Review of Parton Distribution Function

Tie-Jiun Hou
In collaboration with CTEQ-TEA

Southern Methodist University

August 04, DPF2015 at UM, Ann Arbor, MI
CTEQ-TEA group

- CTEQ – Tung et al. (TEA) in memory of Prof. Wu-Ki Tung, who established CTEQ Collaboration in early 90’s.
- Current members:
  Sayipjamal Dulat (Xinjiang U.)
  Tie-Jiun Hou, Pavel Nadolsky (Southern Methodist U.)
  Jun Gao (Argonne Nat. lab.)
  Marco Guuzzi (U. of Manchester)
  Joey Huston, Jon Pumplin, Carl Schmidt,
  Dan Stump, C.-P. Yuan (Michigan State U.)
Hadron Collider Physics

L.D. hadrons

S.D. partons, gauge bosons, new particles

L.D. leptons, hadrons

SM and New physics

(universal) parton Distributions

frag. functions

hadronization models: MC programs

jet algorithms
<table>
<thead>
<tr>
<th>JLab</th>
<th>HERA I+II</th>
<th>Tevatron new W,Z</th>
<th>LHC</th>
<th>di-μ</th>
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<td>HERAPDF2.0</td>
<td>Wichmann</td>
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<tr>
<td>CT14</td>
<td>Nadolski</td>
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<td>MMHT14</td>
<td>Thorne</td>
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<td>CJ12 * (→ CJ15)</td>
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<td>✔ (✓)</td>
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<td>ABM12 **</td>
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<td>✔</td>
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</tbody>
</table>

Table by A. Accardi, DIS’2015 workshop

- NLO only
- No jet data
- but see 1503.05221
- no reconstructed W
CT14 NNLO error PDFs

Available at http://hep.pa.msu.edu/cteq/public/index.html, is being submitted to LHAPDF;

- CT10 includes only pre-LHC data
- CT14 is the first CT analysis including LHC Run 1 data
- CT14 also includes the new Tevatron D0 Run 2 data on W-electron charge asymmetry
- CT14 uses a more flexible parametrization in the non-perturbative PDFs.
Figure 1: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \bar{u}, d, \bar{d}, s = \bar{s}$, and $g$. 
DGLAP evolution for PDFs

- Non-perturbative PDFs are determined at the scale of $Q=1.3$ GeV.
- PDFs at any other scale $Q$ can be obtained from PQCD, via solving DGLAP evolution equations.
- Due to DGLAP evolution, the PDF error band becomes smaller when the energy scale $Q$ increases.
- The evolution effect is large in low $Q$ region, say, from 1.3 GeV to 8 GeV.
The CT14-NNLO global analysis of QCD

**Parametrization** of PDFs $Q = 1.3$ GeV, with 28 parameter values to be chosen; there are from 4 to 6 parameters for each proton type.

**Many data sets**, for short distance interactions.

**Perturbative QCD**, using NNLO approximations whenever available.

Taking account of **experimental errors**, statistical and systematic. (Not so strong on systematic **theoretical errors**).
<table>
<thead>
<tr>
<th>ID#</th>
<th>Experimental data set</th>
<th>$N_{pt}$</th>
<th>$\chi^2_e$</th>
<th>$\chi^2_e/N_{pt}$</th>
<th>$S_n$</th>
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<td>101</td>
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<td>NuTeV $\nu_{\mu\mu}$ SIDIS</td>
<td>38</td>
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<td>20</td>
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<td>H1 $\sigma_b^r$</td>
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<td>6.8</td>
<td>0.68</td>
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<td>Combined HERA charm production</td>
<td>47</td>
<td>59</td>
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<tr>
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<td>HERA1 Combined NC and CC DIS</td>
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<td>591</td>
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<tr>
<td>169</td>
<td>H1 $F_L$</td>
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<td>1.92</td>
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<td>E866 Drell-Yan process, $\sigma_{pd}/(2\sigma_{pp})$</td>
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<td>184</td>
<td>252</td>
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<td>CDF Run-1 electron $A_{ch}, p_{T\ell}$ &gt; 25 GeV</td>
<td>11</td>
<td>8.9</td>
<td>0.81</td>
<td>-0.32</td>
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<tr>
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<td>CDF Run-2 electron $A_{ch}, p_{T\ell}$ &gt; 25 GeV</td>
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<td>14</td>
<td>1.24</td>
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<td>DØ Run-2 muon $A_{ch}, p_{T\ell}$ &gt; 20 GeV</td>
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<td>8.3</td>
<td>0.92</td>
<td>-0.02</td>
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<tr>
<td>240</td>
<td>LHCb 7 TeV 35 pb$^{-1}$ $W/Z$ $d\sigma/dy_\ell$</td>
<td>14</td>
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<td>0.71</td>
<td>-0.73</td>
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<td>DØ Run-2 Z rapidity</td>
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<td>17</td>
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<tr>
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<td>CDF Run-2 Z rapidity</td>
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<td>266</td>
<td>CMS 7 TeV 4.7 fb$^{-1}$, muon $A_{ch}, p_{T\ell}$ &gt; 35 GeV</td>
<td>11</td>
<td>12.1</td>
<td>1.10</td>
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<td>10.1</td>
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<td>-0.06</td>
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<td>ATLAS 7 TeV 35 pb$^{-1}$ $W/Z$ cross sec., $A_{ch}$</td>
<td>41</td>
<td>51</td>
<td>1.25</td>
<td>1.11</td>
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<td>281</td>
<td>DØ Run-2 9.7 fb$^{-1}$ electron $A_{ch}, p_{T\ell}$ &gt; 25 GeV</td>
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<td>2.67</td>
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<tr>
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<td>DØ Run-2 inclusive jet production</td>
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<td>120</td>
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<tr>
<td>535</td>
<td>ATLAS 7 TeV 35 pb$^{-1}$ incl. jet production</td>
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<td>50</td>
<td>0.55</td>
<td>-3.59</td>
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<tr>
<td>538</td>
<td>CMS 7 TeV 5 fb$^{-1}$ incl. jet production</td>
<td>133</td>
<td>177</td>
<td>1.33</td>
<td>2.51</td>
</tr>
</tbody>
</table>

34 data sets used for the CT14-NNLO global analysis.
Experimental uncertainty

An experiment publishes \( N \) measurements,

\[
\{ M_i; i = 1, 2, 3, \ldots, N \}
\]

Each measurement has several parts,

\[
M_i = \{ D_i; \sigma_0i; \{ \sigma_1i, \sigma_2i, \sigma_3i, \ldots \} \}
\]

that is,

\[
D_i = \text{True}_i + \sigma_0i \cdot r_0 + \sum_{k=1}^{N_{sy}} \sigma_{ki} \cdot r_k
\]

Where \( r_0 \) and \( \{ r_k \} \) are random variables (gaussian?). Define

\[
\chi^2_{\text{global}} = \sum_i \left[ \frac{D_i - \sum_k r_k \sigma_{ki} - T_i(a)}{\sigma^2_{0i}} \right]^2 + \sum_k r_k^2.
\]

and minimize with respect to both the normalized systematic shifts \( \{ r_k \} \) and the theory parameters.
Fit well: $\chi^2/N_{pt} = 3252/2947 = 1.10$
CT14 NNLO PDFs

- PDF error bands
  - $u$ and $d$ PDFs are best known
  - currently, no strong constraint for $x$ below 1E-4
  - large error for $x$ above 0.3
  - large sea (e.g. $\bar{u}$ and $\bar{d}$) quark uncertainties in large $x$ region
  - with non-perturbative parametrization form dependence in small and large $x$ regions

- PDF eigensets
  - useful for calculating PDF induced uncertainty
CT14 and CT10 NNLO PDFs

PDF Ratio to Best Fit

$g(x,Q)$ at $Q = 1.3$ GeV 90\%C.L.

- CT14NNLO
- CT10NNLO
- CT10NNLO/CT14NNLO

PDF Ratio to Best Fit

$g(x,Q)$ at $Q = 100.0$ GeV 90\%C.L.

- CT14NNLO
- CT10NNLO
- CT10NNLO/CT14NNLO
CT14 and CT10 NNLO PDFs

$u(x, Q)$

PDF Ratio to Best Fit

$u(x, Q)$ at $Q = 1.3$ GeV 90\% C.L.

CT14NNLO

CT10NNLO/CT14NNLO

$0.5$

$0.6$

$0.7$

$0.8$

$0.9$

$1.0$

$1.1$

$1.2$

$1.3$

$1.4$

$10^{-4}$

$10^{-3}$

$10^{-2}$

$10^{-1}$

$10^{0}$

$x$

$10^{0.2}$

$10^{0.3}$

$10^{0.4}$

$10^{0.5}$

$10^{0.6}$

$10^{0.7}$

$10^{0.8}$

$10^{0.9}$

PDF Ratio to Best Fit

$d(x, Q)$

PDF Ratio to Best Fit

d(x, Q) at $Q = 1.3$ GeV 90\% C.L.

CT14NNLO

CT10NNLO/CT14NNLO

$0.8$

$0.9$

$1.0$

$1.1$

$1.2$

$10^{-4}$

$10^{-3}$

$10^{-2}$

$10^{-1}$

$10^{0}$

$x$

$10^{0.2}$

$10^{0.3}$

$10^{0.4}$

$10^{0.5}$

$10^{0.6}$

$10^{0.7}$

$10^{0.8}$

PDF Ratio to Best Fit

PDF Ratio to Best Fit

d(x, Q) at $Q = 100.0$ GeV 90\% C.L.

CT14NNLO

CT10NNLO/CT14NNLO

$0.8$

$0.9$

$1.0$

$1.1$

$1.2$

$10^{-4}$

$10^{-3}$

$10^{-2}$

$10^{-1}$

$10^{0}$

$x$

$10^{0.2}$

$10^{0.3}$

$10^{0.4}$

$10^{0.5}$

$10^{0.6}$

$10^{0.7}$

$10^{0.8}$
CT14 and CT10 NNLO PDFs

$\bar{u}(x, Q)$

$\bar{d}(x, Q)$
CT14 and CT10 NNLO PDFs

\[ \frac{d(x, Q)}{u(x, Q)} \]

PDF

\[ s(x, Q) \]

PDF Ratio to Best Fit

\[ \frac{s(x, Q)}{s(x, Q)} \text{ at } Q = 1.3 \text{ GeV 90\% C.L.} \]

CT14 NNLO

CT10 NNLO

PDF

\[ \frac{d(x, Q)}{u(x, Q)} \text{ at } Q = 1.3 \text{ GeV 90\% C.L.} \]

CT14 NNLO

CT10 NNLO

PDF Ratio to Best Fit

\[ \frac{s(x, Q)}{s(x, Q)} \text{ at } Q = 1.3 \text{ GeV 90\% C.L.} \]

CT14 NNLO

CT10 NNLO/CT14 NNLO

PDF

\[ \frac{d(x, Q)}{u(x, Q)} \text{ at } Q = 10 \text{ GeV, 90\% c.l.} \]

CT14 NNLO

CT10 NNLO

CJ12 NLO

PDF Ratio to Best Fit

\[ \frac{s(x, Q)}{s(x, Q)} \text{ at } Q = 100.0 \text{ GeV 90\% C.L.} \]

CT14 NNLO

CT10 NNLO/CT14 NNLO

PDF

\[ \frac{d(x, Q)}{u(x, Q)} \text{ at } Q = 10 \text{ GeV, 90\% c.l.} \]

CT14 NNLO

CT10 NNLO

CJ12 NLO

PDF Ratio to Best Fit

\[ \frac{s(x, Q)}{s(x, Q)} \text{ at } Q = 100.0 \text{ GeV 90\% C.L.} \]

CT14 NNLO

CT10 NNLO/CT14 NNLO

PDF
CT14 and CT10 NNLO PDFs

\[
\frac{d(x, Q)}{u(x, Q)} \quad \text{and} \quad \frac{(s + \bar{s})}{(u + d)}
\]

PDF plots at \( Q = 1.3 \) GeV and \( Q = 100.0 \) GeV, 90\% C.L.
NNLO cross section and PDF induced uncertainty for \( gg \rightarrow H \)

\[
\Delta \sigma_{\text{PDF}} = \frac{1}{2} \sqrt{\sum_{i=1}^{n} \left( \sigma_i^{(+)} - \sigma_i^{(-)} \right)^2}, \quad \Delta \sigma_{\alpha_s} = \frac{1}{2} \sqrt{(\sigma(\alpha_s = 0.116) - \sigma(\alpha_s = 0.120))^2}
\]

\[(\Delta \sigma)^2 = (\sigma_{\text{PDF}})^2 + (\sigma_{\alpha_s})^2\]

<table>
<thead>
<tr>
<th>( gg \rightarrow H ) (pb), PDF unc.</th>
<th>8 TeV</th>
<th>13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>68% C.L. (Hessian)</td>
<td>18.7 + 2.1% − 2.3%</td>
<td>42.7 + 2.0% − 2.4%</td>
</tr>
<tr>
<td>68% C.L. (LM)</td>
<td>+2.3% − 2.3%</td>
<td>+2.4% − 2.5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( gg \rightarrow H ) (pb), PDF+( \alpha_s )</th>
<th>8 TeV</th>
<th>13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>68% C.L. (Hessian)</td>
<td>18.7 + 2.9% − 3.0%</td>
<td>42.7 + 3.0% − 3.2%</td>
</tr>
<tr>
<td>68.0% C.L. (LM)</td>
<td>+3.0% − 2.9%</td>
<td>+3.2% − 3.1%</td>
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</tbody>
</table>

Table 1: Uncertainties of \( \sigma_H(gg \rightarrow H) \) computed by the LM method and by the Hessian method, with Tier-2 penalty included. The 68% C.L. errors are given as percentage of the central value, and the PDF-only uncertainties are for \( \alpha_s = 0.118 \).
PDF uncertainties in $gg \to H$

- Most analyses use Hessian Method (n error PDF sets)

\[
(\delta X)^2 = \frac{1}{4} \sum_{k=1}^{n} (X(a_k^+) - X(a_k^-))^2
\]

- Error sets can be used by anyone for any observable
- Assume quadratic and linear dependence of $\chi^2$, $X$ on $a_k$
- Lagrange Multiplier (LM) method is more robust
  - Find best fit for each constrained value of observable $X$
  - No assumptions on dependence of $\chi^2$, $X$ on $a_k$
  - Can validate Hessian method
  - Can display correlations between PDFs and Observable
  - Must calculate separately for each observable
Lagrange Multiplier method in $gg \rightarrow H$

CT14 NNLO 8 TeV, $\alpha_s = 0.118$

$\Delta \chi^2 = 100.0$

$\Delta \chi^2 = 36.96$

CT14 NNLO 13 TeV, $\alpha_s = 0.118$

$\Delta \chi^2 = 100.0$

$\Delta \chi^2 = 36.96$
PDF uncertainties in $gg \rightarrow H$

8 TeV

Gluon–gluon luminosity, $\sqrt{s}=8$ TeV, 68% c.l.

13 TeV

Gluon–gluon luminosity, $\sqrt{s}=13$ TeV, 68% c.l.

Table 2: The Higgs boson production cross sections (in pb) for the gluon fusion channel at the LHC, at 8 and 13 TeV center-of-mass energies, using the CT14, MMHT2014, NNPDF3.0, and CT10 PDFs, with a common value of $\alpha_s(m_Z)$ of 0.118. The errors given are the PDF errors at the 68% confidence level.
Figure 2: Final-state top-quark $p_T$ differential distribution at ATLAS and CMS 7 TeV.
### $t\bar{t}$ cross section

<table>
<thead>
<tr>
<th>$pp \rightarrow t\bar{t}$ (pb), PDF unc.</th>
<th>8 TeV</th>
<th>13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>68% C.L. (top++2.0)</td>
<td>$250 \pm 3.9% - 3.5%$</td>
<td>$820 \pm 2.6% - 2.7%$</td>
</tr>
<tr>
<td>68% C.L. (LM)</td>
<td>$+4.8% - 4.6%$</td>
<td>$+2.9% - 2.9%$</td>
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</table>

<table>
<thead>
<tr>
<th>$pp \rightarrow t\bar{t}$ (pb), PDF+$\alpha_s$</th>
<th>8 TeV</th>
<th>13 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>68% C.L. (Hessian)</td>
<td>$+5.2% - 4.4%$</td>
<td>$+3.6% - 3.5%$</td>
</tr>
<tr>
<td>68% C.L. (LM)</td>
<td>$+5.1% - 4.7%$</td>
<td>$+3.6% - 3.5%$</td>
</tr>
</tbody>
</table>

**Table 3:** The $t\bar{t}$ total inclusive cross sections given in pb are evaluated at LHC center of mass energies of 8 and 13 and TeV with the Top++2.0 code.

Not included in the global fit (and available only at NLO)
$t\bar{t}$ cross section

Figure 3: Theoretical correlation cosine as a function of $x$-gluon for the top-quark $p_T$ distribution in $t\bar{t}$ production in the LHC at $\sqrt{s} = 8$ and 13 TeV.

$$\cos \varphi = \frac{\vec{\nabla} X \cdot \vec{\nabla} Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X \Delta Y} \sum_{i=1}^{N} \left( X_i^{(+)} - X_i^{(-)} \right) \left( Y_i^{(+)} - Y_i^{(-)} \right).$$
Figure 4: Comparison of the CT14 prediction for $W^{\pm} + c$ differential cross sections (left) and for the ratio of $W^{+} + \bar{c}$ to $W^{-} + c$ cross sections (right) from the CMS measurement at 7 TeV.

Not included in the global fit (and available only at NLO)
The $W^\pm + c$ cross sections is more correlated to strange quark for $x = 0.01 - 0.1$. 
Data is already more precise than current PDF uncertainty.

Will help to determine PDFs in small $x$ region.

Most useful for determining $\bar{d}/\bar{u}$. 
PDFs for Future Hadron Colliders

- Parton distribution function $f(x, Q)$
- Given a heavy resonance with mass $Q$ produced at a hadron collider with c.m. energy $\sqrt{s}$
- What’s the typical $x$ value? $\langle x \rangle = \frac{Q}{\sqrt{s}}$ at central rapidity ($y = 0$)
- Generally, $x_1 = \frac{Q}{\sqrt{s}} e^y$, $x_2 = \frac{Q}{\sqrt{s}} e^{-y}$, $x_1 + x_2 = 2 \frac{Q}{\sqrt{s}} \cosh(y)$, therefore $x_1 + x_2 = 1$
Kinematics of a 100 TeV FCC

J. Rojo: kickoff meeting for FCC at CERN, Feb. 2014
On to a 100 TeV SppC

will access smaller $x$, larger $Q^2$

currently have no constraints on PDFs for $x$ values below $1E-4$

poor constraints (still) as well for high $x$ PDFs

at high masses ($Q^2$), rely on DLAP evolution; we know at large $Q^2$, EW effects also become important
On to a 100 TeV SppC

Experimental access to the proton structure
Summary

- PDFs have larger uncertainties in both small $x$ and large $x$ regions.
- The eigen sets is useful for calculating the PDF induced uncertainty, and correlation between observables.
- PDFs will be further determined by LHC data.
The $S_n$ is the effective Gaussian variable which presents the goodness of fit of particular data.