Axion Stars (and Bose-Einstein Condensate Dark Matter)

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with
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For more information, see arXiv: 1412.3430

DPF 2015,
University of Michigan, Ann Arbor

August 6, 2015

Outline

- Motivation: Because Dark Matter (?)
  - More generally, astrophysics of scalar fields
- Boson Stars: Gravitationally bound states of scalar fields
  - History
  - Our Method
  - Results
- Open Questions and Future Work

Original Motivation: Dark Matter

The nature of Dark Matter (DM) is one of the biggest questions in physics.

?

Identity Crisis

Light scalar DM, e.g. axions, a viable paradigm: can form BEC states of large sizes ("Boson Stars") which could compose DM.

What We Want To Know

How large and how massive are these condensates?
Are they stable?
How do they form?
How can we detect them?

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Boson Star History

- Wheeler, "Geons" (1956)
  - Macroscopic bound states of photons

Boson Star History

- Wheeler, "Geons" (1956)
- Kaup, "Klein-Gordon Geon" (1968)
  - Solved Einstein+Klein-Gordon (EKG) equations numerically for a free complex scalar field
  - Maximum mass for bound states: $M_{max}^{free} = .633 M_P^2/m$

Chandrasekhar limit: $M_{max} \sim M_P^3/m^2$

Boson Star History

- Wheeler, "Geons" (1956)
- Kaup, "Klein-Gordon Geon" (1968)
- Ruffini and Bonazzola, "Systems of Self-Gravitating Particles in General Relativity..." (1969)
  - Second-quantized a free real scalar field, put all N particles in ground state $|N\rangle$
  - Ground state expectation value of EKG equations
  - Similar results to Kaup, but method is new (more on this later)

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- Colpi, Shapiro, Wasserman, "Boson Stars: ..." (1986)
  - Solved EKG system for complex scalar with $V_{int} = +\lambda\phi^4$
  - Maximum mass $M_{max}^{int} = .062\sqrt{\lambda}M_P^3/m^2$
  - Very different from Kaup: $M_{max}^{free} = .633M_P^2/m$

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- Barranco and Bernal, "Self-Gravitating System Made of Axions" (2011)
  - Axion potential:
    $V(\phi) = m^2 f^2 \left(1 - \cos(\frac{\phi}{f})\right) \approx \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + \ldots$
  - Used RB method to quantize field, found numerical solutions with sizes $R \sim 1 - 10$ m and masses $M \sim 10^{13} - 10^{14}$ kg.

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  - Used RB method to quantize field, found numerical solutions with sizes $R \sim 1 - 10$ m and masses $M \sim 10^{13} - 10^{14}$ kg.
  - ?: The corresponding Kaup mass is $M \sim 10^{20}$ kg for $m \sim 10^{-5}$ eV. Why are BB solutions so small?

Ruffini-Bonazzola Method

Consider in greater detail the RB method:

1. Begin with a canonically normalized second-quantized scalar field

$$\phi = \sum_{n,l,m} R_{n,l}(r)\left[e^{iE_{n,l}t} Y_l^m(\theta,\phi) a_{n,l,m} + h.c.\right] \tag{1}$$

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$$|N\rangle = \frac{\left(a^\dagger_{1,0,0}\right)^N}{\sqrt{N!}}|0\rangle \tag{2}$$

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3. Evaluate expectation value of EKG equations

$$\begin{aligned} \langle N|G^{\mu\nu}|N\rangle &= \frac{1}{M_P^2}\langle N|T^{\mu\nu}|N\rangle \\ \langle N-1|\left[\Box\phi - \frac{1}{2}W'(\phi)\right]|N\rangle &= 0 \end{aligned} \tag{3}$$

Ruffini-Bonazzola Method

The self-gravity of the scalar field perturbs the metric

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\Omega^2 \tag{4}$$

Find three coupled equations for $R(r) \equiv R_{1,0,0}(r)$, $A(r)$, and $B(r)$:

$$\begin{aligned}
\frac{A'}{A^2\, r} + \frac{A-1}{A\, r^2} &= \frac{1}{M_P{}^2}\left[\frac{E^2\, N\, R^2}{B} + \frac{N\, R'^2}{A} + \langle N|W(\phi)|N\rangle\right] \\
\frac{B'}{A\, B\, r} - \frac{A-1}{A\, r^2} &= -\frac{1}{M_P{}^2}\left[\frac{E^2\, N\, R^2}{B} + \frac{N\, R'^2}{A} - \langle N|W(\phi)|N\rangle\right] \\
\sqrt{N}\, R'' + \sqrt{N}\left(\frac{2}{r} + \frac{B'}{2\, B} - \frac{A'}{2\, A}\right) R' & \\
+ A\left[\frac{\sqrt{N}\, E^2}{B} R - \langle N-1|W'(\phi)|N\rangle\right] &= 0
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Limitations of RB Method

We consider the axion potential, $W(\phi) = m^2 f^2 \Big(1 - cos(\frac{\phi}{f})\Big)$

1. We quantize the field using a flat metric background. Large metric deviations would imply an ill-defined N-particle state.

$\Rightarrow$ We must assume <u>weak GR effects</u>.

Expand metric functions
$A(r) = 1 - \delta a(r)$, $B(r) = 1 - \delta b(r)$ with

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   $$\delta \equiv f^2/M_P^2 \ll 1.$$

2. Quantization is tree-level only, and would be modified by loop effects and pair production if interactions are strong.

   $\Rightarrow$ We must assume small binding energies.

   $$\Delta \equiv \sqrt{1 - E^2/m^2} \ll 1$$

The Infrared Limit

We consider the axion potential, $W(\phi) = m^2 f^2 \Big(1 - cos(\frac{\phi}{f})\Big)$
Need to evaluate the expectation values (with $X(r) \equiv 2\sqrt{N} R(r)/f$):

$$\begin{aligned}\langle N|W(\phi)|N\rangle &= m^2 f^2 \Big(1 - J_0(X)\Big) \\ &= \frac{m^2 f^2}{4}\Big(X^2 - \frac{1}{16}X^4 + \mathcal{O}(X^6)\Big)\end{aligned}$$

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2. Taking $X(r) = \Delta Y(x)$ (with $x = \Delta m r$):
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Similar considerations for KG equation, where we have $\langle N-1|W'(\phi)|N\rangle = m^2 f\, J_1(X) = \frac{m^2 f}{2}\Big(X - \frac{1}{8}X^3 + \mathcal{O}(X^5)\Big)$

Leading Order Equations

To leading order in δ and Δ we find simplified set of equations

What We Solved

$$a'(x) = \frac{x}{2}\,Y(x)^2 - \frac{a(x)}{x}$$

$$b'(x) = \frac{a(x)}{x}$$

$$Y''(x) = Y(x) - \frac{2}{x}Y'(x) - \frac{1}{8}\,Y(x)^3 + \lambda^2\,b(x)\,Y(x) \qquad (6)$$

Depends on only one free parameter, $\lambda \equiv \sqrt{\delta}/\Delta$.

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Note: Leading corrections are $\mathcal{O}(\delta)$ and $\mathcal{O}(\lambda^2\delta)$.

Requiring $\Delta \ll 1$ and $\lambda^2\delta \ll 1$ gives the range of validity:

$$\frac{f}{M_P} \ll \lambda \ll \frac{M_P}{f}$$

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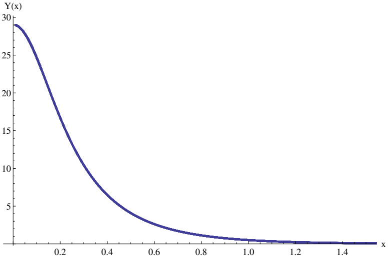
$$10^{-7} \sim \frac{f}{M_P} \ll \lambda \ll \frac{M_P}{f} \sim 10^7$$

for QCD axions

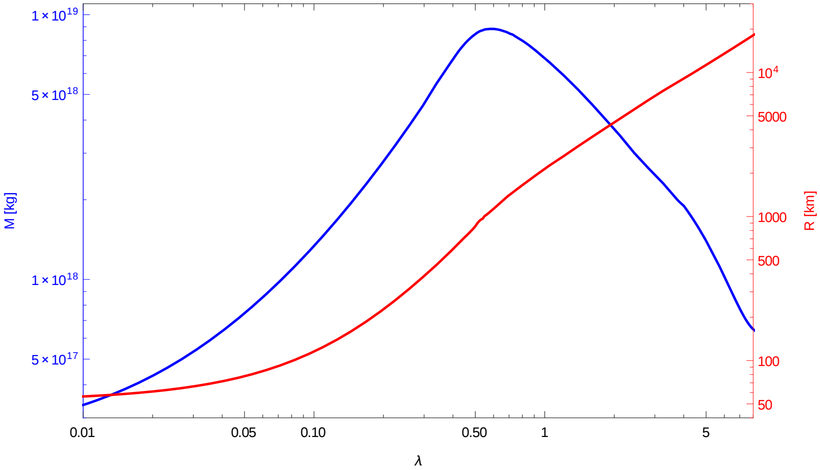
Solutions



λ	M (kg)	R_{99} (km)	d (kg / m^3)	δM (kg)
0.1	1.34×10^{18}	115	207	167000.
0.3	4.61×10^{18}	386	19.1	61700.
0.4	6.78×10^{18}	593	7.74	44700.
0.5	8.44×10^{18}	854	3.24	21700.
0.54	8.74×10^{18}	972	2.27	11100.
0.58	8.84×10^{18}	1076	1.69	1570.
0.62	8.81×10^{18}	1183	1.27	−7160.
0.8	7.98×10^{18}	1652	0.422	−30900.
1	6.85×10^{18}	2145	0.166	−44100.
2	3.71×10^{18}	4499	0.0097	−71200.
4	1.9×10^{18}	9062	0.0006	−11800.
10	7.65×10^{17}	22849	0.000015	−355000.

- We find wavefunction solutions over several orders of magnitude in λ with very similar shape
- The parameter λ uniquely determines M, R, and N (for a given m, f)

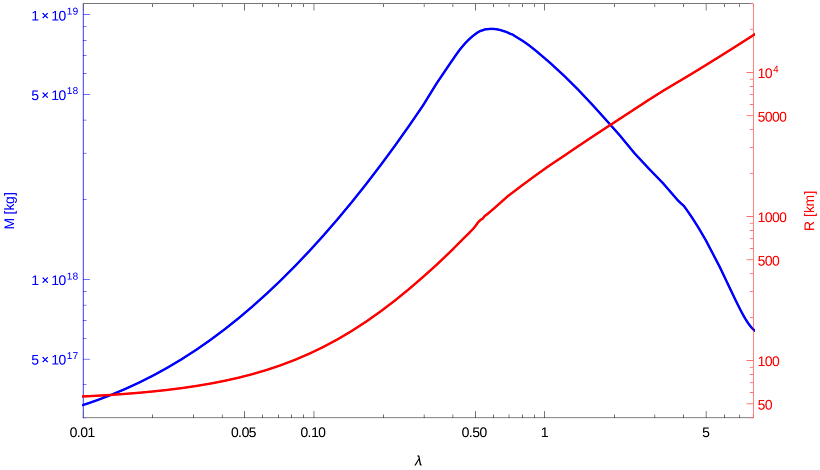
Macroscopic Properties



(For $m = 10^{-5}$ eV and $f = 6 \times 10^{11}$ GeV)

Macroscopic Properties

← Barranco and Bernal solutions correspond to $\lambda \sim 10^{-6}$



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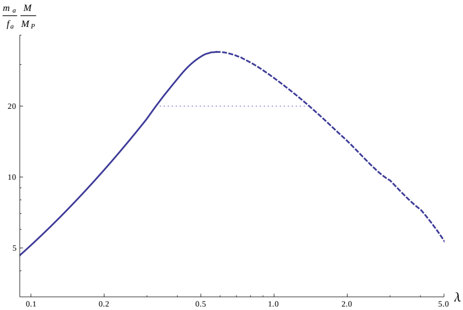
Decays of Axion Stars

!: For a given N, $\exists$ two states S_1 and S_2 with $M_1 < M_2$
$\Rightarrow S_2$ can decay to S_1.



Using $M(\Delta) = Nm\sqrt{1-\Delta^2}$, we can estimate the mass difference

$$\frac{\delta M}{M} \approx \frac{1}{2}\left(\Delta_1^2 - \Delta_2^2\right) = \frac{1}{2}\left(\lambda_1^{-2} - \lambda_2^{-2}\right)\delta$$

Mass difference typically a small fraction of total mass, but still large amount of energy: typically $\gtrsim 1000$ kg!

Conclusions

- Our expansion in δ and Δ elucidates properties of axion stars which were unknown or unclear previously
  - The maximum mass is related to a stable binding energy
  - The RB method is *inherently* limited to $\delta, \Delta \ll 1$
  - Higher-order terms in $V(\phi)$ expansion are suppressed by extra powers of Δ (irrelevant operators)
- The maximum mass of axion stars is $\mathcal{O}(10^{19})$ kg with $R_{99} \sim 1000$ km for $m = 10^{-5}$ eV and $f = 6 \times 10^{11}$ GeV
  - At fixed $mf = \Lambda^2$, masses M change proportionally to f but sizes R are constant with f
- This method can be generalized to many classes of axions (and possibly other potentials), provided $f \ll M_P$ and $(m - E)/m \ll 1$ are satisfied
- Thanks!

References

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Backup Slides

Backup: Strong CP Problem

The θ-term in the QCD Lagrangian violates CP:

$$\mathcal{L}_{QCD} \ni \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu} \tag{7}$$

But the lack of detection of a neutron EDM constrains the free parameter θ severely:

$$\begin{aligned} d_n \approx & 5 \times 10^{-16}\theta \text{ e} \cdot \text{cm} \lesssim 10^{-25} \text{ e} \cdot \text{cm} \\ & \Rightarrow \theta \lesssim 10^{-10} \end{aligned} \tag{8}$$

In principle, $\theta \sim \mathcal{O}(1)$. Why should it be so small?

Backup: Peccei-Quinn Mechanism

Solving the Strong-CP Problem:

- Promote $\theta \to a(x)/f$, a dynamical field, whose potential is minimized at $a(x) = 0$. Naturalness saved!
- The Lagrangian for $a(x)$ has a symmetry, $U(1)_{PQ}$, which is broken at the energy scale f
- The physical axion field is the Goldstone boson of $U(1)_{PQ}$-breaking

ϕ is initially massless, but acquires a mass during the QCD phase transition due to nonperturbative effects:

$$m_a = \frac{\Lambda_{QCD}^2}{f} \tag{9}$$