Review of Parton Distribution Function

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In collaboration with CTEQ-TEA

Southern Methodist University

August 04, DPF2015 at UM,
Ann Arbor, MI

CTEQ-TEA group

- CTEQ – Tung et al. (TEA)
  in memory of Prof. Wu-Ki Tung, who established CTEQ Collaboration in early 90's.
- Current members:
  Sayipjamal Dulat (Xinjiang U.)
  Tie-Jiun Hou, Pavel Nadolsky (Southern Methodist U.)
  Jun Gao(Argonne Nat. lab.)
  Marco Guzzi(U. of Manchester)
  Joey Huston, Jon Pumplin, Carl Schmidt,
  Dan Stump, C.-P. Yuan(Michigan State U.)

Hadron Collider Physics

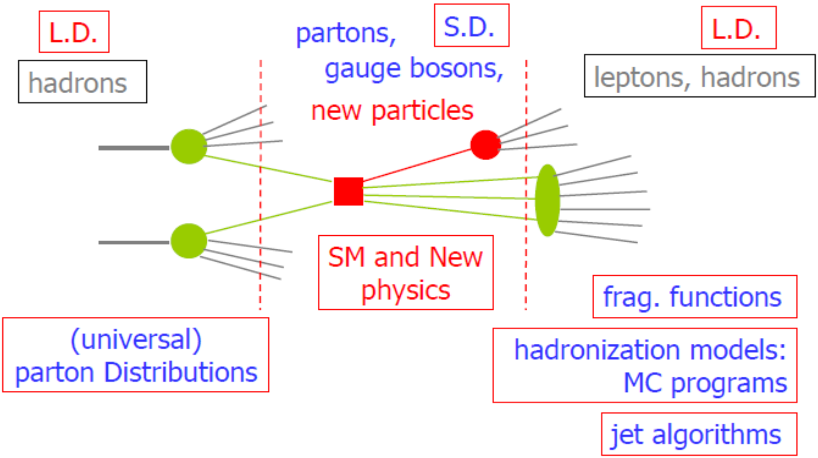


Table by A.Accardi, DIS'2015 workshop

	JLab	HERA I+II *Wichmann*	Tevatron new W,Z	LHC	di-μ	Nucl.	HT TMC	Flex *d*	closure
HERAPDF2.0 → *Myronenko, Brandt*		✓	¤						
CT14 → *Nadolski*			✓ ¤¤	✓	✓			✓	
MMHT14 → *Thorne*			✓ ¤¤	✓	✓	✓			
NNPDF3.0 → *Deans*				✓	✓				✓
[GJR14]	✓			✓	✓	✓	✓		
CJ12 * (→ CJ15) → *Melnitchouk*	✓	(✓)	(✓)		✗	✓	✓	✓	
ABM12 **					✓	✓	✓		

* NLO only   ** No jet data   ¤ but see 1503.05221   ¤¤ no reconstructed W

CT14 NNLO error PDFs

Available at http://hep.pa.msu.edu/cteq/public/index.html, is being submitted to LHAPDF;

- CT10 includes only pre-LHC data
- CT14 is the first CT analysis including LHC Run 1 data
- CT14 also includes the new Tevatron D0 Run 2 data on W-electron charge asymmetry
- CT14 uses a more flexible parametrization in the non-perturbative PDFs.

CT14 NNLO

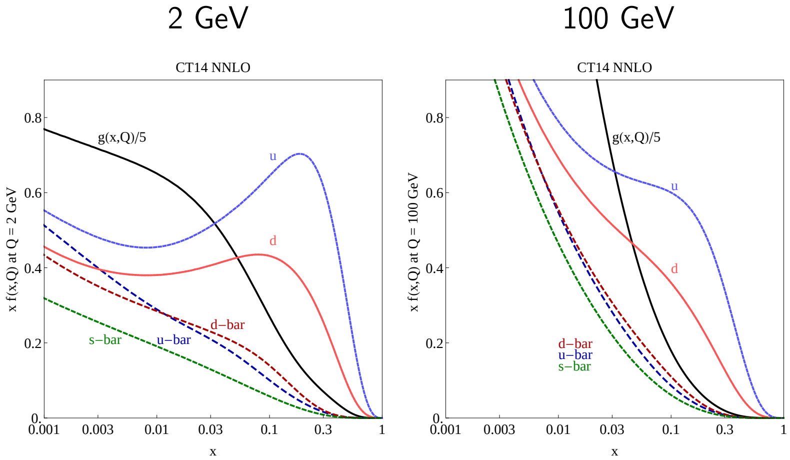


Figure 1: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \overline{u}, d, \overline{d}, s = \overline{s}$, and g.

DGLAP evolution for PDFs

- Non-perturbative PDFs are determined at the scale of Q=1.3 GeV.
- PDFs at any other scale Q can be obtained from PQCD, via solving DGLAP evolution equations.
- Due to DGLAP evolution, the PDF error band becomes smaller when the energy scale Q increases.
- The evolution effect is large in low Q region, say, from 1.3 GeV to 8 GeV.

The CT14-NNLO global analysis of QCD

Parametrization of PDFs $Q = 1.3$ GeV, with 28 parameter values to be chosen; there are from 4 to 6 parameters for each proton type.

Many data sets, for short distance interactions.

Perturbative QCD, using NNLO approximations whenever available.

Taking account of **experimental errors**, statistical and systematic. (Not so strong on systematic **theoretical errors**).

ID#	Experimental data set	N_{pt}	χ_e^2	χ_e^2/N_{pt}	S_n
101	BCDMS F_2^p	337	384	1.14	1.74
102	BCDMS F_2^d	250	294	1.18	1.89
104	NMC F_2^d/F_2^p	123	133	1.08	0.68
106	NMC σ_{red}^p	201	372	1.85	6.89
108	CDHSW F_2^p	85	72	0.85	-0.99
109	CDHSW F_3^p	96	80	0.83	-1.18
110	CCFR F_2^p	69	70	1.02	0.15
111	CCFR xF_3^p	86	31	0.36	-5.73
124	NuTeV $\nu\mu\mu$ SIDIS	38	24	0.62	-1.83
125	NuTeV $\bar{\nu}\mu\mu$ SIDIS	33	39	1.18	0.78
126	CCFR $\nu\mu\mu$ SIDIS	40	29	0.72	-1.32
127	CCFR $\bar{\nu}\mu\mu$ SIDIS	38	20	0.53	-2.46
145	H1 σ_r^b	10	6.8	0.68	-0.67
147	Combined HERA charm production	47	59	1.26	1.22
159	HERA1 Combined NC and CC DIS	579	591	1.02	0.37
169	H1 F_L	9	17	1.92	1.7

ID#	**Experimental data set**	N_{pt}	χ^2_e	χ^2_e/N_{pt}	S_n
201	E605 Drell-Yan process	119	116	0.98	-0.15
203	E866 Drell-Yan process, $\sigma_{pd}/(2\sigma_{pp})$	15	13	0.87	-0.25
204	E866 Drell-Yan process, $Q^3 d^2\sigma_{pp}/(dQdx_F)$	184	252	1.37	3.19
225	CDF Run-1 electron A_{ch}, $p_{T\ell} > 25$ GeV	11	8.9	0.81	-0.32
227	CDF Run-2 electron A_{ch}, $p_{T\ell} > 25$ GeV	11	14	1.24	0.67
234	DØ Run-2 muon A_{ch}, $p_{T\ell} > 20$ GeV	9	8.3	0.92	-0.02
240	LHCb 7 TeV 35 pb^{-1} W/Z $d\sigma/dy_\ell$	14	9.9	0.71	-0.73
241	LHCb 7 TeV 35 pb^{-1} A_{ch}, $p_{T\ell} > 20$ GeV	5	5.3	1.06	0.30
260	DØ Run-2 Z rapidity	28	17	0.59	-1.71
261	CDF Run-2 Z rapidity	29	48	1.64	2.13
266	CMS 7 TeV 4.7 fb^{-1}, muon A_{ch}, $p_{T\ell} > 35$ GeV	11	12.1	1.10	0.37
267	CMS 7 TeV 840 pb^{-1}, electron A_{ch}, $p_{T\ell} > 35$ GeV	11	10.1	0.92	-0.06
268	ATLAS 7 TeV 35 pb^{-1} W/Z cross sec., A_{ch}	41	51	1.25	1.11
281	DØ Run-2 9.7 fb^{-1} electron A_{ch}, $p_{T\ell} > 25$ GeV	13	35	2.67	3.11
504	CDF Run-2 inclusive jet production	72	105	1.45	2.45
514	DØ Run-2 inclusive jet production	110	120	1.09	0.67
535	ATLAS 7 TeV 35 pb^{-1} incl. jet production	90	50	0.55	-3.59
538	CMS 7 TeV 5 fb^{-1} incl. jet production	133	177	1.33	2.51

34 data sets used for the CT14-NNLO global analysis.

Experimental uceratinty

An experiment publishes N measurements,

$$\{M_i; i = 1, 2, 3, ..., N\}$$

Each measuemnt has several parts,

$$\begin{aligned} M_i &= \{D_i; \sigma_{0i}; \{\sigma_{1i}, \sigma_{2i}, \sigma_{3i}, ...\}\} \\ &= \{\text{center value; SD of statistical error;} \\ &\quad\ \text{SDs of correlated systematic errors}\} \end{aligned}$$

that is,

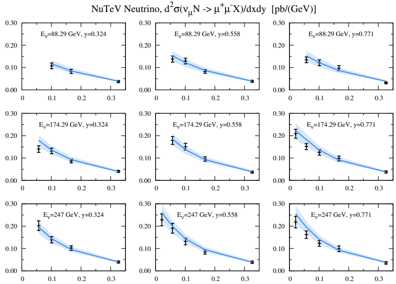
$$D_i = True_i + \sigma_{0i} r_0 + \sum_{k=1}^{N_{sy}} \sigma_{ki} r_k$$

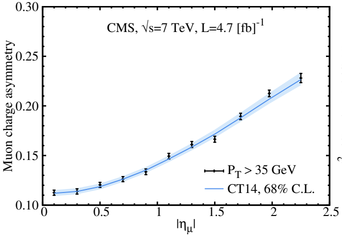
Where r_0 and $\{r_k\}$ are random variables (gaussian ?).
Define

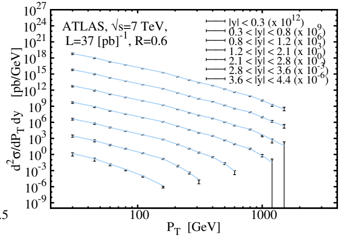
$$\chi^2_{global} = \sum_i \frac{\left[D_i - \sum_k r_k \sigma_{ki} - T_i(a)\right]^2}{\sigma^2_{0i}} + \sum_k r_k^2.$$

and minimize with respect to both the normalized systematic shifts $\{r_k\}$ and the theory parameters.

CT14 NNLO v.s. fitted data





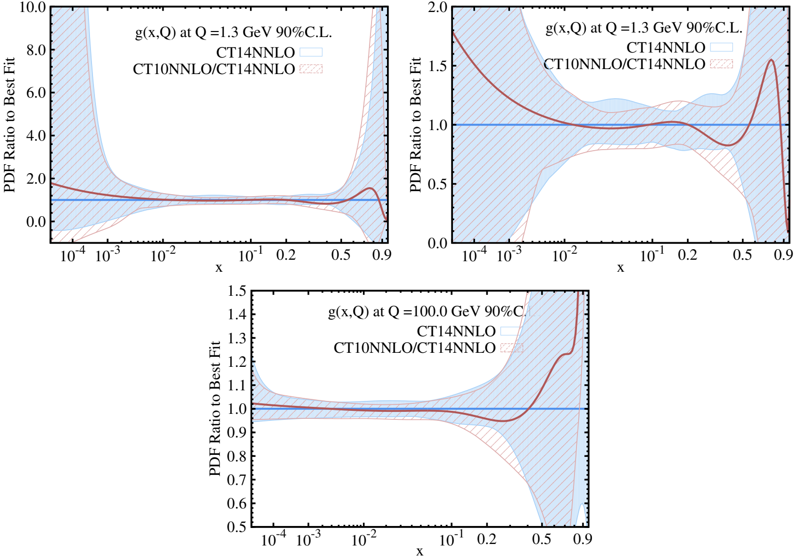


Fit well: $\chi^2/N_{pt} = 3252/2947 = 1.10$

CT14 NNLO PDFs

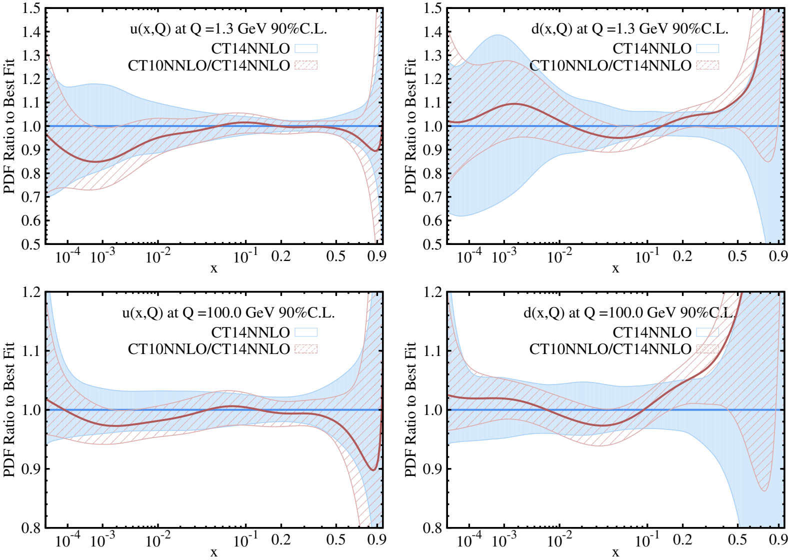
- PDF error bands
  - u and d PDFs are best known
  - currently, no strong constraint for x below 1E-4
  - large error for x above 0.3
  - large sea (e.g. $\bar{u}$ and $\bar{d}$) quark unceratinties in large x region
  - with non-perturbative parametrization form dependence in small and large x regions
- PDF eigensets
  - useful for calculating PDF induced uncertainty

CT14 and CT10 NNLO PDFs



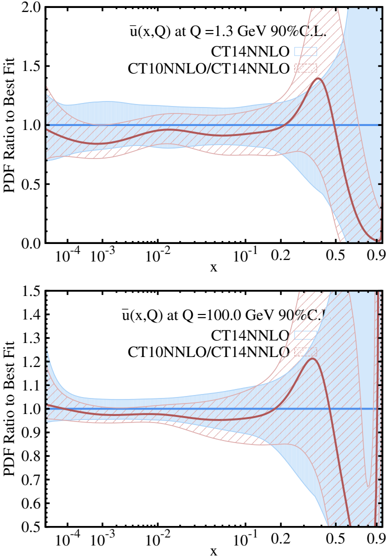
CT14 and CT10 NNLO PDFs

$u(x, Q)$ $d(x, Q)$



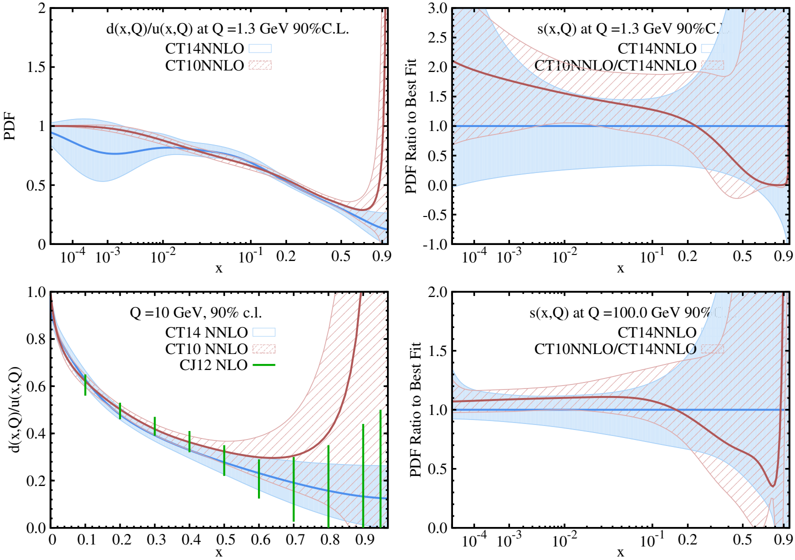
CT14 and CT10 NNLO PDFs

$\bar{u}(x, Q)$ $\bar{d}(x, Q)$



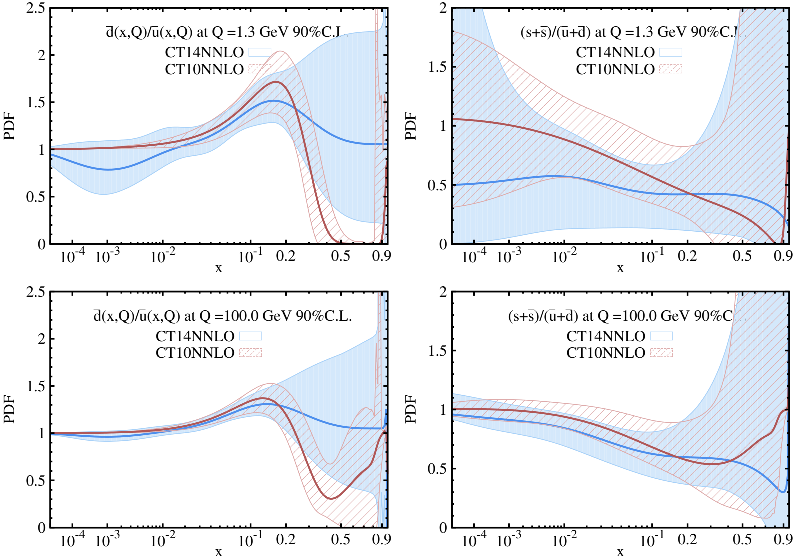
CT14 and CT10 NNLO PDFs

$d(x,Q)/u(x,Q)$ $s(x,Q)$



CT14 and CT10 NNLO PDFs

$\bar{d}(x,Q)/\bar{u}(x,Q)$ $\quad$ $(s+\bar{s})/(\bar{u}+\bar{d})$



NNLO cross section and PDF induced uncertainty for $gg \to H$

$$\Delta\sigma_{\text{PDF}} = \frac{1}{2}\sqrt{\sum_{i=1}^{n}\left(\sigma_i^{(+)} - \sigma_i^{(-)}\right)^2},\ \Delta\sigma_{\alpha_s} = \frac{1}{2}\sqrt{(\sigma(\alpha_s = 0.116) - \sigma(\alpha_s = 0.120)^2}$$

$$(\Delta\sigma)^2 = (\sigma_{\text{PDF}})^2 + (\sigma_{\alpha_s})^2$$

$gg \to H$ (pb), PDF unc.	8 TeV	13 TeV
68% C.L. (Hessian)	$18.7 + 2.1\% - 2.3\%$	$42.7 + 2.0\% - 2.4\%$
68% C.L. (LM)	$+2.3\% - 2.3\%$	$+2.4\% - 2.5\%$
$gg \to H$ (pb), PDF+α_s	8 TeV	13 TeV
68% C.L. (Hessian)	$18.7 + 2.9\% - 3.0\%$	$42.7 + 3.0\% - 3.2\%$
68.0% C.L. (LM)	$+3.0\% - 2.9\%$	$+3.2\% - 3.1\%$

Table 1: Uncertainties of $\sigma_H(gg \to H)$ computed by the LM method and by the Hessian method, with Tier-2 penalty included. The 68% C.L. errors are given as percentage of the central value, and the PDF-only uncertainties are for $\alpha_s = 0.118$.

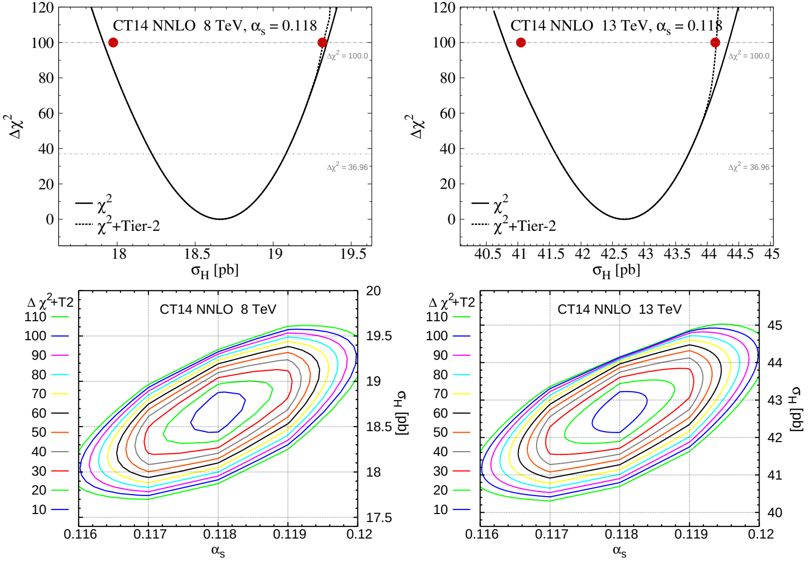
PDF uncertainties in $gg \to H$

- Most analyses use Hessian Method (n error PDF sets)

$$(\delta X)^2 = \frac{1}{4}\sum_{k=1}^{n}(X(a_k^+) - X(a_k^-))^2$$

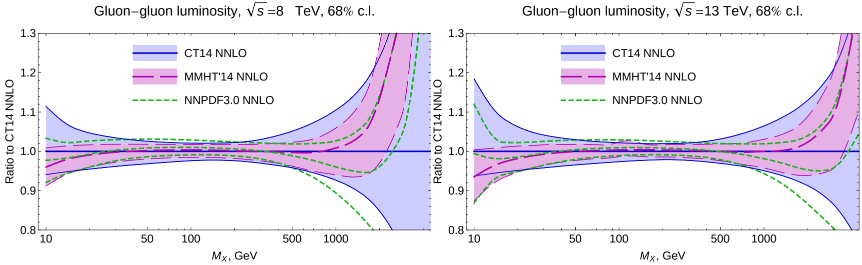
  - Error sets can be used by anyone for any observable
  - Assume quadratic and linear dependence of χ^2, X on a_k
- Lagrange Multiplier (LM) method is more robust
  - Find best fit for each constrained value of observable X
  - No assumptions on dependence of χ^2, X on a_k
  - Can validate Hessian method
  - Can display correlations between PDFs and Observable
  - Must calculate separately for each observable

Lagrange Multiplier method in $gg \to H$



PDF uncertainties in $gg \to H$

8 TeV 13 TeV



	CT14	MMHT2014	NNPDF3.0	CT10
8 TeV	$18.66^{+2.1\%}_{-2.3\%}$	$18.65^{+1.4\%}_{-1.9\%}$	$18.77^{+1.8\%}_{-1.8\%}$	$18.37^{+1.7\%}_{-2.1\%}$
13 TeV	$42.68^{+2.0\%}_{-2.4\%}$	$42.70^{+1.3\%}_{-1.8\%}$	$42.97^{+1.9\%}_{-1.9\%}$	$42.20^{+1.9\%}_{-2.5\%}$

Table 2: The Higgs boson production cross sections (in pb) for the gluon fusion channel at the LHC, at 8 and 13 TeV center-of-mass energies, using the CT14, MMHT2014, NNPDF3.0, and CT10 PDFs, with a common value of $\alpha_s(m_Z)$ of 0.118. The errors given are the PDF errors at the 68% confidence level.

t-$\bar{t}$ cross section

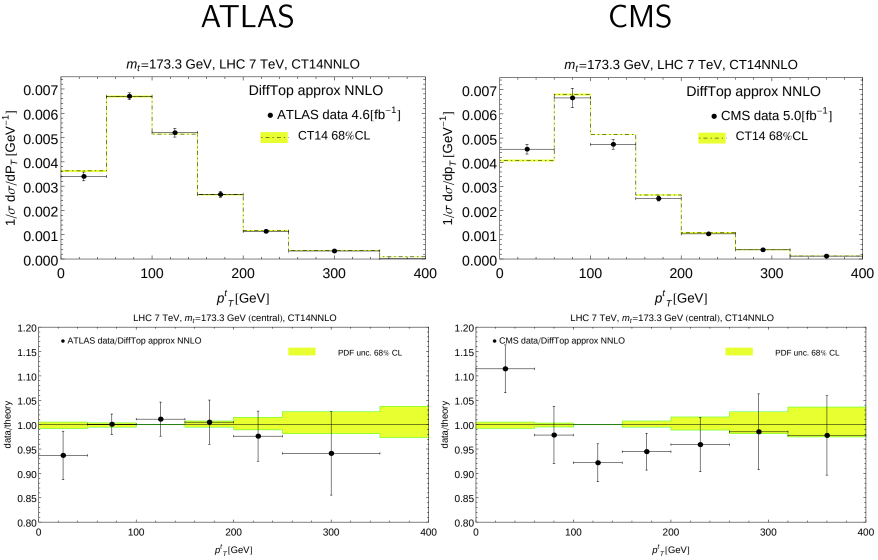


Figure 2: Final-state top-quark p_T differential distribution at ATLAS and CMS 7 TeV.

t-$\bar{t}$ cross section

$pp \to t\bar{t}$ (pb), PDF unc.	8 TeV	13 TeV
68% C.L. (top++2.0)	$250 + 3.9\% - 3.5\%$	$820 + 2.6\% - 2.7\%$
68% C.L. (LM)	$+4.8\% - 4.6\%$	$+2.9\% - 2.9\%$
$pp \to t\bar{t}$ (pb), PDF+α_s	8 TeV	13 TeV
68% C.L. (Hessian)	$+5.2\% - 4.4\%$	$+3.6\% - 3.5\%$
68% C.L. (LM)	$+5.1\% - 4.7\%$	$+3.6\% - 3.5\%$

Table 3: The $t\bar{t}$ total inclusive cross sections given in pb are evaluated at LHC center of mass energies of 8 and 13 and TeV with the TOP++2.0 code.

Not included in the global fit (and available only at NLO)

t-$\bar{t}$ cross section

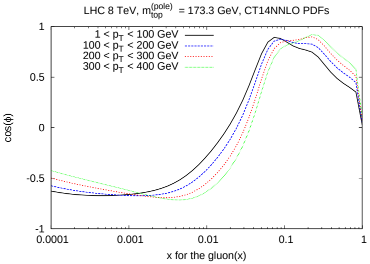


Figure 3: Theoretical correlation cosine as a function of x-gluon for the top-quark p_T distribution in $t\bar{t}$ production at the LHC at $\sqrt{s} = 8$ and 13 TeV.

$$\cos\varphi = \frac{\vec{\nabla} X \cdot \vec{\nabla} Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X\,\Delta Y} \sum_{i=1}^{N} \left(X_i^{(+)} - X_i^{(-)}\right)\left(Y_i^{(+)} - Y_i^{(-)}\right).$$

CMS W+c

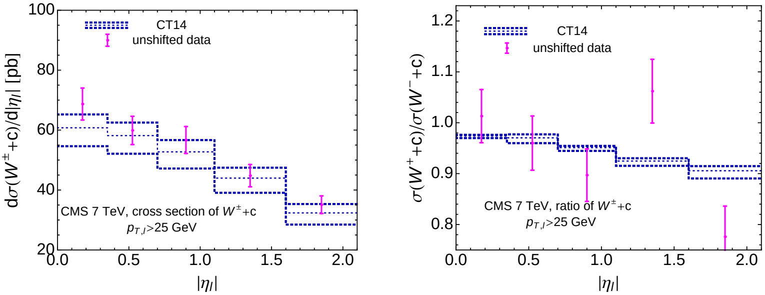
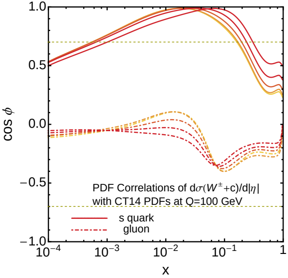
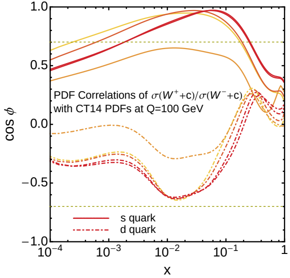


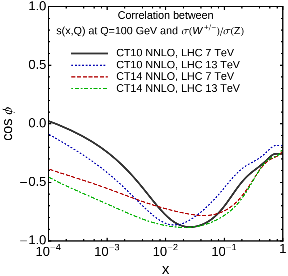
Figure 4: Comparison of the CT14 prediction for $W^{\pm} + c$ differential cross sections (left) and for the ratio of $W^{+} + \bar{c}$ to $W^{-} + c$ cross sections (right) from the CMS measurement at 7 TeV.

Not included in the global fit (and available only at NLO)

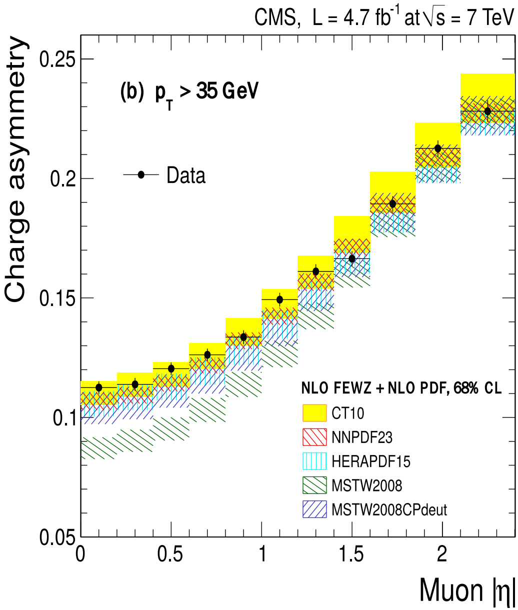
CMS W+c







The $W^{\pm} + c$ cross sections is more correlated to strange quark for $x = 0.01 - 0.1$.



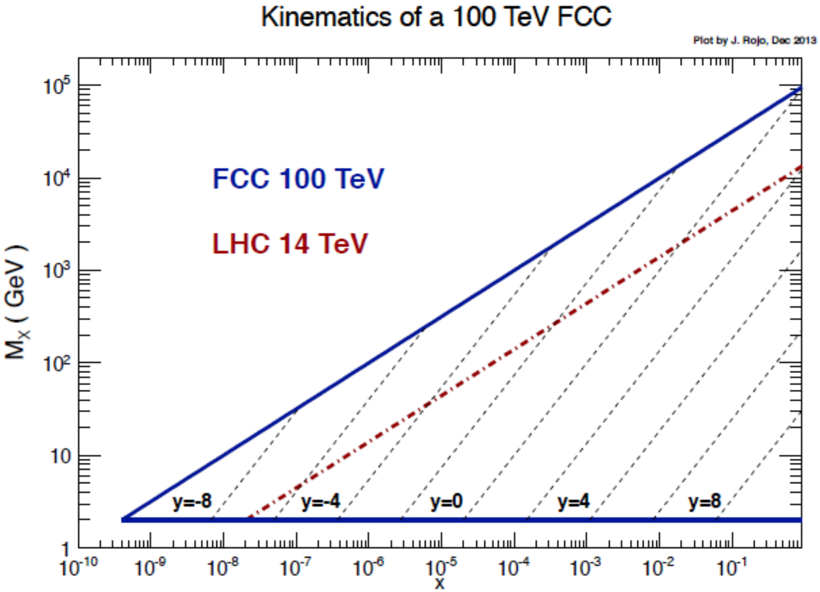
Data is already more precise than current PDF uncertainty.

Will help to determine PDFs in small x region.

Most useful for determining $\bar{d}/\bar{u}$.

PDFs for Future Hadron Colliders

- Parton distribution function $f(x, Q)$
- Given a heavy reasonance with mass Q produced at hardron collider with c.m. energy $\sqrt{s}$
- What's the typical x value? $\langle x \rangle = \frac{Q}{\sqrt{s}}$ at central rapidity ($y = 0$)
- Generally, $x_1 = \frac{Q}{\sqrt{s}} e^{y}$, $x_2 = \frac{Q}{\sqrt{s}} e^{-y}$, $x_1 + x_2 = 2\frac{Q}{\sqrt{s}} \cosh(y)$, therefore $x_1 + x_2 = 1$



J. Rojo: kickoff meeting for FCC at CERN, Feb. 2014

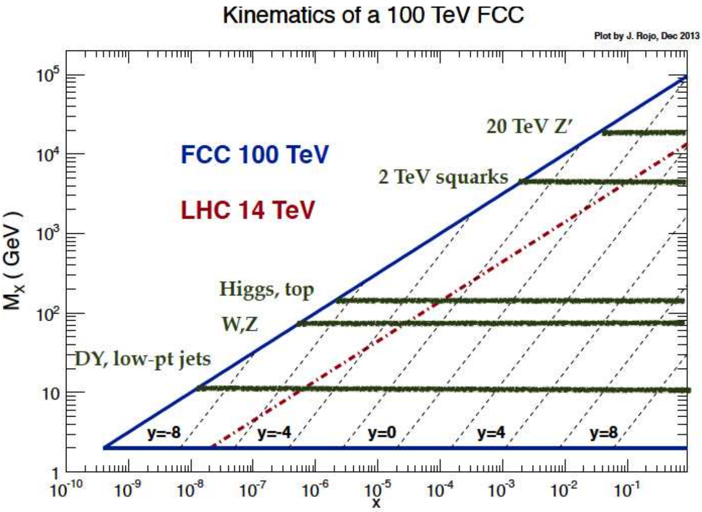
On to a 100 TeV SppC

will access smaller x, larger Q^2

currently have no constraints on PDFs for x values below 1E-4

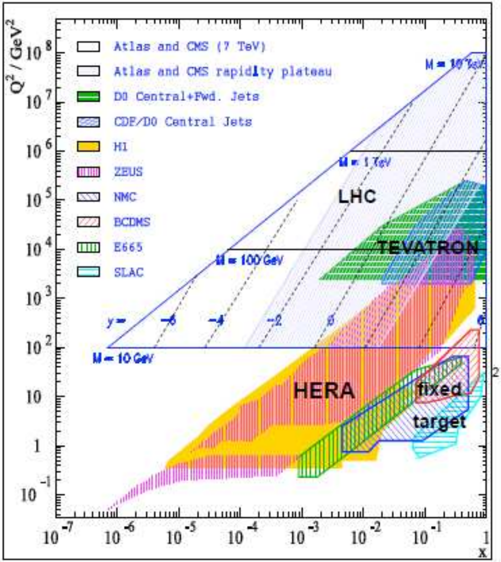
poor constraints (still) as well for high x PDFs

at high masses (Q^2), rely on DLAP evolution; we know at large Q^2, EW effects also become important

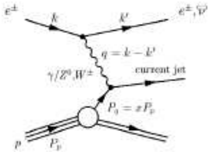


On to a 100 TeV SppC

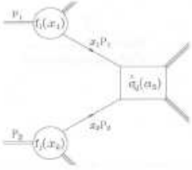
Experimental access to the proton structure



HERA: low and medium x



LHC: important constraints on g(x), flavour separation

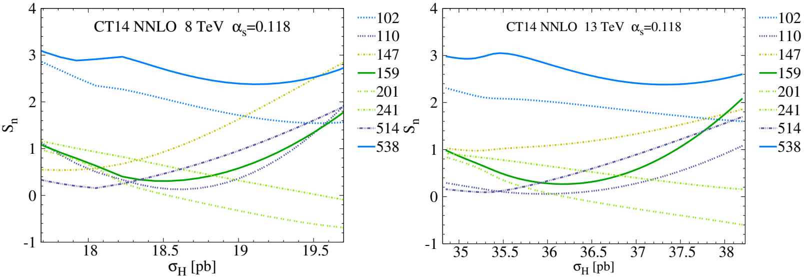


Fixed Target: high x, nuclear PDFs

Summary

- PDFs have larger uncertainties in both small x and large x regions.
- The eigen sets is useful for calculating the PDF induced uncertainty, and correlation between observables.
- PDFs will be further determined by LHC data.

Backup



The S_n is the effective Gaussian variable which present the goodness of fit of particular data.