improvements to get final answer
$q=2 \pi / L N_{\text {prop }}=81000 \longmapsto \longmapsto$ $q=0 N_{\text {prop }}=26568 \longmapsto \bigcirc$


- Prospect of unambiguous detection of dark matter is very exciting
- Typical DM detector: look for recoil in large bucket of nuclei
- Post-detection: aim to determine the nature of the interaction with nucleus
- Understand target dependence and effects

http://www.hep.ucl.ac.uk/darkMatter/
- Requires SM calculations of nuclear matrix elements of possible currents to distinguish them
- Potentially understand seemingly conflicting positive (DAMA/ CoGeNT, CDMS-Si) and negative signals (LUX, XENON,...)
- Important goal of LBNF/DUNE: extraction of neutrino mass hierarchy and precise mixing parameters
- Requires knowing energies/fluxes to high accuracy
- Neutrino scattering on argon target
- Nuclear axial \& transition form factors
- Resonances
- Neutrino-nucleus DIS
- $\sim 10 \%$ uncertainty on oscillation parameters [C Mariani, INT workshop 2013]
- Nuclear matrix elements are not well understood
- Gamow-Teller transitions in nuclei are a stark example of problems (analogue of neutron decay)
- Best nuclear structure calculations are systematically off by 20-30\%
- Nuclei $(30<A<60)$ where spectrum is well described
- QRPA, shell-model,...
- Correct for it by arbitrarily "quenching" axial charge in nuclei
- Can NP become rigorous?


$$
T(G T) \sim \sqrt{\sum_{f}\langle\boldsymbol{\sigma} \cdot \boldsymbol{\tau}\rangle_{i \rightarrow f}}
$$

$$
\langle\boldsymbol{\sigma} \boldsymbol{\tau}\rangle=\frac{\langle f|\left|\sum_{k} \boldsymbol{\sigma}^{k} \boldsymbol{t}_{ \pm}^{k} \| i\right\rangle}{\sqrt{2 J_{i}+1}}
$$

- Nuclei arise from SM, so can be addressed using LQCD
- In practice: a hard problem
- Multiple exponentially difficult challenges
- Recent progress shows light nuclei can be studied rigorously
- Spectroscopy
- Matrix elements
- Connect to larger nuclei through nuclear effective field theory [Weinberg 199 I;Kaplan Savage Wise I995,...]


## Magnetic moments of nuclei

[NPLQCD PhysRevLett. II 3 (2014) 25, 25200I]

- First QCD study of structure of nuclei
- Magnetic moments and polarisabilities for A<5
- Patterns very interesting but calculations not physical
- Similar studies underway for weak current, scalar currents,...


In units of appropriate nuclear magnetons (heavy $\mathrm{M}_{\mathrm{N}}$ )

- Future ILC will determine Higgs couplings to unprecedented accuracy: further quantitative tests of SM
- Requires precise SM inputs and calculations
- High order pQCD calculations
- Commensurately precise b, c quark masses, strong coupling
- Recent study [Lepage, Mackenzie \& Peskin 1404.0319 ] of prospects for reaching requisite precision for relative uncertainties in Higgs partial widths

$$
\delta_{A}=\frac{1}{2} \frac{\Delta \Gamma(h \rightarrow A \bar{A})}{\Gamma(h \rightarrow A \bar{A})} \square \begin{aligned}
\delta_{b} & =1 . \cdot \delta m_{b}(10) \oplus(-0.28) \cdot \delta \alpha_{s}\left(m_{Z}\right) \\
\delta_{c} & =1 . \cdot \delta m_{c}(3) \oplus(-0.80) \cdot \delta \alpha_{s}\left(m_{Z}\right) \\
\delta_{g} & =1.2 \cdot \delta \alpha_{s}\left(m_{Z}\right)
\end{aligned}
$$

## Precision Higgs physics

- Consider expected growth in computing resources in ILC timeframe [1404.0319]

|  | $\delta m_{b}(10)$ | $\delta \alpha_{s}\left(m_{Z}\right)$ | $\delta m_{c}(3)$ | $\delta_{b}$ | $\delta_{c}$ | $\delta_{g}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| current errors $[10]$ | 0.70 | 0.63 | 0.61 | 0.77 | 0.89 | 0.78 |
| +PT | 0.69 | 0.40 | 0.34 | 0.74 | 0.57 | 0.49 |
| +LS | 0.30 | 0.53 | 0.53 | 0.38 | 0.74 | 0.65 |
| $+\mathrm{LS}{ }^{2}$ | 0.14 | 0.35 | 0.53 | 0.20 | 0.65 | 0.43 |
|  |  |  |  |  |  |  |
| $+\mathrm{PT}+\mathrm{LS}$ | 0.28 | 0.17 | 0.21 | 0.30 | 0.27 | 0.21 |
| $+\mathrm{PT}+\mathrm{LS}{ }^{2}$ | 0.12 | 0.14 | 0.20 | 0.13 | 0.24 | 0.17 |
| $+\mathrm{PT}+\mathrm{LS}^{2}+\mathrm{ST}$ | 0.09 | 0.08 | 0.20 | 0.10 | 0.22 | 0.09 |
|  |  |  |  |  |  |  |
| $\mathrm{ILC}^{2}$ goal |  |  |  | 0.30 | 0.70 | 0.60 |

- Allows finer lattice spacings (LS, LS²) and higher statistical precision (ST) (also requires higher order lattice perturbation theory matching)
- Will be an important contribution to Higgs program
- LQCD has come of age in the last decade and is a precision tool for many quantities
- Flavour physics, charged leptons, nuclear effects, SM parameters
- Not discussed: thermodynamics, hadron spectroscopy and structure, strongly interacting models of EWSB, composite DM,...
- Progress on many fronts
- Increase precision (deal with newly relevant systematics: QED, isospin breaking)
- Ways to compute new observables (nuclei, multi-body decays, nonlocal operator matrix elements
- New computational tricks to get more from less flops
- Lots more to come

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